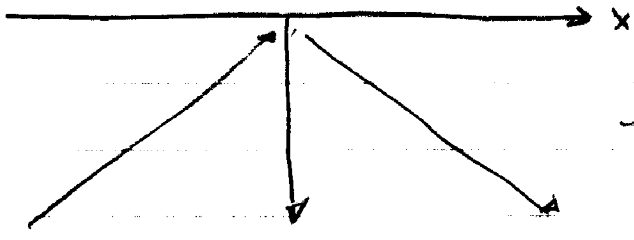
Trapped Waves



liquid of velocity V

$p_x^2 + p_z^2 = 1/V^2$

$$p = \acute{A} \sin(\omega p_x x - \omega t - \omega p_z z) + \grave{A} \sin(\omega p_x x - \omega t + \omega p_z z)$$

$p(z=0)=0$ implies $\acute{A} = -\grave{A} = A$

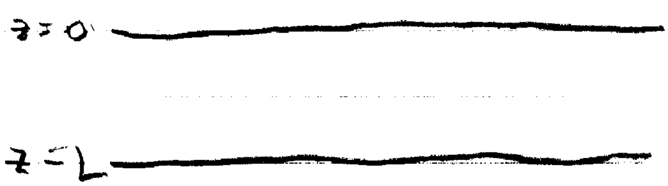
also $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

$a = \omega p_x x - \omega t \qquad b = \omega p_z z$

$$p(x,z,t) = A \sin(\omega p_z z)\cos(\omega p_x x - \omega t)$$

Put bottom on it



or put bottom on
$p=0$ on this surface, to

either $A=0$ or $\sin(\omega p_z L) = 0$

implies $\omega p_z L = N\pi$
$N = 1, 2, 3$

$p_z = \frac{N\pi}{\omega L} \qquad p_x = \sqrt{\frac{1}{V^2} - p_z^2}$

$$= \sqrt{\frac{1}{V^2} - \frac{N^2\pi^2}{\ldots}} = P(\omega)$$

"dispersion" horizontal slowness depends on frequency

$$P_x(\omega) = \sqrt{\frac{1}{V^2} - \frac{N^2\pi^2}{\omega^2 L^2}}$$

lowest frequency: $\frac{1}{V^2} = \frac{N^2\pi^2}{\omega^2 L^2}$

$$\frac{\omega^2 L^2}{N^2\pi^2} = V^2$$

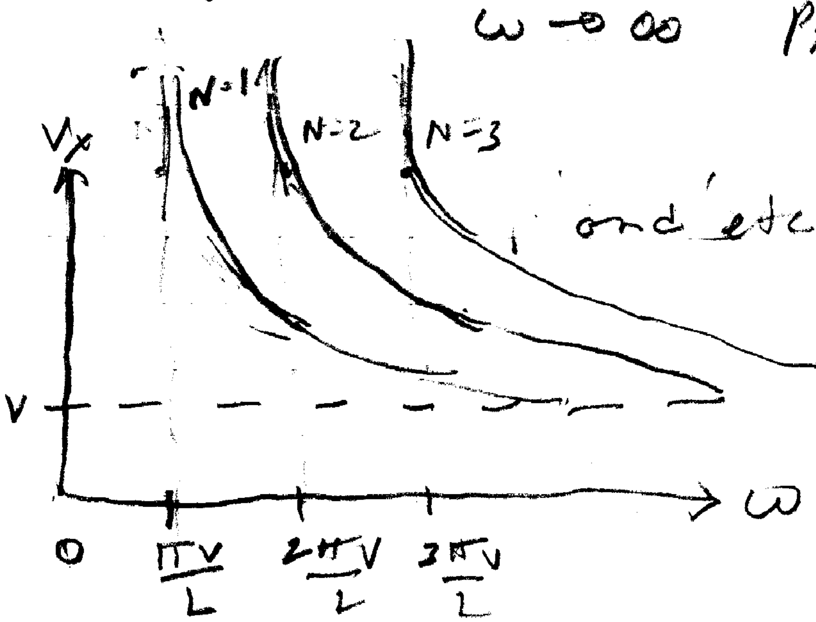
$$\omega_0 = \frac{N\pi V}{L}$$

$P_x(\omega_0) = 0 \quad V_x(\omega_0) = \infty$

wave bouncing up and down in layer

highest frequency:

