MARKET AND CONTRACT DESIGN FOR CATASTROPHIC LOSSES

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Classical theory of insurance contracting

The classical theory is depicted in the following boxes. It starts with a population who are risk averse but having endowed risk. Capital is allocated to insurers who are able to pool. By underwriting many policies and exploiting the law of large numbers, insurers can provide a low risk return on capital. A departure from the simple classical model arises with frictional costs. The most widely analyzed are information asymmetry and moral hazard (*ex ante* and *ex post)*. Second best contract design (self selection mechanisms, ex post settling, etc) are selected by competitive insurers and contracts are closed.

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TIME 0

Parties exposed to known risk
Investors provide risk bearing capital
Risk transfers impeded
        moral hazard
        adverse selection
Insurers design second best contract with
        incentive mechanisms
        revelation mechanisms
Contracts closed & premiums paid
Insurer write spread of risk
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TIME 1

Risk resolved
Losses observed & paid
or
Losses not observed and settled according to
incentive compatible contract features
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The classical theory and catastrophic losses.

Large losses, such as natural disasters and terrorist attacks, create a number of problems for insurance markets. The first of these, lack of independence, is well known and intensively analyzed. While perfect risk spreading is not possible, a second best solution can be achieved through decomposition of risk. Diversifiable risk can still be distributed widely through the insurance pool with undiversifiable risk left with the policyholders. This process, mutualisation, was first prescribed by Karl Borch in 1962 and forms the basis for organizing other markets such as the capital market.

THE MUTUALISATION MODEL

RISK

DIVERSIFIABLE   UN-DIVERSIFIABLE

INSURANCE

Claim hedges diversifiable risk

Dividend passes through undiversifiable risk

POLICYHOLDER
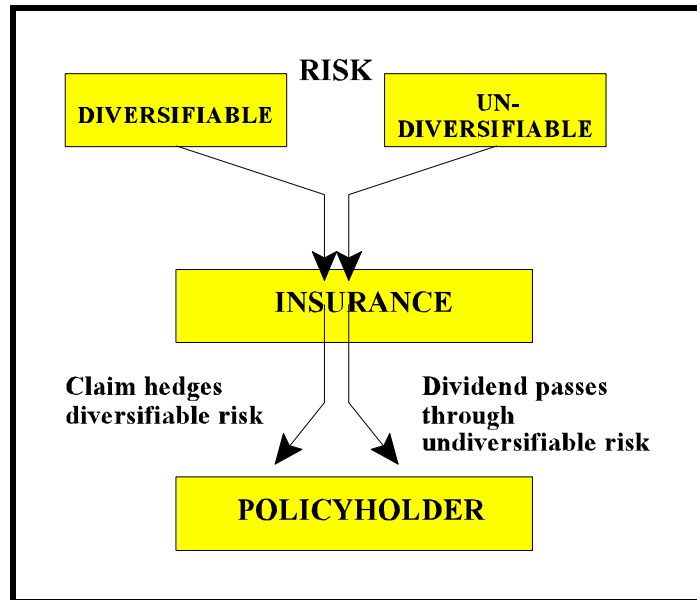
A second and closely related issue is ambiguity. There may be more or less statistical information from which to estimate the loss distribution for the event itself, and the process by which the economic cost from a well defined physical event is distributed, is not well understood or easy to predict. We can think of these two problems as *ex ante* and *ex post* ambiguity.

Ex ante and ex post ambiguity

For many natural hazards, the *ex ante* ambiguity problem recedes as models of these events evolve. In addition, whether or not a property owner has undertaken structural mitigation or other preparedness activities that could affect ultimate losses may be largely unobservable. The current generation of models for wind and shake events are based on scientific, engineering and economic mechanisms and are estimated with volumes of data. Some of this data is, itself, a source of ambiguity as their may be conflicting data, or errors in the data, on the nature of structures, their locations and the character of geological determinants of damage; all of these drive ex ante ambiguity.

