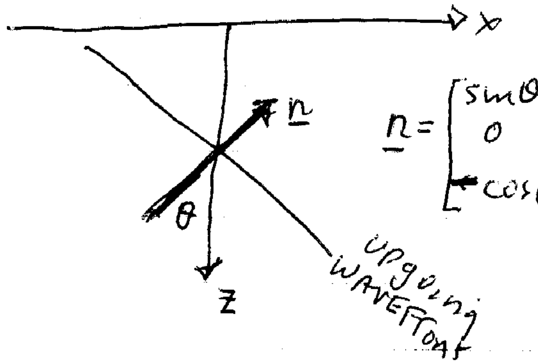
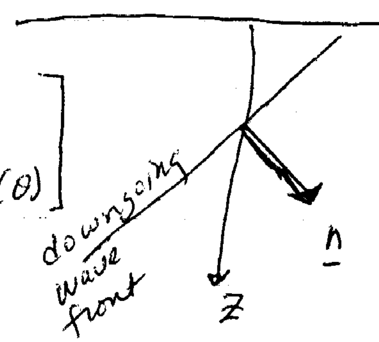
slowness:

$$u_i = A_i \cos(k\hat{n}\cdot x - \omega t)$$

$\frac{\omega}{k}$ = velocity = V $\quad \hat{n}$ = direction $\quad \theta$ = angle



$$n = \begin{bmatrix} \sin\theta \\ 0 \\ -\cos(\theta) \end{bmatrix}$$



$$n = \begin{bmatrix} \sin \\ 0 \\ \cos \end{bmatrix}$$

$$u_i = A_i \cos\left(\omega\left(\frac{k n}{\omega}\right)\cdot x - \omega t\right)$$

