Problem Set 7
(Due Apr 17, 2008)

1. At mid-latitudes, there is a strong separation of time scales between gravity and Rossby waves. At the equator, however, this separation is much weaker. Demonstrate this by finding the minimum frequency of equatorial gravity waves \( \omega_{\text{min}} \), and the maximum frequency of equatorial Rossby waves \( \omega_{\text{max}} \), as a function of the mode number \( n \). Compute the ratio \( \omega_{\text{min}} / \omega_{\text{max}} \) for \( n = 1 \). Now, repeat this exercise for mid-latitude gravity and Rossby waves for the first baroclinic mode of the ocean.

2. At the equator, the Coriolis parameter \( f \) vanishes, and its \( y \) dependence cannot be neglected. The dispersion relation for waves near the equator is therefore rather complicated. However, in analogy with its mid-latitude \( f \)-plane counterpart, we may approximate the dispersion relation for Poincare waves by

\[
\omega^2 = f(y)^2 + c^2 (k^2 + l^2),
\]

where \( f(y) = \beta y \), \( c \) is a shallow water gravity wave speed, and \( k \) and \( l \) are the zonal and meridional wavenumbers, respectively.

(a) Find the \( x \) and \( y \) components of the group velocity.

(b) Following a “slowly varying” wave packet, discuss how the frequency and wavenumber components vary in space and time.

(c) Calculate, sketch, and discuss the trajectory of a wave packet (ray path) in the \( x-y \) plane. (There is an analytical solution for the trajectory, \( y(x) \).) What happens at the value of \( y_c \), such that \( f(y_c)^2 = \omega^2 - c^2 k^2 \)?