Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2005

Problem Set 6

(Due April 7, 2005)

1. *Poincare waves on a β-plane*. Consider the shallow water equations on a β-plane. For simplicity assume a flat bottom. In class we showed that the gravity wave solution, namely the Poincare waves, are only slightly modified by the presence of β. Quantify this by deriving the lowest order *correction* to the dispersion relation. Give a numerical estimate of this correction for a mid-latitude (45°N) baroclinic plane wave of zonal wavenumber $k\lambda_d = 3$, and meridional wavenumber $l\lambda_d = 0$. Assume a first baroclinic phase speed of 3 ms^{-1}. *Hint*: You may find it convenient to work with the cubic equation for ω that we derived in class. In particular, observe (graphically) what happens to the gravity wave root as $\beta \rightarrow 0$, and then approximate accordingly.

2. *Rossby wave dispersion*. Derive expressions for the group velocity and zonal phase speed (ω/k) for shallow water Rossby waves in a fluid of constant depth. For $l = 0$, make a plot of ω as a function of k. (Nondimensionalize axes in a sensible manner.) Indicate on the figure the direction in which phase and energy propagate. Assuming a mean stratification of $N = 2 \times 10^{-3}$ and a depth of 4300 m, provide numerical estimates of the zonal group and phase velocity for mid-latitude (45°) first baroclinic mode Rossby waves in the ocean.

3. *Doppler shifted dispersion relation*. In the presence of a mean zonal current (or wind) U, the governing equation (linearized about the background flow) for low frequency motions is given by:
$$(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x})(\nabla^2 - \frac{1}{\lambda_d^2})\eta + \beta\frac{\partial \eta}{\partial x} = 0.$$
(This is the equation one gets if the momentum equations are linearized, not about a state of rest as we have done until now, but about a zonal background flow.)

   (a) Derive and physically interpret the dispersion relation for small amplitude waves governed by this equation.

   (b) The figure below shows a longitude-time plot of 500 mbar geopotential height anomalies averaged over 45-50°N. The data are for a 25 day period beginning on January 1, 1997. The mean zonal winds at this latitude and for that period were roughly 20 ms^{-2}. Suppose we were to interpret the pattern of height anomalies as that associated with barotropic Rossby waves. (These are all gross approximations, but adequate for the purpose at hand.) Based on the results of problem 2 above, what is the most striking feature of this plot? Use part (a) above to interpret these observations. A qualitative explanation will do, and probably the best one can do given the not atypically noisy data. But this should not stop you from suspending disbelief and plugging in some numbers to see if your qualitative argument make quantitative sense. (You can make your own plots and animations here: `http://www.cdc.noaa.gov/map`.)