The *ex post* ambiguity can be illustrated by separating the direct and indirect economic consequences of the event. For example, the cost of repairing well defined hurricane damage to a given structure may be estimated with tolerable accuracy but the distribution of that cost amongst parties by litigation or the business interruption resulting from the physical damage, can be subject to considerable ambiguity.

For terrorism losses, the issues are the same but the magnitudes not. *Ex ante* ambiguity for large terrorism events is considerable. On the one hand there is relatively little data. Small scale terrorist events are not unusual but large events are scarce, so, in principal, tail fitting methods such as extreme value theory can be used. (see Gordon Woo for other ideas). But to make such methods feasible requires drawing of data from a variety of political jurisdictions, (e.g. combining data from U.S., Israel, Northern Ireland, etc) and, since terror by its nature is partly jurisdiction specific, it is difficult to use statistical control. This is related to the second problem which is the nature of the "game". While natural hazards can be thought of as a game against nature, terrorism is a game against an intelligent and competing player who, as Gordon Woo has pointed out, is not merely capricious but positively malicious. While game theory offers a framework to understand this process, translation of any conceptual understanding into a usable loss distribution is a formidable task. Thus, *ex ante* ambiguity is large for terrorism losses and will remain so for the foreseeable future.

The events of September 11th, provide a good illustration of the *ex post* ambiguity. We now have pretty accurate idea of the magnitude of the event in terms of physical damage and loss of life. But, ranges of insurers estimates of their loss are subject to enormous margins of error. Part of this is the coverage issue (is there a war coverage exclusion, did the WTC loss(es) comprise one or two occurrences,, etc). Secondly, there is the business interruption issue which takes time to work through. But, perhaps most important is the liability issue. Despite the Trial Lawyers Association calling for a moratorium on lawsuits, and the Federal Government's "victims' compensation fund", the stage seems to be set for an open ended round of litigation. Fitting a distribution around such outcomes is sheer guesswork.

The mutualisation principle and correlation

As mentioned in the introduction, the optimal design of an insurance market for correlated risks involves mutualisation. This idea dates from Borch and has been developed by many others since (see for example, Marshall, Dionne and Doherty, Chichilnisky and Heal, Gollier). The basic idea is to define an insurance pool as widely as possible to take advantage of geographical diversification. Organizationally, this is partly achieved through reinsurance which pools losses from many parts of the world. However, there are limits to geographical risk spreading and there are transaction costs associated with reinsurance. Thus, primary insurers are still left with significant undiversifiable risk. Under the mutualisation principle, this remaining risk is borne by the policyholders who assume the role of implicit owners of the insurance pool. This can be done organizationally (through a mutual insurance company), contractually (by having retroactive

assessment to policyholders that depend on the poll's loss experience) or implicitly (by having future premiums, or coverage terms, vary with pool loss experience).

The mutualisation principal and ex ante ambiguity

Ambiguity imposes a behavioural impediment to hedging (e.g. Kunreuther et al. (1995)). But while it is true that individuals make decisions, organizational and contractual structures sometimes evolve to offset behavioural distortions. Several writers have pointed out that the same mutualisation mechanism can be used when there *is ex ante* ambiguity (Doherty and Schlesinger, etc. [1]). The idea is simply that ambiguity is not a barrier to pooling *per se*; rather it is a barrier to setting an advance premium. Indeed, ambiguity essentially implies that there is a risk that the aggregate premiums will not cover the aggregate losses even if there is a large number of independent losses. However, if the size of the insurer's losses can be verified *ex post*, then the realization of losses resolves the ambiguity and the premiums to be paid by policyholders can be conditioned on the aggregate loss. Practically, the policyholder pays a deposit premium and receives a dividend or pays an assessment when the insurer's total losses are known. The size of the up-front deposit premium can be varied to control for the risk that policyholders might default on the assessment.

