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Short Note

Constant Stress Drop from Small to Great Earthquakes in Magnitude-Area Scaling

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Abstract Earthquakes span a tremendous range of scales, more than 5 orders of magnitude in length. Are earthquakes fundamentally the same across this huge range of scales, or are the great earthquakes somehow different from the small ones? We show that a robust scaling law seen in small earthquakes, with stress drops being independent of earthquake size, indeed holds for great earthquakes as well. The simplest hypothesis, that earthquake stress drops are constant from the smallest to the largest events, combined with a more thorough treatment of the geometrical effects of the finite seismogenic layer depth gives a new magnitude-area scaling that matches the data well and matches the data better over the whole magnitude range than the currently used scaling laws, which have nonconstant stress-drop scaling. This has significant implications for earthquake physics and for seismic hazard estimates.

Introduction

How the slip in an earthquake scales with rupture length, or, alternatively, how the magnitude scales with rupture area, has profound implications for earthquake physics, and also, it turns out, seismic hazard estimates. For small earthquakes, constant stress-drop scaling has been robustly observed (Aki, 1972; Hanks, 1977), whereby slip scales linearly with rupture length, or equivalently magnitude scales linearly with the log of rupture area. What happens for large earthquakes, when the rupture length is large compared to the depth of the seismogenic layer, has been a source of much debate and controversy (Scholz, 1982; Romanowicz, 1992, 1994; Scholz, 1994; Wells and Coppersmith, 1994; Bodin and Brune, 1996; Pegler and Das, 1996; Henry and Das, 2001; Shaw and Scholz 2001; Hanks and Bakun, 2002). The leading magnitude-area scaling relations (Hanks and Bakun, 2002; Working Group on California Earthquake Probabilities [WGCEP], 2003), used in the most recent U.S. national seismic hazard maps, all assume a breakdown of the scaling seen in small earthquakes, with stress drops increasing for the largest earthquakes. This poses a huge challenge for earthquake physics and for seismic hazard estimates. What is different in the physics of great earthquakes as compared to their smaller brethren? Even more problematically from the point of view of seismic hazard estimates, how do we use and extrapolate from the much more numerous moderate and destructive large earthquakes to the rare and devastating great earthquakes if the physics differs? With a gaping sparsity of direct measurements for the great earthquakes, yet an essential need to design for their consequences, extrapolating from measurements of more numerous smaller events remains a necessary task. In this article we show that the simplest hypothesis, that earthquake stress drops are constant from the smallest to the largest events, when combined with a more thorough treatment of the geometrical effects of the finite seismogenic layer depth gives a magnitude-area scaling that matches the data well and matches the data better over the whole magnitude range than the currently used scaling laws, which have nonconstant stress-drop scaling.

The two leading magnitude-area scaling laws, given equal 50% weighting in the 2007 U.S. National Seismic Hazard maps, both imply increasing stress drops for the largest earthquakes. The so-called Ellsworth B model (WGCEP, 2003) is the simplest parameterization, extrapolating the constant relationship between moment magnitude **M** (Hanks and Kanamori, 1979) with the log of the rupture area A to the largest events (though with a different offset than is generally used for smaller events [Wells and Coppersmith, 1994]):

$$\mathbf{M} = \log A + 4.2. \tag{1}$$

For an infinitely long rupture with a finite width W and a constant stress drop, slip should scale as W, so moment scales with area A as AW and moment magnitude, which is $2/3$ of the log of the moment, should scale with A as $\frac{2}{3}\log A$ for fixed W. Then to get the $\log A$ magnitude-area relationship in equation (1) for fixed W we would need stress drop $\Delta\sigma$ to scale as $\Delta\sigma \sim A^{1/2}$. The second scaling relation, the Hanks–Bakun relation (Hanks and Bakun, 2002), used a bilinear scaling relationship of

$$\mathbf{M} = \begin{cases} \log A + 3.98 & A \leq 557\ \text{km}^2, \\ \frac{4}{3}\log A + 3.07 & A > 557\ \text{km}^2. \end{cases} \tag{2}$$

Based on a scaling of slip continuing to increase linearly with rupture length L, the so-called L model (Scholz, 1982), this gives an even stronger stress-drop scaling for the largest events, with $\Delta\sigma \sim A$.

