EESC UN3201
Solid Earth Dynamics
Spring 2023

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Instructional Team

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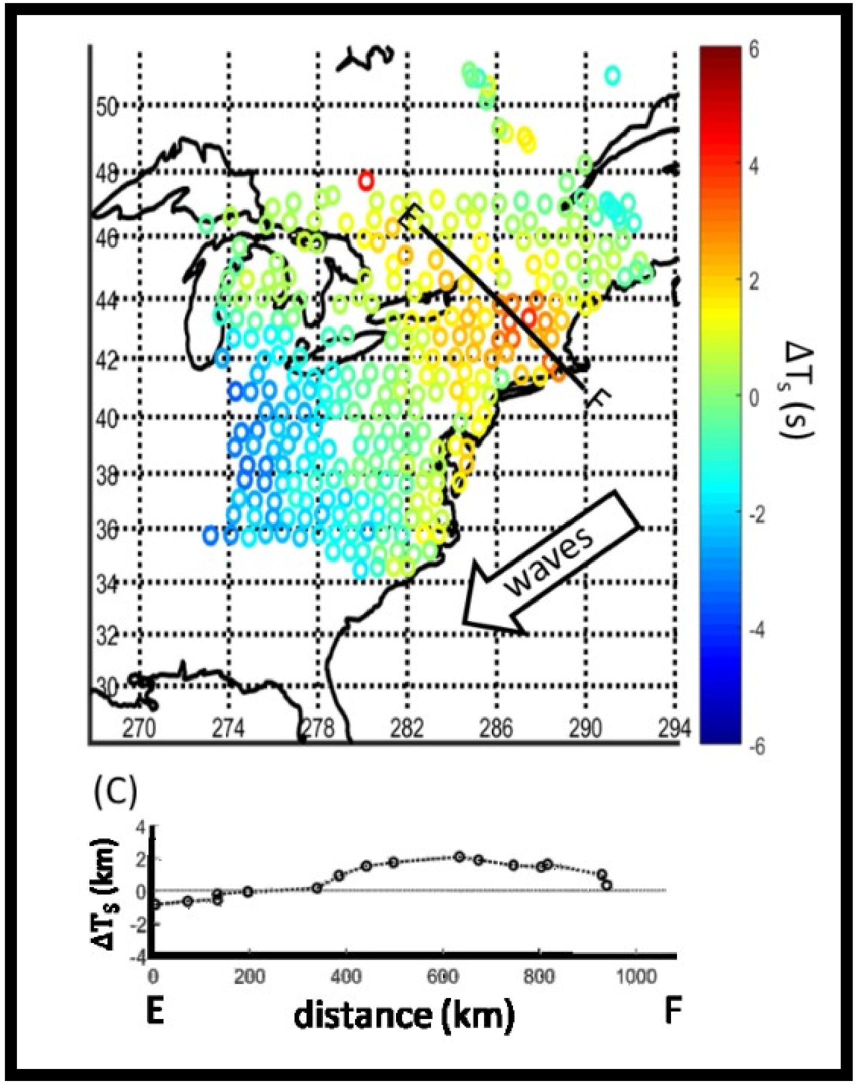
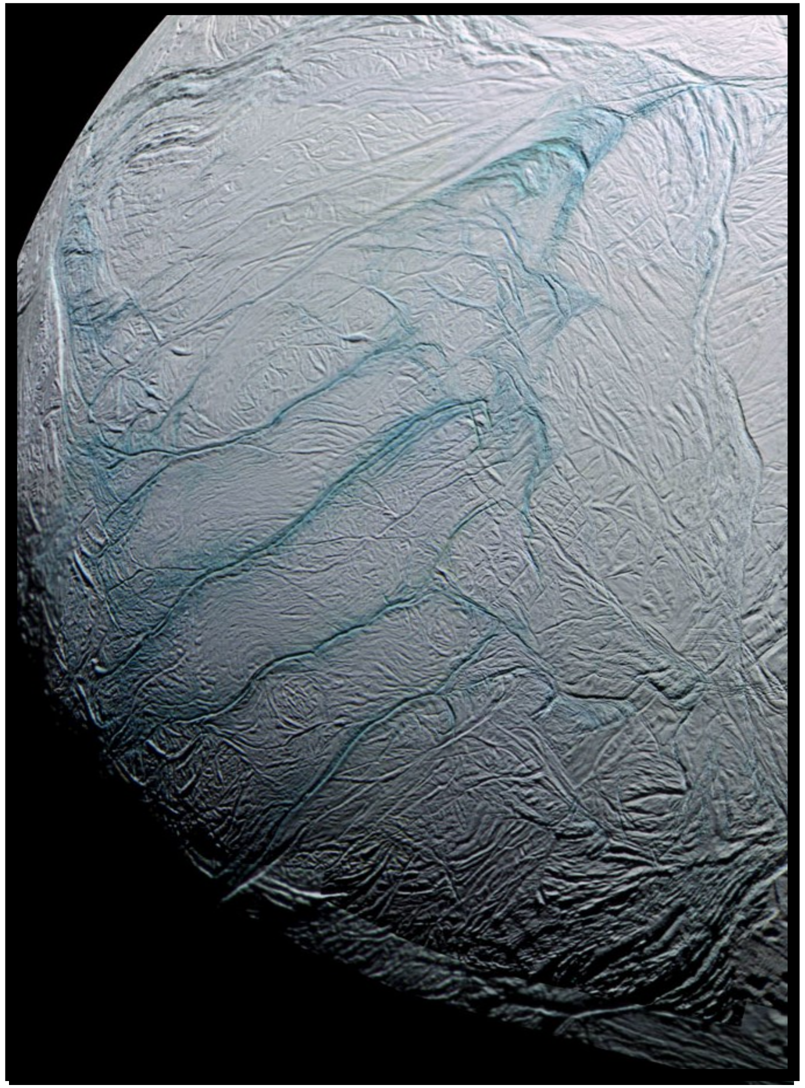
What We Study



Geo-Tomography

APPLIED TO THE EARTH'S UPPER MANTLE



Ice Physics

APPLIED TO ICY PLANETARY BODIES

Please Read My Policies On

grading
collaboration
disabilities
class absences
confidentiality

at

www.ldeo.columbia.edu/users/menke/gradingpolicy.html

grading

10 % Class Participation

30% HW

30% Midterm

30% Final (mostly focused on last half of semester)

Intellectual Goals

(1) Understand the fundamental principles of dynamics in an intuitive way

(2) Be able to apply the fundamental principles of dynamic to Solid Earth phenomena

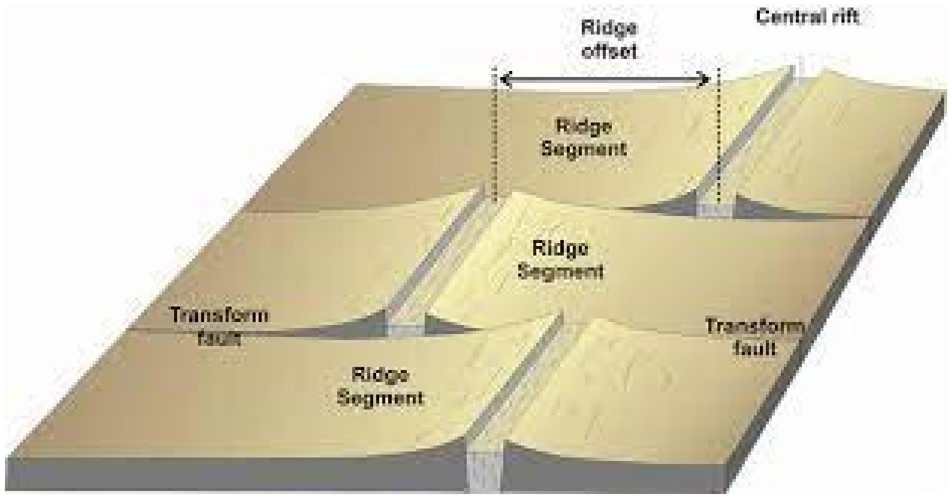
What Keywords Does “Dynamics” Suggest to You?

Here’s My “Dynamics” Keywords

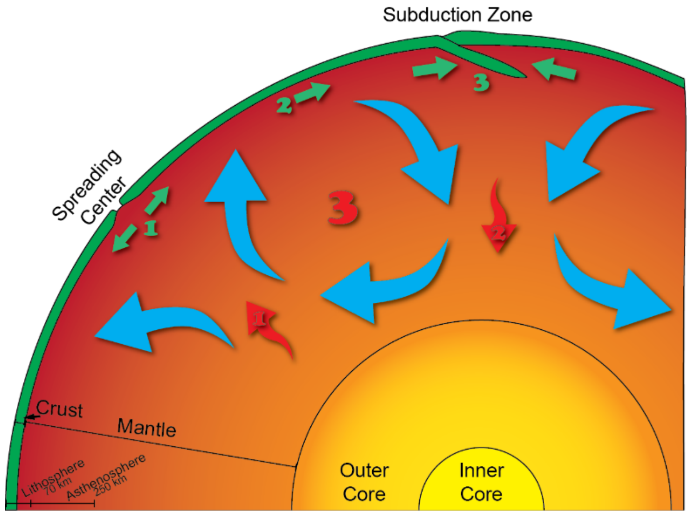
cause and effect
prediction
reservoirs
conservation (energy, mass, momentum)
fluxes
sources and sinks
feedbacks
directionality (vectors)
characteristic times
delays & advances

