

# The Edges of Large Earthquakes and the Epicenters of Future Earthquakes: Stress-induced Correlations in Elastodynamic Fault Models

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*Abstract*—Fault models can generate complex sequences of events from frictional instabilities, even when the material properties are completely uniform along the fault. These complex sequences arise from the heterogeneous stress and strain fields which are produced through the dynamics of repeated ruptures on the fault. Visual inspection of the patterns of events produced in these models shows a striking and ubiquitous feature: future events tend to occur near the edges of where large events died out. In this paper, we explore this feature more deeply. First, using long catalogues generated by the model, we quantify the effect. We show, interestingly, that it is an even larger effect for future small events than it is for future large events. Then, using our ability to directly measure all aspects of the model, we find a physical explanation for our observations by examining the stress fields associated with large events. Looking at the average stress field we see a large stress concentration left at the edge of the large events, out of which the future events emerge. Further, we see the smearing out of the stress concentration as small events occur. This indicates why the epicenters of future small events are more correlated with the edges of large events than are the epicenters of future large events. Finally, we discuss how results from our simple model may be relevant to the more complicated case of the earth.

**Key words:** Earthquake dynamics, stress interactions, seismology, spatial correlations.

## 1. Introduction

Earthquake prediction has remained a long sought, but highly elusive goal. Despite decades of effort, a debate still rages on whether or not it is even possible. Implicit in most of this discussion is the goal of predicting when earthquakes will happen. If, instead, we think about other scientific and useful predictions that can be made, we see that this is only part of the story. Thus, even if the goal of when future earthquakes will occur remains unachieved, it would nevertheless be very useful to be able to predict what will occur and where it will occur. In this paper, we present measurements in simple dynamical earthquake models which show strong correlations between past large model events and the locations of epicenters of future model events. Specifically, we find that the edges of where large events

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died out produce large stress concentrations, which then tend to be the most stressed regions and therefore the epicenters of events which occur in the future. We discuss as well, the applicability of these results from this simple model to the more complex case of the earth.

Previous work on earthquakes has examined a number of the issues we discuss in this paper. PEREZ and SCHOLZ (1997) inspected the epicenters of events before and after large events for numerous cases globally. They found an increase in activity around the ends of future large events both preceding and following the large events. They also pointed out the stress concentrations left at the edges of large events. Availability of data, however, limited the discussion to mainly qualitative answers. OLSEN *et al.* (1997) noted large stress concentrations left at edges of the inferred 1992 Landers M7.4 earthquake, and referred to them as likely nucleation centers of future seismicity.

There is extensive literature detailing that stress perturbations from large earthquakes trigger future earthquakes (HARRIS and SIMPSON, 1992; JAUME and SYKES, 1992; STEIN *et al.*, 1992; KING *et al.*, 1994; HARRIS *et al.*, 1995). Good agreement between calculated stress perturbations and the numbers of aftershocks as a function of location has been demonstrated (DIETERICH, 1994). Long-term correlations of stress increments and future seismicity have also been shown (DENG and SYKES, 1996). Collectively, this work has solidified the central role that stress plays in earthquake mechanics. In all these cases, however, there remains an ambiguity of what the absolute stress levels are, since the initial stress to which the stress increments are added remains unknown. Further, there are many more locations where we have insufficient information about past large events to make sufficiently accurate estimates of stress perturbations to be used for future seismicity forecasts. Thus, finding general patterns which relate past earthquakes to future seismicity could be quite useful.

We have studied a number of very simple scalar elastodynamic models of earthquake faults, and found what seems to be a quite general pattern emerging from the long catalogues generated in these models. Specifically, we find that the edges of where past large events die out tend to be the locations where epicenters of future events occur. By investigating the guts of the model, we find a physical origin of the effect in terms of the stress field. Understood in this way, the results become less surprising; they can be used, however, to assist our means of extrapolation from these simple models to the more complex earth. The rest of the paper is organized as follows. In the forthcoming Section 2 we describe the simple models. We then present in Section 3 the new results for the correlations and the origin of the correlations. Finally, in Section 4, we discuss how these results might apply to the more complicated case of the earth.

## 2. The Model

Elastodynamic models of earthquake faults have been shown to produce remarkably vast complexity, with a wide distribution of sizes of events, even with completely uniform properties along the fault (CARLSON and LANGER, 1989). Complex solutions are a legitimate outcome of the continuum of elastodynamic equations (SHAW and RICE, 2000), although the richest population of events occurs only over restricted regions of frictional parameter space (SHAW and RICE, 2000). These models spontaneously generate a heterogeneous stress distribution from repeated ruptures along the fault, and sequences of events on this complex attractor produce chaotic, complex behavior. While complex, the behavior is clearly not random: we see quite systematic aspects to it, one of which we will focus on in this paper. First, however, we present the model we will use to exhibit this systematic feature.