The mutualisation principal and ex post ambiguity - the implicit contract model

A difficulty arises in using the retrospective premiums (or dividends of assessments) if the insurer's aggregate losses cannot be independently verified by all parties. For example, following 9/11, insurers can estimate their losses but these estimates cannot be verified independently. And while insurers will make adjustments to loss reserves over time as more information arises, much discretion is exercised in deciding when to make such adjustments, how much to adjust and to which policy year the adjustment are made. Thus, while in principal, premiums could be conditioned *ex post* on the insurer's *estimated* losses, the fuzziness of these estimates allows considerable room for wealth transfers between stakeholders. For example, a stock company with participating policies could easily exaggerate the estimated loss and thereby reduce the policyholder dividend thereby transferring wealth from policyholders to shareholders. Accordingly, an efficient contract needs a mechanism permit the policyholders to absorb the undiversifiable risk, while still imposing a cost on insurers. A mechanism to do this was derived for liability insurance (Doherty and Posey) from implicit labor contract design and can be used for cat losses having *ex post* ambiguity. The model rests on some degree of lock in of insurance contracts (i.e., there are switching costs to changing insurers). Under this mechanism, prices can be raised by the insurer if its estimate of aggregate losses are high. But to signal that it is not lying, it will also ration coverage offered at these higher prices.

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[1]See also Chilchilnisky and Heal.

Some comments on terrorism hedging

These comments on ambiguity and extreme events allow some comment on how the insurance market has responded to 9/11. Terrorism insurance seems to be in high demand and low supply (Jaffee and Thomas (2002), Kunreuther, 2002)). Insurers, where permitted by regulation, have severely reduced the amount of terrorism insurance available. This has been promoted by reinsurer, who are not compelled to offer such coverage, also withdrawing much of their coverage. Thus, we have a situation of excess supply at the prices being charged. In the meantime, some capital has been flowing back into the industry, though still considerable less than the amount lost in 9/11 losses. This, in indeed what one would expect from the implicit contract model. Thus, we can interpret the availability crunch, not as a market failure, but rather as a mechanism for which the undiversifiable risk is shared amongst those commonly at risk and given the scarcity of insurer capital. One must also bear in mind that, to expand terrorism coverage given the depleted capital base would simply undermine the availability of other non catastrophic coverage. But while one cannot point to "failure" of the insurance market, one can still conclude that the degree of *ex post* risk sharing though rationing would be lower if the capital base was expanded.

This thinking leads to three directions.

● One is simply to expand the capital base of insurers. This may be inefficient given the widespread belief that the industry was over-capitalized before 9/11 and bearing in mind the costs of locking up capital against tail events.

● Another solution widely discussed is Federal Terrorism Insurance which in principle leads to the widest possible mutualisation of risk (across the whole tax base), but has the usual problems associated with government risk transfer programs (they can warp into transfer programs and thereby erode incentives, and they can crowd out private provision). Cummins and Doherty (2002).

● A third avenue is to securitize this risk thus tapping directly into the much larger capital base of financial markets. This course has been explored to a limited extent for natural catastrophe risk with the issue of catastrophe bonds, catastrophe contingent equity and similar instruments. The attractiveness of the securitization model is that it not only provides a relief of the capital constraint, but is an appropriate vehicle for addressing *ex post* ambiguity. Many catastrophe securitizations have not been indemnity contracts but based on an indexed or parameterized trigger. The latter is of interest here. For example, in some cat bonds, debt forgiveness is triggered by a physical description of the loss (e.g. an earthquake in a given area reaching a point on the Richter scale. This type of trigger has the advantage that it is correlated with the loss even though we know the loss cannot be measured quickly and accurately. Thus, basis risk is accepted in exchange for *ex post* ambiguity. The trick with terrorism hedging, is to find equivalent, parameterized triggers that are (relatively) free of ambiguity and are correlated with the actual loss.

