EVENT IDENTIFICATION FRAMEWORK FOR TELESEISMIC AND REGIONAL DISCRIMINANTS

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ABSTRACT

Seismic monitoring for nuclear explosions answers three questions: Where is the seismic event located? What is the source type for the event? If an explosion, what is the yield of the event? Under the Threshold Test Ban Treaty (TTBT), treaty verification involved a seismic analysis processing seismograms of strong events, whose path was largely in the mantle, that is, teleseismic events. This paper reviews the seismic/mathematical development of a general event identification framework. This framework is extensible to include both regional and teleseismic discriminants.

The developed framework consists of two fundamental steps, or components. First, for each discriminant (teleseismic or regional) a probability model is formulated under a general null hypothesis of explosion characteristics. The veracity of the hypothesized model for each discriminant is measured with a calculation that is exactly, or analogous to, a p-value and ranges between zero and one. A value near zero indicates inconsistency with explosion characteristics, and a moderate to large value indicates consistency with explosion characteristics. The hypothesis test formulation ensures that seismic phenomenology is tied to the interpretation of the p-value.

In the second component, the p-values are transformed into standardized discriminants that also possess predictable statistical properties. They also range between zero and one, their interpretation is analogous to that of p-values, and they are approximately Gaussian. Therefore, established Gaussian discrimination methods can be used to formulate a unified decision from standardized discriminants.

The framework components are modular, and the mathematics to aggregate standardized discriminants is operationally independent of the construction of discriminant hypothesis tests. To develop and integrate new discriminants into the framework 1) the seismic theory of the discriminant must be integrated into an appropriate probability model, 2) a hypothesis test must be formulated from the model with an explosion characteristics null hypothesis and, 3) the result of the test must be summarized as a p-value calculation. The aggregation mathematics does not need to change when new discriminants and/or source types are added.

OBJECTIVES

This research and development effort provided a mathematical statistics formulation of established teleseismic discriminants, mapped these discriminants to a p-value calculation and constructed a general mathematical framework for teleseismic discrimination using p-values that is fully extensible to regional discriminants.

RESEARCH ACCOMPLISHED

Four core teleseismic discriminants are depth from travel time (TT), presence of long-period surface energy (LP), depth from reflective phases (PP), and polarity of first motion (FM). The mathematical statistics formulation of these follows, all under a general null hypothesis of H0: explosion characteristics.

TT Discriminant. Elements of location estimation as discriminants are intuitive and logically simple. For example, the combined costs and limitations of mining and drilling technology make deep underground nuclear explosions (deeper than 5 km or 10 km) very unlikely. Also, underground nuclear tests have associated support infrastructure or hole/tunnel construction. These illustrations indicate how epicenter and depth estimation contribute to event identification. The TT discriminant is mathematically formulated with the hypotheses H0: Depth $\leq \xi$ and HA: Depth $> \xi$. If H0 is true, then a test statistic, based on the sum of squares of fit, can be derived from non-linear regression theory that has an approximate F distribution with degrees of freedom equal to 1 and n–4. Here, n is the number of defining stations used in the hypocenter estimation. However, the hypotheses H0 and HA have directionality, that is, a test is needed that determines if H0: Depth $\leq \xi$ is consistent with the data. If H0 is true, then the statistic T = sign(depth estimate–ξ)Sqrt(F) has a central Student's T-distribution with n–4 degrees of freedom. Large values of T are inconsistent with H0 because the F statistic measures adequacy of fit under H0. The p-value is simply the right tail probability of a central Student's T-distribution calculated from the observed value of T. A small p-value implies a large observed T, which leads to the inference inconsistent with H0. A large p-value implies a small observed T, which leads to the inference consistent with H0 (Figure 1).

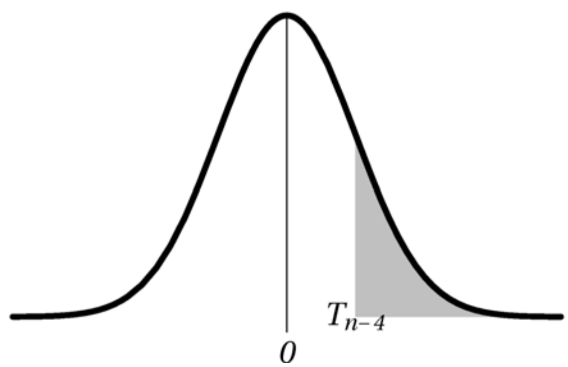

Figure 1. TT p-value calculation.

