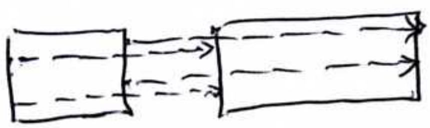
<u>translation</u> no deformation u_x and u_y constant

$$\frac{du_x}{dx} = \frac{du_y}{dy} = \frac{du_x}{dy} = \frac{du_y}{dx} = 0$$

<u>horizontal stretching</u>



$$\frac{du_x}{dx} > 0 \qquad \frac{du_x}{dy} = 0$$

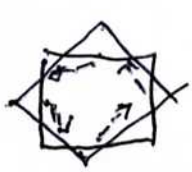
$u_y = 0$ so $\frac{du_y}{dx} = \frac{du_y}{dy} = 0$

<u>shearing</u> (horizontal

$$\frac{du_x}{dx} = 0 \qquad \frac{du_x}{dy} > 0$$

$u_y = 0$ so $\frac{du_y}{dx} = \frac{du_y}{dy} = 0$

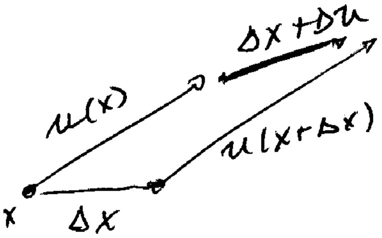
counterclockwise rotation by 45°



$$\frac{du_x}{dx} = 0 \qquad \frac{du_x}{dy} < 0$$

$$\frac{du_y}{dx} > 0 \qquad \frac{du_y}{dy} = 0$$

last case, rotation, not deformation, so $\frac{du_i}{dx_j} \neq 0$ not limited to deformation



Has its length changed

before $l_1 = |\Delta x|$

after $l_2 = |\Delta x + \Delta u|$

$$l_2^2 = |\Delta x + \Delta u|^2 = \left|\Delta x + \frac{du}{dx} dx\right|^2$$

$$= \left(\Delta x_i + \frac{du_i}{dx_j}\Delta x_j\right)\left(\Delta x_i + \frac{du_i}{dx_j}\Delta x_j\right)$$

now let $\dfrac{du_i}{dx_j} = \varepsilon_{ij} + \Omega_{ij}$

ε_{ij} = symmetric $= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

Ω = antisymmetric $= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_i}{\partial x_j}\right)$

$$l_2^2 = (\Delta x_i + \varepsilon_{ij}\Delta x_j + \Omega_{ij}\Delta x_j) \times (\Delta x_i + \varepsilon_{ij}\Delta x_j + \Omega_{ij}\Delta x_j)$$

$$\approx (\Delta x_i + 2\varepsilon_{ij}\Delta x_i \Delta x_j + \cancel{2\Omega_{ij}\Delta x_i \Delta x_j})$$

0 because Ω antisym while $\Delta x_i \Delta x_j$ sym

now let v_i = direction of Δx_i

$\Delta x_i = l_1 v_i$

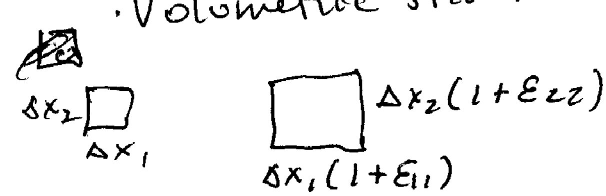
$$l_2^2 = l_1^2 + 2 l_1^2 \varepsilon_{ij} v_i v_2 = l_1^2(1 + 2\varepsilon_{ij} v_i v_i)$$

but $(1+\varepsilon)^2 \approx 1 + 2\varepsilon$ so

$$l_2 = l_1(1 + \varepsilon_{ij} v_i v_j)$$

$$\frac{l_2 - l_1}{l_1} = \frac{\Delta l}{l} = \varepsilon_{ij} v_i v_j$$

Volumetric strain



$V_1 = \Delta x \Delta y \Delta z$

$V_2 = \Delta x \Delta y \Delta z (1+\varepsilon_{11})(1+\varepsilon_{22})(1+\varepsilon_{33})$

$\Delta V / V \approx \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_{ii}$

stress induced by deformation

~~Eij~~ analog of hooks law

$$f = k u$$

T_{ij} linear related to ε_{ij}

<u>general case</u>

$$T_{ij} = \sum_p \sum_q C_{ijpq} \varepsilon_{pq}$$

every cmp T depends on all 6 independent cmpts of ε. 36 independent coefficients

$C_{ijpq} = C_{jipq}$ T sym.

$C_{ijpq} = C_{ijqp}$ ε sym

$C_{ijpq} = C_{pqij}$ $\rightarrow 36 \rightarrow 21$

21 elasticity components!

<u>special case:</u> isotropic material 2 constants

$$C_{ijpq} = \lambda \delta_{ij} \delta_{pq} + \mu (\delta_{ip} \delta_{jq} + \delta_{iq} \delta_{jp})$$

$$T_{ij} = \lambda \delta_{ij} \delta_{pq} \varepsilon_{pq} + \mu \delta_{ip} \delta_{jq} \varepsilon_{pq} + \mu \delta_{iq} \delta_{jp} \varepsilon_{pq}$$

$$T_{ij} = \lambda \delta_{ij} \varepsilon_{pp} + 2\mu \varepsilon_{ij}$$

volumetric strain

Pressure: inviscid force per unit area acting normal ~~on~~ to surface, independent on direction of surface

$$T_i = \tau_{ij} \cdot n_j \qquad \tau_{ij} = -p\,\delta_{ij}$$

~~ideal gas law~~

Newton's Law for solid $\quad$ ~~f=m~~ $\quad ma = f$

$$\int_{\text{volume}} \rho \ddot{u}_i \, dV = \int_{\text{surface}} T_i \, dA = \int_{\text{surface}} \tau_{ij} n_j \, dA = \int_{\text{volume}} \text{div}(\tau_{ij})\, dV$$

true for any volume

$$\rho \ddot{u}_i = \text{div}(\tau_{ij}) = \frac{d}{dx_j} \tau_{ij}$$

$$\rho \ddot{u}_i = \frac{d}{dx_j} \tau_{ij} = \frac{d}{dx_j}\left[C_{ijpq}\, \varepsilon_{pq}\right] =$$

$$= \frac{d}{dx_j}\left[\lambda \delta_{ij} \varepsilon_{pp} + 2\mu \varepsilon_{ij}\right] \qquad \text{assumed } \lambda, \mu \text{ const.}$$

$$= \frac{d}{dx_j}\left[\lambda \delta_{ij} \frac{du_p}{dx_p} + \mu \frac{\partial u_i}{\partial x_j} + \mu \frac{\partial u_j}{\partial x_i}\right]$$

$$\rho \ddot{u}_i = \lambda \frac{d}{dx_i}\frac{d}{dx_p} u_p + \mu \frac{d}{dx_j}\frac{d}{dx_j} u_i + \mu \frac{d}{dx_i}\frac{d}{dx_j} u_j$$

$$= (\lambda + \mu) \frac{d}{dx_i}\frac{d}{dx_j} u_j + \mu \frac{d}{dx_j}\frac{d}{dx_j} u_i$$