Subjects Covered

Heat flow



subsidence of the seafloor

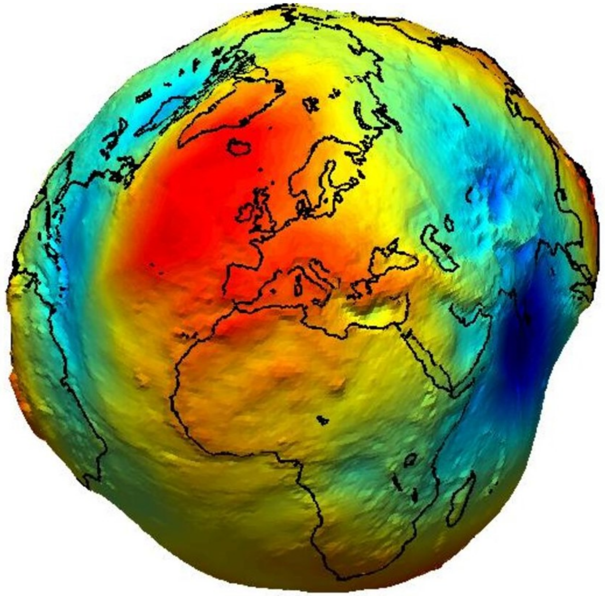


mantle convection

Subjects Covered

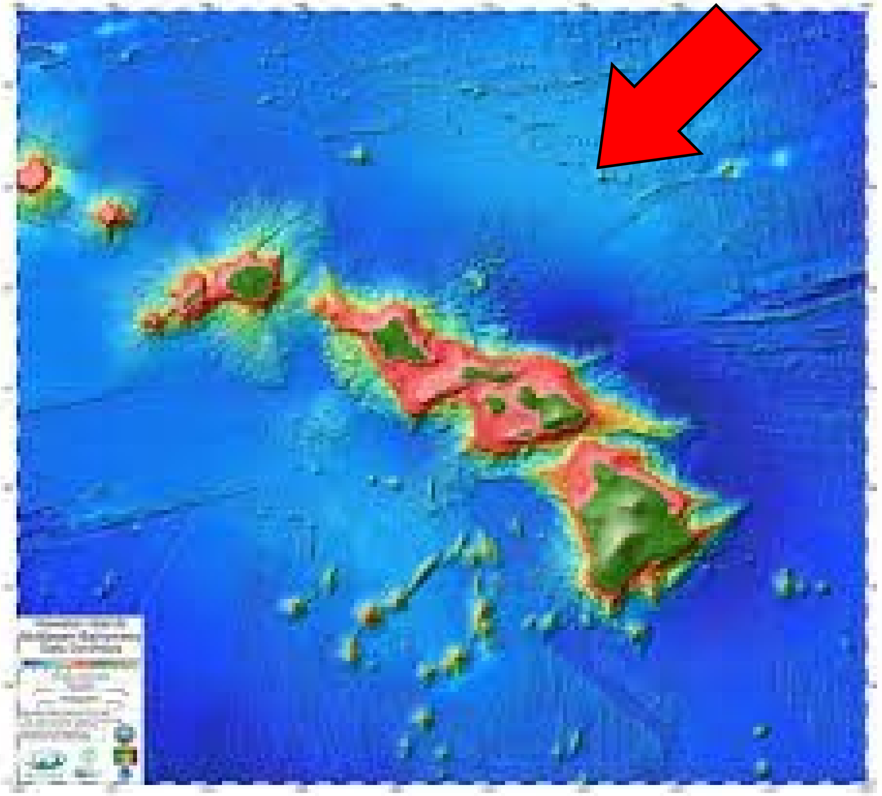
Heat flow
Gravity

orbit of
planets

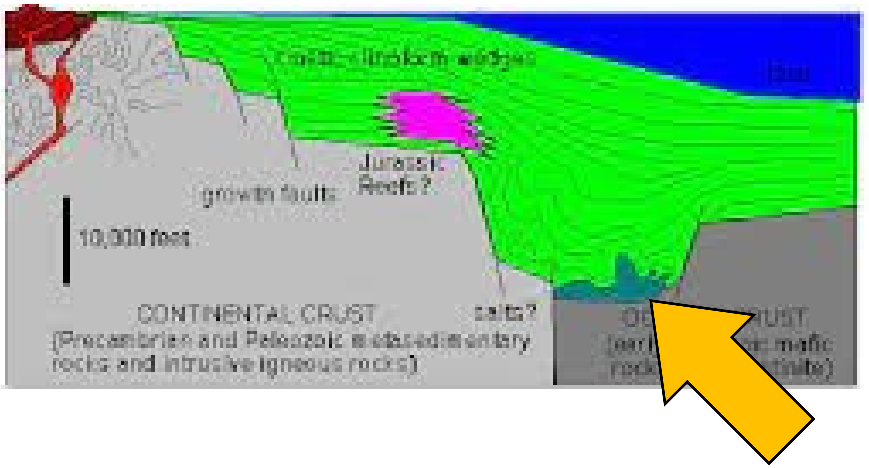


shape of
Earth

Heat flow
Gravity
Isostasy
Deformation



peripheral
bulge

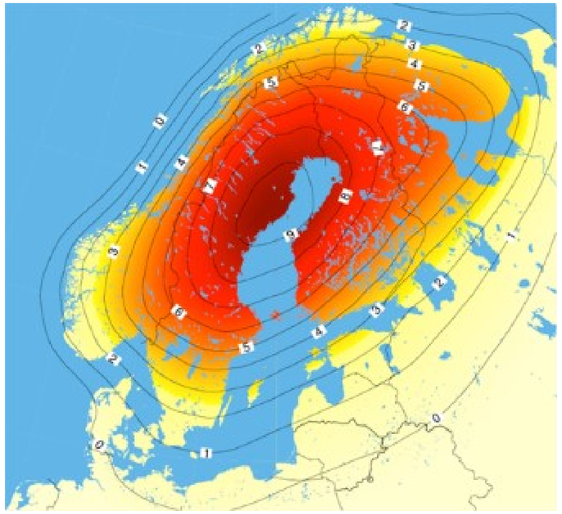


sediment
subsidence

Subjects Covered

Heat flow
Gravity
Isostasy
Deformation

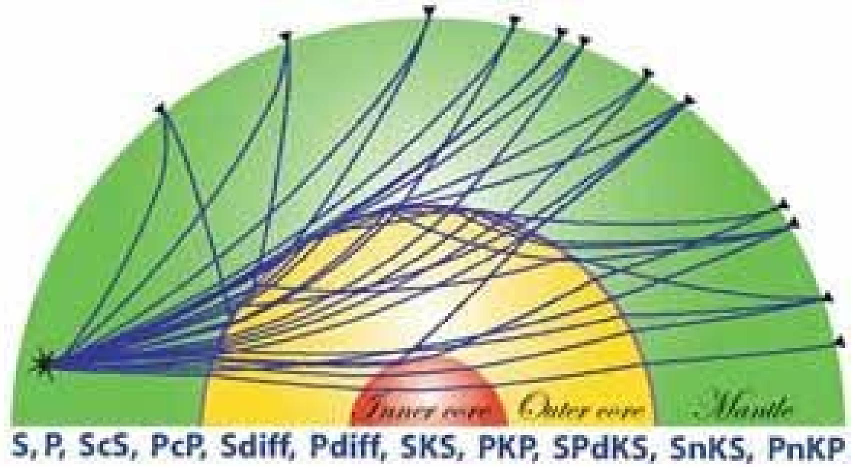
co-seismic subsidence



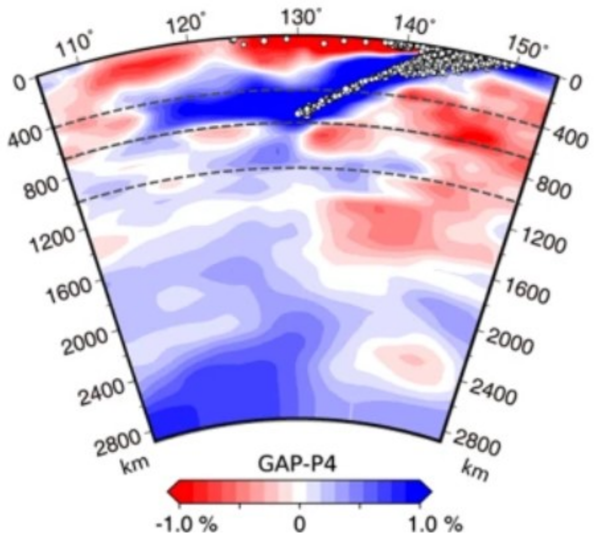
post-glacial uplift

Subjects Covered

Heat flow
Gravity
Isostasy
Deformation
Seismic Waves



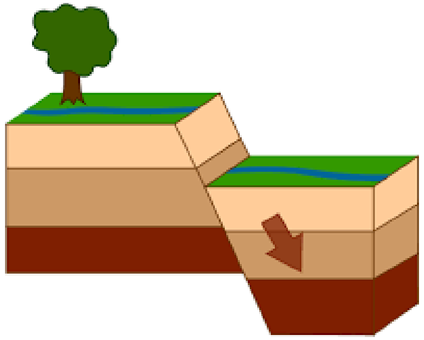
reflection
and
refraction



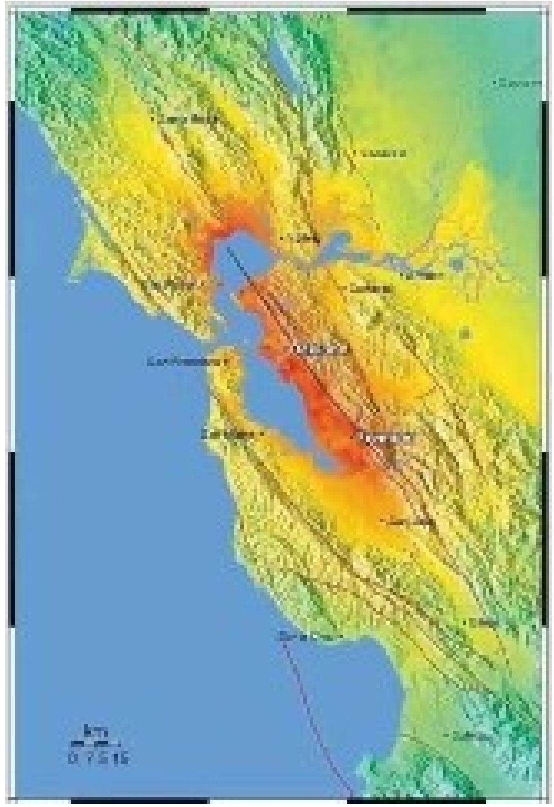
geotomography

Subjects Covered

Heat flow
Gravity
Isostasy
Deformation
Seismic Waves
Earthquakes



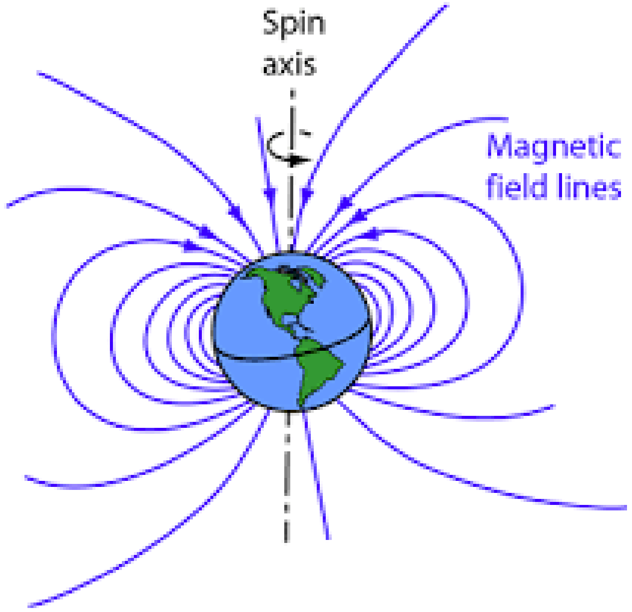
faulting



shaking

Subjects Covered

- Heat flow
- Gravity
- Isostasy
- Deformation
- Seismic Waves
- Earthquakes
- Geomagnetism



Earth's magnetic field

Magnetic reversals

