Supplementary Online Material:

Evolution of subglacial overdeepenings in response to sediment redistribution and glaciohydraulic supercooling

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S1. Longitudinal sections

Rather than work with exclusively idealized model geometries, we base our examination of sediment transport and supercooling on simplified representations of Matanuska Glacier. Matanuska stands out as the best-studied example of glaciohydraulic supercooling with glaciofluvial sediment transport.

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We use ice–bed configurations (Figs. 4a,b) that are inspired by longitudinal sections of Storglaciaren (pers. comm. P. Jansson, 2006) and Matanuska Glacier [Lawson et al., 1998]. These sections, however, have less structure in the bed and surface slopes than those glaciers. Each of the sections has a constant surface slope and a bed elevation that is given by,

$$z_b = \begin{cases} 
  z_l & \text{for } x_u < x \leq x_l, \\
  C_x (x - x_u)^{2c_b} + z_l & \text{for } x_w < x \leq x_u, \\
  2C_x c_b (x_w - x_u)^{2c_b-1} (x - x_w) + C_x (x_w - x_u)^{2c_b} + z_l & \text{for } x_0 \leq x \leq x_w.
\end{cases} \quad (S1)$$

where $x_0$ is the outlet location, $x_l$ is the inlet location, $x_u$ is the start of the adverse slope, $x_w$ is the start of the adverse slope with constant value, $c_b$ and $C_x$ are constants, and $z_l$ is the inlet elevation of the bed. Values for terms are given in Table S1. The elevation of the ice surface is simply a line of constant slope. Other details can be found in Creyts and Clarke [2010].
Table S1. Parameters used to create the synthetic glacier sections in Figure 4.

<table>
<thead>
<tr>
<th>Figure location</th>
<th>Section at twice the critical threshold</th>
<th>Section at the threshold for supercooling</th>
<th>Section at half the critical threshold</th>
<th>Flat-bedded section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outlet location:</td>
<td>(x_0) m</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Inlet location:</td>
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<tr>
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</tr>
<tr>
<td>Outlet ice thickness:</td>
<td>(Z_0) m</td>
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<td>0.00</td>
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<td>Inlet ice thickness:</td>
<td>(Z_l) m</td>
<td>35.99</td>
<td>46.86</td>
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<td>Bed to surface slope ratio: (R)</td>
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<td>-3.40</td>
<td>-1.70</td>
<td>-0.85</td>
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<tr>
<td>Surface slope:</td>
<td>(\tan \alpha_r)</td>
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<tr>
<td>Bed slope:</td>
<td>(\tan \alpha_w)</td>
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<tr>
<td>Transition index:</td>
<td>(c_b)</td>
<td>-</td>
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<td>1.0</td>
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<tr>
<td>Coefficient:</td>
<td>(C_x) m(^{1-2c_b})</td>
<td>0.002</td>
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</table>

Table S2. Supplementary parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu)</td>
<td>(1.781 \times 10^{-3}) Pa s</td>
<td>Viscosity of water; equation (S10)</td>
<td></td>
</tr>
<tr>
<td>(\kappa_s)</td>
<td>0.4</td>
<td>-</td>
<td>von Karman’s constant; equation (S26)</td>
</tr>
<tr>
<td>(D_{50})</td>
<td>0.001 m</td>
<td>Median grain diameter; eqs. (S10), (S18), and (S19)</td>
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</tr>
<tr>
<td>(D_{90})</td>
<td>0.0018 m</td>
<td>Grain diameter of the 90th percentile; equation (S31)</td>
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</tr>
<tr>
<td>(n_{s,a})</td>
<td>0.5 [unitless]</td>
<td>Mobile fraction of the bed; equation (S21)</td>
<td></td>
</tr>
</tbody>
</table>
S2. Sediment balance in the flowing water

Here, we give an abridged derivation of the sediment balance in the flowing water. The total mass of sediment in the flowing water $M_{sw}$ is simply the sum of the suspended component $M_{sw:sus}$ and the bed component $M_{sw:bed}$,

$$M_{sw} = M_{sw:sus} + M_{sw:bed},$$

where the time rate of change of equation (S2) also leads to the summation of the individual rates.

