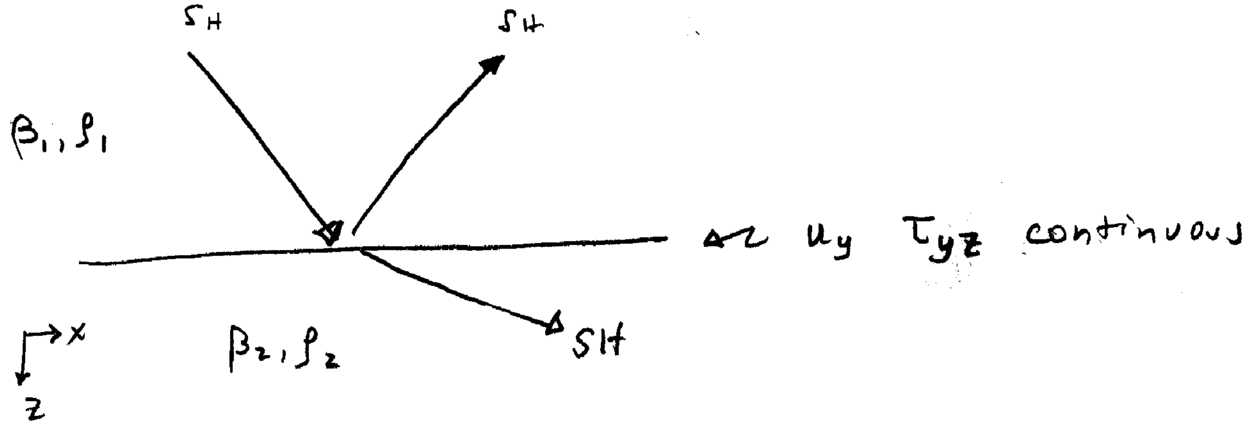


TOP $$u_y^{(1)} = \grave{C}_1 \cos(\omega p_x x + \omega p_z^{(1)} z - \omega t) + \acute{C}_1 \cos(\omega p_x x - \omega p_z^{(1)} z - \omega t)$$

BOT $$u_y^{(2)} = \grave{C}_2 \cos(\omega p_x x + \omega p_z^{(2)} - \omega t)$$

p_x equal for all waves. p_z's given by snell's law

$$p_z^{(1)} = \sqrt{\beta_1^{-2} - p_x^2} \qquad p_z^{(2)} = \sqrt{\beta_2^{-2} - p_x^2}$$

since $\frac{\sin\theta_1}{\beta_1} = p_x^{(1)}$ and $\frac{\cos\theta_1}{\beta_1} = p_z^{(1)}$ square and add to get

$$\frac{1}{\beta_1}(\sin^2\theta + \cos^2\theta) = p_x^2 + p_z^{(1)2}$$

$$\tau_{ij} = \lambda \delta_{ij} \sum_p \varepsilon_{pp} + 2\mu \varepsilon_{ij} \qquad \tau_{yz} = 2\mu \varepsilon_{yz}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i}\right) \qquad \frac{1}{2}\frac{du_y}{dz} = \varepsilon_{yz}$$

and $\tau_{yz} = \mu \frac{du_y}{dz}$

TOP: $-\mu_1 \grave{C}_1 \omega p_z^{(1)} \sin(\cdot) + \mu_1 \acute{C}_1 \omega p_z^{(1)} \sin(\cdot)$

BOT $-\mu_2 \grave{C}_2 \omega p_z^{(2)} \sin(\cdot)$

continuity of displacement $u_y^{(1)}(z=0) = u_y^{(2)}(z=0)$

$$\grave{C}_1 \cos(\omega p_x x - \omega t) + \acute{C}_1 \cos(\omega p_x x - \omega t) = \grave{C}_2 \cos(\omega p_x x - \omega t)$$

$$\grave{C}_2 - \acute{C}_1 = \grave{C}_1$$

continuity of traction $\tau_{yz}^{(1)}(z=0) = \tau_{yz}^{(2)}(z=0)$

$$-\mu_1 \grave{C}_1 \omega p_z^{(1)} \sin(\omega p_x x - \omega t) + \mu_1 \acute{C}_1 \omega p_z^{(1)} \sin(\cdot) = -\mu_2 \grave{C}_2 \omega p_z^{(2)} \sin(\cdot)$$

$$+\mu_2 p_z^{(2)} \grave{C}_2 + \mu_1 p_z^{(1)} \acute{C}_1 = +\mu_1 p_z^{(1)} \grave{C}_1$$