Subjects Covered

Heat flow
Gravity
Isostasy
Deformation
Seismic Waves
Earthquakes
Geomagnetism
Glaciers

Glacial flow

Crevassing

The Typical Week

Reading Assignment
Tuesday – Thursday Lectures
Homework
Discussion Section

The Typical Week

Reading Assignment
Tuesday – Thursday Lectures
Homework
Discussion Section

required

optional but highly recommended

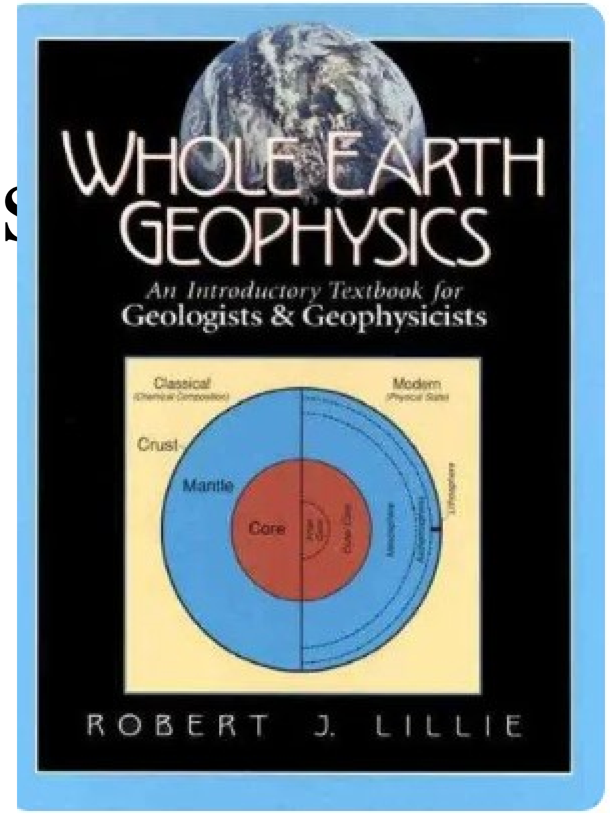
The Typical Week

Reading Assignment

Tuesday – Thursday Lectures

Homework

Discussion Section



The Typical Week

Reading Assignment } in Files

Tuesday – Thursday Lectures

Homework } In Assignments, PDF

None this week

Discussion Section } 2-3 important topics plus Q&A.

The Typical Week

Reading Assignment

Tuesday – Thursday Lectures

Homework

Discussion Section

Due 11:59 on Fridays

When to hold?

Today's Discussion

The water level is rising as I sit here fishing

Why Rivers?

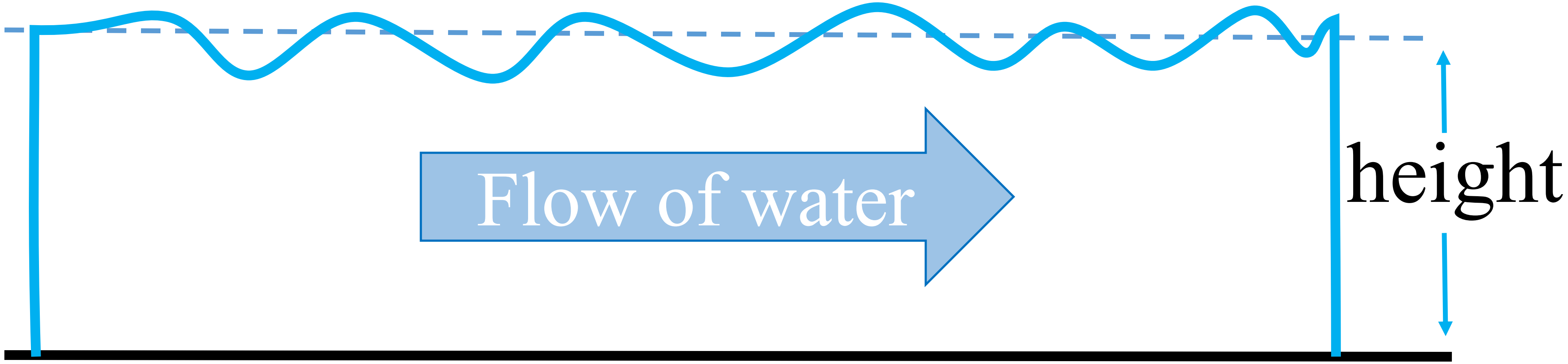
Athanasius Kircher,
Mundus Subterraneous, 1641

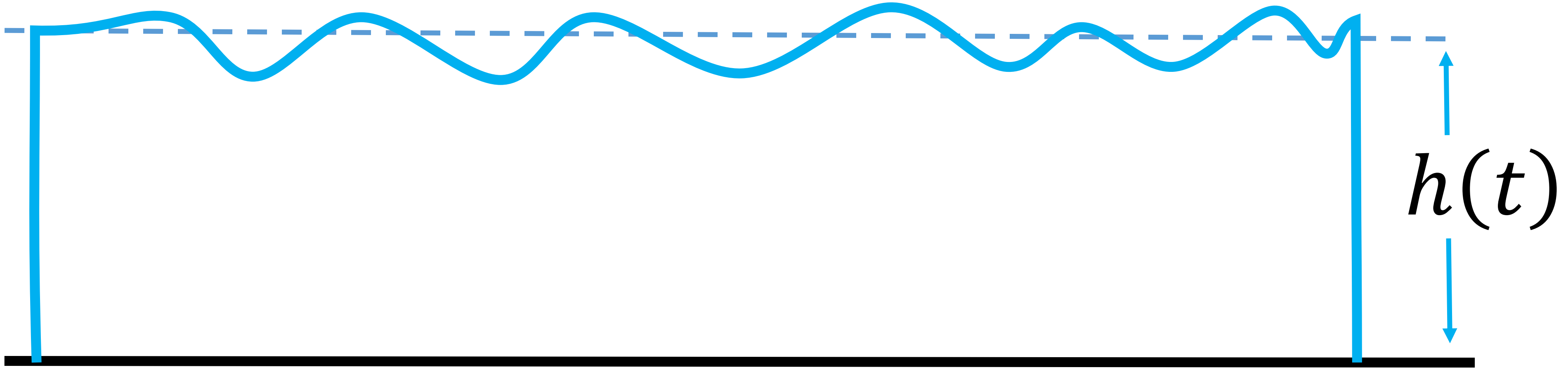
because the ancients got their dynamics all wrong!

