Spherical cavity in fluid of velocity v

$$\ddot{p} = c^2\left(\frac{d^2p}{dx^2} + \frac{d^2p}{dy^2} + \frac{d^2p}{dz^2}\right) = v^2 \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dp}{dr}\right)$$

try solution $p(r,t) = \frac{A}{r}\cos(kr-\omega t) + \frac{B}{r}\sin(kr-\omega t)$

"spherical wave" propagating outward with amplitude diminishing at rate $1/r$ (spherical spreading).

cosine only:

$$p = Ar^{-1}\cos(kr-\omega t) \qquad \ddot{p} = -Ar^{-1}\omega^2\cos(kr-\omega t)$$

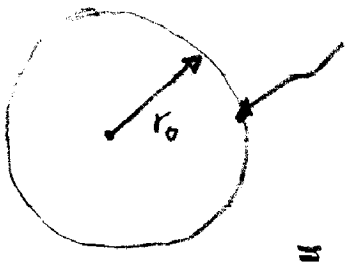
$$\frac{dp}{dr} = -Ar^{-2}\cos(kr-\omega t) - Ar^{-1}k\sin(kr-\omega t)$$

$$r^2\frac{dp}{dr} = -A\cos(kr-\omega t) - Ark\sin(kr-\omega t)$$

$$\frac{d}{dr}\left(r^2\frac{dp}{dr}\right) = +Ak\sin(kr-\omega t) - Ak\sin(kr-\omega t) - Ark^2\cos(kr-\omega t)$$

$$r^2\frac{d}{dr}\left(r^2\frac{dp}{dr}\right) = -\frac{A}{r}k^2\cos(kr-\omega t)$$

so l.h.s. = r.h.s. if $\frac{\omega}{k} = v$ (usual rule relating ω, k).
Similar calc. demonstrates sin term also satisfies eqn.



pulsating cavity

what are values of A, B?

$$p(r_0) = p_0\cos(-\omega t) = \frac{A}{r_0}\cos(kr_0-\omega t) + \frac{B}{r_0}\sin(kr_0-\omega t)$$

$$= \frac{A}{r_0}\cos(kr_0)\cos(-\omega t) - \frac{A}{r_0}\sin(kr_0)\sin(-\omega t)$$

$$+ \frac{B}{r_0}\cos(kr_0)\sin(-\omega t) + \frac{B}{r_0}\sin(kr_0)\cos(-\omega t)$$

note: used $\cos(a+b) = \cos(a)\cos(b) - \sin a \sin b$

to cancel out $\sin(-\omega t)$ terms:

$$\frac{A}{r_0}\sin(kr_0) = \frac{B}{r_0}\cos(kr_0)$$

$$B = \frac{\sin(kor)}{\cos(kor)}A$$

$$p(r_0) = \frac{A}{r_0}\left\{\cos(kr_0) + \frac{\sin^2(kr_0)}{\cos(kr_0)}\right\}\cos(-\omega t) = P_0\cos(-\omega t)$$

$$\frac{\cos^2(kr_0)}{\cos(kr_0)} + \frac{\sin^2(kr_0)}{\cos(kr_0)} = \frac{\sin^2(kr_0)+\cos^2(kr_0)}{\cos(kr_0)} = \frac{1}{\cos(kr_0)}$$

so $A = P_0 r_0 \cos(k_0 r)$

$B = P_0 r_0 \sin(k_0 r)$

$$p(x,t) = P_0\frac{r_0}{r}\left\{\cos(\underbrace{kr_0}_{b})\cos(\underbrace{kr-\omega t}_{a}) + \sin(\underbrace{k_0 r}_{b})\sin(\underbrace{kr-\omega t}_{a})\right\}$$

