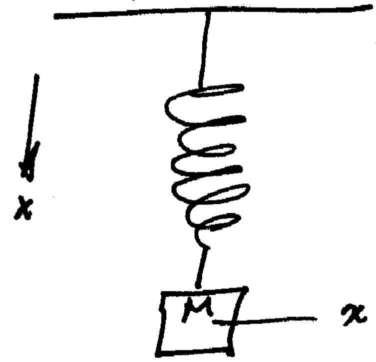
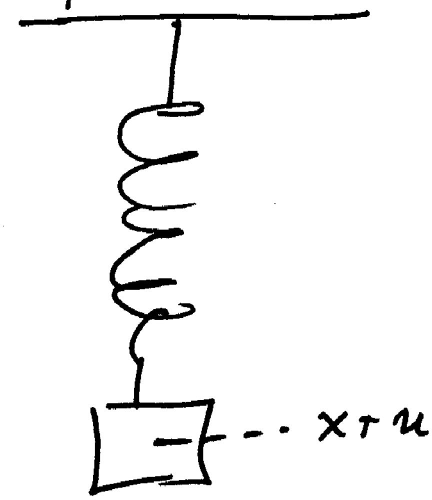
Motion of mass on spring (no gravity)

reference state



deformed state



u = displacement

Newton's Force law: $F = Ma$
connects force and acceleration

k = spring constant

Hooke's Law $F = -ku$
connects force and deformation

$$\cdot M \frac{d^2}{dt^2} u = -k u$$

$$\ddot{u} = -\frac{K}{m} u$$

$$u = C \sin(wt)$$

oscillatory motion
amplitude C, freq. ω

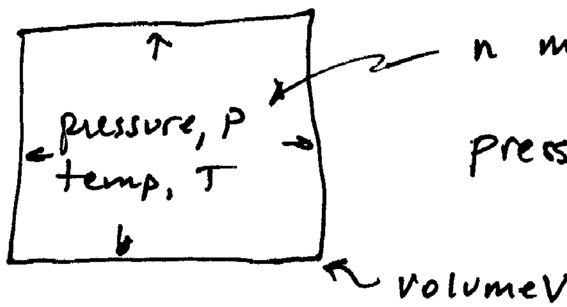
$$-MC\omega^2 \sin(\omega t) = -\frac{KC}{\cancel{M}} \sin(\omega t)$$

C = anything

$$\omega = \sqrt{K/M}$$

VIBRATIONS IN AN IDEAL GAS
"Sound"

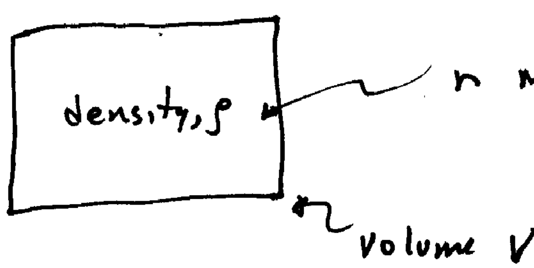
① ideal gas law $PV = nRT$



n moles in here

pressure: outward force per unit area on surface of box.

② convert from Volume – "Bulk property" to density – "point property"



n moles in here

$M = \text{mass} = \rho V$

$n = \dfrac{M}{m}$ where m = molar mass, moles per kg

$\;\; = \dfrac{\rho V}{m}$ N_2: 28 grams/mol $\doteq$ 0.028 kg/mol

$PV = nRT = \rho V RT / m$

$P = \dfrac{RT}{m}\rho$ let $K = \dfrac{RT}{m}$

$P = K\rho$

Nitrogen:

$R = 8.3 \dfrac{m^2\,Kg}{s^2\,K\,mol}$

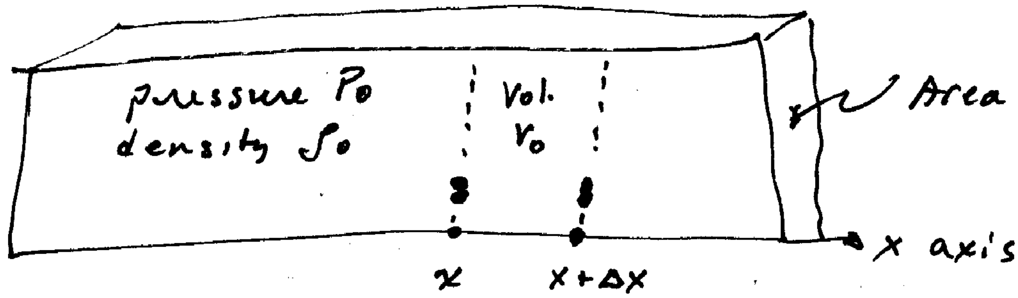
$T = 300\,K$

$m = 0.028$ Kg/mol

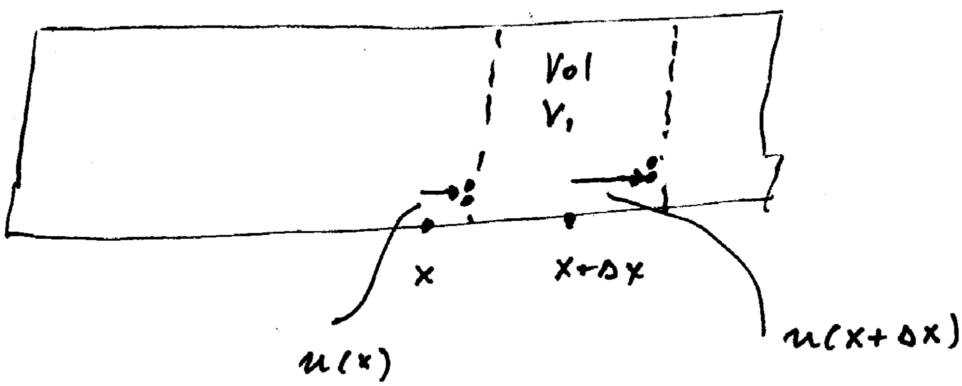
$K = RT/m = (298\ m/s)^2$

(2) 1-D deformation in a gas

reference state



deformed state



deformation related to displacement u.