Today's Discussion

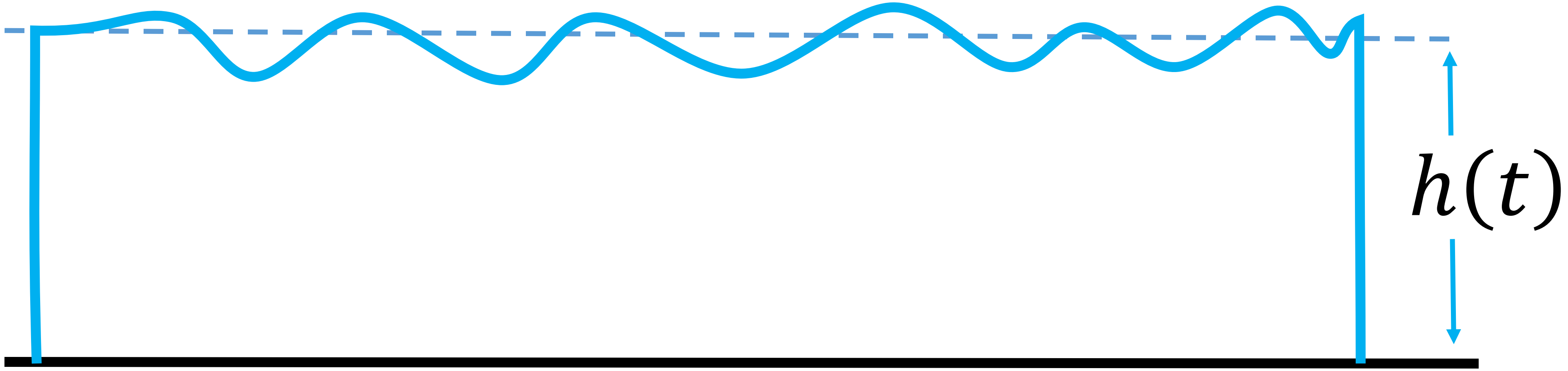
The water level is rising as I sit here fishing

Draw a “conceptual diagram” (very important!)

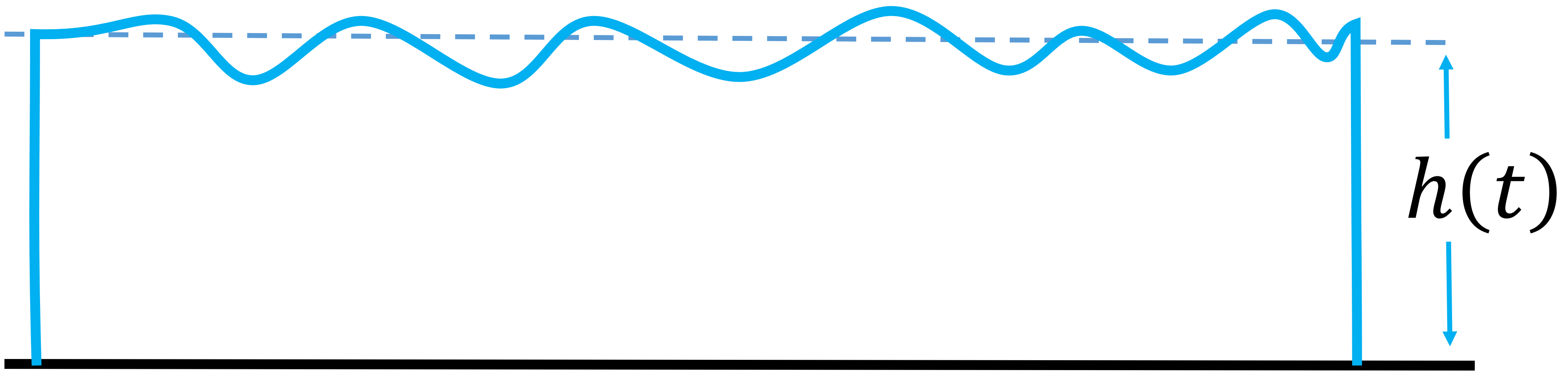




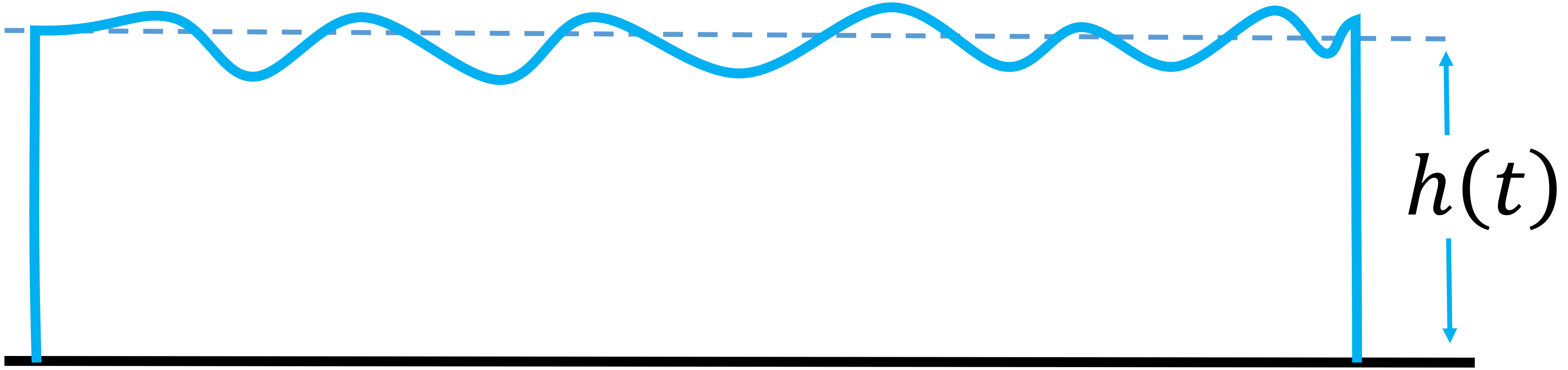
with time, t



What does “rising” mean in terms of $h(t)$?



$h(t)$ is increasing with time, t



which is bigger, $h(t)$ or $h(t+\Delta t)$?

where Δt is a small increment in time logical thinking steps

$h(t + \Delta t)$ bigger than $h(t)$

logical thinking steps

$h(t + \Delta t)$ bigger than $h(t)$

$h(t + \Delta t) - h(t)$ greater than zero

logical thinking steps

$h(t + \Delta t)$ bigger than $h(t)$

$h(t + \Delta t) - h(t)$ greater than zero

$$\frac{h(t + \Delta t) - h(t)}{\Delta t}$$ greater than zero

since dividing each by a positive amount doesn't change their relative sign

logical thinking steps

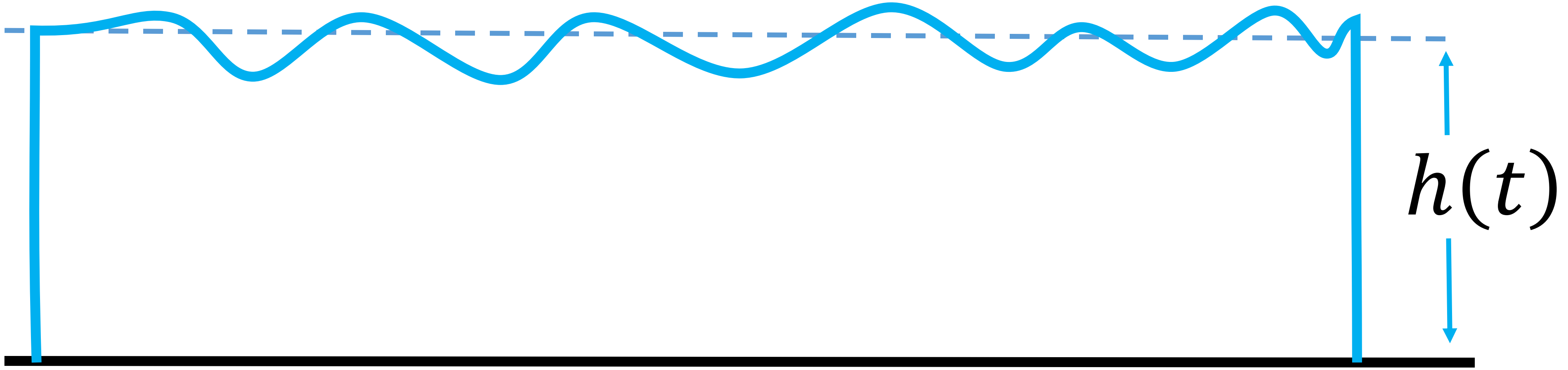
$h(t + \Delta t)$ bigger than $h(t)$

$h(t + \Delta t) - h(t)$ greater than zero

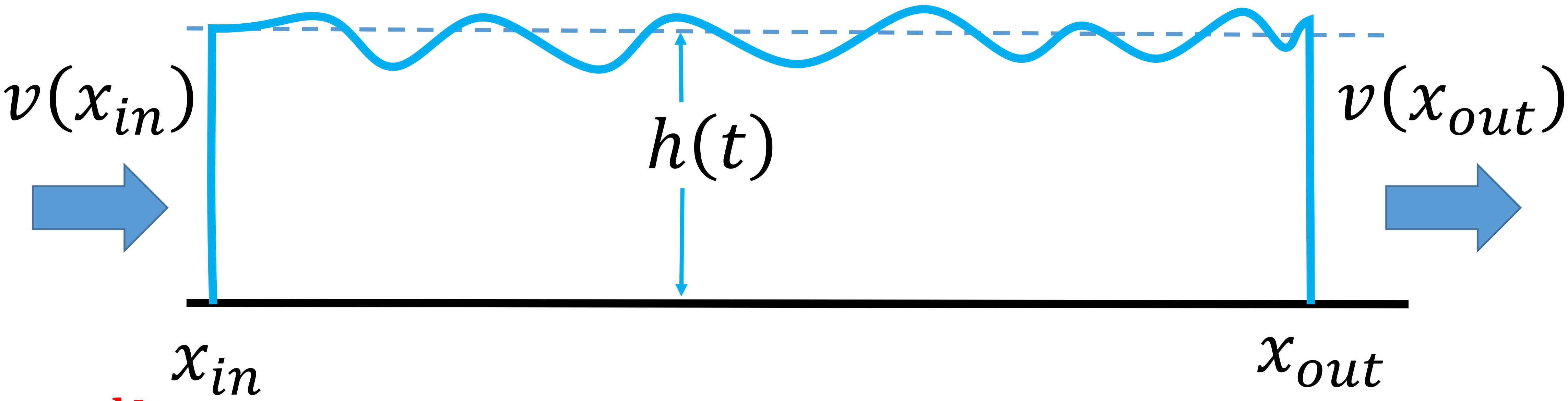
$\frac{h(t + \Delta t) - h(t)}{\Delta t}$ greater than zero