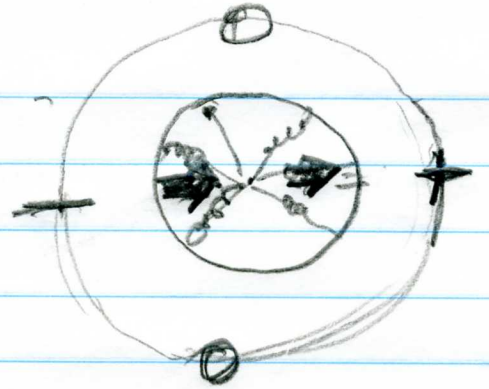
$\cos(a-b) = \cos(a)\cos(b) + \sin a \sin b$

$$p(x,t) = P_0\frac{r_0}{r}\cos\left(k(r-r_0) - \omega t\right)$$

decay with distance

phase of wave matches phase of surface at $r = r_0$

outward propagation



Equivalent force approximation

$$\rho \ddot{u}_i = \sum_j \frac{d}{dx_j} \tau_{ij} + f_i$$

f_i: force per unit volume

$$\tau_{ij} = -p\delta_{ij}$$

$$\sum_j \frac{d}{dx_j}(-p\delta_{ij}) = -\sum_j \frac{dp}{dx_i}\delta_{ij} = -\nabla p$$

$$\rho \ddot{u}_i = -\nabla p + \nabla \cdot f$$

take $\nabla\cdot$ of both sides

$$\nabla \cdot u = -\kappa^{-1} \dot{p}$$

ideal gas law

$$-\frac{\rho}{\kappa}\ddot{p} = -\nabla p + \nabla \cdot f$$

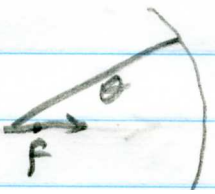
$$\boxed{\ddot{p} = \frac{\kappa}{\rho}\nabla^2 p - \frac{\kappa}{\rho}\nabla \cdot f}$$

Acoustic eqn with force

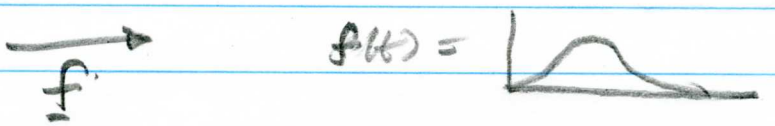
$$\underline{f} = A\hat{n}\delta(\underline{x})\cos(-\omega t)$$

easier to solve equation with force than with complicated B.C.

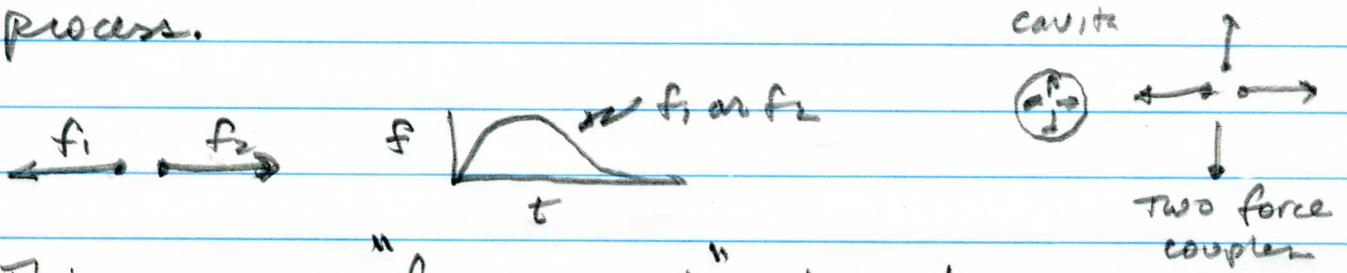
Radiation Pattern $\rightarrow$ $p \sim \frac{\cos(\theta)}{r}\cos(kr - \omega t)$



equivalent force – might help w/ intuition.
issue: conservation of momentum



transfers finite momentum to earth. OK for meteorite impact. Not good for internal process.



This one, a "force couple", is ok.

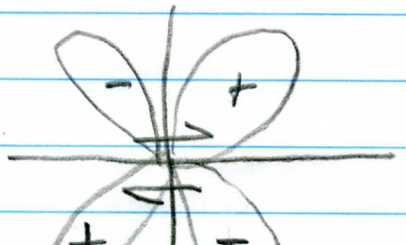
But this one,  , also a force couple, is not!