LP Discriminant. The body-wave magnitude (mb) versus surface-wave magnitude (Ms) discriminant is based on the physics that an earthquake excites relatively more surface-wave energy than a single-shot explosion. This means the ratio of Ms to mb generally will be larger for an earthquake than for an underground explosion with equivalent body-wave energy. In practice, this discriminant is formed from the difference of network average of surface-wave and body-wave magnitudes. Subtracting the historical average of this difference, when the seismic source is an explosion and then dividing by the standard error gives standard z-score statistic. The common variance for mb and Ms in the standard error is calculated from historical data and is assumed known. From established statistical theory, the test statistic z has a standard Gaussian distribution. Extreme negative values of z are inconsistent with H0; therefore, the p-value is simply the left tail probability of a standard Gaussian distribution calculated from the observed value of z. A small p-value, calculated from an extreme negative value of Z, leads to the inference inconsistent with H0. A large p-value implies consistency with H0 (Figure 2).

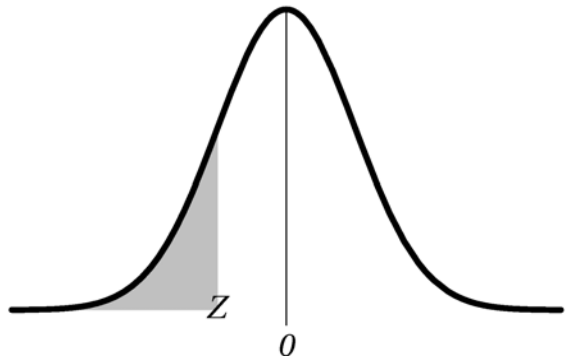

Figure 2. LP p-value calculation.

PP Discriminant. A reliable depth estimate can be obtained from the difference in arrival times of the compression waves P and pP. The P-wave travels directly from a seismic disturbance to a seismometer. In contrast, the pP-wave is a reflected P-wave that travels from the seismic disturbance to the earth's surface before being reflected to a seismometer. The time difference between the arrival of P and pP-waves (Δt) is a function of the depth of the seismic source and the epicentral distance from the source to the seismometer. This difference is predominantly dependent on the depth of a seismic disturbance when the focus is less than approximately 100 km deep.

For an event, stepout (denoted as r) is the observed change in Δt from the nearest station to the farthest station. Estimation of focal depth with reflected wave arrival times requires an a priori epicenter estimate. Physical phenomenology indicates it is highly unlikely that observed reflected waves of high quality (good signal-to-noise ratio and azimuthal distribution) could exhibit stepout if those waves were not correctly associated depth waves. Scenarios where this claim fails includes events that are analyzed with an inadequate earth model or spurious associations. Should the observed values of Δt for the closest and most distant seismometers be significantly and systematically different, then stepout is indicated, which implies that the event is deep.

Developing a mathematical formulation of the PP discriminant, and associated hypothesis H0 and p-value, requires that mathematical statistics theory yield obeisance to physical basis. The mathematical statistics formulation of the PP discriminant is a compound probability distribution of two measurements - the number of observed depth phases (number of observed pP) from an event, and a measurement of stepout. The number of observed pP is modeled with a Poisson distribution - appropriate because this number is rare under the null hypothesis. Stepout given a number of observed pP is modeled with statistical methods to relate Stepout to epicentral angle.

The null hypothesis is H0: No observed pP (Explosion Characteristics). Inconsistency with H0 is indicated when the number of observed pP is large or when observed stepout is large. The mathematical formulation of PP will give a small p-value when good-quality depth phases are seen; however, solid inconsistency with H0 requires observed stepout. For example, the formulation provides solid inconsistency with H0 with only two observed pP and strong stepout. In contrast, many observed pP with weak stepout indicates only marginal inconsistency with H0. The p-value calculation for the PP discriminant is illustrated in Figure 3.

