Department of Earth and Environmental Science
Columbia University

EESC G9810. Mathematical Earth Science Seminar: Vibrations and Waves

Spring 2003

Problem Set 2

(Due Feb 17, 2003)

1. Problem 4-1 in French

2. Problem 4-6 in French

3. Problem 4-11 in French

4. Transient beats: Consider a forced, damped harmonic oscillator $\ddot{x}+\gamma\dot{x}+\omega_o^2 x = (F_o/m)\cos\omega t$. Suppose the damping is weak and the forcing frequency ω is close to the frequency of the free oscillations, $\omega_1 = \omega_o(1-(\gamma/2\omega_o)^2)$. Assuming $x(t=0) = 0$ and $\dot{x}(t=0) = 0$, compute the *complete* solution $x(t)$. Also compute the total stored energy $E(t)$ of the system. Make plots of $x(t)$ and $E(t)$. You will find that unless the driving frequency ω is equal to the free-oscillation frequency ω_1, the energy does not build up smoothly to its steady-state value. Instead the system undergoes "beats". Find this beat frequency. Give a qualitative explaination for this phenomenon of "transient beats". If the system is even slightly damped, the system will eventually settle into its steady-state behavior. How long does it take to reach this steady-state.

5. Design of a seismometer: Explain why seismic instruments are designed with a low Q value.

6. Nonlinearities: Suppose the restoring force on a particle is $F = -kx + \epsilon m x^2$, where m is the mass of the particle, k the spring constant, and ϵ a "small" parameter. Write down the equation of motion of this particle. If $\epsilon = 0$ we know how to solve this problem. Now suppose $\epsilon \neq 0$ but $\ll 1$. Try a solution of the form $x(t) = x_o\cos(\omega_o t) + \epsilon x_1(t)$. Neglect all terms with ϵ^2 and higher orders, and find the approximate solution which should contains oscillations at both the fundamental frequency ω_o and its second harmonic. Note that this "perturbation" procedure breaks down completely when the nonlinearity is $-\epsilon m x^3$. The perturbation approach yields a solution which is, in the language of mathematics, not "uniformly valid".