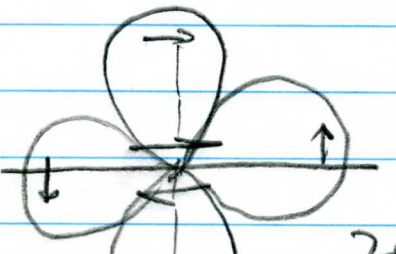
changes spin (angular momentum) of earth.

fault

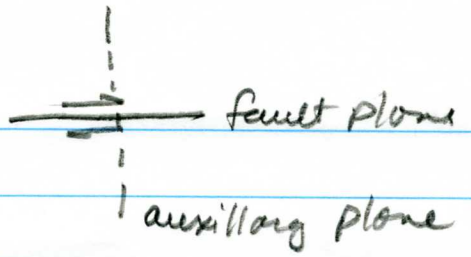
can't be right equivalent force

This turns out to be right one



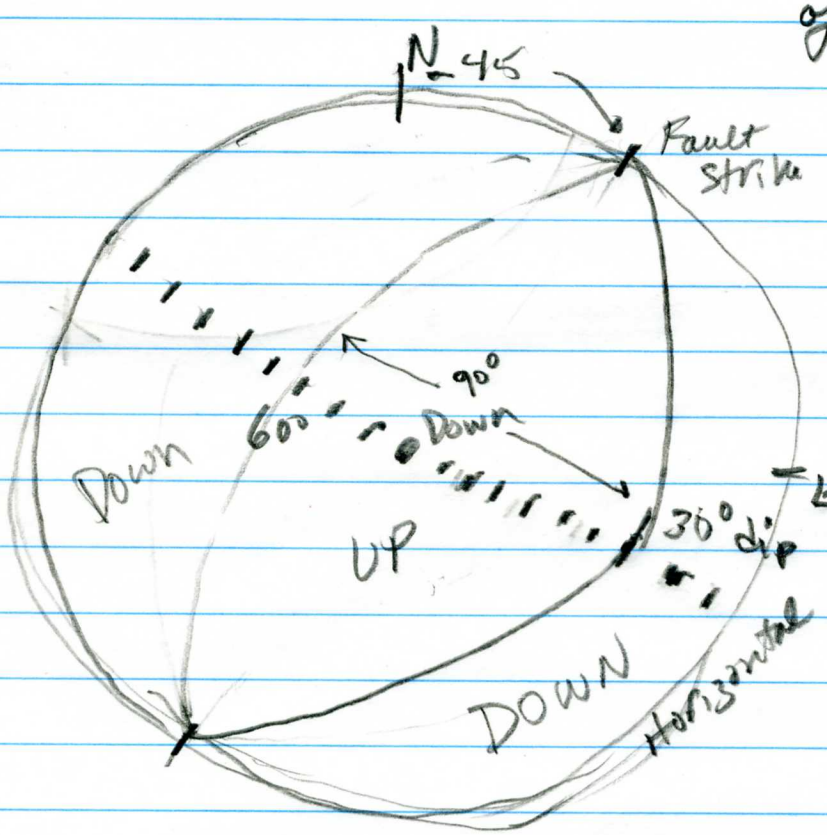


s

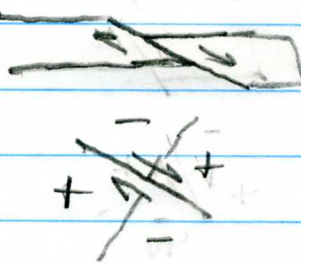
note: 