Our new scaling builds on the $\Delta\sigma \sim A^0$ scaling seen for more than 5 orders of magnitude of A in small earthquakes, by assuming this continues to hold for large earthquakes. We then combine this with finite rupture length L and width W effects, and their interaction with the seismogenic depth H and the free surface. We build our new magnitude-area scaling relations by looking at how average slip D scales with rupture dimensions assuming stress drop is constant. Slip times stiffness is stress, and taking for stiffness the inverse lengths in the vertical and horizontal directions of the rupture, Shaw and Scholz (2001) obtained

$$D \sim \begin{cases} \frac{1}{\frac{1}{L}+\frac{1}{L}} & L \leq 2W, \\ \frac{1}{\frac{1}{L}+\frac{1}{2W}} & L > 2W, \end{cases} \tag{3}$$

with the factor of 2 coming from the free surface. Shaw and Scholz (2001) presumed that rupture width W equaled seismogenic depth H and noted the discrepancy of the observed data with the expected scaling, although their dynamic model, which had a scale invariant physics, matched the observations well. Manighetti *et al.* (2007) examined the fit of this scaling to the data when W was generalized as a fitting parameter. We can write this as:

$$D \sim \begin{cases} \frac{1}{\frac{1}{L}+\frac{1}{L}} & L \leq 2\xi H, \\ \frac{1}{\frac{1}{L}+\frac{1}{2\xi H}} & L > 2\xi H. \end{cases} \tag{4}$$

Fitting the data for a single best value of ξ, Manighetti *et al.* (2007) found $\xi = 3$, though they argued that using multiple values of ξ were a better match and argued for this fitting based on fault segmentation, and not based on fault width. We propose an alternative interpretation of ξ as an effective width. But what might be the cause of such a large effective width? It has generally been presumed that large earthquakes break coseismically only down to the seismogenic depth H, but theoretical work is calling this assumption into question. Developing an idea originally proposed by Das (1982), King and Wesnousky (2007) recently showed that if one relaxed the assumption that large earthquakes broke only down to the bottom of the seismogenic layer, and instead kinematically had ruptures penetrating deeper into the stably sliding lower fault, constant stress-drop scaling could be maintained while matching observed slip-length scaling relations for large strike-slip earthquakes. Shaw and Wesnousky (2008) went further and showed that three-dimensional scalar elastodynamic models, which had previously been seen to match slip-length scaling observations (Shaw and Scholz, 2001), indeed dynamically chose this deep penetrating slip behavior. They found that while W increased systematically with L, a fit as in equation (4) with a single value $\xi = 3$ was a good fit to the data. Significantly, this effective single ξ fit was larger than any individual values of W/H—the increase of W/H with L mapped onto a larger effective single ξ. Based on this work, we treat this as a fitting parameter to be compared with the data, and now convert to magnitude-area scaling. Before proceeding, we note that current observations provide insufficient constraints on this question of the depth-of-coseismic slip. King and Wesnousky (2007) discuss in particular the lack of constraints from Global Positioning System (GPS) data due to poor depth resolution; seismic inversions as well are poorly constrained in depth. In short, while substantial coseismic slip below the seismogenic layer has not been directly observed, it has also not been ruled out by observations.

Converting to moment-area scaling, we continue to examine only the seismogenic rupture lengths and area, because the actual downdip widths W of ruptures remain difficult to observe seismologically, whereas the seismogenic depth H can be constrained by seismicity (Nazareth and Hauksson, 2004). Then, for seismogenic area A and moment $\mathcal{M}$, for now just that associated with that area,

$$\mathcal{M} \sim AD \sim \frac{A}{\frac{1}{L} + \max(\frac{1}{L}, \frac{1}{\beta H})} = \frac{AL}{1 + \max(1, \frac{L}{\beta H})}, \tag{5}$$

where we have used a new scaling parameter $\beta \equiv 2\xi$, and we have also combined the two different scalings depending on L relative to W into one expression using the max functional. Note that by seismogenic area A, we mean the area within the seismogenic width H and do not include any area in W below the seismogenic depth H, which may nevertheless rupture coseismically. We consider only the seismogenic area because this is observable, whereas any deeper area that may slip is not unambiguously imageable with current methods. In this way, we make direct contact as well with current seismic hazard estimates that use only seismogenic areas. Translating equation (5) into magnitude $\mathbf{M}$ and writing this as a correction to $\log A$ scaling, we have

$$\begin{aligned} \mathbf{M} &= \frac{2}{3}\log_{10}\mathcal{M} \\ &= \log_{10} A + \frac{2}{3}\log_{10}\frac{L/A^{1/2}}{[1+\max(1, \frac{L}{\beta H})]/2} + \text{const.} \end{aligned} \tag{6}$$