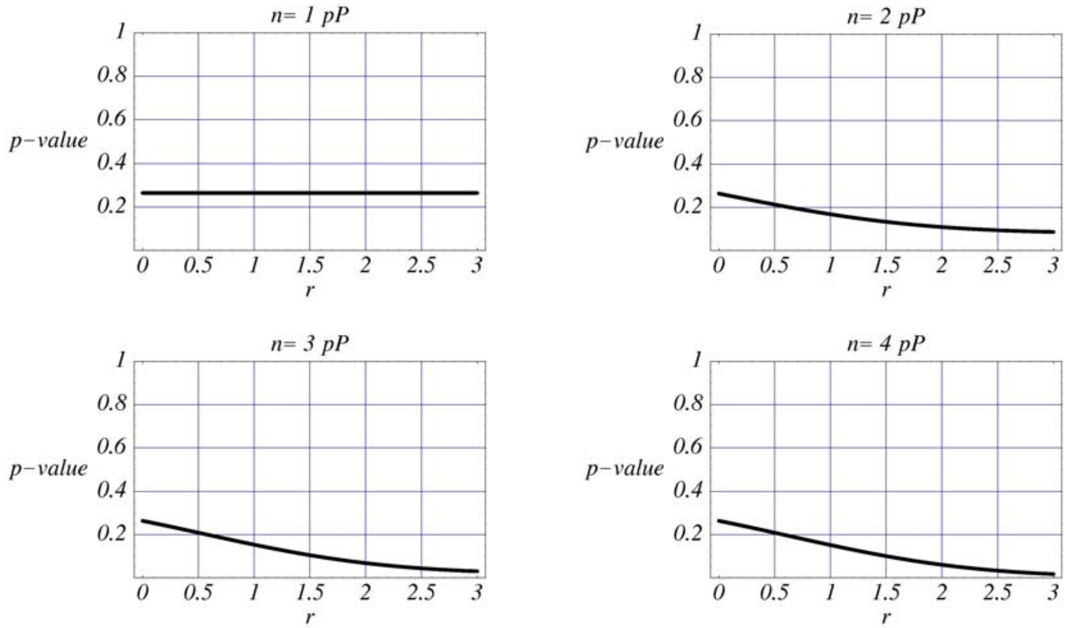

Figure 3. PP p-value calculation.

FM Discriminant. Polarity of first motion, as a discriminant, is based on the fact that an underground or underwater nuclear detonation produces mostly P-wave energy. Excluding pathological cases, a seismogram from an explosion exhibits the initial earth movement of the P-wave as upward or positive, regardless of the location of the seismometer. In contrast, an earthquake is caused by relative movements of adjacent blocks of the earth due to tectonic forces. This movement creates P-waves with opposite initial movements in different directions. Across a well-dispersed global seismic monitoring network, the initial P-waves from an earthquake will appear positive at some stations and negative at others. As a discriminant, if the polarity of first motion is negative at some stations, then the seismic disturbance is unlikely to be an explosion. If the polarity of first motion is positive at all stations, then the seismic disturbance might be the result of an explosion. The ambiguity under unanimous positive first motion is potentially caused by an inadequate distribution of seismic stations, that is, no earthquake P-waves with negative first-motion polarity travel to areas with seismic network coverage, or poor signal-to-noise ratio (inability to observe the P-wave signal because it is too small compared to background noise). A clear limitation of this discriminant is its dependence on the accurate identification of the first arrival of a seismic disturbance. In addition, shallow earthquakes of a type that produce vertical uplifts (thrusts or reverse mechanisms) often radiate energy with exclusively positive polarity. The first motion from some seismic disturbances is often difficult to distinguish from background noise. If the true onset of a seismic disturbance is poorly estimated, then the polarity of the first motion may be incorrectly assigned. For these reasons, the FM discriminant is only used as an explosion rejecter.