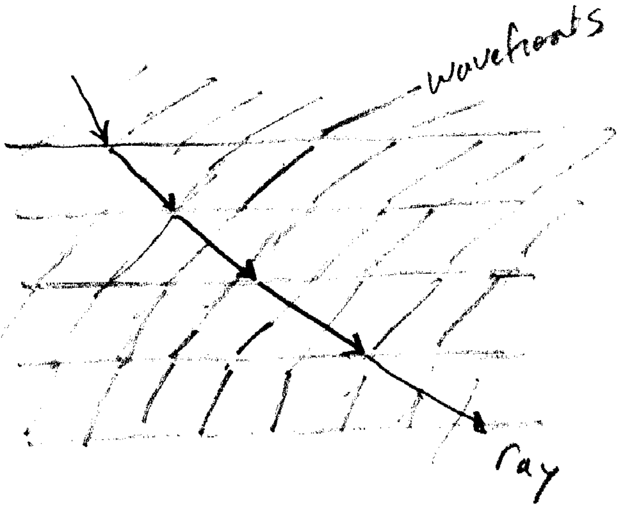
$$\begin{bmatrix} 1 & -1 \\ \mu_2 p_z^{(2)} & \mu_1 p_z^{(1)} \end{bmatrix} \begin{bmatrix} \grave{C}_2 \\ \acute{C}_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 p_z^{(1)} \end{bmatrix} \grave{C}_1$$

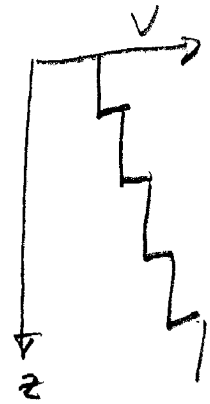
$$\det = \mu_1 p_z^{(1)} + \mu_2 p_z^{(2)}$$

$$\begin{bmatrix} \grave{C}_2 \\ \acute{C}_1 \end{bmatrix} = \frac{1}{\det} \begin{bmatrix} \mu_1 p_z^{(1)} & 1 \\ -\mu_2 p_z^{(2)} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \mu_1 p_z^{(1)} \end{bmatrix} \grave{C}_1$$

$$= \begin{bmatrix} 2\mu_1 p_z^{(1)} / [\mu_1 p_z^{(1)} + \mu_2 p_z^{(2)}] \\ [\mu_1 p_z^{(1)} - \mu_2 p_z^{(2)}] / [\mu_1 p_z^{(1)} + \mu_2 p_z^{(2)}] \end{bmatrix} \grave{C}_1$$

Vertically-stratified media.