Defining a new parameter to be the aspect ratio $\lambda \equiv L/H$ gives the scaling relation

$$\mathbf{M} = \log_{10} A + \frac{2}{3}\log_{10}\frac{\lambda^{1/2}}{[1+\max(1, \lambda/\beta)]/2} + \text{const} \tag{7}$$

or, written just in terms of area A, and noting $\lambda = 1$ for small events, we can write the scaling relations as

$$\mathbf{M} = \log_{10} A + \frac{2}{3} \log_{10} \frac{\max\left(1, \sqrt{\frac{A}{H^2}}\right)}{[1 + \max(1, \frac{A}{H^2 \beta})]/2} + \text{const.} \quad (8)$$

Thus far we have only considered the moment from the seismogenic layer. If there is rapid coseismic slip below the seismogenic depth H, there will be additional moment coming from this deeper slip. Dynamic modeling studies suggest such slip may occur but also that the character of the slip may be somewhat different from slip in the seismogenic layer, being much more low frequency in nature and depleted in high frequency (Shaw and Wesnousky, 2008). From the point of view of moment, however, only the low-frequency coseismic motion matters. We can write the additional contribution as

$$\mathbf{M} = \log_{10} A + \frac{2}{3} \log_{10} \frac{\max\left(1, \sqrt{\frac{A}{H^2}}\right)}{[1 + \max(1, \frac{A}{H^2 \beta})]/2} \left(1 + \frac{\mathcal{M}_{z>H}}{\mathcal{M}_{z\leq H}}\right) + \text{const}, \quad (9)$$

where $\mathcal{M}_{z>H}$ is the moment from slip at depths z greater than the seismogenic depth H, and $\mathcal{M}_{z\leq H}$ is the seismogenic depth moment we have been considering up to this point. Further parameterizations of the ratio $\mathcal{M}_{z>H}/\mathcal{M}_{z\leq H}$ enable more detailed transitions from the small event regime where $\mathcal{M}_{z>H}/\mathcal{M}_{z\leq H} \approx 0$ to the great event regime where $\mathcal{M}_{z>H}/\mathcal{M}_{z\leq H}$ approaches a constant. Such behavior was seen in dynamic models (Shaw and Wesnousky, 2008). We have explored a few such parameterizations of the ratio but have found that the data limitations, the scatter and the limited number of data points and the limited magnitude range over which the ratio changes much, make the additional parameters needed insufficiently distinguishable and not statistically significant in terms of added information content. Instead, we find it most useful to simply absorb this additional ratio into the β parameter, so that β now encompasses both the additional slip in the seismogenic layer due to increased W and the additional moment below the seismogenic layer. In this way, we maintain a minimal parameterization that connect the small event scaling where $\mathbf{M} \sim \log A$ and the great event scaling where $\mathbf{M} \sim 2/3 \log AW \sim 2/3 \log AH\beta$, and sacrifice some potential details of the interpolating large events connecting the two regimes, which are unresolvable with the data. Thus, we return back to equation (8) as our proposed scaling for magnitude versus seismogenic area but with a generalized interpretation of the parameter β. Equation (8) generalizes the Wells and Coppersmith (1994) and Ellsworth B (WGCEP, 2003) relations with two additional parameters β and H. The value of the constant term is set by fitting at the small events. If we assume we know H through other means (e.g., hypocentral depths), then we can fix H, and we would have only one additional parameter; otherwise we can set this by fitting to the data. The parameter β is set by fitting the data.

Data Fitting

Because the most revealing data are for large aspect ratio L/W events, we examine strike-slip earthquakes, which have the largest aspect ratios of any mechanism earthquakes. The magnitude-area data are from Hanks and Bakun (2008). The Hanks and Bakun (2008) data (and other previously used data sets [Wells and Coppersmith, 1994; Hanks and Bakun, 2002; WGCEP, 2003]) do not have error bars, so we assume errors in all data points are the same. Doing a least-squares fitting and assuming errors in $\log A$ are the same size as errors in magnitude, we find a best fitting of equation (8) for parameter values $H = 15.6$ km and $\beta = 6.9$ to the complete data set.

