Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2005

Problem Set 2

(Due Feb 10, 2005)

1. *Surface gravity waves*. Consider a homogeneous (constant density) layer of depth D.
   (a) Neglecting rotation, write down the linearized fluid equations (momentum and continuity).
   (b) Show that the perturbation pressure p' satisfies Laplace's equation.
   (c) Derive the linearized boundary conditions p' is subject to at the bottom, $z = -H$, and the top, $z = 0$. (The bottom is easy; to derive the condition at the top, note that the surface of the fluid is a material surface.)
   (d) Substituting a plane wave solution of the form $p' = R(z) \exp i(kx + ly - \omega t)$, find the gravity wave dispersion relation and the vertical structure $R(z)$.
   (e) Aligning the wave vector with the the x direction, i.e., $\mathbf{k} = (k, 0)$, make a plot of the ratio of group velocity to phase speed as a function of kD.

2. As we saw in the film in class, surface gravity waves generated by storms can travel extraordinarily large distances in the ocean. Indeed, in the field experiment described in the film, waves generated in the Indian Ocean south of New Zealand were tracked all the way across the Pacific Ocean to Alaska.

   One of the principal goals of the experiment was to use the dispersive property of surface waves, i.e., the fact that group velocity is a function of wavenumber, to locate their point of origin. (If the distance from the source region is large, then the source can be treated as a single point.) At a fixed point in space, dispersion manifests itself as a change over time in the observed frequency of waves. A beautiful example of this phenomenon is evident in the figure below. The top panel shows the observed energy spectrum of a wave train at Honolulu at roughly 12 hour intervals. (Ignore the lower two panels.) At any instant, there is a peak in the spectrum corresponding to the dominant frequency of the wave group. This peak shifts to higher frequencies over time. (The frequency is in millicycles per second or mHz.) Use the information in the plot to estimate the distance of the wave source from Honolulu. Give your answer in degrees. To simplify the calculation, recall that the wavelength of these waves is much smaller than the depth of the ocean, i.e., we are considering "deep water" waves. (You only need to provide a rough estimate!)

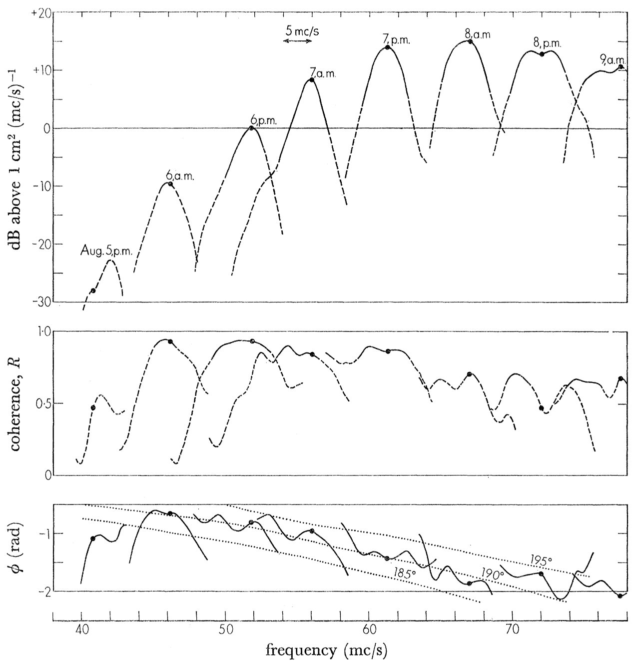

FIGURE 19. Successive Honolulu spectra for the event of 1·9 August. The results for 6 August p.m. are shown in more detail in figure 9. The dots correspond to the chosen ridge line for this particular event (see figure 16), and they are positioned relative to the bottom frequency scale. The spectra to either side are drawn on a compressed frequency scale to avoid overlap; the width of a 5 mc/s band is shown by the arrow. Solid curves correspond to a signal/background ratio above 10 dB, and a coherence above $0{\cdot}9R_0$ (see text). The dotted curves on the lower plot show $\phi(f)$ for various offshore directions as read from figure 14.