With adequate signal-to-noise at each station, the polarity of first arrival can be accurately identified with high probability (even with good signal-to-noise, first arrival polarity can be hard to identify in some instances). Uncertainty in identifying first arrival polarity motivates a mathematical statistics construction of the FM discriminant. The null hypothesis is H0: The source mechanism is single-point explosive. Under H0, the probability of positive first motion at a station is composed of two component probabilities; the probability of positive first motion from the source, and the probability that first motion polarity is correctly determined given positive first motion from the source. Under H0, the first component, the probability of positive first motion from the source, equals one. There may be pathological cases where this is not true; however, they are assumed to be negligible for this development. The second component probability is governed by many factors including signal-to-noise, analyst training and experience, and accurate P arrival pick. For this development, only signal-to-noise is considered in the determination of the second component probability, and with good signal-to-noise at all stations, this probability is modeled as a constant. This reasoning is succinctly summarized as P(+ first motion observed at a station)=P(+ first motion from source) × P(first motion polarity correctly identified | + first motion from source)=1 × θ. From this formulation, there will be a positive first motion (or not) at each station -- a binary random variable with P(+ first motion observed at a station)= θ. Assume that stations are probabilistically independent. Therefore, for M stations forming an event, the number of stations (N = n) under H0 that have positive first motion has a binomial distribution with parameters M and θ. For observed N = n, the FM p-value is simply the binomial cumulative distribution function.

The parameter P(+ first motion observed at a station) = θ will be near one under H0, and so a large number of stations with positive first motion will give a large p-value, leading to the inference consistent with H0. However, note again that the FM discriminant cannot offer confirmatory evidence for an explosion unless the station constellation fully covers the focal sphere. Illustrative plots of p-value versus the number of stations (N=n) that have positive first motion are given in Figure 4 for a suite of values M.

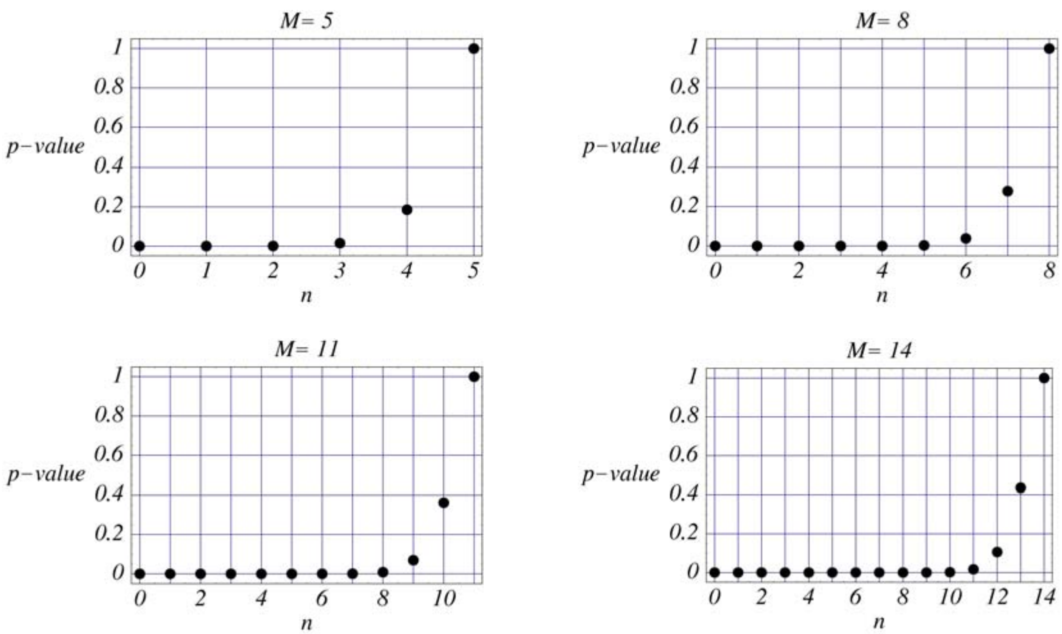

Figure 4. FM p-value calculation.

Regularized Discriminant Analysis-based Framework for Seismic Event Identification. Regularized Discriminant Analysis (RDA) (Friedman 1989) is fundamentally a Gaussian likelihood-based method. It therefore simplifies to a Mahalanobis distance as the basis for identification. For inputs into a general RDA-based discrimination framework, p-values are transformed into standardized discriminants that possess predictable statistical properties. Like p-values they range between zero and one, their interpretation is analogous to that of p-values, and they are approximately Gaussian. At an intuitive level, the Mahalanobis distance is very clear and appealing -- it measures the membership of a suite of standardized discriminants in source distributions (see Figure 5). In doing this membership analysis, the Mahalanobis distance accounts for the potentially different covariances that sources may have. RDA naturally grounds source identification as an empirical analysis, yet it is model-based.