Contracting under Conditions of Ambiguity

Contracting theory enjoys a rich literature in economics, law and related disciplines. The classical contracting approach assumes complete and common knowledge about the probability distribution governing uncertain states of the world. Based on this distribution, and on the preferences and actions of the parties to the contract, optimal contingent actions and payments are the subject of the resulting contract design problem. This model has been advanced in important ways in the Principal-Agent literature (e.g., Holmstrom (1979), Laffont and Tirole (1988)), and has been a subject of continuing interest in the insurance literature (for a recent review, see Doherty and Mahul (2001)). This was further enriched in the literature on law and economics, e.g., through the work of Williamson (1975) and Klein, Crawford and Alchian (1978). This literature has made great progress in clarifying the incentives for participating in risk sharing and in investments in joint activities among contracting parties.[2] However, while this work addresses issues of opportunism (a.k.a. moral hazard) arising from unobservability and information problems, the impact of ambiguity has not received much attention in the literature on the theory of contracts, either in general or in the insurance context of interest here. Rather, the standard assumptions are that parties share a common-knowledge distribution that captures entirely all uncertainties of interest. This may be a reasonable presumption when the transactions of interest involve repeated opportunities for learning, feedback and exchange of information about the valuations held by the parties, but it is unlikely to be reasonable in the area of catastrophic risk, where the processes and actions of the parties giving rise to losses are complex and often unobservable, and where by their very nature feedback in the form of real events is relatively rare. From what we know about the consequences of such ambiguity on real behavior in insurance markets and in the laboratory (e.g., Kunreuther et al. (1993, 1995), Heath and Tversky (1991), Fox and Tversky (1995)), it seems a useful exercise to examine the likely consequences of the existence of significant ambiguity on contracting behavior for risk sharing involving catastrophic losses.

In the Technical Appendix we develop a model along the following lines. Two parties, denoted I and R, wish to contract to share the losses that might result from a risk. We think of I as a direct insurer and R as a reinsurer. The losses L are a function of the uncertain state of the world "s" and are denoted by $L(x, y, s)$, where $x = (x_a, x_p)$ is a vector ex ante and ex post investments by the original bearer of the risk, and "y" is the ex ante investment by a second party to whom some or all of this risk might be transferred.[3] Parties are assumed to bargain about the nature of the risk transfer and payments and responsibilities before the fact. If they are unable to

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[2] See also Gul (2001) for a review of recent work on the impact of observability on the outcome of bargaining problems. Gul analyzes, in particular, the consequences of observability and the structure of the bargaining process on the efficiency of outcome-enhancing investments.

[3] The investment "y" may be thought of as risk management expertise, e.g., in the form of catastrophe modeling expertise or other idiosyncratic investment, that the reinsurer might bring to bear in assisting the insurer in the cost-effective management of its portfolio of insured risks.

reach agreement, no risk transfer occurs. If they do, then we assume that the contract they sign is completely enforceable ex post. However, some ex post moral hazard is assumed in that the insurer I can influence the magnitude of the actual losses ex post through exerting effort x_p (in the insurance context, one might think of this as the effort to accurately assess and control the cost of claims). Both I and R are assumed to be ambiguity averse in a sense to be made more precise below. We examine is the impact of ambiguity on the contracting behavior and the efficiency of agreements between I and R about risk mitigation and risk transfers. We only consider here a simple case in which there are only two states of the world, "loss" or "no loss", in order to keep the level of mathematical detail to a manageable level. The basic results obtained are that differences in ambiguity aversion will affect both the ex ante likelihood of coming to agreement between I and R and the level of investments they make, ex ante and ex post, in controlling and mitigating losses.

To understand the nature of the model development, it is important to note the approach we use to represent ambiguity and the preference structure of our contracting agents in the face of such ambiguity.[4] The study of ambiguity began with Frank Knight's distinction between conditions of certainty, risk and uncertainty, with the latter reflecting imperfect knowledge even of the distribution that might reflect unknown states of the world when decisions are made. The term "ambiguity" became prominent with the well-known paper of Ellsberg (1961). Over the intervening years, ambiguity has come to be understood as reflecting a decision context where any of a set of probability distributions might reasonably be argued to govern the outcomes of random states of the world, and where experts or other sources of information relating to the context are unable to provide accurate information about which of these probability distributions is the "right one". Just as in Savage's extension of von Neumann-Morgenstern expected utility theory, the central question in axiomatic foundations of decision making under uncertainty is linking preference representation to beliefs, where in this case beliefs encompass the degree of subjective uncertainty about which probability distribution from the set of feasible such distributions might obtain. The simplest such axiomatic approach will be pursued here, namely that following Schmeidler (1989) in which a non-additive probability measure is associated with the uncertain states. Schmeidler's theory can be shown to imply that the decision maker will undertake a von Neumann-Morgenstern-Savage expected utility evaluation of alternatives, but will do so at the least favorable distribution from among those believed to be reasonable. This calculus can be captured in various ways in general, but we will only be concerned with a two-state example here, so the theory is easy to state and understand.   In the two-state world of interest, the effect of the ambiguity theory considered here is that I and R weight the ex post consequences of loss