We focus our attention on the dynamics of a single fault, and a very simplified fault at that. There are three main reasons for studying such a simplified system. First, the simplest systems generally allow the most minimal parameterizations, thus the fewest features must be specified and the parameter space can be most fully explored. Secondly, our simplifications, particularly the low dimensionality of the fault we study, are computationally very fast, and allow for long sequences on large faults to be studied. Thirdly, we want to understand why our system is behaving as it does, and the simpler systems make this understanding easier to achieve.

The model we study here is a two-dimensional model of a fault (SHAW, 1997; SHAW and RICE, 2000). The two-dimensional fault has the advantage over a one-dimensional fault (BURRIDGE and KNOPOFF, 1967) of preserving long-range elastic interactions, and the advantage over a full three-dimensional fault of being vastly less expensive numerically. We consider only scalar motions, consequently we have the wave equation in the bulk. The coupling to the stably sliding lower fault, which provides the main driving both in our model and in real faults, is represented by a stiff boundary sliding at a constant creep rate. A long strip of the wave equation connects this stably sliding lower fault to the unstably sliding seismogenic fault; on the seismogenic fault we have a frictional boundary condition, and sliding occurs in episodic stick-slip events. We study a system with uniform material properties, so that all of the irregularities which develop are dynamic in origin.

In equations, we have, for the bulk, the wave equation for the scalar displacement field  $U$ :

$$\frac{\partial^2 U}{\partial t^2} = \nabla^2 U \quad (1)$$

where  $t$  is time, and  $\nabla^2$  is the two-dimensional Laplacian operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

for the directions  $x$ , which we take to be along the fault, and  $y$  which is then the direction perpendicular to the fault. Along the fault, located at  $y = 0$ , we have the boundary condition that the strain equals the traction, here the friction  $\Phi$  on it:

$$\left. \frac{\partial U}{\partial y} \right|_{y=0} = \Phi. \quad (2)$$

We will return to a discussion of  $\Phi$  shortly; first let us specify the other boundary conditions. Away from the fault, a distance of a crust depth (normalized to unity) away, the displacement creeps along at a constant steady slow rate  $v$ , the plate velocity:

$$\left. \frac{\partial U}{\partial t} \right|_{y=1} = v \quad (3)$$

with  $v \ll 1$ . Along the fault, we use periodic boundary conditions:

$$U(x + L_x) = U(x). \quad (4)$$

To complete the description of the model, we must specify the friction  $\Phi$ . All of the nonlinearity in our model is contained in the friction. We specify the friction through a constitutive relation between motions on the fault, and the traction there which resists the motion.

### *The Friction*

We present here a somewhat detailed discussion of the physical basis of the friction we use, to motivate the resulting constitutive equations. The results we present in this paper are not specific to the physical mechanism of frictional weakening discussed. Rather, they arise more generally as a consequence of dynamic stress heterogeneities. Nevertheless, we find it useful to see the physical basis of the friction we use. The reader less interested in the specifics of the friction can skip to the next part which treats numerics.

The friction we use is motivated by a physical idea which reverts to SIBSON (1973). Simple estimates of the heat generated by sliding at usual values of friction suggest that the fault would melt (MCKENZIE and BRUNE, 1972); since this is not generally observed in exhumed faults, something else must be transpiring. SIBSON (1973) noted an interesting feedback mechanism: if there are pore fluids present, then the heat generated would raise the temperature and pressure of the fluids, thereby reducing the effective normal stress and thus the friction. A simple mathematical quantification of this physical idea was presented by SHAW (1995) as follows. The friction  $\Phi$  is the product of the coefficient of friction  $\mu$  and the effective normal stress  $N$ :

$$\Phi = \mu N. \quad (5)$$

The effective normal stress is reduced by the pore fluid pressure, which increases with temperature:

$$N = N_0 - \alpha Q, \quad (6)$$

where  $N_0$  is the ambient normal stress,  $\alpha$  is a proportionality constant, and  $Q$  is the heat. The heat  $Q$  evolves in two ways. It is produced by frictional sliding and, in our approximation, is dissipated over some timescale:

$$\frac{\partial Q}{\partial t} = -\gamma Q + \Phi V, \quad (7)$$

where  $V$  is the velocity on the fault and  $\gamma$  is some dissipation inverse timescale. Two limiting cases were found for this friction; in the case where the dissipation was slow compared to the rupture timescale,  $\gamma \ll 1$ , slip-weakening results, a case LACHENBRUCH (1980) had examined long ago. In the opposite limit, when the dissipation is fast compared to the rupture timescale, when  $\gamma \gg 1$ , velocity-weakening results (SHAW, 1995). In between, some mixture of slip- and velocity-weakening occurs.

