Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2004

Problem Set 9

(Due April 12, 2004)

1. Consider the statement for conservation of potential vorticity (PV) for a shallow layer of fluid (see figure):
$$\frac{D}{Dt}\left(\frac{\xi + f}{H}\right) = 0.$$
   (a) Show that, in the limit of small Rossby number, this statement is equivalent to the quasigeostrophic potential vorticity equation (QGPVE).
   (b) Show that the dimensional QGPVE can be written as:
$$\frac{\partial q}{\partial t} + J(\psi, q) = 0,$$
   where, the potential vorticity q is given by
$$q = \nabla^2\psi - \frac{1}{\lambda_d^2}\psi + \beta y + f_o\frac{h_B}{H_o},$$
   and, $\psi = g\eta/f_o$ is the streamfunction. (The fact that the O(1) flow is nondivergent allows us to define a streamfunction in this manner.)

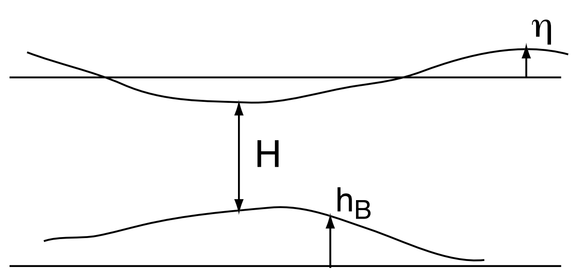

Figure 1: Definition sketch for problem 1.

2. Show that a single plane Rossby wave is an exact solution of the nonlinear QGPVE (see problem 1). What about a *sum* of plane waves?

3. Write down the linearized version of the QGPVE. Assuming that the ambient potential vorticity, $Q = \beta y + f_o h_B/H_o$ has a constant *gradient*, find the dispersion relation for quasi-geostrophic Rossby waves.