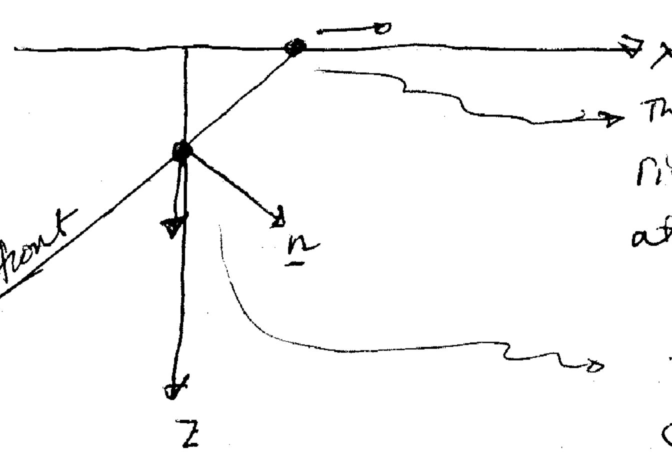
$\rightarrow p$ = slowness vector

$$= A_i \cos(\omega(p\cdot x - t))$$

$$p = \frac{k}{\omega} n = n/V$$

V big, p small so p called slowness

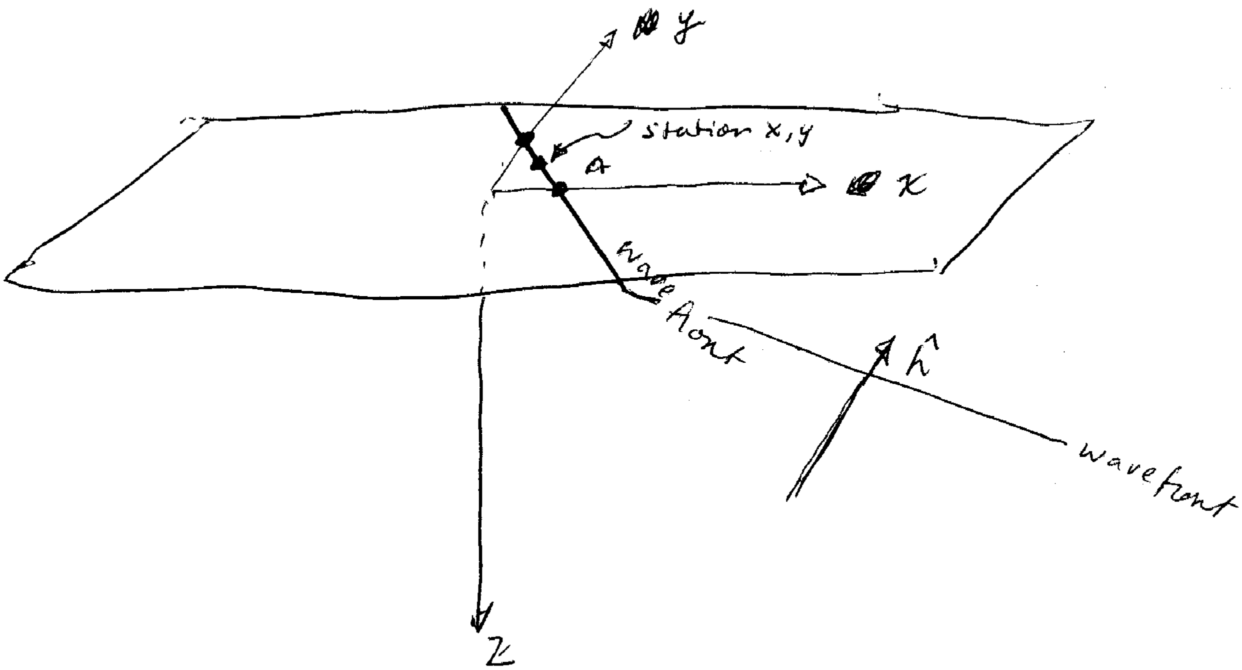
if $n = \begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix}$ Then $p = \frac{1}{V}\begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix}$



This intersection point moves right at slowness $p_x = \frac{\sin\theta}{V}$ at apparent velocity $V_x = V/$

This intersection point m down at slowness $p_z = \frac{\cos}{V}$ at apparent velocity $V_z =$

note (p_x, p_y, p_z) is a vector. (V_x, V_y, V_z) is not a ve



suppose wavefront crosses origin at $t = 0$

Then it crosses a station at (x, y) at a time $p_x x + p_y y = \frac{x}{V_x} + \frac{y}{V_y}$

so you can estimate p_x, p_y from arr time at 3 stations, A, B, C

$$T_B = T_A + p_x (X_B - X_A) + p_y (y_B - y_A$$

$$T_C = T_A + p_x (X_C - X_A) + p_y (y_C - y_A$$

2 equations in 2 unknowns.

$$\begin{pmatrix} (X_B - X_A) & (y_B - y_A) \\ (X_C - X_a) & (y_C - y_a) \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} T_B - T_A \\ T \; - T_A \end{pmatrix}$$

Issues

$(\text{lon}, \text{lat}) \rightarrow (x, y)$ flat earth approx

$$x = 111.12 \cdot \text{lon} \cdot \cos(\text{typical latitude})$$
$$y = 111.12 \cdot \text{lat}$$

invert 2x2 matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad M^{-1} = \frac{1}{AD - BC} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}$$

azimuth from north from p_x, p_y

$$\text{azimuth} = \tan^{-1}(p_x / p_y)$$

horizontal slowness in direction of azimuth $\sqrt{p_x^2 + p_y^2}$

horizontal apparent velocity $1 / \sqrt{p_x^2 + p_y^2}$

vertical slowness:

use $p_x^2 + p_y^2 + p_z^2 = 1/v^2$

$$p_z^2 = 1/v^2 - p_x^2 - p_y^2$$
$$p_z = \sqrt{1/v^2 - p_x^2 - p_y^2}$$

but is unknown

angle of incidence

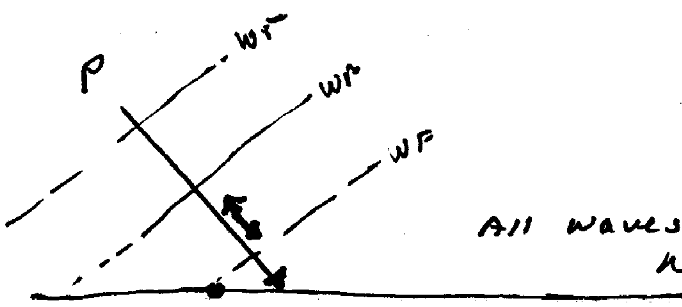
use $p_z = \cos\theta / v$

$$\theta = \cos^{-1}(v p_z)$$

Two uniform half-spaces in contact

medium 1 — plane P, S waves here with velocity α_1, β_1

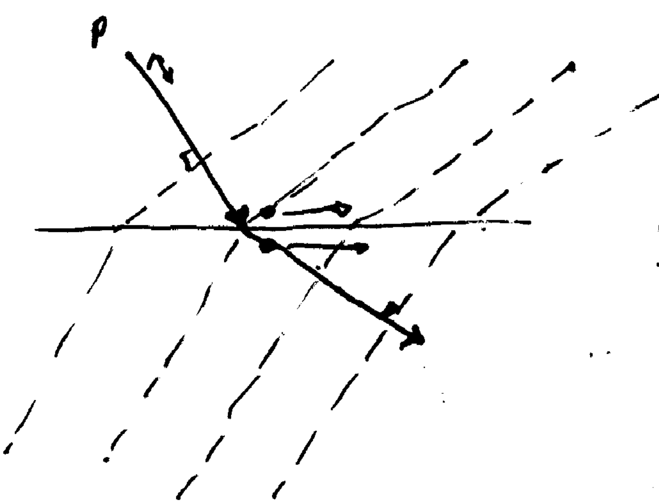
medium 2 — Plane P, S waves here with velocity α_2, β_2



All waves must have this horizontal apparent velocity

→ waves on other side of boundary must have this apparent velocity

Displacement, traction continuous across interface ⇒ wavefronts must not tear as they go across the interface



Horizontal apparent velocity (horizontal slowness) must be equal on two sides

$$p_x^{top} = p_x^{bottom}$$

$$\frac{\sin\theta^{Top}}{\alpha^{Top}} = \frac{\sin\theta^{Bottom}}{\alpha^{Bottom}}$$

Snell's law.

Displacement continuous: boundary remains "welded"

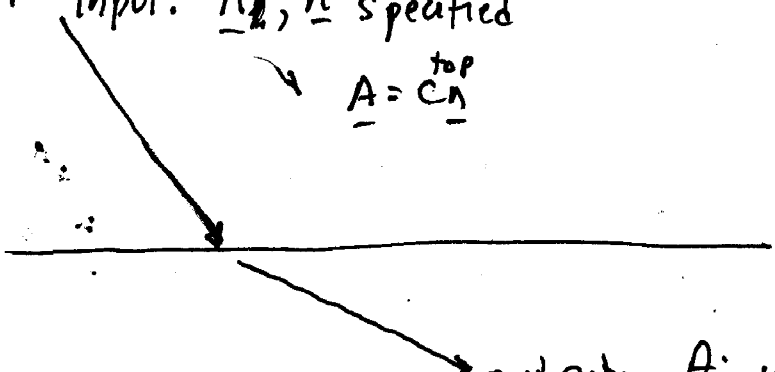
Tractions continuous: no infinite ~~forces~~ accelerations on small masses near boundary

Six conditions

$$u_x^{top} = u_x^{bot} \qquad \tau_{xz}^{top} = \tau_{xz}^{bot}$$
$$u_y^{top} = u_y^{bot} \qquad \tau_{yz}^{top} = \tau_{yz}^{bot}$$
$$u_z^{top} = u_z^{bot} \qquad \tau_{zz}^{top} = \tau_{zz}^{bot}$$