One physical process neglected in this friction is the possibility of hydrofracturing, which can occur at high pore fluid pressures, and would limit the drop in friction to some nonzero value. SHAW (1995) considered various possibilities of how this might affect the friction, and examined how different nonlinear saturations might affect the dynamics. The conclusion was that the initial linear regime was the most important feature to the resulting complexity, and the nonlinear saturation was much less important. Therefore we use a simplified friction presented by SHAW (1997) which retains the basic heat-weakening physics, and saturates at a finite value. We use:

$$\Phi = \phi(V(t'), t' \leq t)H(V) - \eta \nabla_{\parallel}^2 V. \quad (8)$$

Here  $\phi$  depends on the past history of slip. The function  $H$  is the antisymmetric step function, with

$$H = \begin{cases} \frac{\partial \hat{S}}{\partial t} & \frac{\partial S}{\partial t} \neq 0; \\ |H| < 1 & \frac{\partial S}{\partial t} = 0, \end{cases} \quad (9)$$

where  $\widehat{\partial S / \partial t}$  is the unit vector in the sliding direction. Thus  $H$  represents the stick-slip nature of the friction, being multivalued at zero slip rate.

The parameter  $\eta$  is the strength of the viscous-like boundary dissipation, with  $\nabla_{\parallel}^2 = \partial^2 / \partial x^2$  being the fault-parallel Laplacian operator. This term usefully provides stability to the smallest lengthscales (LANGER and NAKANISHI, 1993; SHAW, 1997).

The history dependent  $\phi$  we use in this paper is given by

$$\phi = \Phi_0 - \frac{\alpha Q}{1 + \alpha Q} - \Sigma. \quad (10)$$

The first term  $\Phi_0$  is a constant, the threshold value of sticking friction, which, as long as it is large compared to the maximum friction drop, becomes an irrelevant parameter in the problem. The second term in (10) generates the same initial linear decrease with heat  $Q$  as in the full nonlinear case (Eqs. (5)–(7)), but here has a different nonlinear saturation; a difference expected to be unimportant. It is normalized so that it drops at most a stress of unity. The last term in (10)  $\Sigma$  represents the drop in friction as we go from sticking to slipping; it corresponds with what the standard rate and state friction (DIETERICH, 1979; RUINA, 1983) would contribute. Here we simplify and consider a  $\Sigma$  which depends instead only on time:

$$\Sigma(t) = \begin{cases} \sigma \frac{t - t_s}{\tau} & t - t_s < \tau; \\ \sigma & t - t_s \geq \tau, \end{cases} \quad (11)$$

where  $\tau$  is a timescale for the drop to occur, and  $t_s$  is the time since last sticking. With this nucleation mechanism there is instantaneous healing upon sticking. This simplification of the friction allows for a dramatic speed-up of the numerics. Fortunately, these two-dimensional models with the friction we consider are, interestingly, quite insensitive to many of the details of the nucleation term  $\Sigma$ ; for example, a slip-weakening  $\sigma$  and this time dependent  $\sigma$  produce the same large-scale results (SHAW and RICE, 2000). Thus, while our  $\Sigma$  is a drastic simplification of what occurs, the results are quite insensitive to it, and it manifest a tremendous speedup.

### *Numerics*

In the simulations, beginning from any nonuniform initial condition, the system settles down to an attractor which has a statistically steady state. Two general types of attractors are seen, depending on the friction parameter values—principally the heat-weakening parameter  $\alpha$ . For  $\alpha$  below a critical value, a simple attractor with only large events which span the whole system size is seen. For  $\alpha$  above the critical value, a complex attractor with a distribution of large events is seen. Close to the critical value, a rich population of small events occurs, when  $\sigma$  is not too large compared to unity. This population of small events exhibits power-law distribution of sizes and displays a remarkable richness of behavior; while it is a legitimate outcome of the continuum equations (SHAW and RICE, 2000), it does, however, only occur over a quite restricted range of parameter space (SHAW and RICE, 2000). Because we are interested in how the small events correlate with past large events,

we will focus our attention on the restricted parameter range where the small events occur.