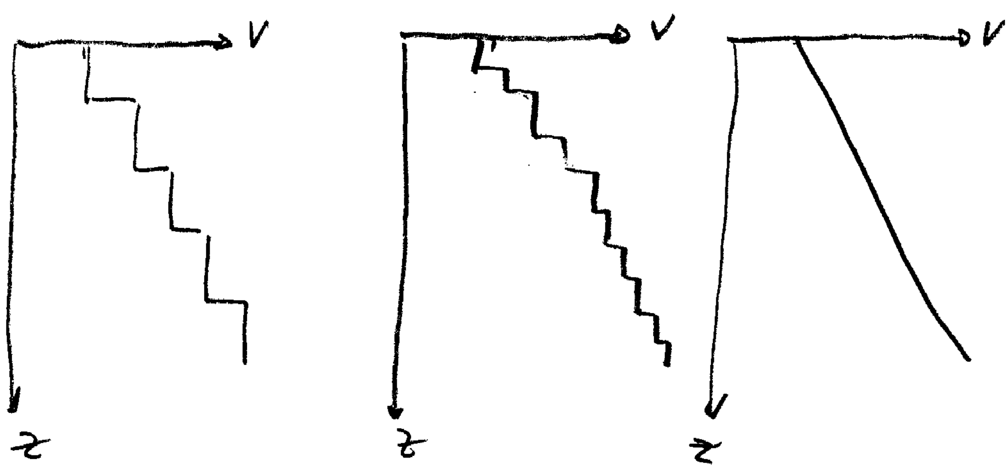


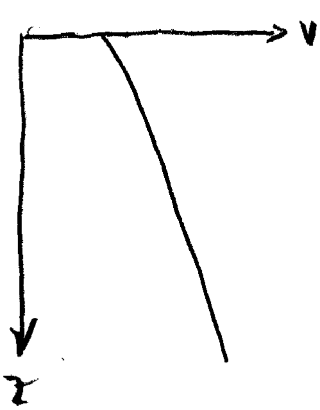
p_x equal in all layers

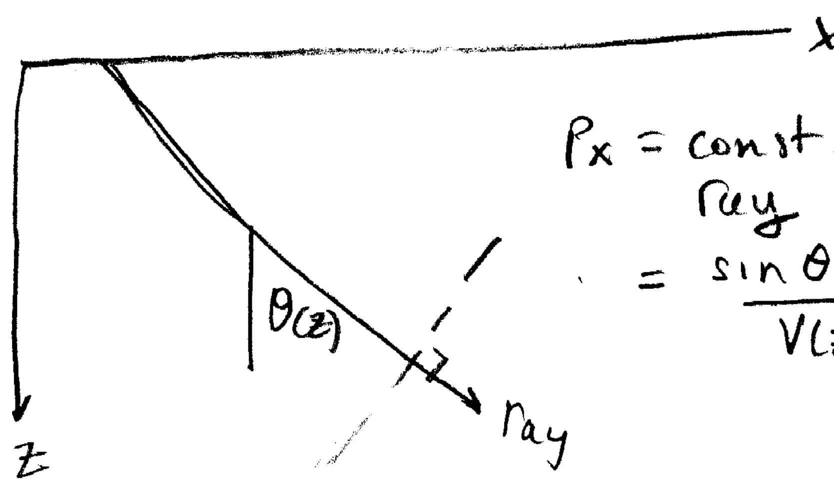
just draw $\underline{n}$'s connect to make "ray"

p_x constant along the "ray".