An ad hoc derivation is for mass of suspended sediment and bed load sediment in the flowing water are,

$$M_{sw:sus} = \int_x \int_W \int_{z_b}^{z_w} \lambda_{sw:sus}(V, t) \rho_s \, dV,$$
$$M_{sw:bed} = \int_x \int_W \int_{z_b}^{z_b+\delta_b} \lambda_{sw:bed}(V, t) \rho_s \, dV,$$

where $W$ is flow width, $dV$ is a unit volume of flowing water along the bed, $z_b$ and $z_w$ are the top and bottom of the water layer as defined in Figure 2a, and $\delta_b$ is the bed load layer height shown in Figure 2b in the main body of the text.

Moving to a local balance gives,

$$\rho_s \frac{\partial}{\partial t} (\lambda_{sw:sus}H) + \rho_s \frac{\partial}{\partial s} (u \lambda_{sw:sus}H) = \dot{\rho}_s \left( H \frac{\partial \lambda_{sw:sus}}{\partial t} + \lambda_{sw:sus} \frac{\partial H}{\partial t} \right) + \rho_s \frac{\partial q_{sw:sus}}{\partial s},$$
$$= \dot{\Psi}_{b:sus} + \dot{\Psi}_{i:sus},$$

$$\rho_s \frac{\partial}{\partial t} (\lambda_{sw:bed}H) + \rho_s \frac{\partial}{\partial s} (u_{bed} \lambda_{sw:bed}H) = \dot{\rho}_s \left( H \frac{\partial \lambda_{sw:bed}}{\partial t} + \lambda_{sw:bed} \frac{\partial H}{\partial t} \right) + \rho_s \frac{\partial q_{sw:bed}}{\partial s},$$
$$= \dot{\Psi}_{b:bed} + \dot{\Psi}_{i:bed},$$

where we have assumed that sediment is incompressible (i.e., $\partial \rho_s/\partial t = 0$), and sediment sources are either from the underlying bed ($\dot{\Psi}_{b:sus}$ and $\dot{\Psi}_{b:bed}$) or the overlying ice ($\dot{\Psi}_{i:sus}$ and $\dot{\Psi}_{i:bed}$). We
define the 1D fluxes of sediment as,

\[ q_{sw} = q_{sw:sus} + q_{sw:bed}; \]  
\[ q_{sw:sus} = u_{sw:sus} H; \]  
\[ q_{sw:bed} = u_{bed} \lambda_{sw:bed} \delta_{bed}; \]

where the bed load velocity \( u_{bed} \) is not necessarily equal to the water velocity \( u \). As is common, we assume that the suspended load travels at the water velocity. Similarly, because the suspended and bed load mass sums in equation (S2), the supply terms must also sum,

\[ \tilde{\Psi}_l = \tilde{\Psi}_{l:bed} + \tilde{\Psi}_{l:sus}; \]  
\[ \tilde{\Psi}_b = \tilde{\Psi}_{b:bed} + \tilde{\Psi}_{b:sus}. \]

Rewriting equations (S4a) and (S4c) for the time rate of change for sediment concentration gives,

\[ \frac{\partial \lambda_{sw:sus}}{\partial t} = \frac{1}{\rho_s H} \left( -\rho_s \lambda_{sw:sus} \frac{\partial H}{\partial t} - \rho_s \frac{\partial q_{sw:sus}}{\partial s} + \tilde{\Psi}_{b:sus} + \tilde{\Psi}_{i:sus} \right), \]  
\[ \frac{\partial \lambda_{sw:bed}}{\partial t} = \frac{1}{\rho_s H} \left( -\rho_s \lambda_{sw:bed} \frac{\partial H}{\partial t} - \rho_s \frac{\partial q_{sw:bed}}{\partial s} + \tilde{\Psi}_{b:bed} + \tilde{\Psi}_{i:bed} \right), \]

that are exactly equations (4a) and (4b) in the original manuscript.