It is satisfying that the best-fitting value of H is so close to the generally presumed default worldwide values of H. This validates in a fundamental way the physical foundation of our scaling. There is, though, a very broad minimum around this best-fitting value, and the constraints on H and β are not tight; using the likelihood function and assuming Gaussian errors, a one standard deviation contour in the parameters gives bands of $H = 15.6^{+5}_{-3}$ km and $\beta = 6.9^{+3}_{-1.5}$. Assuming the errors in $\log A$ are somewhat larger or somewhat smaller than the errors in $\mathbf{M}$ changes slightly as well the best-fitting H and β parameters, though their product $H\beta$ remains nearly constant. The more robust metric of absolute value of the distance misfit, as opposed to the square distance, gives a best-fitting $H = 16.0$ km and $\beta = 5.9$.

The fit of the scaling laws to the data are shown in Figure 1. The data, shown with gray circles, is seen to be very well fit by the new scaling law, shown with a solid thick line. The leading other scaling relations are also plotted: Wells and Coppersmith (1994) with a solid thin line, Ellsworth B (WGCEP, 2003) with a dash-dot thin line, and Hanks and Bakun (2002) with a dashed thick line.

The new scaling relation appears an excellent fit by eye, and indeed is measured to have a smaller standard deviation than the fits to the other curves (0.1418, 0.1456, 0.1927, for equations 8, 2, and 1, respectively). It does, however, have more parameters and, thus, must beat them sufficiently to make up for that. Using Akaike Information Crtiterion (Akaike, 1974) to penalize for the extra parameters, we find the new scaling is nevertheless still better (-90.2, -87.5, -39.0 for equations 8, 2, and 1, respectively). An alternative measure of fitting, the Fischer F test also shows the new scaling to be superior at the 5% significance level ($F(1, 86) = 4.65$ for equation 8 relative to equation 2 and $F(2, 86) = 36.43$ for equation 8 relative to equation 1). From a hazard point of view the larger events are more important, and some of the scaling laws have been developed to fit just that region (e.g., Ellsworth B (WGCEP, 2003); for completeness we have therefore examined the scaling relations as we fit only above a magnitude threshold and exam-

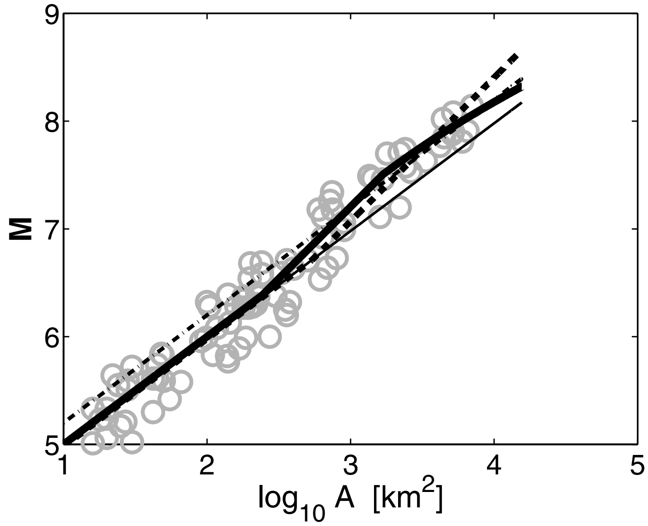

Figure 1. Magnitude-area relations for large strike-slip events. Gray circles denote magnitude and area of events from the Hanks and Bakun (2008) database. The solid thin line is the linear Wells and Coppersmith (1994) magnitude-area relation, and the dash–dot thin line is the linear Ellsworth B (WGCEP, 2003) magnitude-area relation, equation (1). The dashed thick line is the Hanks and Bakun (2002) bilinear relation, equation (2). The solid thick line is our new proposed scaling relation, equation (8), with least-squares best-fitting parameters $H = 15.6$ km, $\beta = 6.9$. Note the excellent agreement of the solid thick line with data across the whole range of magnitudes.

ine the relative fit as the magnitude threshold is increased. With loss of data points and the narrowing of the range over which the scalings differ, the differences in the fits for just the large events are, however, within the one sigma contour in likelihood and thus not significant. They, therefore, do not exceed a significance test, which penalizes extra parameters if we restrict the magnitude range to be only above **M** 6.5 for which equation (1) was developed. Because current seismic hazard analysis considers events down to **M** 5.5, however, a lower cutoff magnitude is needed. The prospects for more data helping reduce uncertainties are most promising at the lower magnitudes, where modern relative relocation techniques (e.g., Waldhauser and Ellsworth, 2000) could be brought to bear, and many well-recorded events that have occurred have not been analyzed for inclusion in the data set.