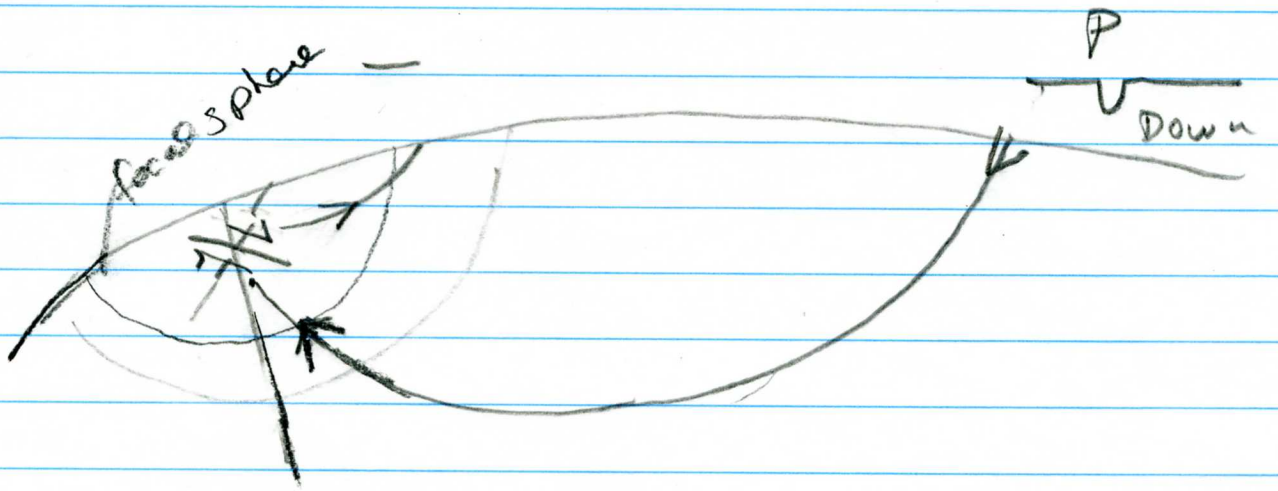
radiation the same for slip on aux. plane.

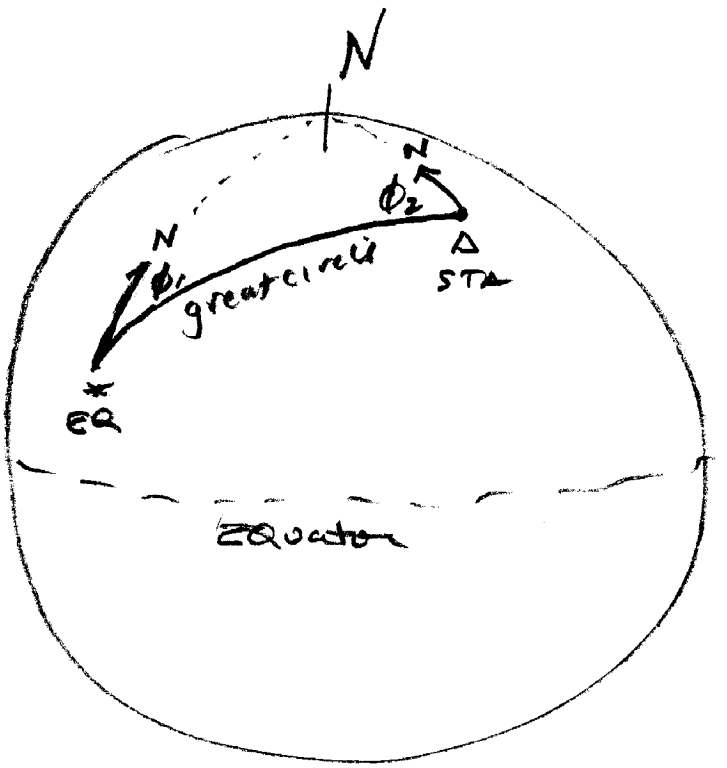
non-uniqueness of seismic data.