**S2.1. Exner equation in one dimension**

Now, setting \( \partial \lambda_{sw:sus}/\partial t = \partial \lambda_{sw:bed}/\partial t = 0 \) and \( \partial H/\partial t = 0 \) for steady state conditions, and ignoring supply from the overlying ice (\( \tilde{\Psi}_{i:sus} = \tilde{\Psi}_{i:bed} = 0 \)), we manipulate equations (S7a) and (S7b) to obtain,

\[ \tilde{\Psi}_{b:bed} = \rho_s \frac{\partial q_{sw:bed}}{\partial s}; \]
\[ \tilde{\Psi}_{b:sus} = \rho_s \frac{\partial q_{sw:sus}}{\partial s}. \]
so that combining these with equation (S6b) gives,

\[ \tilde{\Psi}_b = \rho_s \frac{\partial q_{sw}}{\partial s} . \]  

(S8c)

Substituting this equation with equation (5) in the main body of the text gives,

\[ \frac{\partial z_b}{\partial t} = -\frac{1}{(1 - n_b)\rho_s} \tilde{\Psi}_b , \]  

(S9a)

\[ = -\frac{1}{(1 - n_b)} \frac{\partial q_{sw}}{\partial s} . \]  

(S9b)

This is the 1D Exner equation discussed in the main body of the text [e.g., Henderson, 1966, Chap. 10] that is used in the analytic solutions of subglacial sediment transport. Thus, to recover the 1D Exner equation, no additional information is needed other than the general balance rules and an assumption about supply from the ice and bed.

S2.2. Nonequilibrium sediment load

The nonequilibrium sediment erosion and deposition as formulated in equation (7) always drives the instantaneous sediment load back towards equilibrium conditions [e.g., Einstein, 1968]. Equation (7) is not the downstream divergence of total sediment flux because it is not a spatial derivative nor is it related to the balance rules laid out above for sediment. It is a heuristic way of calculating the rate of sediment exchange between the flowing water and underlying bed. Equation (6) drives both the bed load and suspended flux calculated from equations (S5) toward the semiempirical values calculated from van Rijn’s equations. The sediment concentration in the water is calculated from equations (4a) and (4b) in the manuscript.

S3. Sediment transport

In what follows, we briefly review van Rijn’s [1984a; 1984b] formulas that we use to determine the equilibrium sediment flux \( q_{sw,e} \) for both bed load and suspended load. The aim of this
appendix is to present the transport relations in the form in which we implement them. We present the simple formulation followed by the detailed procedure. The derivations use theoretical and empirical considerations, and we refer the reader to the original papers for these derivations.

S3.1. Conditions necessary for sediment transport

van Rijn [1984a] used two nondimensional numbers to describe sediment motion: the particle parameter and the transport stage parameter. The particle parameter describes the nondimensional grain size,

\[ D_\ast = D_{50} \left\{ \frac{\rho_s - 1}{\rho_w \nu^2} \right\}^{\frac{1}{3}}, \tag{S10} \]

where \( \nu \) is the kinematic viscosity of water. This equation assigns the length scale for the nondimensional grain size as the median grain size \( D_{50} \) of the mobile bed. \( D_\ast \) is a ratio of the gravity forces on a particle in fluid relative to the viscous forces. The second nondimensional number is transport stage parameter, which characterizes an excess shear stress available to move sediment,

\[ T = \frac{(u'_s)^2 - (u_{s,cr})^2}{(u_{s,cr})^2}, \tag{S11} \]

where \( u'_s \) is the shear velocity on the grains, and \( u_{s,cr} \) is a critical bed shear velocity. In (S11),

\[ u'_s = u \left( \frac{f'_b}{8} \right)^{1/2}, \tag{S12} \]

is the bed shear velocity related to the grains using the Darcy-Weisbach formulation. The bed shear velocity is

\[ u_s = \sqrt{\frac{\tau_0}{\rho_w}}, \tag{S13} \]

where \( \tau_0 \) is the shear along the hydraulic perimeter (equation (12)).
In equation (S11), \( u_{*,cr} \) is the critical bed shear velocity from the Shields criterion

\[
\theta_{cr} = \frac{u_{*,cr}^2}{\left( \frac{\rho_s}{\rho_w} - 1 \right) gD}, \tag{S14}
\]

where \( \theta_{cr} \) is a critical mobility parameter. The Shields threshold criterion is commonly used to express the critical shear stress necessary for motion,

\[
\tau_{cr} = (\rho_s - \rho_w) gD\theta_{cr}. \tag{S15}
\]