$$\frac{dh}{dt} > 0$$

Here's the critical question …

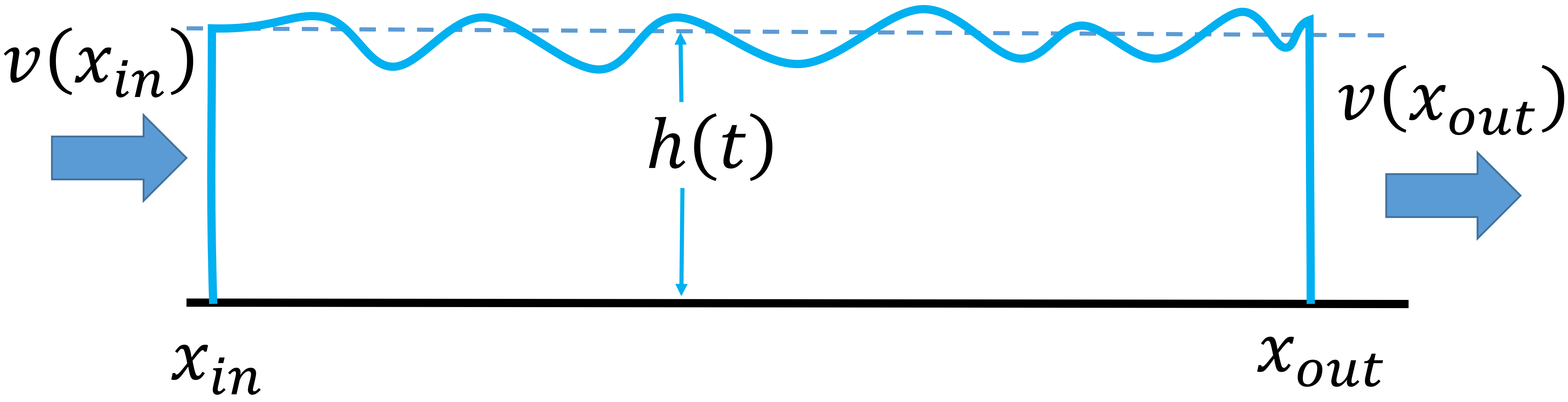


OK $\frac{\mathrm{d}h}{dt} > 0$ … why ?



For $\frac{dh}{dt} > 0$,

what must be relationship between $v(x_{in})$ and $v(x_{out})$?



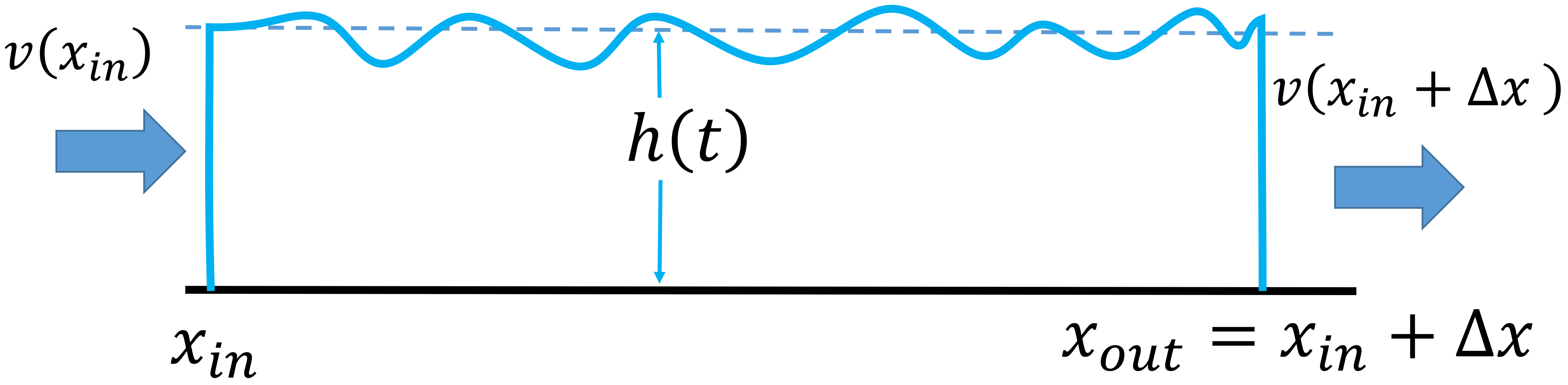
for $\frac{\mathrm{d}h}{dt} > 0$, $v(x_{in})$ must be bigger than $v(x_{out})$

What principle did we just invoke?

conservation of volume

=

conservation of mass at constant density



for $h(t) > 0$, $v(x_{in})$ must be bigger than $v(x_{in} + \Delta x)$

logical thinking steps

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) > v(x_{in} + \Delta x)$$

logical thinking steps

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) > v(x_{in} + \Delta x)$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) - v(x_{in} + \Delta x) > 0$$

logical thinking steps

$$\frac{dh}{dt} > 0 \qquad \text{when} \qquad v(x_{in}) > v(x_{in} + \Delta x)$$

$$\frac{dh}{dt} > 0 \qquad \text{when} \qquad v(x_{in}) - v(x_{in} + \Delta x) > 0$$

$$\frac{dh}{dt} > 0 \qquad \text{when} \qquad v(x_{in} + \Delta x) - v(x_{in}) < 0$$

flip direction of equality

logical thinking steps

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) > v(x_{in} + \Delta x)$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) - v(x_{in} + \Delta x) > 0$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in} + \Delta x) - v(x_{in}) < 0$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad \frac{v(x_{in} + \Delta x) - v(x_{in}) < 0}{\Delta x}$$

logical thinking steps

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) > v(x_{in} + \Delta x)$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in}) - v(x_{in} + \Delta x) > 0$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad v(x_{in} + \Delta x) - v(x_{in}) < 0$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad \frac{v(x_{in} + \Delta x) - v(x_{in}) < 0}{\Delta x}$$

