EESC G9810. Mathematical Earth Science Seminar: Waves **Spring 2003**

Lecture #0: Organizational Meeting Jan 17 , 2003

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Syllabus

1. Discrete Systems
   (a) The simple harmonic oscillator
   (b) Forced oscillations and resonance
   (c) Coupled oscillators and normal modes
   (d) Dispersion I: the beaded string
2. Math Review I: Linear PDEs
   (a) Derivation of heat and wave equations
   (b) Separation of variables
   (c) Fourier series
   (d) Sturm-Liouville theory and eigenfunctions
3. Waves on a String
   (a) The classical wave equation
   (b) Energy and momentum transport
   (c) Dispersion II: the Klein-Gordon equation
   (d) Phase and group velocity
4. Surface Gravity Waves (Non-Rotating)
   (a) Equations of motion
   (b) Energy and momentum transport
   (c) Dispersion III
5. Math Review II
   (a) Fourier and Laplace transforms
   (b) Initial value problems
   (c) The stationary phase approximation

   (d) Applications to surface waves

6. Shallow Water Equations (Non-Rotating)
   (a) The hydrostatic approximation
   (b) Adjustment under gravity
   (c) Energy transport

7. Effect of Stratification
   (a) Internal gravity waves
   (b) Normal modes for layered and continuously stratified fluids
   (c) Normal modes and the WKBJ approximation

8. Shallow water equations (Rotating)
   (a) Poincare and Kelvin waves
   (b) Effect of boundaries

9. Geostrophic Adjustment
   (a) The Rossby adjustment problem
   (b) Energetics

10. Rossby Waves
   (a) Dispersion IV
   (b) Energy transport

Textbooks

1. Vibrations and Waves, A. P. French (Norton).
2. Vibration and Waves in Physics, I. Main (Cambridge).
3. Atmosphere-Ocean Dynamics. A. E. Gill (Academic Press).
4. Elementary Applied Partial Differential Equations, R. Haberman (Prentice Hall).
5. Geophysical Fluid Dynamics, J. Pedlosky (Springer).
6. Mathematical Methods in the Physical Sciences, M. Boas (Wiley).
7. Methods of Mathematical Physics. Jeffreys and Jeffreys (Cambridge; 3d ed.).

8. Physics of Waves, W. C. Elmore and M. A. Heald (Dover).
9. Vibration and Sound, P. M. Morse (Acoustical Society of America).
10. Waves in the Ocean, P. LeBlond and L. Mysak (Elsevier).

Note: I highly recommend buying the book by French. I will assign problems from it. Plus it is worth having. If you don't own a decent book on PDEs, I suggest getting Haberman. Morse is also worth having. It is easily one of the most sophisticated (mathematically and physically) books on the subject of "vibration and waves". For waves in ocean and atmosphere we will primarily use Gill. Although Pedlosky is somewhat more systematic, Gill is more in the spirit of this class. French, Main, Gill, and Haberman will be on **reserve in the Geoscience library**.

Homework and Grading

Grading will be based on your performance on weekly homeworks. There will be no mid-term or final exams.

Questionnaire

I need to get a sense of the level of preparation of those attending. Please take a few minutes out to fill out this questionnaire. Most questions simply require a “yes” or “no” answer.

1. I know at least one method to solve a system of algebraic equations $\mathbf{A}\mathbf{x} = \mathbf{b}$
2. My favorite method to solve $\mathbf{A}\mathbf{x} = \mathbf{b}$ is ...
3. I know what
   (a) a linear vector space is
   (b) the nullspace and rank of a matrix are
   (c) eigenvectors and eigenvalues are
4. Solutions of $\mathbf{A}\mathbf{x} = \mathbf{b}$ form a vector space if and only if
5. $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a unique solution if and only if $\mathbf{A}$ is ...
6. Two vectors $\mathbf{x}$ and $\mathbf{y}$ are orthogonal if ...
7. I know how to solve linear, ordinary differential equations.
8. $y'' + ky = 0$ (primes denote derivative) has ?? independent solutions.
9. The solution(s) of (8) are ...
10. The equation $y'' + y(x) = \sin(x)$ illustrates the concept of ...
11. I know how to solve linear PDEs by
    (a) separation of variables,
    (b) Fourier transforms
    (c) Laplace transforms
12. I know what
    (a) a Fourier series is
    (b) the Gibbs phenomenon is
    (c) eigenfunctions and eigenvalues of differential operators are
13. The classical wave equation with phase speed c is ...
14. The general solution to $y_{tt} = c^2 y_{xx}$ (subscripts are partial derivatives) can be written as ...
15. An example of a dispersive wave equation is ...

16. If the dispersion relation (between frequency ω and wavenumber k) is $\omega = \Omega(k)$, the

   (a) phase speed is ...

   (b) group velocity is ...