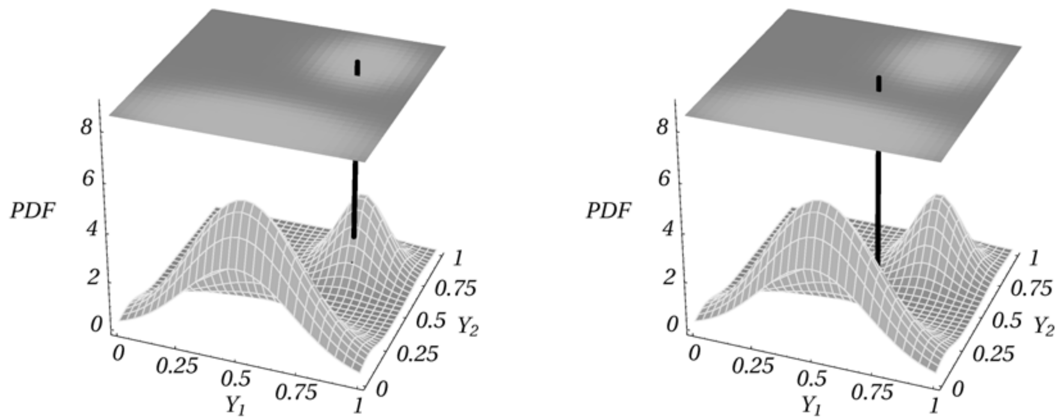

Figure 5. Illustrations of RDA source membership.

Inherent in the RDA identification approach is the concept that events that differ from historical source data are flagged for further analysis. This means that an event with individual p-values that strongly indicate explosion can be flagged for further analysis if, in the aggregate, the standardized discriminants are inconsistent with a historical explosion model. This property of RDA also implies that unusual natural events may be flagged for further analysis. Events that are in fact natural, yet inconsistent with all sources may be a new source type and appropriately merit further analysis. However, these events may be clear earthquakes from an inspection of individual p-values.

RDA is remarkably robust in its performance and offers a parametric transition between Linear Discriminant Analysis (LDA) and Quadratic Discriminant Analysis (QDA). It provides an operational identification solution even in settings with minimal calibration data. It is model-based, yet requires its parameters to be calibrated with data, and this balance implies that RDA will not over-fit calibration data. It provides the mathematics to perform identification in cases with missing discriminants. RDA offers the flexibility to include standardized discriminants that are co-linear.

CONCLUSIONS AND RECOMMENDATIONS

A mathematical framework for teleseismic discrimination has been completed. Teleseismic discriminants have been appropriately formulated for inclusion in the framework. The framework is mathematically extensible to regional discriminants. The framework should be calibrated and populated with regional discriminants.

REFERENCES

D. N. Anderson, S. R. Taylor, and K. K. Anderson (1999), Discrimination information in phase amplitude thresholds with application to Western China regional data, *Bulletin of the Seismol. Soc. Am.* 89: (3), 648-656.

D. N. Anderson and S. R. Taylor (2002), Application of regularized discrimination analysis to regional seismic event identification," *Bulletin of the Seismological Society of America* 92: 2391-2399.

L. Dumbgen and L. Homke (2000), *P-Values for Discriminant Analysis*. Preprint series: Schriftenreihe der Institute fuer Informatik/Mathematik, Bericht A-00-14, University of Lubeck, Germany.

P. J. DiPillo (1976), The application of bias to discriminant analysis, *Communications in Statistics - Theory and Methods A* 5(9):843-854.

P. J. DiPillo (1977), Further application of bias to discriminant analysis, *Communications in Statistics - Theory and Methods A* 6: (10):933-943.

P. J. DiPillo (1979), Biased Discriminant Analysis: Evaluation of the Optimum Probability of Misclassification, *Communications in Statistics - Theory and Methods A* 8: (14):1447-1457.

J. H. Friedman (1989), Regularized Discriminant Analysis, *Journal of the American Statistical Association* 84: 165-175.

E. A. Maharaj (2000), “Clusters of Time Series,” *Journal of Classification* 17: 297-314.

R. K. Smidt and L. L. McDonald (1976), *Ridge Discriminant Analysis*. University of Wyoming Department of Statistics, Technical Report No. 108, Laramie, Wyoming.