Shear stresses below this critical value will not cause motion of the sediments. While arguments against this criterion exist [e.g., Yang, 1996, p. 22–23], no relation is in more common use for the initial motion of sediments. Following van Rijn [1984a], a piecewise approximation for the critical mobility parameter is

\[
\theta_{cr} = \begin{cases} 
0.24 D_{*}^{-1} & \text{for } D_* \leq 4.5, \\
0.14 D_{*}^{-0.64} & \text{for } 4.5 < D_* \leq 10, \\
0.04 D_{*}^{-0.10} & \text{for } 10 < D_* \leq 18, \\
0.013 D_{*}^{0.29} & \text{for } 18 < D_* \leq 144, \\
0.055 & \text{for } 144 < D_*,
\end{cases} \tag{S16}
\]

where we have modified the original form to make the curve smoother without changing its overall effect. A value of 0.055 for \( D_* > 144 \) indicates tightly packed, uniform beds.

![Shields diagram](image)

**Figure S1.** Shields diagram. An illustration of equation (S16).
S3.2. Equilibrium bed load flux

van Rijn assumed that most relatively fine-grained ‘bedload’ (i.e., sand) moves by saltation and developed a physical model based on first principles. He then compared model simulations against experimental data of saltation along the bed. His model reproduced observed jump lengths and jump heights of sand grains reasonably well. The jump height varies from a few millimeters to about a centimeter. The jump length varies from a few millimeters to a few centimeters [Abbott and Francis, 1977; Lee and Hsu, 1994; van Rijn, 1984a]. He then scaled this model to create a bed load transport relation. The relation is valid for grain sizes in the range \(0.2 \leq D \leq 2.0\) mm. van Rijn [1984a] supported his theory with examples from experimental data.

Equilibrium bed load flux \(q_{sw,\text{bed}}\), is

\[
q_{sw,\text{bed}} = u_{\text{bed}} \delta_{\text{bed}} \lambda_{sw,\text{bed}}, \tag{S17}
\]

where \(u_{\text{bed}}\) is the mean bed load sediment velocity, \(\delta_{\text{bed}}\) is the thickness of the bed load layer, and \(\lambda_{sw,\text{bed}}\) is the volumetric bed load sediment concentration.

The bed load velocity is assigned via

\[
u_{\text{bed}} = 1.5 \mathcal{T}^{0.6} \left( \frac{\rho_s}{\rho_w} - 1 \right) g D_{50}^{0.5} . \tag{S18}
\]

Transport occurs in the bed load layer with thickness

\[
\delta_{\text{bed}} = 0.3 D_{\star}^{0.7} \mathcal{T}^{0.5} D_{50} \tag{S19}
\]

From the discussion above, \(\delta_{\text{bed}}\) is approximately one centimeter for sand. Finally, the volumetric sediment concentration in the bed load layer is

\[
\lambda_{sw,\text{bed}} = 0.18 \lambda_{sw,0,\text{bed}} \frac{\mathcal{T}}{D_{\star}}, \tag{S20}
\]
where $\lambda_{sw,0:bed}$ is a reference concentration for the bed,

$$
\lambda_{sw,0:bed} = n_{s,a}(1 - n_b),
$$

(S21)

and $n_{s,a}$ is the mobile fraction of the bed. van Rijn [1984a] considered the entire bed mobile and gave $\lambda_{sw,0:bed}$ as 0.65. Because the closure relation requires larger, immobile clasts to be held static by the bed [Creyts and Clarke, 2010], we set the mobile fraction of the bed $n_{s,a} = 0.56$.

This term states what areal fraction of the bed material is mobile.

Substituting equations (S18) to (S20) into equation (S17) yields the final form of the bed load flux,

$$
q_{sw,e:bed} = 0.081\lambda_{sw,0:bed} \frac{T^{2.1}}{D_{*}^{0.3}} \left( \frac{\rho_s}{\rho_w - 1} \right) g \left\{ \frac{1}{D_{50}^{1.5}} \right\}^{0.5}.
$$

(S22)