old position, x new position $x + u(x)$

$x + \Delta x$ $\quad$ $x + \Delta x + u(x+\Delta x)$

1-D means u is horizontal

reference volume $V_0 = A \Delta x$

deformed volume $V = V_0 + A\,(u(x+\Delta x) - u(x))$

$$\Delta V = V - V_0 = A\,(u(x+\Delta x) - u(x)) = A \frac{du}{dx} dx = V_0 \frac{du}{dx}$$

$\frac{V - V_0}{V_0}$ = fractional change in volume $= \frac{du}{dx} =$

$\frac{\Delta V}{V_0}$ = "volometric strain" convert volume to density

$$V_0 = \frac{M}{\rho_0} \qquad \frac{\Delta V}{V_0} = \frac{\Delta \frac{1}{\rho}}{\frac{1}{\rho_0}} = \rho_0 \, \Delta \frac{1}{\rho}$$

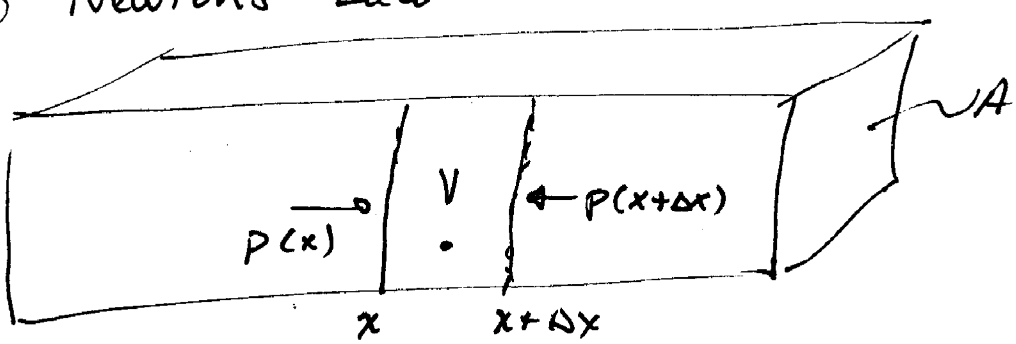
but $\Delta \rho^{-1} = -\rho_0^{-2} \Delta \rho$ so

$$\frac{\Delta V}{V_0} = -\frac{\Delta \rho}{\rho_0} = \frac{du}{dx}$$

By Ideal gas law $p = K\rho$ so $\Delta P = K \Delta \rho$

and $\frac{\Delta V}{V_0} = -\frac{\Delta P}{P_0}$

③ Newton's Law $F = Ma$



$$F = -A\left[\, p(x+dx) - p(x) \,\right] = -A\,\Delta x \,\frac{dp}{dx}$$

$M = \rho_0 A \Delta x$

$a \approx \ddot{u}(x)$ sloppy!

$F = Ma$

$$-A\,\Delta x \,\frac{dp}{dx} = \rho_0 A \Delta x \,\ddot{u} \qquad \text{or} \qquad \rho_0 \ddot{u} = -\frac{dp}{dx}$$

let $p = p_0 + \Delta p$

$\rho = \rho_0 + \Delta\rho$

with reference state constant in x, t

so $\dfrac{dp}{dx} = 0 + \dfrac{d\,\Delta p}{dx}$

$$\frac{d\rho}{dx} = 0 + \frac{d\,\Delta\rho}{dx}$$

now differentiate w.r.t. x ~~Erth~~ $\rho_0 \dfrac{d^2}{dt^2} u = -\dfrac{dp}{dx}$

$$\rho_0 \frac{d^2}{dt^2} \frac{du}{dx} = -\frac{d^2p}{dx^2}$$

sub in $\dfrac{du}{dx} = -\dfrac{\Delta\rho}{\rho_0}$

$$-\rho_0 \frac{d^2}{dt} \frac{\Delta\rho}{\rho_0} = -\frac{d^2p}{dx^2} = -\frac{d^2\Delta p}{dx^2}$$

$$\frac{d^2}{dt}\Delta\rho = \frac{d^2\Delta p}{dx^2} = K^{-1}\frac{d^2}{dt^2}\Delta p$$

$$\boxed{K^{-1}\frac{d^2}{dt^2}\Delta p = \frac{d^2}{dx^2}\Delta p}$$

velocity

$\Delta p = \cancel{\sin(t - vt)}$ $C\sin(x - vt)$

$$-K^{-1} C v^2 \sin(x - vt) = -C\sin(x - vt)$$

$C =$ anything

$v^2/K = 1$ or $v = \sqrt{K}$ $\quad \sim 300\,m/s$ for air