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[4]For a review of the normative theory of choice under conditions of ambiguity, see Schmeidler (1989) and Dow and Werlang (1992). For a review and synthesis of the descriptive and experimental literature on ambiguity, see Einhorn and Hogarth (1986) and Fox and Tversky (1995). For applications of ambiguity in the insurance context, see Kunreuther et al. (1993, 1995). Our approach applies the Schmeidler theory to the insurance setting in a bilateral bargaining setting between an assumed insurer (or property owner) and reinsurer (or insurer) who bargain about the terms of sharing an ambiguous risk.

scenarios more heavily than they would absent ambiguity. This is intuitive since the model adopted here is that ambiguity gives rise to an assessment based on the least favorable distribution. However, the consequences of this higher weighting of ex post effects is more subtle than might be suspected at first. It affects not only the ex ante investments in loss reduction, but also interacts with ex post moral hazard as well as the likelihood that I and R will actually be able to come to a profitable agreement about the terms of risk transfer in the first place. The reader is referred to the appendix for details.

Some Concluding Puzzles and Questions for Discussion

We have explored a number of aspects of ambiguity in the specific context of catastrophic risks, in which feedback and other aspects of learning and experimentation are not likely to be successful in eliminating ambiguity. We record here some of the key questions we have explored, partial answers for which have been suggested by our theoretical analysis, but all of which remain fascination open issues for research.

1. How does ambiguity aversion (AA) affect the level of mitigation (ex ante and ex post) of the insurer and that of the insurer (ex ante only)?

2. How does ambiguity aversion (AA) affect the possibility that the insurer and reinsurer will close a contract for risk transfer?

3. How do differences in ambiguity aversion between the insurer and reinsurer affect the level of mitigation (ex ante and ex post) of the insurer and that of the insurer (ex ante only)?

4. How do differences in ambiguity aversion between the insurer and reinsurer ambiguity aversion (AA) affect the possibility that the insurer and reinsurer will close a contract for risk transfer?

5. How does ambiguity affect the "mutuality" principle. This model is not yet set up well to consider catastrophic risk (this will require specification of the aggregate loss and exploration of the correlation between the losses of different insurers)?

6. Our model considers ex ante mitigation (not moral hazard) and ex post moral hazard. We also would like to explore how ambiguity affects ex ante moral hazard?

Technical Appendix

We assume a two-state world ("loss" or "no loss"), so we need only define the measure of interest on these two states S = {LOSS, NO LOSS}. Thus, suppose a loss L may occur with probability "p" and no loss with probability 1-p, but where the value of "p" is uncertain, with ambiguity captured by a non-additive probability measure P_c defined as follows. We assume for some p O (0, 1) and c O [0, 1] that :

$$P_c(S) = 1; \qquad P_c(LOSS) = (1-c)p; \quad P_c(NO\ LOSS) = (1-c)(1-p) \qquad (1)$$

where the parameter "c", which is specific to the decision maker, reflects the degree of ambiguity that decision maker's beliefs. Note that the probability measure P_c is non-additive when c > 0, since $P_c(LOSS) + P_c(NO\ LOSS) = (1-c) < 1$. To keep matters simple, we assume that both I and R are risk-neutral agents with possibly different degrees of ambiguity (i.e., different parameters "c") about the resulting loss. Applying the Schmeidler theory to the probability distribution (1), a risk-neutral agent whose beliefs are represented by (1) will maximize a weighted average of the worst possible outcome and the expectation of the additive distribution that arises when there is no ambiguity, with the weight given by (1-c) and c. Thus, if the final wealth under the "loss" and "no-loss" states are W_L and W_N, respectively, then the certainty equivalent of this ambiguous lottery with beliefs captured in (1) would be:

