Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2005

Problem Set 3
(Due Feb 17, 2005)

1. *Boussinesq approximation.* In class we introduced the Boussinesq approximation in the context of the equation for the vertical velocity w. We showed that when the vertical scale on which ρ_o varies is much greater than that on which w varies, the term

$$\frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o \frac{\partial w}{\partial z})$$

simplifies to

$$\frac{\partial^2 w}{\partial z^2}.$$

The Boussinesq approximation can also be stated as follows: in the momentum equations, variations in density may be neglected except when computing buoyancy forces, i.e., wherever density is coupled to gravity. Thus, in the x-momentum equation

$$\rho_o \frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x},$$

we may replace ρ_o with $\overline{\rho}$, where $\overline{\rho}$ is a *mean* density. The z-momentum equation can then be written as:

$$\overline{\rho}\frac{\partial w}{\partial t} = -\frac{\partial p'}{\partial z} - \rho^{'} g.$$

Thus, rather than neglecting density variations (in certain terms) in the w equation (as we did in class) we can equivalently make the Boussinesq approximation at the outset in the momentum equations.

Now for the problem. Write down the linearized governing equations for a stratified fluid in the Boussinesq limit. Manipulating them as we did in class, derive a single equation for w and show that it is indeed the same as the simplified equation for w derived in class.

2. *Internal gravity waves*. Derive an expression for the group velocity of internal gravity waves. Show that the group velocity is perpendicular to the wave vector. Since particle motion is generally in the direction of the group velocity, this implies that internal waves are transverse waves.