limit of lots of fine layers



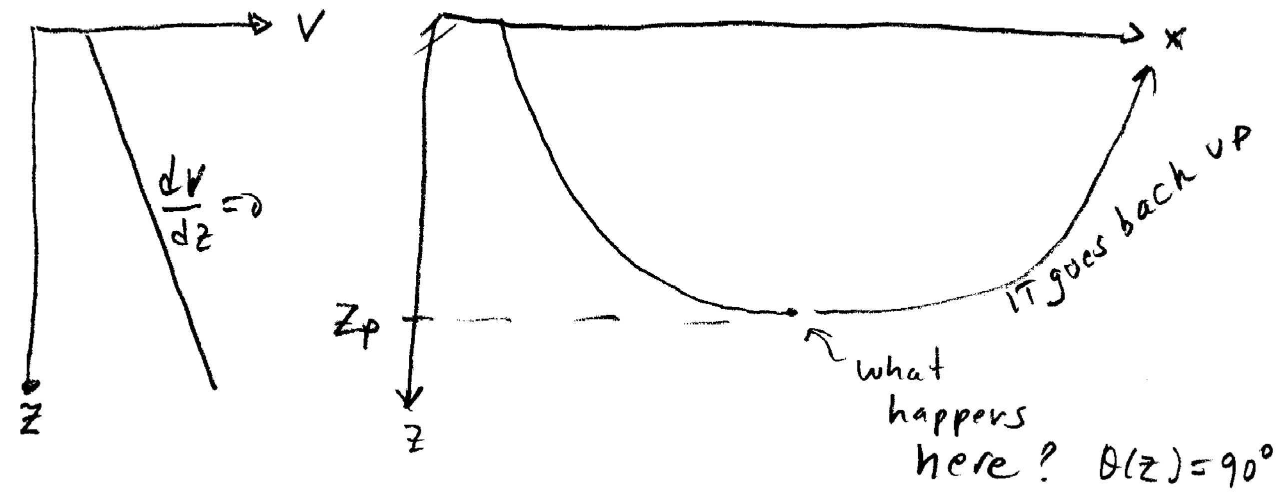




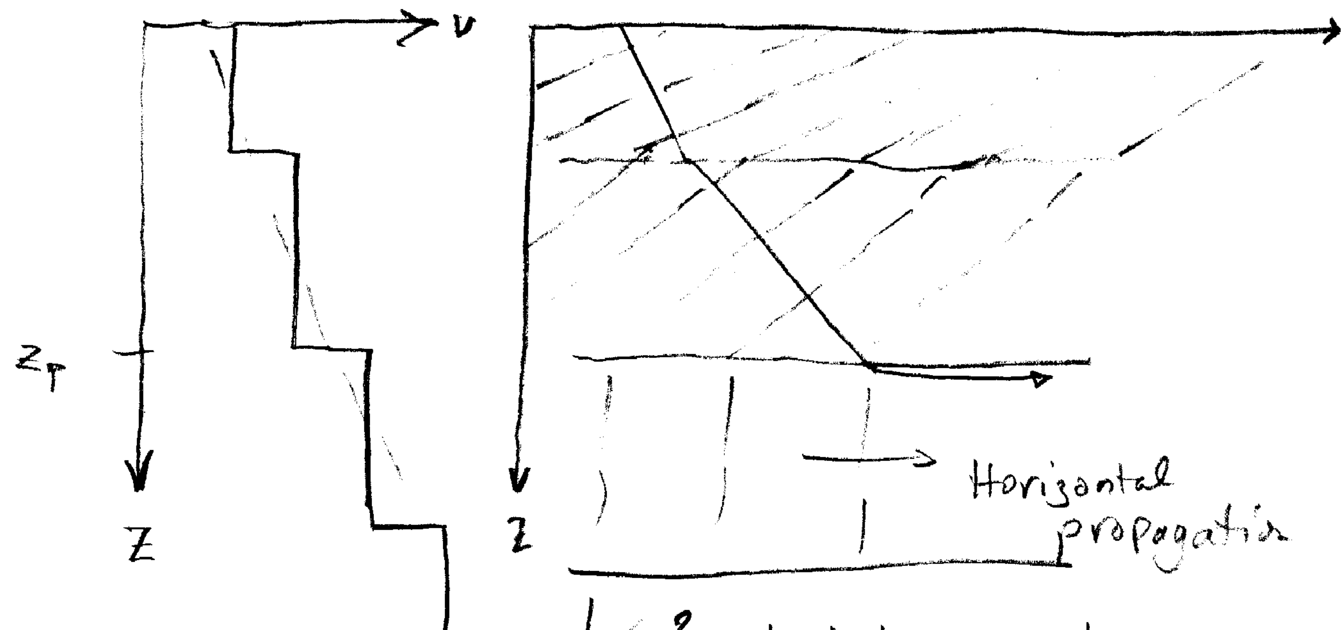
$$p_x = \text{const along ray} = \frac{\sin\theta(z)}{V(z)}$$

wavefronts normal to ray

Case: velocity increase with depth, rays curve up



$$P_x = \frac{\sin(\theta(z))}{V(z)} \qquad P_x = V^{-1}(Z_P) \text{ turning point of ray}$$

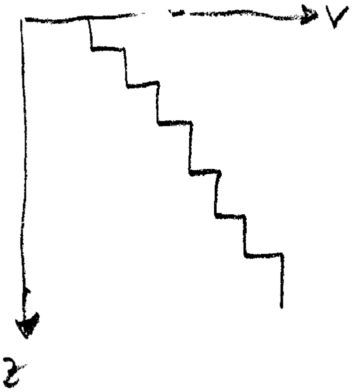


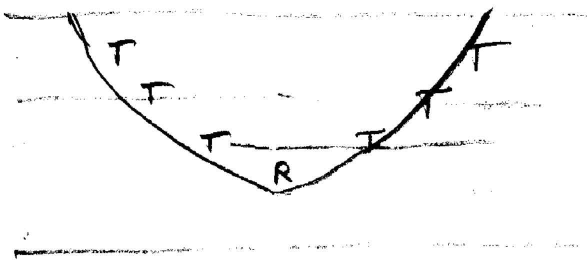
? what happens here

$$P_z = \sqrt{1/V^2 - P_x^2} \quad \text{is imaginary}$$

$$\cos(\omega P_z z) = \cosh(i\omega P_z z)$$

exponential decay





most important waves
Transmitted, except for
lowest layer in which
case wave field is
reflected.