We solve the equations with a second-order explicit finite difference technique. The displacement and velocity are integrated forward in time on a non-staggered grid. This technique can provide some long-term drift relative to the continuum, however numerical evidence suggests we are shadowing the attractor, and thus the drift is principally along the attractor, rather than away from it. We have confirmed that an alternative formulation which follows velocity and stress (VIRIEUX and MADARIAGA, 1982) and the formulation used here produces the same statistics on their shadowed attractors (SHAW and RICE, 2000). Grid independence of the large scales is a principal signature of the continuum. Dispersion of the waves in the bulk at small wavelengths, an inevitable consequence of the finite difference (ALFORD *et al.*, 1974), interestingly does not appear to significantly affect the fault attractor, which seems to be relatively insensitive to the dispersion in the bulk away from the fault (SHAW and RICE, 2000).

What does the model behavior look like? Figures 1 and 2 show examples of the attractors for two different sets of parameters, both in the range where numerous small events are seen. Two different views of the attractor are shown in each picture. At the top, we plot the times at which various parts of the fault have slipped. On the bottom, we plot the net slip at each point of the fault. Because the small events slip considerably less than the large events, they are difficult to see on the bottom plots, but are much more apparent on the top plots. The detailed sequence of events is in fact chaotic (CRISANTI *et al.*, 1992; DE SOUSA VIEIRA, 1999; SYKES *et al.*, 1999), and thus is highly sensitive to the initial conditions. The statistics of the attractor, however is stationary, and we see long-term regularities in the patterns. Visually, we see considerable order and structure to the patterns, amidst this chaos. It is our purpose in this paper to discuss one such general, striking regularity of these patterns: how the edges of large events influence where future events occur.

### 3. Results

Inspection of Figures 1 and 2 confirms a remarkable regularity of the model behavior: observing where most events initiate, we see that the vast majority occur near the edges of past large events. This is most easily seen by looking at the small events in (a) of the figures, and mentally tracing backwards in time at the locations where they occurred: most frequently they line up very close to an edge of a previous large event. (Despite the epicenters not being directly indicated in the figure, this effect can be easily seen with the small events because of their limited spatial extent which bounds the epicenter location.) This basic effect, which seems apparent from visual inspection, transcends many aspects of the models, including

dimensionality of the models (it is also true in the one-dimensional models), geometry of the two-dimensional models (alternative coupling to the stably sliding layer, e.g., through the bulk which gives a Klein-Gordon equation (MYERS *et al.*,

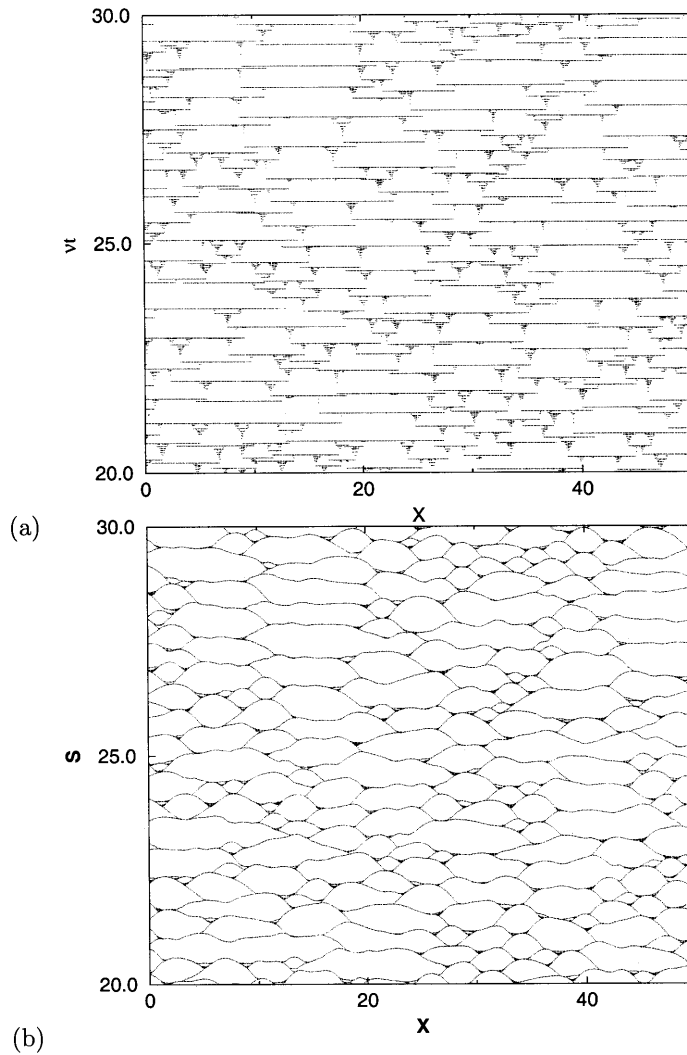


Figure 1

Two different representations of the attractor produced by the model for the parameter range where numerous small events occur. The horizontal axis is distance along the fault, measured in units of the brittle crust depth. The material properties are uniform along the fault. (a) We plot the times at which various parts of the fault break—a standard space-time plot in seismology, only here for very many loading cycles, whereas only a fraction of a loading cycle is generally available for real data. (b) We plot the cumulative slip along the fault following the event, for the same set of events as in (a). The friction parameters used are  $\alpha = 3$ ,  $\sigma = .05$ ,  $\gamma = 1$ ,  $\tau = .1$ , and  $\eta = .00001$ .