The relatively large value of β found has a few origins. First, dynamic models (Shaw and Wesnousky, 2008) fitting the slip-length scaling, equation (4), found larger single best-fitting values when there was some L dependence to W. Second, we use the seismogenic area for A in equation (5) because seismic hazard estimates use this area, because area estimates typically presume this area, and because seismogenic H is much better defined than the difficult to measure W. Additional moment and area below the seismogenic zone will map onto our effective β value. That is, for very large A, $\mathbf{M} \sim 2/3 \log A + 2/3 \log H\beta$ so underestimates in A get mapped onto larger β values if H is constrained by the initial increase in magnitude in transitioning from small to large events.

Discussion

Our new scaling relations have interesting connections with prior work. We get $\log A$ scaling at small magnitudes ($A/H^2 < 1$) as all the scaling relations have, L scaling of $4/3 \log A$ for moderately large events ($1 < A/H^2 < \beta$) as Hanks and Bakun (2002) found and, finally, a new regime of W scaling of $2/3 \log A$ for the very largest events ($\beta \ll A/H^2$).

Our scaling is based on seismogenic depth H being fixed. Observations of seismicity along major faults suggest, however, it can vary along strike (Nazareth and Hauksson, 2004). To test the generality of our scaling relation, we have extended our analysis to include a seismogenic depth varying along strike $H(x) = H_0 + \epsilon(x)$ and examined the impact of varying $H(x)$ in the scaling relations. To first order perturbation theory, for $\epsilon(x)/H_0 \ll 1$, scaling H_0 so $\langle \epsilon(x) \rangle_x = 0$ we find that we can replace H in the scaling relation by its average $H_0 = \langle H \rangle_x$. This shows a robustness of the results and extends the utility of our relation.

The maximum deviation of our relation from $\log A$ scaling, occurring at $A = H^2\beta$ (ignoring magnitudes too large to be observed on real faults), is only $1/3 \log \beta$, which is $\approx$0.28. This may seem a small amount, but from the point of view of seismic hazard, where recurrence times scale with slip, this means a factor of 2 difference.

It was satisfying that best-fitting measured values of H correspond with seismogenic depths. How do we understand measured values of β? As noted before, the measured parameter β can be understood as a combination of two effects: the additional slip at seismogenic depths due to the effective W being larger than H and the additional moment below the seismogenic depth. From the scaling of the largest events, we get

$$\beta \approx \frac{2W}{H}\left(1 + \left.\frac{\mathcal{M}_{z>H}}{\mathcal{M}_{z\leq H}}\right|_{A/H^2 \gg 1}\right). \tag{10}$$

Some sense of the rough size of $\mathcal{M}_{z>H}/\mathcal{M}_{z\leq H}$ can be obtained from dynamic models. Shaw and Wesnousky (2008) found ratios around 0.5 for this ratio for the frictions they studied. Because it is the sum of this ratio plus unity that matters, we are relatively insensitive to the exact value and thus can take the value from Shaw and Wesnousky (2008) to get a rough estimate for W. Then $W \approx \beta H/(2 \times 1.5) \approx 35$ km. This, interestingly, corresponds roughly with typical values of the depth of the crust.

Magnitude-area scaling relations are one of the key branchpoints in seismic hazard calculations and gives one of the dominant uncertainties in hazard estimates (WGCEP, 2007). Here, we have shown that a physical hypothesis of constant stress drop across all magnitudes gives a scaling with a better fit to the whole range of the magnitude-area data. This work also raises additional questions. The question of whether slip penetrates coseismically deeply below the seismogenic depth remains an open question, and observations that can unambiguously constrain this would be extremely valuable. Revisiting the question of area measurements at moderate magnitude events is also called for, particularly with new precise locations of aftershock available (Waldhauser and Ellsworth, 2000). Finally, if earthquake stress drops are constant across all magnitudes, as this work suggests, the question remains: Why?

Data Resources

All data used in this article came from published sources listed in the references.

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