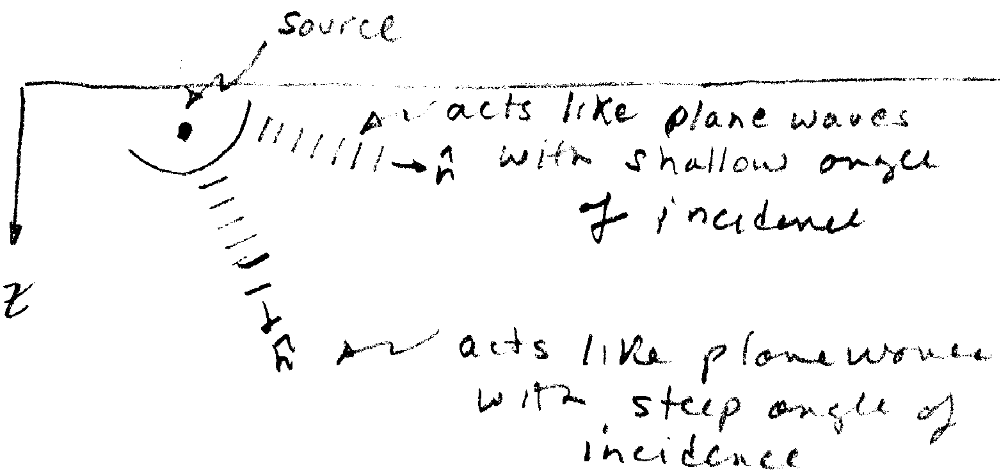
Ray Theory: smoothly varying media
not necessarily vertically stratified
λ short compared to
scale-length of velocity
variability.

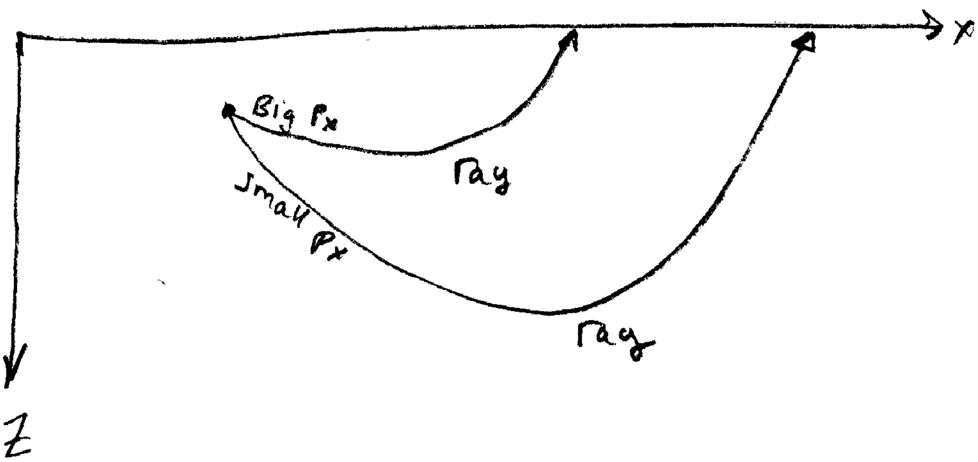
Guiding Principle: "Almost planar waves"

locally, the wave front moves
parallel to itself at a speed
given by the local velocity, v

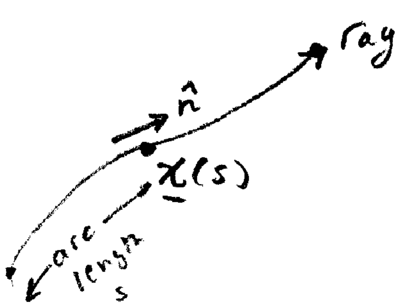
$T(\underline{x}, \underline{x}_s)$ traveltime from $\underline{x}_s$ to $\underline{x}$.

$$\nabla_{\underline{x}} T = \frac{\hat{n}}{v}$$





The equation for the ray is completely specified by the condition $\nabla T = \hat{n}/v$



proof

start with $\frac{\hat{n}}{v} = \nabla T$

dot with self $\frac{1}{v^2} = \nabla T \cdot \nabla T$

take $\frac{d}{ds}$ of $\frac{\hat{n}}{v} = \nabla T$

note $\hat{n} = \frac{dx}{ds}$

$$\frac{d}{ds}(v^{-1}\hat{n}) = \boxed{\frac{d}{ds}\left(v^{-1}\frac{dx}{ds}\right)} = \frac{d}{ds}\nabla T = \hat{n}\cdot\nabla(\nabla T$$

$$= \frac{dx}{ds}\cdot\nabla(\nabla T) = v\nabla T\cdot\nabla\nabla T = \frac{v}{2}\nabla(\nabla T^2)$$