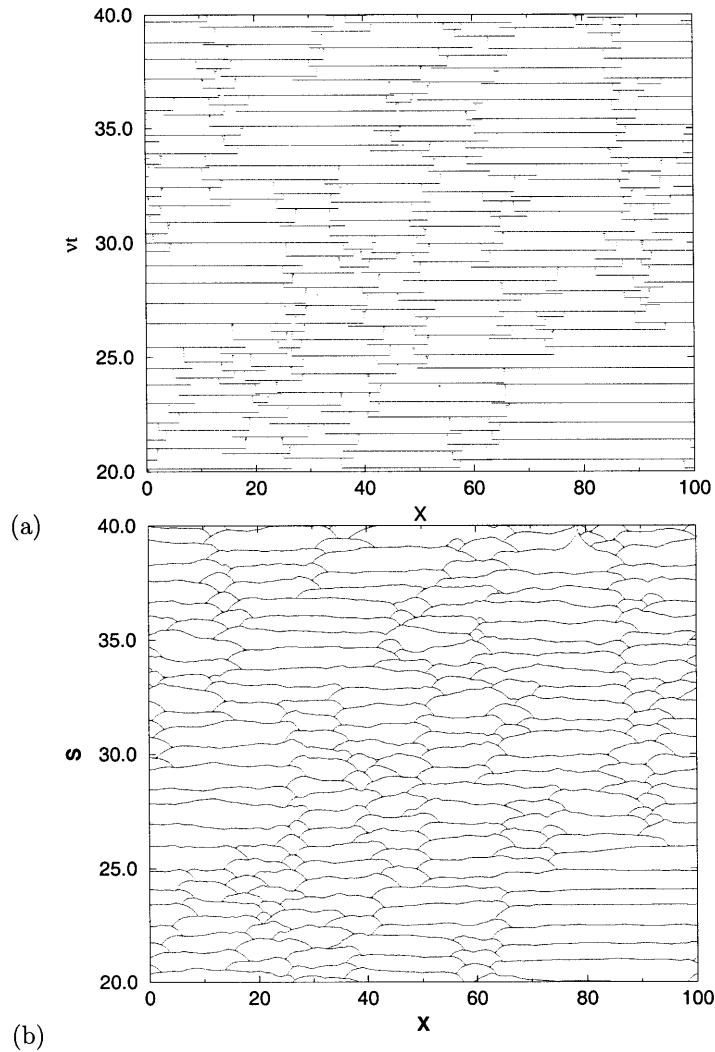


Figure 2

Two different representations of the attractor produced by the model for a somewhat larger value of the weakening parameter. The same type of plots as in Figure 1 is shown, although with the axes rescaled somewhat. (a) The times at which various parts of the fault break. (b) The cumulative slip along the fault following an event, for the same set of events as in (a). The same set of parameters as in Figure 1 are used, except now for  $\alpha = 8$ .

1996), also shows this effect), and a variety of frictional instabilities (slip-weakening and velocity-weakening to name two). It is a basic feature of the dynamic complexity set-up by the uniform fault models: it is the edges of where the large events die out which create the largest strain heterogeneities in the model, and are the main sites where future events initiate.

Let us explore this effect more quantitatively with the model, and assure we are not being misled by apparent visual correlations. Figure 3 quantifies this effect, and affirms its significance. Figure 3a uses events of Figure 1, while Figure 3b uses events of Figure 2. The measurement is made as follows. We begin by defining large events, which we take as all events which break a length of the fault which is larger than some cutoff length  $L_0$  (typically taken to be a crust depth in length, so  $L_0 = 1$ ). Then, at the edge of the patch that broke, we mark the spot, along the fault. Within any patch that broke, we overwrite any previously marked edges. If an event breaks the fault in multiple disconnected patches, we execute this procedure with any patch that is longer than the minimum  $L_0$ . In this way, we conclude with a list of edge marks along the fault, which is updated on the part of the fault where a new large event occurs. The next step is easy: each time an event occurs, we measure the distance of its epicenter to the nearest edge from the list we just described of previous large event edges. This produces a probability density function of distance from epicenters to previous edges. Figure 3 shows the cumulative of this probability density function (PDF) as a function of distance, showing on the vertical axis the probability of the distance being less than the distance plotted on the horizontal axis. We add further information to the plot by grouping events according to the size of the events. The solid line corresponds with the large events. The top short-dashed line corresponds with the small events. Interestingly, we see that small-sized events are even more correlated with the edges of past large events than are the large events. One last curve helps show the size of the effect. The long-dashed curve illustrates what this cumulative PDF would be if the event epicenters occurred with completely randomness in space. We calculate this by, every time an event occurs which is counted by the solid curve, calculating a distance to the nearest edge from a randomly chosen point along the fault. This generates a control distribution for the null hypothesis of uncorrelated epicenters with respect to previous large event edges. We see that the clustering is a substantial effect: close in it is a factor of many times higher than that of the cumulative PDF for the random case. Thus we have strong correlations of the large event edges with future event epicenters.