$\omega \to \infty \quad P_x \to \frac{1}{V} \quad V_x \to V$



Dispersion Curve

pressure equation

$$v^{-2}\ddot{p} = \frac{d^2 p}{dx^2} + \frac{d^2 p}{dz^2}$$

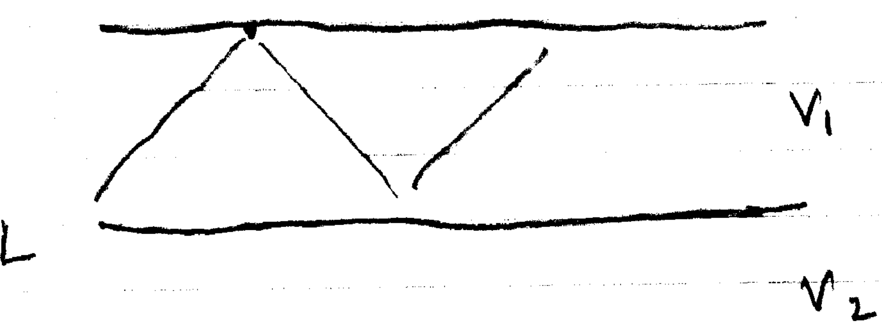
$$p(x,z,t) = e^{-\omega q_z z}\cos(\omega p_x x - \omega t)$$

$$-\frac{\omega^2}{v^2} = -\omega^2 p_x^2 + \omega^2 q_z^2$$

$$p_x = \sqrt{q_z^2 - \frac{1}{v^2}}$$

so decay const q_z similar to

vertical wave number p_z.



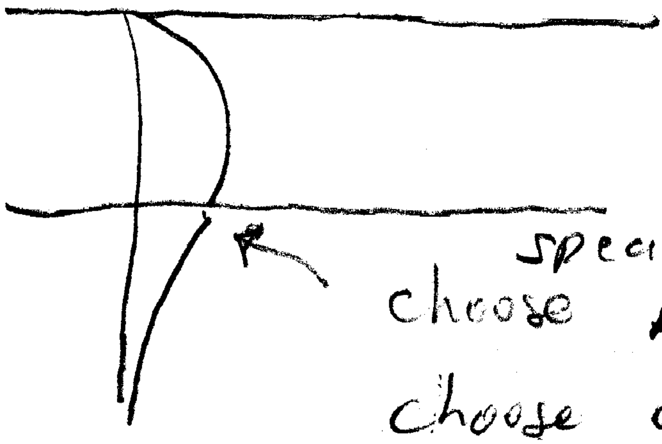
~~$p = A\sin(\omega p_z z)$~~

Layer 1: $p_1 = A\sin(\omega p_z z)\cos(\omega p_x x - \omega t)$

satisfies top BC's

Halfspace 2: $p_2 = B\exp(-\omega q_z z)\cos(\omega p_x x - \omega t)$

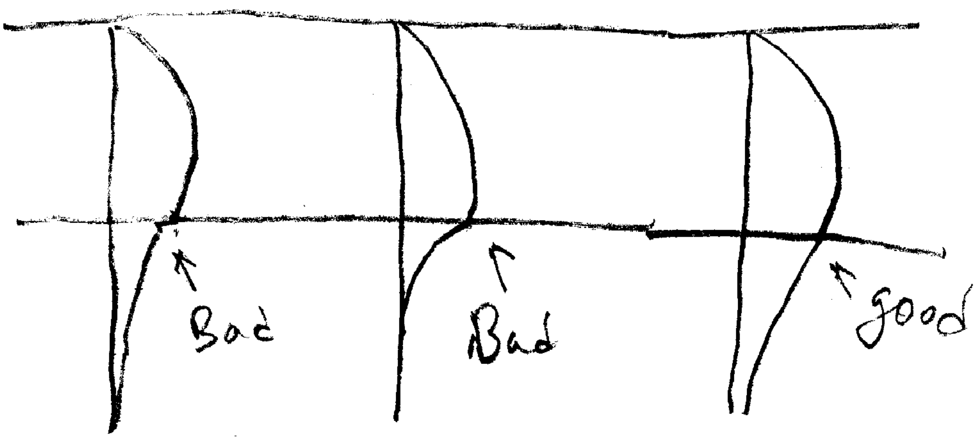
2 BC's. $p(L) =$ $|\, u(L=0)$ continuous



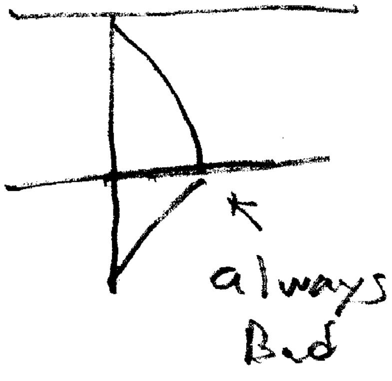
specify P_0

choose B/A so p = continuous

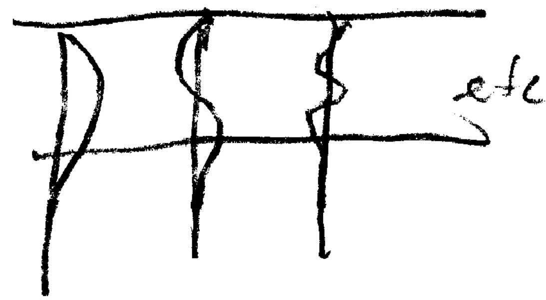
choose ω so $\frac{dp}{dz}$ = continuous



Note: must be a minimum freq.
since sin must "turn" to match BC's



Note. Higher modes exist



Love & Rayleigh Waves
"G" "R"

Love: SH...

Requires surface LVZ

linear, transverse particle moti

Rayleigh P–SV

Exists in halfspace

elliptical, retrograde particle motion