Fault strike N45°E
dip 30° to west





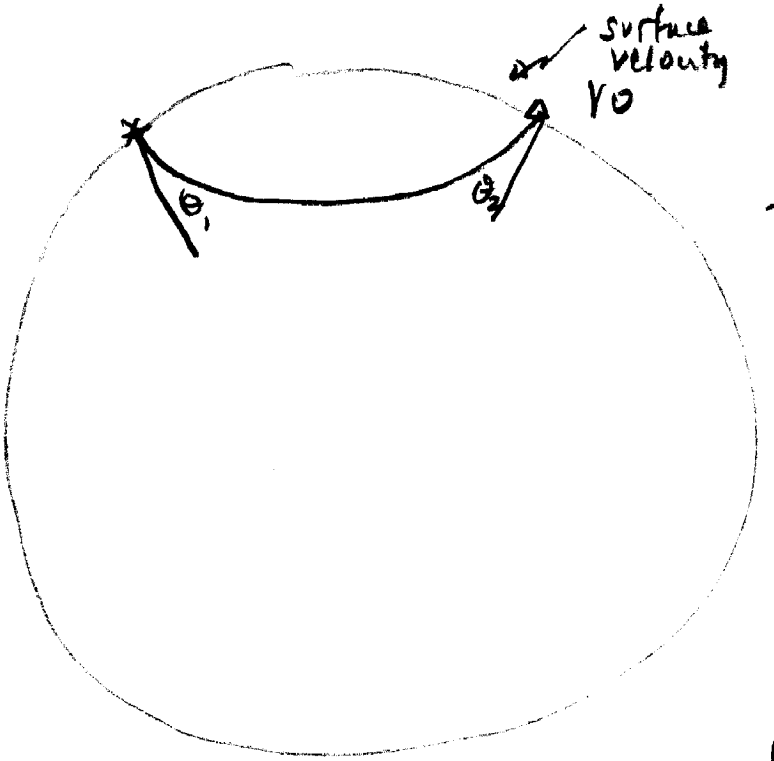


Earth

ϕ_1 azimuth from EQ to STA

ϕ_2 backazimuth from STA to EQ

$$\phi_1 \neq \phi_2$$



$\theta_1 \approx \theta_2$ for shallow EQ

$$\theta_1 = \sin^{-1}(P_x V_0)$$

$$P_x = \frac{dT}{dx}$$

measure or estimate from T.T. table

Numerical TABLE AK135

Focal mechanism plot:

need for each earthquake

Azimuth, ϕ – based on sta location and eq location

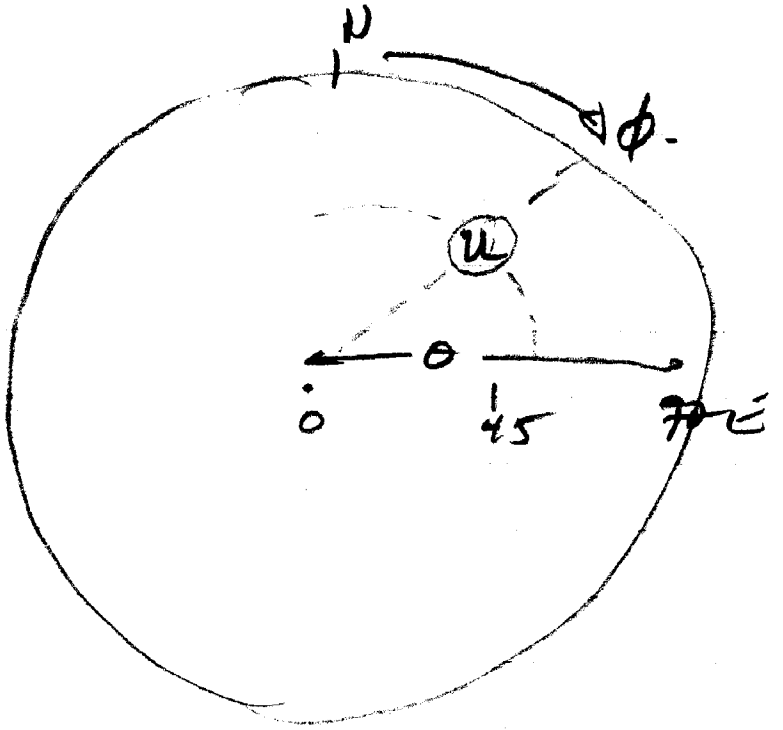
angle of incidence, θ – based on TT Table and V_0

Polarity of P wave, P

up

down

(nodal)



eg. $\theta = 50$

$\phi = 45$

P = up

Stereonet