How do the results change with parameter values? Using larger values of the weakening parameter  $\alpha$  only increases the correlation. Comparing (a) where  $\alpha = 3$  with (b) where  $\alpha = 8$ , both the small event curve and the large event curve move up with increasing  $\alpha$ , being even more closely clustered at the large event edges, while the control curve moves down, as the large event lengths elongates. Thus larger values of  $\alpha$  correspond with stronger correlations. Changing other parameters has only relatively small effects. In all cases, the correlations are strong.

Why do these correlations occur? Thinking in terms of stress, we gain a clear physical understanding. Large events tend to die out in regions of low stress and while propagating into regions that are even less stressed ahead. The propagating ruptures carry stress concentrations with them, which are able to break through

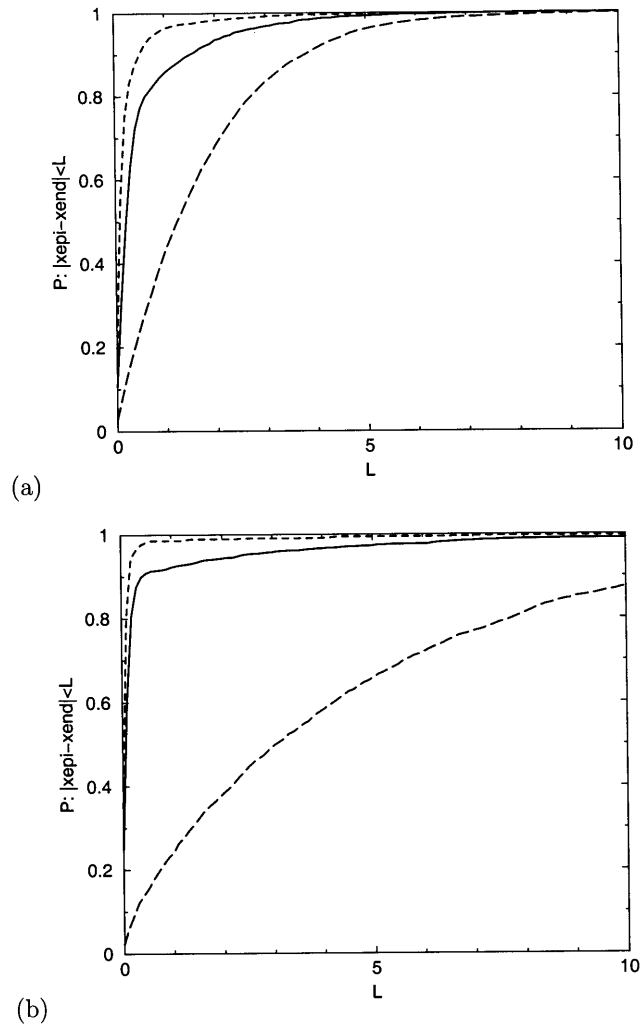


Figure 3

Epicenter correlations with large event edges. The vertical axis is the cumulative probability density function of the distance from an epicenter to a previous large event edge being less than the distance on the horizontal axis. Distances are in units of the brittle crust depth. Within each figure three curves are shown. The upper small dashed curve is the epicenters of the small and medium events. The middle solid curve is the epicenters of the large events. The lower long-dashed curve is the control for the case where the epicenters are randomly distributed. The curves lying far above this control case show clearly the epicenters are closely clustered near the large event edges. Note that the small and medium epicenter curve is even more clustered than the large event epicenter curve. The two different figures correspond to two different values of the weakening mechanism, with  $\alpha = 3$  in (a), using the events of Figure 1, and  $\alpha = 8$  in (b), using the events of Figure 2.

