Department of Earth and Environmental Science
Columbia University
EESC G9810. Mathematical Earth Science Seminar: Vibrations and Waves
Spring 2003

Problem Set 1

(Due Feb 3, 2003)

1. Problem 1-6 in French
2. Problem 1-8 in French
3. Problem 1-9 in French
4. Problem 3-1 in French
5. Problem 3-15 in French
6. Problem 3-18 in French. This problem is on waves in fluids. In particular, it asks you to calculate the period of the "sloshing" mode. (You don't need to know any fluid dynamics; the book leads you through the derivation.) As French points out, the derivation is only *approximate*, but in a few weeks you will be able to simply write down the *exact* solution mentioned in the book. This mode is often referred to as a seiche and is typically excited by wind bursts. Lamont's Ken Hunkins has done some interesting work on seiches in Lake Champaign.
7. Exact solution of pendulum problem. The equation for large amplitude oscillations of a pendulum is nonlinear:
$$\ddot{\theta} = -(g/l)\sin\theta$$
Amazingly, this equation has an *exact* solution. In particular, the period of the pendulum is given by
$$T(\text{exact}) = T_o\left[1 + \frac{1}{4}\sin^2(\frac{\theta_o}{2}) + \frac{9}{64}\sin^4(\frac{\theta_o}{2}) + \ldots\right],$$
where $T_o = 2\pi\sqrt{l/g}$ is the period in the linear limit. The term in brackets (up to a constant prefactor) is known as the "complete elliptic integral of the first kind". To satisfy your curiosity look this up in your favorite math book (Chap. 11 of Boas is relatively comprehensible). Suppose the amplitude $\theta_o = 0.2$ radian. Find the approximate relative error in the period computed using the linear relation.

8. Tidal friction. The rotational energy in the Earth-Moon system is continuously lost through frictional forces. Oceanic tides make a large contribution to this dissipation. Given a global tide energy of 4×10^5 TJ (1 TJ=$10^{1}2$ J), and a tidal dissipation rate of ≈ 2.5 TW, calculate (a) the time it takes to renew the tidal energy, and (b) the Q of the ocean (for the barotropic tide).