Problem Set 9
(Due April 12, 2004)

1. Consider the statement for conservation of potential vorticity (PV) for a shallow layer of fluid (see figure):

\[ \frac{D}{Dt} \left( \frac{\xi + f}{H} \right) = 0. \]

(a) Show that, in the limit of small Rossby number, this statement is equivalent to the quasigeostrophic potential vorticity equation (QGPVE).

(b) Show that the dimensional QGPVE can be written as:

\[ \frac{\partial q}{\partial t} + J(\psi, q) = 0, \]

where, the potential vorticity \( q \) is given by

\[ q = \nabla^2 \psi - \frac{1}{\lambda_d^2} \psi + \beta y + f_o \frac{h_B}{H_o}, \]

and, \( \psi = g \eta / f_o \) is the streamfunction. (The fact that the O(1) flow is nondivergent allows us to define a streamfunction in this manner.)

2. Show that a single plane Rossby wave is an exact solution of the nonlinear QGPVE (see problem 1). What about a sum of plane waves?

3. Write down the linearized version of the QGPVE. Assuming that the ambient potential vorticity, \( Q = \beta y + f_o h_B / H_o \) has a constant gradient, find the dispersion relation for quasigeostrophic Rossby waves.