$$\frac{dh}{dt} > 0 \quad \text{when} \quad \frac{dv}{dx} < 0$$

Now let’s apply

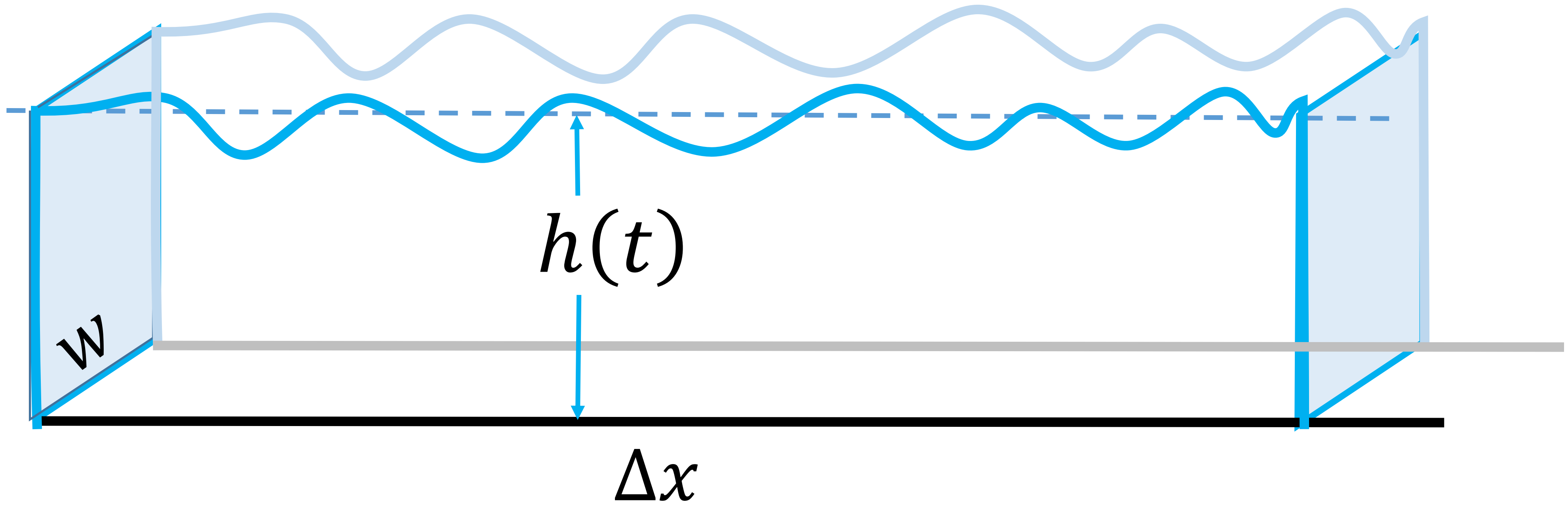
conservation of volume

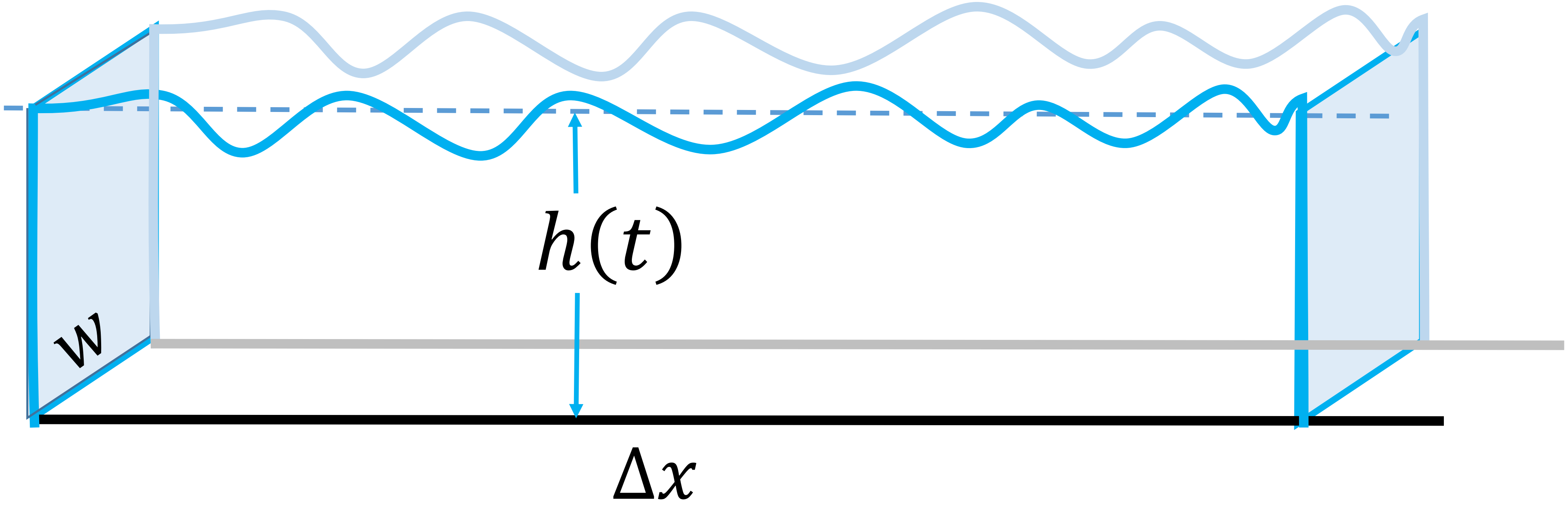
directly

Start by thinking about the amount of water stored in the stream

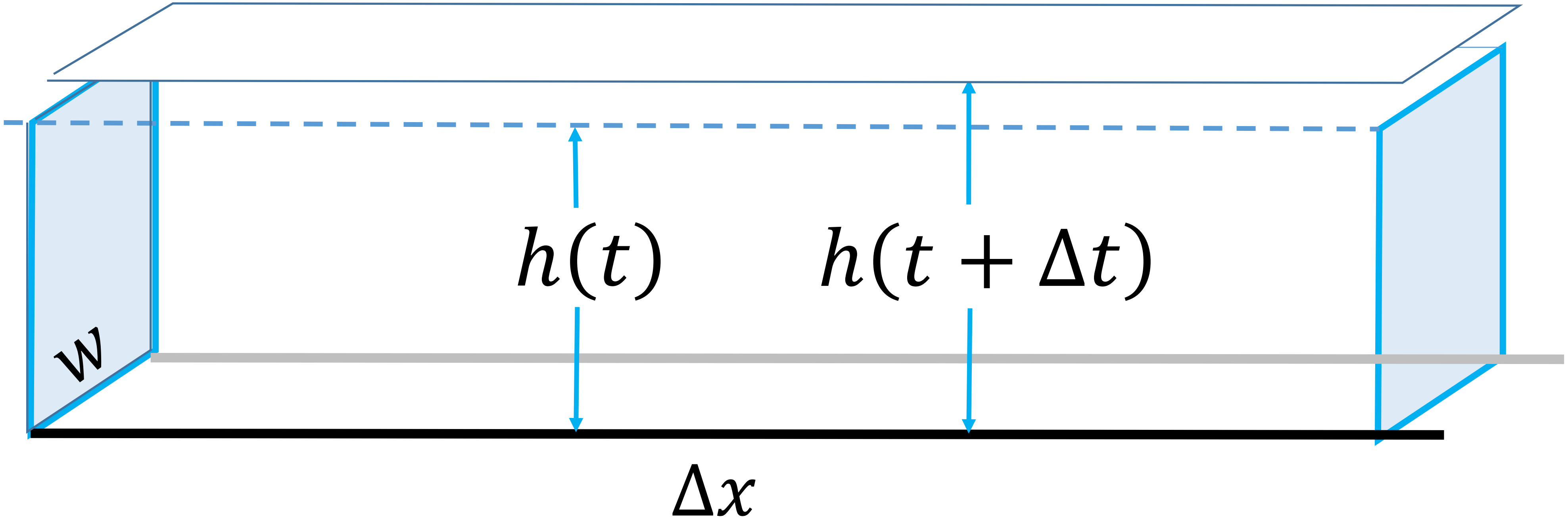
"the reservoir"

How much does the volume change when the height changes?



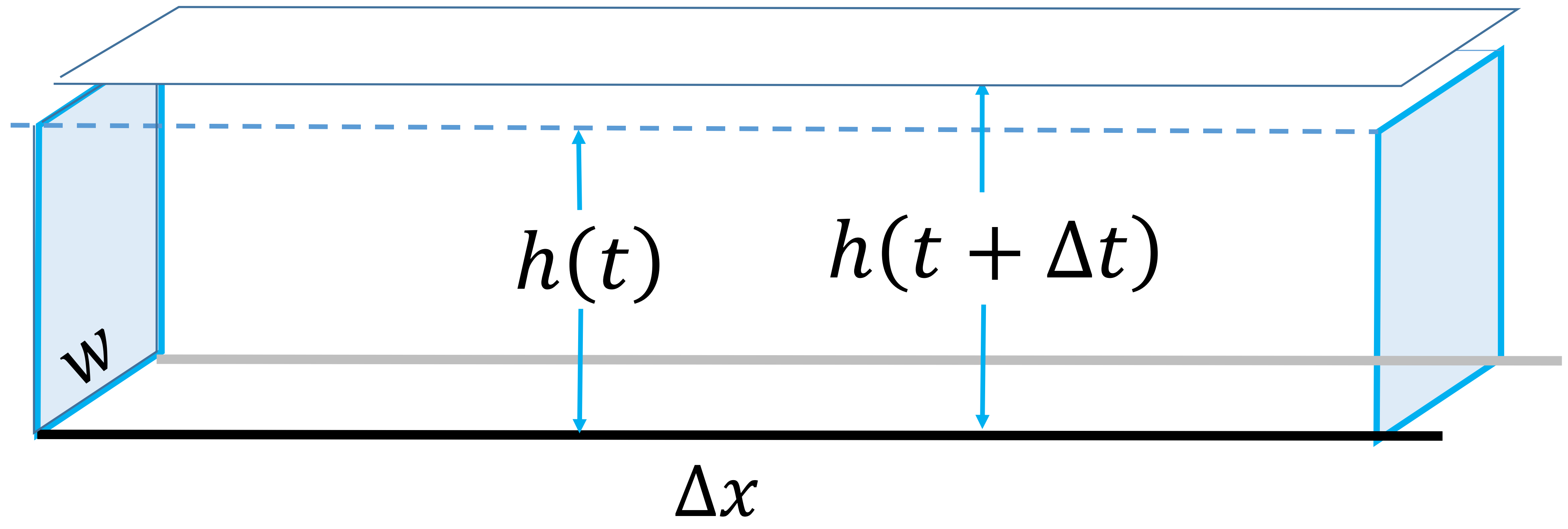


volume at time, t is $V(t) = w \;\Delta x\; h(t)$



volume at time, t is $\quad V(t) = w\ \Delta x\ h(t)$

volume at time, t is $\quad V(t+\Delta t) = w\ \Delta x\ h(t+\Delta t)$



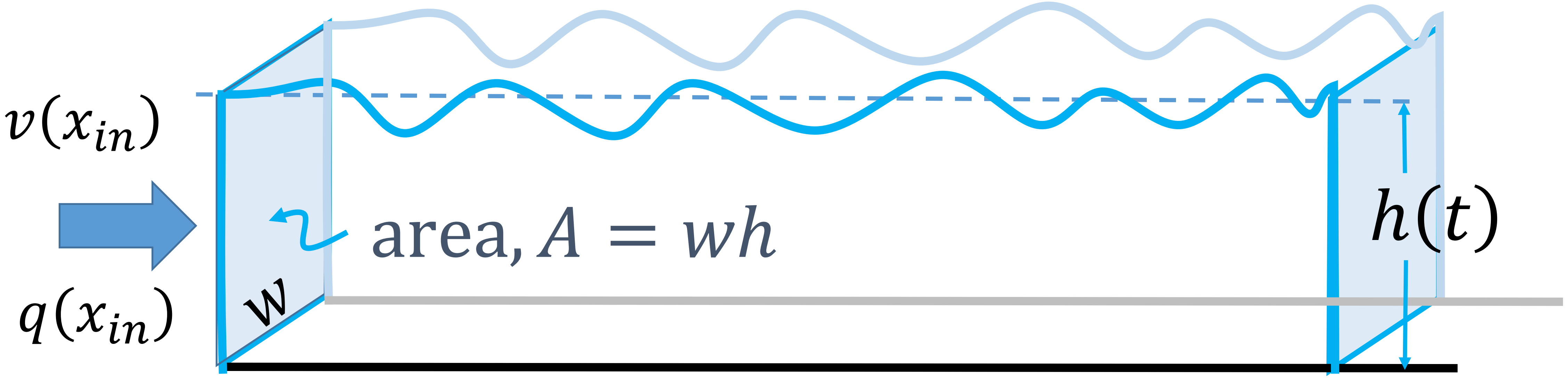
$$\left.\begin{array}{l} \text{volume at time, } t \text{ is} \quad V(t) = w \;\; \Delta x \;\; h(t) \\ \text{volume at time, } t \text{ is} \quad V(t+\Delta t) = w \;\; \Delta x \;\; h(t+\Delta t) \end{array}\right\} \begin{array}{l}\text{subtract and} \\ \text{divide by } \Delta t\end{array}$$

$$\frac{V(t+\Delta t) - V(t)}{\Delta t} = w \;\; \Delta x \frac{h(t+\Delta t) - h(t)}{\Delta t} \longrightarrow \frac{dV}{dt} = w \;\; \Delta x \frac{dh}{dt}$$