**Bed Load Flux Procedure**

1. Compute the particle parameter $D_*$ using equation (S10).

2. Compute the critical bed-shear velocity according to the Shields criterion (eq. (S16), Fig. S1, eq. (S14)).

3. Compute the effective bed shear velocity using equation (S12).

4. Compute the transport stage parameter $T$ (S11).

5. Compute the equilibrium bed load transport $q_{sw,e:bed}$ using equation (S22).

**S3.3. Suspended load flux**

The formula for the suspended load is

$$
q_{sw,e:sus} = uH\lambda_{sw,e:sus},
$$

(S23)

where $\lambda_{sw,e:sus}$ is the mean equilibrium suspended sediment concentration. This equation assumes that all suspended load transport occurs at the water velocity and that the concentration is
constant across the water depth. van Rijn [1984b] notes that his relation is valid for grain sizes in the range \((0.1 < D < 0.5 \text{ mm})\). Grain sizes smaller than 0.1 mm, silt and finer particles, undoubtedly enter suspension. For these grain sizes, the suspension velocity is less than the entrainment velocity, so they act as a ‘wash’ load. The wash load is entirely governed by supply. The division is usually in the fine sand range, but there is no simple hydraulic formulation for wash load. We therefore assume that these grain sizes act as fine sand.

**Formulation**

The initiation of suspended load is commonly given in terms of the fall velocity of a sediment particle as well as the shear velocity of the water. Based on experimental results, van Rijn [1984b] formulated a criterion for the initiation of suspended load,

\[
  u_{*,crs} = \begin{cases} \frac{4w_s}{D_s} & \text{for } 1 < D_s \leq 10, \\ 0.4w_s & \text{for } 10 < D_s, \end{cases} \tag{S24}
\]

where \(u_{*,crs}\) is the critical shear velocity necessary for entrainment into suspension, and \(w_s\) is the fall velocity of a grain. For large grain sizes, \(u_{*,crs}\) takes the value 0.4\(w_s\). The sediment fall velocity \(w_s\) is

\[
  w_s = \begin{cases} \left( \frac{\rho_s}{\rho_w} - 1 \right) \frac{gD^2}{18\nu} & \text{for } D \leq 0.1 \text{ mm}, \\ 10\frac{\nu}{D} \left[ \frac{0.01 \left( \frac{\rho_s}{\rho_w} - 1 \right) gD^3}{1 + \frac{\nu^2}{\rho_w}} \right]^{\frac{1}{2}} - 1 & \text{for } 0.1 < D \leq 1.0 \text{ mm}, \\ 1.1 \left( \frac{\rho_s}{\rho_w} - 1 \right) \frac{gD}{\nu^2}^{\frac{1}{2}} & \text{for } 1.0 < D, \end{cases} \tag{S25}
\]

where the first case is Stoke’s law, the last case is a turbulent settling law, and the middle case represents a transition between the two.

A suspension parameter \(Z\) relates the downward fall of sediments to upward turbulent motions,

\[
  Z = \frac{w_s}{\beta_s \kappa_s u_s}, \tag{S26}
\]
where $\kappa_s$ is von Karman’s constant and $\beta_s$ describes how the sediments interact with the individual turbulent eddies. Values of $\beta_s$ greater than unity indicate that the sediments are propelled to the outside of the individual eddies. Sediments at the outside of the eddies mix more readily in the flow [van Rijn, 1984b]. The $\beta_s$ parameter is

$$\beta_s = 1 + 2 \left( \frac{w_s}{u_*} \right)^2 \quad \text{for} \quad 0.1 < \frac{w_s}{u_*} < 1. \quad (S27)$$

If the fall velocity is greater than the bed shear velocity in equation (S27), then the particle cannot be in suspension. Particles in the flow will occupy space, damp turbulence, and reduce the particle fall velocity. As a result, a simple correction to the suspension parameter is introduced,

$$Z' = Z + \varphi, \quad (S28)$$

where $\varphi$ is the correction for these additional effects. van Rijn [1984b] used this parameter to fit theoretical values to measured vertical profiles of sediment concentration. This correction is

$$\varphi = 2.5 \left( \frac{w_s}{u_*} \right)^{0.8} \left( \frac{\lambda_{sw,0:sus}}{\lambda_{sw,0:bed}} \right)^{0.4} \quad \text{for} \quad 0.01 \leq \frac{w_s}{u_*} \leq 1, \quad (S29)$$

where $\lambda_{sw,0:sus}$ is the sediment concentration at reference level $\delta_s$, and $\lambda_{sw,0:bed}$ is the maximum sediment concentration of the bed. This maximum concentration is equivalent to the solid fraction of the bed, $(1 - n_b)$. The reference value for suspended sediment is

$$\lambda_{sw,0:sus} = 0.015 \frac{D_{50}}{\delta_s} \frac{T^{3/2}}{D_*^{0.3}}, \quad (S30)$$

where $D_{50}$, $D_*$, and $T$ were defined for bed load.