somewhat understressed regions, but not overly understressed regions. The regions they leave behind are even less stressed after they have broken. Thus we end up with a stress minimum behind the ruptures, a stress concentration at the edge of the rupture, and a less stressed region ahead of the rupture; all of which leaves the edge of the event where the rupture died out as the highest stressed region locally. Subsequently, when the system is uniformly reloaded, this region is the closest to threshold, and the first to break. Figure 4 shows this effect quantitatively where we plot the average stress before and after a stack of large events. There are four curves in each plot, all showing the average stress as a function of distance from the edge of a large event, averaged over many events. A fifth vertical thin solid line demarcates where the large events died out, at  $x = 0$ , about which all the stress stacks are made. The thick long-dashed curve delineates the average stress before the large events occurred, events which then break through  $x < 0$  and die at  $x = 0$ . The thick solid curve shows the average stress after the large events occurred. Both of these thick curves show all of the effects we just described above: the events died propagating into less stressed regions; they left behind destressed regions; and added large stress concentrations at their edges. The last two curves in the plots, the two thin-dashed curves, indicate additional information. They show the average stress just before the next large event ruptures through, measured at the same points where the thick solid line is taken, though now at later times. The upper thin-dashed curve shows the stress just before the next large event ruptures through. The lower dashed thin curve shows the same data, but with a constant stress subtracted from it; this is done so as to allow comparison with the thick solid curve, which shows the stress at the same locations at an earlier time. We see something interesting here: the stress concentration at the large event edges has been lessened and smeared out onto the neighboring regions.

This is a result of the small events which occurred at the stress concentration. They have two effects: they lower the stress where they break, and they also shunt this stress onto the neighboring regions. The net effect is a smearing of the stress concentration. This provides an explanation as to why the epicenters of the future small events in Figure 3 were more correlated with the edges of the large events than were the epicenters of the future large events: by the time the fault has reloaded enough for a large event to occur again, the stress concentration has been somewhat eroded and smeared out by the smaller events.

Completing our examination of Figure 4, looking to the sides of the stress concentration we see to the left the ruptured interior of the last large event, the shifted thick-dashed curve overlays the original thick solid curve, indicating this least stressed previously broken region has generally not rebroken yet. Again, stress levels are consistent with activity, with more stressed regions being more active.

The effect of changing parameters reappears in the stress field, and again explains the correlations. Two weakening values, again  $\alpha = 3$  in (a), and  $\alpha = 8$  in (b), illustrate this. In Figure 3 we noted that larger values of  $\alpha$  increased the