$$CE \ ' \ (1 \& c)[pW_L \% (1 \& p)W_N] \ \% \ cW_L \qquad (2)$$

We will apply this very simple structure throughout to capture the beliefs of I and R about the results of various contracts. When c = 0, the certainty equivalent CE in (2) reduces to the expected value of the lottery at the base probability distribution determined by "p".

As per the motivating description in the text, we imagine that I and R are engaged in bargaining about the terms and conditions that should apply to the transfer or sharing of the ambiguous risk with loss L(x, y) and non-additive probability measure in (1). We model this process via the Nash Bargaining Solution with the default outcome (i.e., the outcome resulting if no agreement results from this bargaining) as the payoffs arising when I retains the entire risk. Otherwise, we assume a perfectly enforceable contract obtains, which specifies the investments that are to be made ex ante by I (x_a) and R (y). It is assumed here that both parties understand that ex post I will choose investments x_p that maximize I's profits, given the (perfectly enforceable) terms of the contract.

The form of the contract we consider is as follows. I will transfer a fixed share s O [0, 1] of the realized loss L(x, y) to R, in exchange for a payment to R of a state contingent fee T(?), where we denote T_L as the payment when there is a loss and T_N when there is no loss. Assuming initial wealth levels are denoted w_i, i O {I, R}, the resulting Nash Bargaining Solution is characterized as the solution to the following problem.

$$\textit{Maximize} \ [G_I \ \& \ G_{I0}] \times [G_R \ \& \ G_{R0}] \tag{3}$$

where the ambiguity-adjusted expected profits and default options are given by

$$G_I \ ' \ (1 \& c_I)\Big(w_I \ \& \ x_a \ \& \ p\,[(1\&s)L(x,y)\%x_p\%T_L] \ \& \ (1\&p)T_N\Big)$$
$$\% \ c_I\Big(w_I \ \& \ x_a \ \& \ [(1\&s)L(x,y)\%x_p\%T_L]\Big) \tag{4}$$

$$G_R \ ' \ (1 \& c_R)\Big(w_R \ \& \ y \ \% \ p\,[T_L\&sL(x,y)] \ \% \ (1\&p)T_N\Big)$$
$$\% \ c_R\Big(w_R \ \& \ y \ \% \ T_L \ \& \ sL(x,y)\Big) \tag{5}$$

$$G_{I0} \ ' \ \textit{Maximum}_{x_a\$0}\Big[(1\&c_I)\big(w_I\&x_a\&p\,[x_{p0}\%L(x,0)]\big) \ \% \ c_I\big(w_I\&x_a\&x_{p0}\&L(x,0)\big)\Big] \tag{6}$$

$$G_{R0} \ ' \ w_R \tag{7}$$

where, in (4), x_{pa} is the level of ex post investment by I in the event of a loss when ex ante investments by I and R are x_a and y. We assume that these are unobservable investments, and therefore are chosen by I to minimize overall costs ex post, i.e.,

$$x_{pa} \ ' \ x_{pa}(x_a,y) \ ' \ arg\ min_{x_p\$0}\Big[x_p \ \% \ (1\&s)L(x_a,x_p,y)\Big] \tag{8}$$

Similarly, in (6), x_{p0} is the level of ex post investment by I in the default state, so that

$$x_{p0} \ ' \ x_{p0}(x_a) \ ' \ arg\ min_{x_p\$0}\Big[x_p \ \% \ L(x_a,x_p,0)\Big] \tag{9}$$

Note in (4)-(5) that we are assuming that the worst case outcome is in the loss state. Note also that the default profits for I in (6) are the payoffs to I in the event that I is unable to contract for any risk transfer with R, in which case "y" is then 0.