The suspended sediment reference level can take a reasonable value near the bed. van Rijn [1984b] suggested that the reference level could be the bedform height $\Delta_s$, or the Nikuradse sand bed roughness $k_s$. Little information is known about the reference height in subglacial water flows. Based on the discussion presented by van Rijn [1984b], we arbitrarily modify the reference
level to be
\[ \delta_s = \max(0.01H, D_{90}) \]  
(S31)
such that \( \min(\delta_s) = 0.01H \).

The mean size of the suspended load will likely differ from the mean grain size of the bed. To account for this difference, \( D_s \) is a characteristic suspended grain size,
\[ D_s = D_{50} \left[ 1 + 0.011 (\sigma_s - 1) (T - 25) \right]. \]  
(S32)
where \( \sigma_s \) is the geometric standard deviation of the sediment distribution. We adopt the standard deviation as \( \sigma_s = 2.5 \) following van Rijn [1984b].

Because suspended sediment travels at the mean flow velocity \( u \), the suspended sediment flux is
\[ q_{sw,\text{csus}} = uH\lambda_{sw,\text{csus}}. \]  
(S33)
where the volumetric sediment concentration is
\[ \lambda_{sw,\text{csus}} = F_s \lambda_{sw,0;\text{csus}}. \]  
(S34)
The correction factor \( F_s \) accounts for a nonuniform vertical distribution of suspended sediment,
\[ F_s = \frac{\left( \frac{\delta_s}{H} \right)^{2'} - \left( \frac{\delta_s}{H} \right)^{1.2}}{\left( 1 - \frac{\delta_s}{H} \right)^{2'} \left( 1.2 - Z' \right)}. \]  
(S35)

**Suspended load flux procedure**

van Rijn [1984b] develops a method of calculating the suspended sediment load for particles in the size range 0.1–0.5 mm. The procedure presented here is taken from van Rijn [1984b].

1. Compute the particle parameter \( D_s \) using equation (S10).
2. Compute the critical bed-shear velocity \( u_{*,cr} \) using equations (S14) and (S16).
3. Compute the transport stage parameter $T$ (equation S11).

4. Compute the reference level $\delta_s$ using equation (S31).

5. Compute the reference concentration $\lambda_{sw,0:sus}$ using equation (S30).

6. Compute the particle size of suspended sediment using equation (S32) or another method.

7. Compute the fall velocity of suspended sediment $w_s$ using equation (S25).

8. Compute the $\beta_s$ factor (equation S27).

9. Compute the overall bed shear velocity $u_s$ using equation (S13).

10. Compute the $\varphi$ factor using equation (S29).

11. Compute the suspension parameters $Z$ and $Z'$ using equations (S26) and (S28), respectively.

12. Compute the correction factor $F_s$ using equation (S35).

13. Compute the volumetric concentration of suspended sediment $\lambda_{sw,csus}$ using equation (S34).

14. Compute the suspended load transport $q_{sw,csus}$ using equation (S33)
S3.4. Sediment transport in the flowing water

Here, we briefly present results for two components of sediment in transport: bed load (Fig. S2) and suspended load (Fig. S3). The along path derivative of the total sediment concentration (Column 1 in both Figures S2 and S3) is related to the rates of bed elevation change in Figures 7 and 10. Overall, total sediment transport (Column 1 in both Figures S2 and S3) is dominated by the suspended load component (Column 2 in both Figures S2 and S3), but the bed load component is not insignificant. The constant recharge simulations have lower sediment transport than daytime conditions in the diurnally varying recharge simulations. The differences vary by about a factor of three, so that total sediment transported over a 24 h period in the diurnal case is significantly larger over the course of a 100 d simulation.

