Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2004

Problem Set 11

(Due May 6, 2004)

1. In this problem you will complete the solution of the Eady baroclinic instability problem. To refresh your memory: we are considering quasigeostrophic motions of a uniformly rotating (f-plane), uniformly stratified (N constant) fluid bounded by two horizontal surfaces at $z = 0$ and $z = H$. The instability problem considers small perturbations about a background state consisting of a steady, uniformly sheared, zonal flow $\overline{u}(z) = (U_o/H)z$. This flow is in thermal wind balance with a horizontal density (temperature) gradient.

   (a) Make a contour plot of the growth rate ($\mathrm{Im}\,\omega$) as a function of k and l. (Nondimensionalize appropriately.) Note that the fastest growth occurs when $l = 0$. Wave motion is then purely in the meridional direction, i.e., down the mean temperature gradient, and the release of available potential energy is maximized.

   (b) In class we showed that short wavelength perturbations, i.e., waves for which $\mu H = \kappa_{\mathrm{H}}\lambda_d \gg 1$, the disturbances are stable and that there are two real roots to the dispersion relation. Show that in the limit of large μH, these roots correspond to the "non interacting" boundary wave solutions we found in class. What is the vertical structure of these limiting solutions? Here, and in what follows, it is convenient to write $\hat{\psi}(z) = |\hat{\psi}(z)| \exp i\alpha(z)$. Make plots of the amplitude $|\hat{\psi}(z)|$ and phase $\alpha(z)$ for $\mu H = 6$.

   (c) In the Eady problem, long waves, i.e., waves with μH less than a critical value, are unstable (c is complex).

      i. Find the full solution $\psi'(x, y, z, t) = \mathrm{Re}\,\hat{\psi}(z) \exp i(kx + ly - \omega t)$ for the unstable waves. (Write $\hat{\psi}(z)$ as above.)

      ii. For μH corresponding to the most unstable wave, make plots of $|\hat{\psi}(z)|$ and $\alpha(z)$ for both the growing and decaying solutions. Indicate in which direction the phase lines tilt with height for the two solutions. Note that $|\hat{\psi}(z)|$ takes on a minimum value at $z_c = H/2$. At this height, known as the *steering level*, the phase speed is equal to the local mean flow, i.e., $c_r = \overline{u}(z_c)$.

      iii. Calculate the meridional heat flux, $\overline{v'\theta'}$, associated with the disturbance. Show that a wave with phase lines tilting westward with height is associated with poleward heat flux, that is, a growing disturbance transports heat poleward (as it must if it is to draw upon the potential energy stored in the mean flow).

iv. For the most unstable wave, make contour plots (in the x-z plane) of the pressure field (ψ'), the temperature perturbation (θ'), and the meridional velocity (v'). Note the characteristic tilt of the phase lines. Also note that at the surface ($z = 0$), there is a phase shift between the temperature and pressure perturbations associated with the disturbance, with warm air just ahead (westward) of the pressure trough. This is in fact what is seen in observations. What is the magnitude of this phase shift?

v. For the most unstable wave, make a plot (in the y-z plane) of the velocity field (v',w'). (You will need to pick a particular value of x.)

(d) For a disturbance with $k = l$ (a so called "square Eady wave"), find the maximum growth rate and wavelength of the most unstable perturbation. Assuming a buoyancy frequency of $N = 10^{-2}$s^{-1}, what is the e-folding time (in days) for growth for this wave?