The basic time line for decisions envisaged in the above bargaining problem is this:

1. x_a and y are chosen, simultaneously with agreement or default; if default occurs, $s = y = 0$, no transfers occur ($T_L = T_N = 0$), and I sets x_a through the maximization problem in (6).

2. The state of the world ? materializes and losses, if any, are realized.

3. In the "loss" state, x_p is chosen according to (8) or (9). If a contract is signed, the fact that (8) determines x_p is known to both I and R.

4. Transfers and shares take place according to the contract.

The above bargaining framework provides a number of insights about the impact of ambiguity on the risk transfer contract between I and R. To do so, we optimize (3) in the standard fashion,[5] subject to the individual rationality constraints $G_i \geq G_{i0}$, i $\in$ {I, R}. Rather than prove these formally, we simply state these as intuitive results here and interpret them in terms of the framework presented earlier. To do so, and to provide the intuition behind these results, we first note the following characterizing result for the Nash Bargaining Solution (NBS) to (3). Using (4)-(7), we note that the Nash product in (3) can be written in the form:

$$Maximize\ [f \& aT_N] \times [g\ \%\ bT_N] \tag{10}$$

where a = (1 - c_I)(1-p) and b = (1-c_R)(1-p), and where f and g are all the remaining terms in (3), which we note do not depend on T_N. Thus,

$$f\ '\ G_I\ \&\ G_{I0}\ \%\ (1\&c_I)(1\&p)T_N; \quad g\ '\ G_R\ \&\ G_{R0}\ \&\ (1\&c_R)(1\&p)T_N \tag{11}$$

It is easily verified that the solution for T_N maximizing (10) is unique and is given by:

$$T_N\ '\ \frac{bf\ \&\ ag}{2ab}\ '\ \frac{\left(w_I\ \&\ x_a\ \&\ [c_I\ \%\ (1\&c_I)p]\,[(1\&s)L(x,y)\ \%\ x_p\ \%\ T_L]\ \&\ G_{I0}\right)}{2(1\&c_I)}$$
$$\%\ \frac{\left(w_R\ \&\ y\ \%\ [c_R\ \%\ (1\&c_R)p]\,[T_L\ \&\ sL(x,y)]\ \&\ G_{R0}\right)}{2(1\&c_R)} \tag{12}$$

Substituting this optimal value of T_N back into (10), we see that (10) is of the form:

$$Maximize\ [f \& aT_N] \times [g\ \%\ bT_N]\ '\ [f\ \&\ a\frac{bf\&ag}{2ab}] \times [g\ \%\ b\frac{bf\&ag}{2ab}]\ '\ \frac{(bf\%ag)^2}{4ab} \tag{13}$$

Thus, we see that the NBS must itself must be the solution is characterized by (12) for the optimal value of T_N and, from (13), as the solution to maximizing bf + ag for the other contract variables. Using (4)-(7), and the definitions above of the parameters "a" and "b", we see that the NBS must solve:

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[5] The formal derivation assumes only that L(x, y) is jointly convex and strictly decreasing in (x, y), reasonable assumptions of the impact of mitigating investments on losses.

$$\text{Maximize } (1\&c_R)(1\&c_I)\Big(w_I \ \& \ x_a \ \& \ p[(1\&s)L(x,y)\%x_p\%T_L]\Big)$$
$$\% \ (1\&c_R)c_I\Big(w_I \ \& \ x_a \ \& \ [(1\&s)L(x,y)\%x_p\%T_L]\Big) \ \& \ (1\&c_R)G_{I0}$$
$$\% \ (1\&c_I)(1\&c_R)\Big(w_R \ \& \ y \ \% \ p[T_L \ \& \ sL(x,y)]\Big) \tag{14}$$
$$\% \ (1\&c_I)c_R\Big(w_R \ \& \ y \ \% \ T_L \ \& \ sL(x,y)\Big) \ \& \ (1\&c_I)G_R$$

subject to the ex post minimizing condition (8) characterizing $x_p = x_{pa}$.