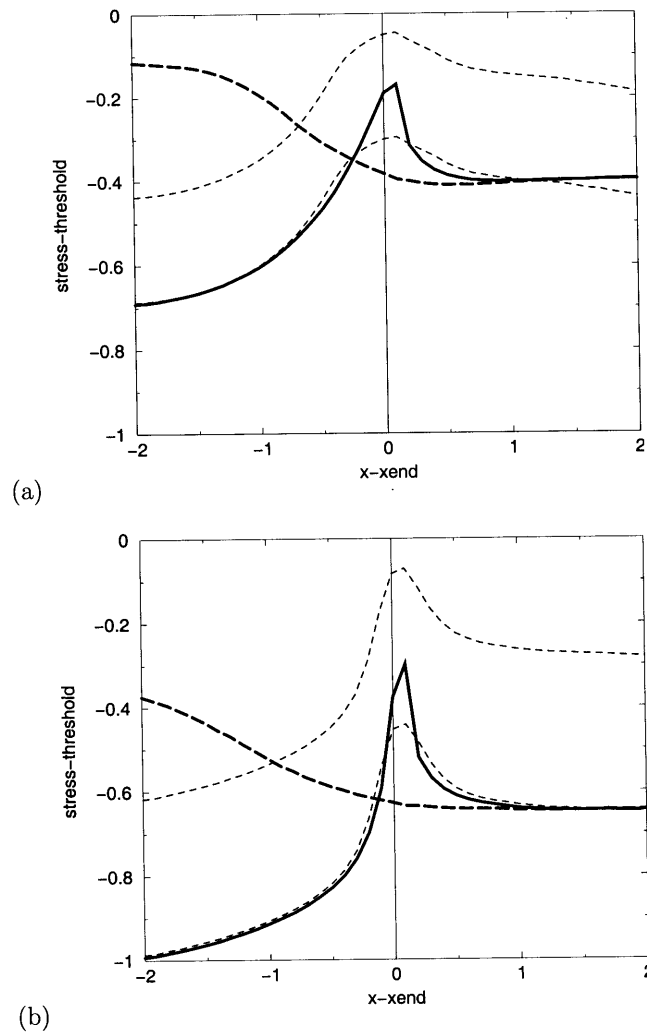


Figure 4

Stress averages. The stress state relative to the edges of large events, averaged over many events. The stack is made with respect to the end of a large event which ruptured  $x < x_{\text{end}}$  and died out at  $x - x_{\text{end}} = 0$ . The thin solid line shows this edge. Within each figure four curves are shown. The long-dashed curve shows the stress before the large event. The thick solid curve shows the stress after the large event ruptured. The upper thin-dashed line shows the stress just before the next large event ruptures through the previous edge; the lower thin-dashed line shows this same stress but shifted down by a constant, to compare with the thick solid line. Note the stress concentration left by the large event peak in the thick solid line at the edge. Note further the smearing of the stress concentration with time, evidenced by the smearing of the peak in the thin-dashed line. The two different figures correspond to two different values of the weakening mechanism, with  $\alpha = 3$  in (a), using the events of Figure 1, and  $\alpha = 8$  in (b), using the events of Figure 2.

correlations. Examining the stress fields in Figure 4 demonstrate why: larger values of  $\alpha$  lead to larger stress concentrations, with the peak at  $x = 0$  being higher and the averages off to the sides of the peak being lower. Qualitatively, the picture is the same: quantitatively, we see and can understand the differences.

#### *4. Discussion and Conclusion: Extrapolating Results to Real Earthquakes*

While the models show the large event edges to be a dominant effect on future events, we are most interested in how these results might apply to the case of real earthquakes. Here, there are a number of issues to consider.

First, one significant approximation of the model is that we have considered a one-dimensional fault, as opposed to a more realistic two-dimensional fault surface. Thus, actual fault edges are not merely points, but lines, and potentially quite complicated lines at that. Further, the bottom of the seismogenic depth will not have broken in a large earthquake, so the closest edge will be at most a crust depth away, if we include vertical distance. At a minimum, this would have substantial quantitative effects on the results. Nevertheless, if we confine our results to qualitative statements, we can still make certain observations. To reiterate, first, large event edges have the highest residual stress concentrations, and consequently are the first locations to be reactivated. Second, small events smear out the stress concentrations. Thus with time a broader area becomes activated. Eventually, a large event comes ripping through, and the cycle is repeated. These statements seem quite general. Thus, the qualitative underpinning of our results seem to carry over, even if the quantitative aspects would be substantially modified.

A second geometry complication is that faults exist not as isolated individuals faults, as considered in our simple model, but as parts of systems of faults. Here, if one thinks in the language of stress, which involves, in the elastic earth case, considerations of tensor aspects of stress and failure, the basic ideas still carry over. However, rather than simple distance, one must consider direction as well. Still, again, the qualitative ideas ought to transfer. In this context, DENG and SYKES (1996) have made some interesting observations of seismicity in Southern California. They noted, what they have euphemistically called a "great wall," marking a change in the level of seismic activity where the large M8.1 1857 Fort Tejon earthquake was believed to have died out. Below this end the activity is much higher than behind, in the region estimated to be shadowed by the stress relief from the events. This, along with the observations of PEREZ and SCHOLZ (1997), provide some of the strongest observational evidence for the qualitative picture being presented here (though the suggestion of PEREZ and SCHOLZ (1997) of enhanced activity at the edges preceding the large events raises other questions).

An additional complication of real earthquakes not present in the simple model is the existence of aftershocks and foreshocks for earthquakes. In the simple

frictions we have considered, we do not have this intermediate timescale. The re-emergence of future events in the model is more akin to what would be called “preshocks” in seismology, which occur over fractions of the loading cycle, and indicate an emergence out of the low stress regime following a large event. If the model were to have an aftershock timescale, however, some of this signal would likely appear as aftershocks as well. Thus, to look for the signature of the effect being described here, counting aftershocks would be appropriate. The fact that aftershocks are the most numerous around the edges of the rupture support the picture presented here.

The fact that earthquakes eventually stop requires heterogeneities to exist in the Earth. A central question which remains unanswered is the role of relatively fixed material and geometrical heterogeneities and the role of changing, dynamic, stress heterogeneities. In the simple models we have examined here, we have completely neglected any fixed heterogeneities and studied a completely uniform fault. All of our heterogeneities are stress heterogeneities. In this way, we have provided a baseline context with which to compare the data. Deviations from, or similarities with, the simple picture emerging from these models would speak to these heterogeneity questions: to what extent does seismicity concentrate on the edges of large events? To what extent is seismicity tied to an absolute location (fixed in space) as opposed to a relative location (relative to previous events)?

This work suggests immediate practical prescriptions: to heighten monitoring at the edges of known previous large ruptures, and to identify and locate more accurately the previous large event edges. Even with all the other additional complications of the real earth, on long faults where great earthquakes occur there should be an enhanced probability of events occurring at the edges of previous large events; what remains to be determined is the strength of this enhancement.

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