P input: $\underline{A}^{top}$, $\underline{n}^{top}$ specified

$\underline{A} = C\underline{n}^{top}$

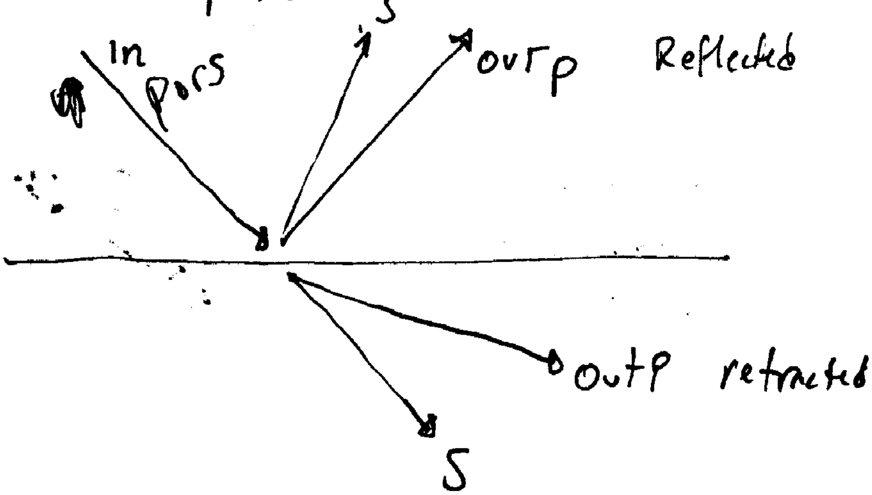


output: A_i unknown $= Cn^{bot}$ bottom

n_i^{bot} given by snell's law

One output not enough to satisfy 6 conditions

Need 6 outputs



S

note 2 S waves in each layer

SV: Polarization in plane of paper

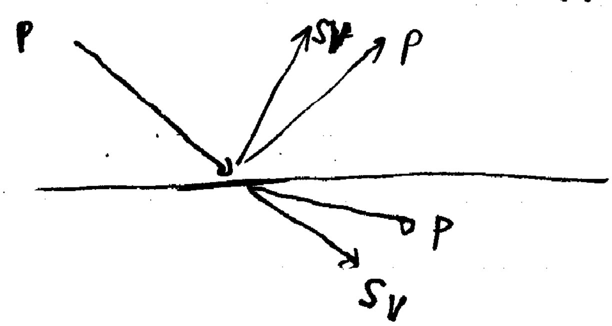
SH Polarization ⊥ " " "

Special cases:

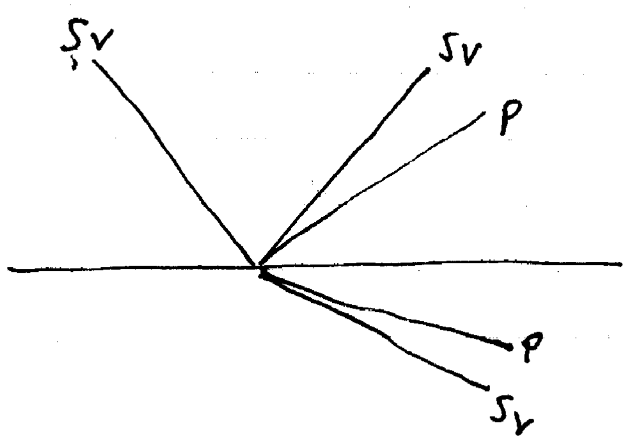
P^{TOP}_{in}: no motion $\perp$ to plane of page

P^{TOP}_{out}, P^{Bot}_{out} $S^{Bot}_{V\,out}$ $S^{TOP}_{V\,out}$

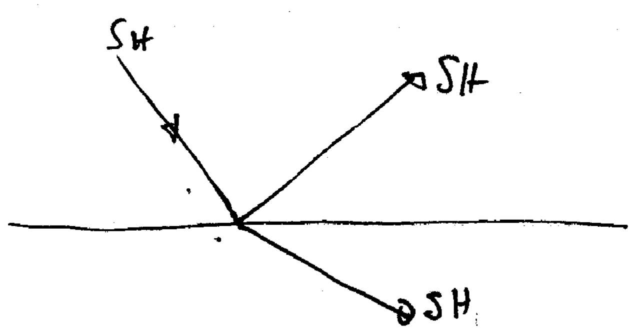
but no $S^{Bot}_{H_{out}}$ or $S^{TOP}_{H_{out}}$ due to symmetry



$S^{top}_{V\,in}$:



$S^{top}_{H\,in}$



note "S" always steeper than "P", due to snell's law

$$\frac{\sin\theta_p}{\alpha} = \frac{\sin\theta_S}{\beta} = p_x = \text{horizontal slowness}$$

and $\alpha > \beta$