Then think about the amount of water entering and leaving the stream

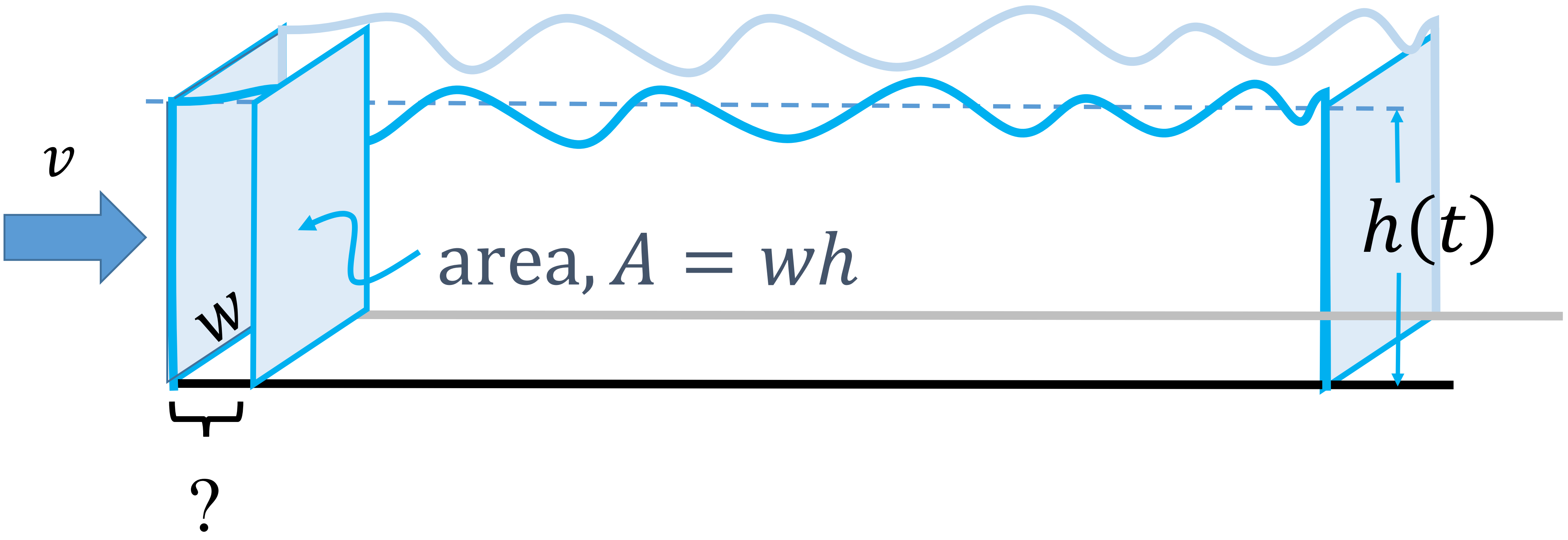
“the fluxes”

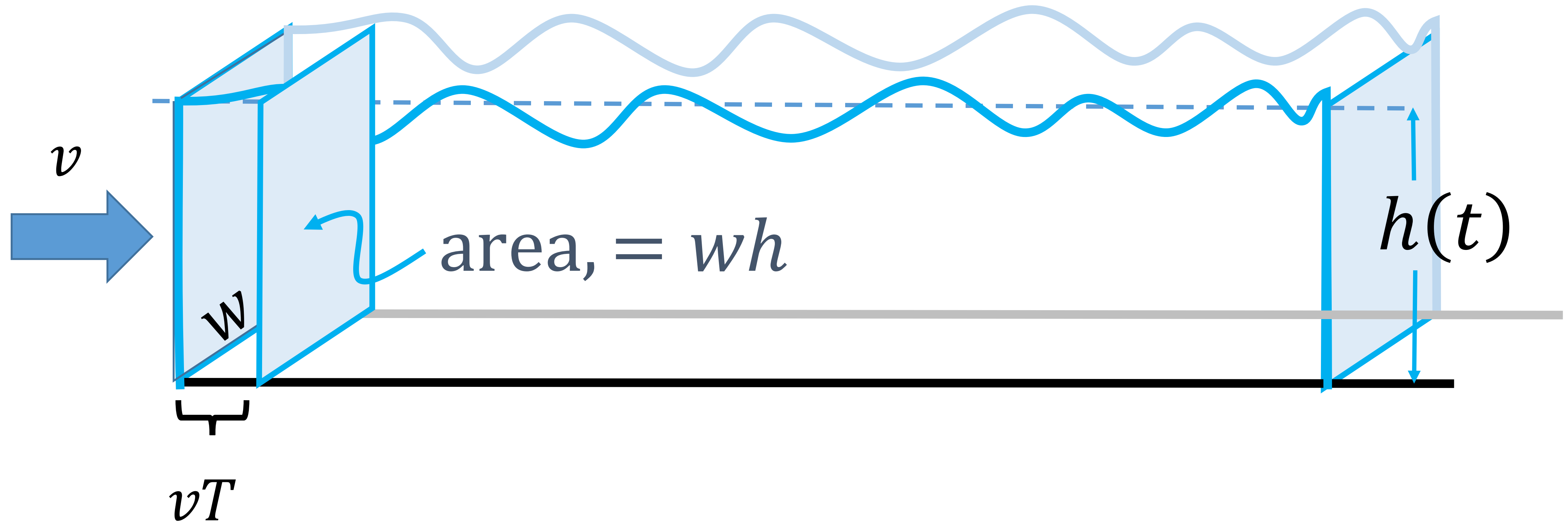
q = volume flux = volume per unit time



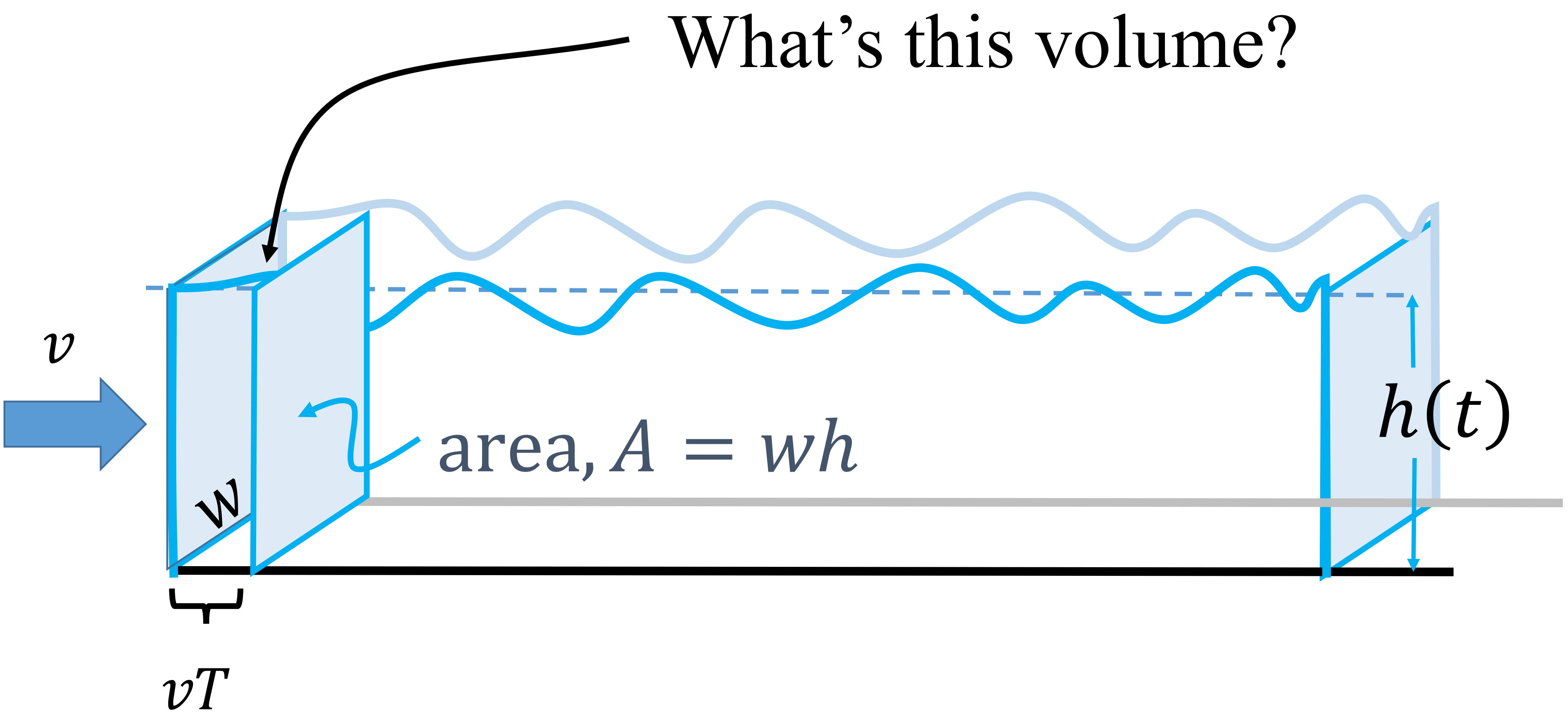
what is the relationship between $v(x_{in})$ and $q(x_{in})$?