Supplemental notation

\[ C_x \] Coefficient for the analytic expression for the bed.
\[ c_b \] Transition index for the analytic expression for the bed.
\[ D \] Sediment grain diameter.
\[ D_s \] Nondimensional sediment particle parameter.
\[ D_{50} \] Median grain diameter of the mobile fraction of the bed.
\[ D_{90} \] Grain diameter of the 90th percentile of the grain size distribution along the bed.
\[ F_s \] Suspended sediment correction factor for nonuniform vertical distribution.
\[ k_s \] Nikuradse sand-bed roughness.
\[ n_{s,a} \] Mobile fraction of the bed.
\[ q_{sw,e:bed} \] Equilibrium bed load sediment flux.
\[ q_{sw,e:sus} \] Equilibrium suspended sediment flux.
\[ T \] Transport stage parameter.
\[ u_s \] Water shear velocity.
\[ u_s' \] Water shear velocity on the grains.
\[ u_{s,cr} \] Critical shear velocity necessary for entrainment.
\( u_{\text{bed}} \) Average bed load velocity.
\( u_{s,crs} \) Critical shear velocity necessary for entrainment into suspension.
\( w_s \) Grain fall velocity through water.
\( x_u \) Start of the overdeepening along flow.
\( x_w \) Start of the region of constant bed slope.
\( x_0 \) Location of outflow end of the overdeepening.
\( Z \) Suspension parameter: ratio of fall velocity to upward turbulent motion.
\( Z' \) Corrected suspension parameter \((= Z + \varphi)\).
\( z_l \) Entrance elevation for the bed along the longitudinal section.
\( \beta_s \) Parameter describing the interactions between suspended sediments and turbulent eddies.
\( \Delta_s \) Bedform height.
\( \kappa_s \) von Karman’s constant.
\( \lambda_{sw,e:bed} \) Equilibrium bed load concentration.
\( \lambda_{sw,0:bed} \) Reference bed load concentration.
\( \lambda_{sw,e:sus} \) Equilibrium suspended sediment concentration.
\( \lambda_{sw,0:sus} \) Reference suspended sediment concentration.
\( \nu \) Kinematic viscosity of water \((= \mu / \rho_w)\)
\( \varphi \) Suspended load correction for particle-particle and particle-water interactions.
\( \tau_0 \) Shear stress along the hydraulic perimeter.
\( \tau_{cr} \) Critical shear stress required for grain motion.
\( \theta_{cr} \) Critical nondimensional shear stress.
\( \sigma_s \) Standard deviation of the bed sediment grain size distribution.
Figure S2. Sediment transport for simulations with constant water forcing through 100 d. Rows are the same simulations as in Figure 5 in the manuscript. Column 1: upper blue axes: Water discharge through the subglacial system. Inflow from the upstream feeder crevasse is represented by dark blue lines. Light dashed lines are the effluent water flow at the downstream end of the grid. lower black axes: Water elevation in the feeder crevasse. Gray dashed lines are the flotation and glacier surface elevations. Column 2: Instantaneous total sediment concentration. Column 3: Bed load component of sediment concentration. Column 4: Suspended load component of total sediment transport. Contour interval is $2.5 \times 10^{-5}$. Smallest contour ($2.5 \times 10^{-5}$) is given by the thick white contour in each. Color scale saturates at $6 \times 10^{-4}$. This contour interval is one quarter that of the equivalent panels in Figure S3.
**Figure S3.** Sediment transport for daily forcing through two days. Rows are the same simulations as in Figure 8 in the manuscript. **Column 1:** upper blue axes: Water discharge through the subglacial system. Inflow from the upstream feeder crevasse is represented by dark blue lines. Light dashed lines are the effluent water flow at the downstream end of the grid. lower black axes: Water elevation in the feeder crevasse. Gray dashed lines are the flotation and glacier surface elevations. **Column 2:** Instantaneous total sediment concentration. **Column 3:** Bed load component of sediment concentration. **Column 4:** Suspended load component of total sediment transport. Contour interval is $1 \times 10^{-4}$. Smallest contour ($5 \times 10^{-5}$) is given by the thick white contour in each. Color scale saturates at $15 \times 10^{-4}$. 

Water discharge $Q^w (m^2 s^{-1})$

Total sediment concentration $\lambda_{sw}$ (unitless)

Bed load concentration $\lambda_{sw:bed}$ (unitless)

Suspended load concentration $\lambda_{sw:sus}$ (unitless)

Sediment concentration (volume of sediment per volume of water)
References