Deleting constant terms and rearranging, we obtain the following equivalent problem characterizing (with (12)) the NBS:

$$\text{Minimize } H(x,y,s,T_L) \ ' \ (1\&c_R)\Big(x_a \ \% \ [c_I\%(1\&c_I)p][(1\&s)L(x,y)\%x_p\%T_L]\Big)$$
$$\% \ (1\&c_I)\Big(y \ \% \ [c_R\%(1\&c_R)p][sL(x,y) \ \& \ T_L]\Big) \tag{15}$$

It is useful to analyze the NBS solution to (15) under several headings. First note that the coefficient of T_L in (15) is

$$Coefficient(T_L) \ ' \ (1\&c_R)[c_I\%(1\&c_I)p] \ \& \ (1\&c_I)[(c_R\%(1\&c_R)p] \tag{16}$$

A bit of algebra shows that this coefficient has the sign of (c_I - c_R). Thus, minimizing H in (15) implies that $T_L = 0$ if $c_I > c_R$; $T_L = 0$ if $c_I = c_R$; and T_L is a maximum if $c_I < c_R$, where the maximum value for T_L is the value that would completely indemnify the reinsurer in the event of a loss, i.e. $T_L = sL(x, y)$. At this value, it is interesting to note from (12) that when $c_I < c_R$ (and therefore $T_L = sL(x, y)$), we have[6]

$$T_N \ ' \ \frac{\Big(w_I \ \& \ x_a \ \& \ [c_I\%(1\&c_I)p][L(x,y) \ \% \ x_p] \ \& \ G_{I0}\Big)}{2(1\&c_I)} \ \% \ \frac{\Big(w_R \ \& \ y \ \& \ G_{R0}\Big)}{2(1\&c_R)} \tag{17}$$

Thus, the mutualisation process described earlier is very much in evidence here, but how the optimal transfer of risks between I and R, through contract features and transfer payments, depends fundamentally on the relative perceived ambiguities of these parties to the risk. The party with relatively less ambiguity makes lower transfers to the other party in the loss state, and relatively higher transfers in the "no-loss" state. The magnitude of the transfers is further influenced by level of investments made by the parties, both ex ante and ex post.

Concerning the level of investments in loss reduction, we see from (15) that if an agreement can be reached by I and R for risk transfer, then such investments will be made to

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[6] Note from (6)-(7) that the apparent dependence of T_N here on w_I and w_R is specious.

minimize an ambiguity-adjusted sum of total investment costs and resulting losses. These investments are, of course, subject to the ex post moral hazard of actions of I (e.g., in claims adjustment). To gain some insight on the impact of ambiguity, consider the case where I and R share sufficient information on the risks involved (e.g., via catastrophe modeling) that $c_I = c_R = c$. In this case, dividing by (1 - c), the NBS characterizing function H in (15) can be expressed as:

$$h(x,y) \; ' \;\; x_a \;\% \; y \;\% \; [c \,\% \,(1 \& c)p]\,[L(x,y) \;\% \; x_p] \tag{18}$$

In this case, the NBS solution leads to minimizing total ex ante investment costs plus an ambiguity adjusted measure of ex post losses plus investment costs. Note that the coefficient $[c + (1-c)p] = p$ when ambiguity is 0, i.e. when $c = 0$, and otherwise exceeds p. As expected, he impact of ambiguity is, thus, to focus greater attention on ex post losses and investment costs.   At the same time, ex post moral hazard becomes relatively more important under ambiguity as well.

Finally, it is worth noting that the above analysis has proceeded entirely on the basis of the presumption that I and R are able to reach agreement, rather than defaulting. It should be clear from the nature of the problem here that the key elements leading to agreement on feasible transfers are the value added of R in reducing losses (i.e., the impact of "y" on L(x, y)), the magnitude of moral hazard by I (as reflected in the difference between x_{pa} and x_{p0}) and the difference in relative ambiguity aversion. As a benchmark case, if $c_I = c_R = c$ and "y" has no effect on L(x, y), then it is intuitive that no agreement can take place. The fundamental drivers of whether efficient risk transfer can take place here are differences in the ambiguity of I and R and the productivity of "y" in reducing losses.

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