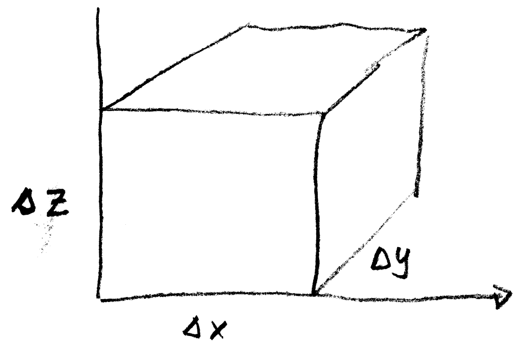

$M a_i = f_i$ = Sum of tractions times areas on 6 faces

$M \to \rho \Delta x \Delta y \Delta z = \rho \Delta V$, $\quad a_i \to \ddot{u}_i$

$$\left(T_i^{\text{right}} + T_i^{\text{left}}\right) \Delta y \Delta z =$$

note $T(-\underline{n}) = -T(\underline{n})$

$$\left(T_i(\Delta x, \hat{x}) + T_i(0, -\hat{x})\right) \Delta y \Delta z =$$

$$\left(T_i(\Delta x, \hat{x}) - T_i(0, \hat{x})\right) \Delta y \Delta z =$$

$$\frac{d}{dx} T_i(0, \hat{x}) \, \Delta x \Delta y \Delta z =$$

$$\Delta V \frac{d}{dx} \tau_{ij} \hat{x}_j = \Delta V \frac{d}{dx} \tau_{ix}$$

similarly

$$\left(T_i^{\text{back}} + T_i^{\text{front}}\right) \Delta x \Delta z = \Delta V \frac{d}{dy} \tau_{iy}$$

$$\left(T_i^{\text{top}} + T_i^{\text{bottom}}\right) \Delta x \Delta y = \Delta V \frac{d}{dz} \tau_{iz}$$

so $$f_i = \Delta V \left(\frac{d \tau_{ix}}{dx} + \frac{d \tau_{iy}}{dy} + \frac{d \tau_{iz}}{dz} \right) = \Delta V \sum_j \frac{d}{dx_j} \tau_{ij}$$

and newton's law becomes

$$\rho \ddot{u}_i = \sum_j \frac{d}{dx_j} \tau_{ij} = \left[\mathrm{div}(\underline{\underline{\tau}}) \right]_i$$