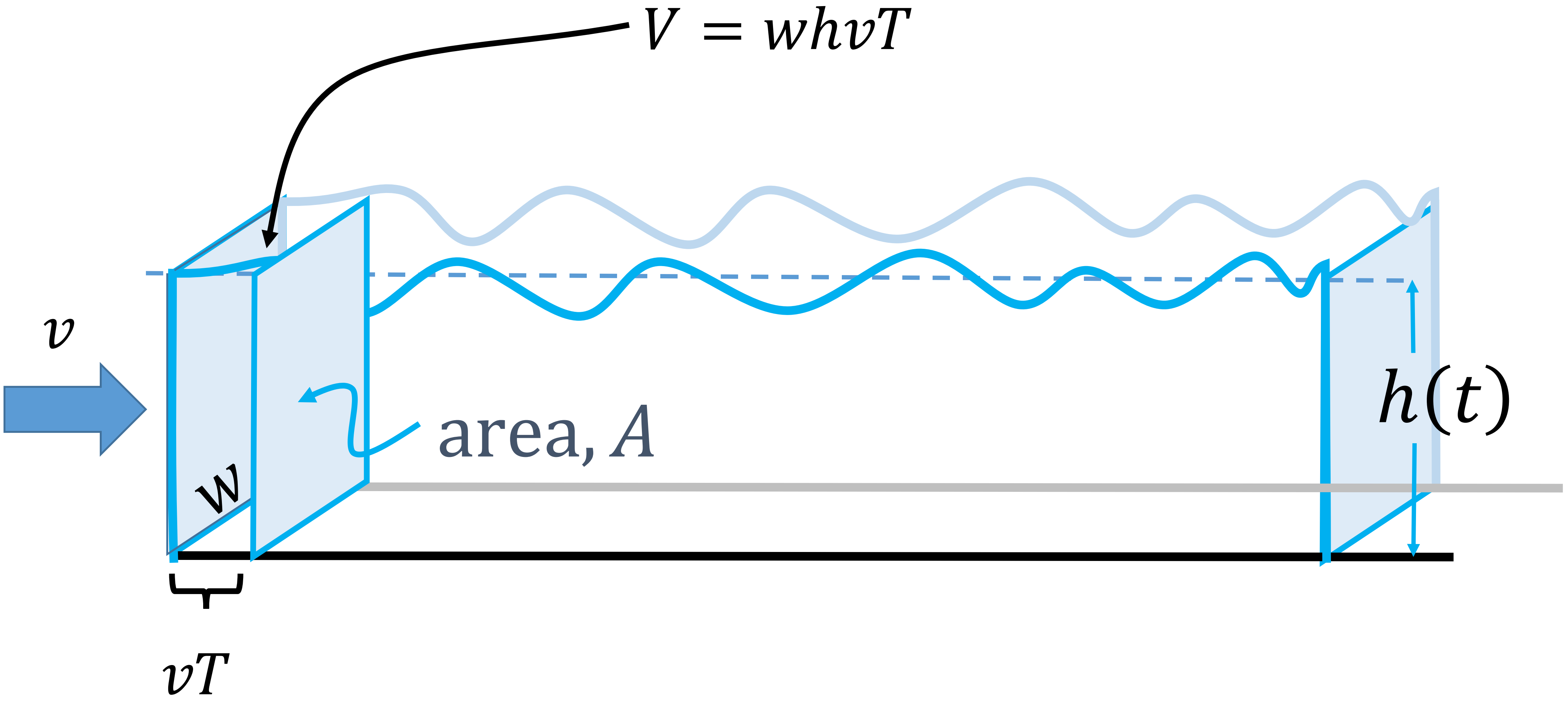
after time, T, water has moved ?



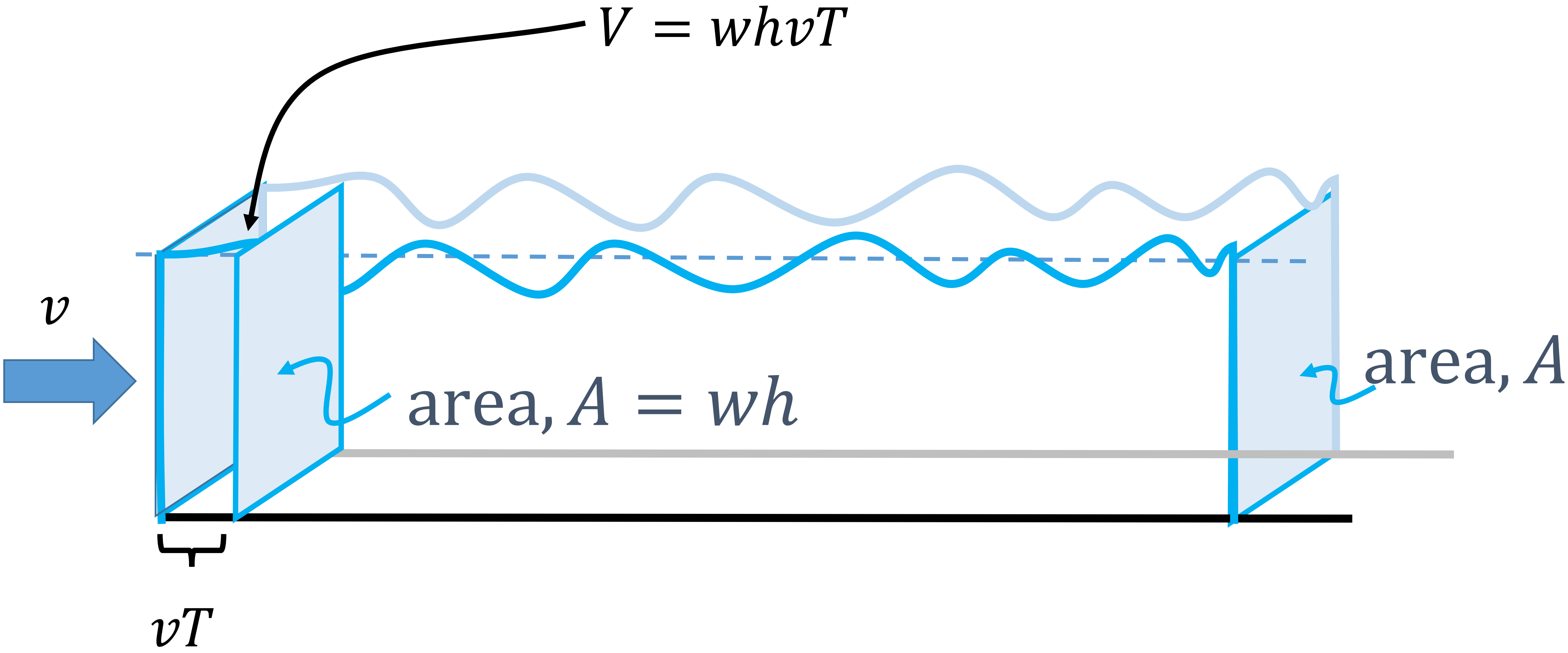


v
area, $= wh$
w
$h(t)$
vT


What's this volume?
v
w
area, $A = wh$
$h(t)$
vT



$V = whvT$
v
area, A
w
$h(t)$
vT



so the volume flux is, $q = \frac{V}{T} = whv$

Now apply “conservation of volume”

increase in volume with time
equals
flux of volume in
minus
flux of volume out

Now apply "conservation of volume"

$$\frac{dV}{dt} = q(x_{in}) - q(x_{in} + \Delta x)$$

Now apply "conservation of volume"

$$\frac{dV}{dt} = q(x_{in}) - q(x_{in} + \Delta x)$$

substitute in $\frac{dV}{dt} = w\Delta x \frac{dh}{dt}$ and $q = whv$

$$\frac{dV}{dt} = q(x_{in}) - q(x_{in} + \Delta x)$$

$$w\Delta x \frac{dh}{dt} = \mathrm{wh}\{v(x_{in}) - v(x_{in} + \Delta x)\}$$

rearrange

$$w\Delta x\,\frac{dh}{dt} = w\mathrm{h}\{v(x_{in}) - v(x_{in} + \Delta x)\}$$

$$\frac{1}{h}\frac{dh}{dt} = \left\{\frac{v(x_{in}) - v(x_{in} + \Delta x)}{\Delta x}\right\}$$

$$\frac{1}{h}\frac{dh}{dt} = -\frac{dv}{dx}$$

“conservation of volume”
in a river

$$\frac{1}{h}\frac{dh}{dt} = -\frac{dv}{dx}$$

fractional rate of change in height	=	negative of gradient in velocity

The river is rising

because the velocity is faster on the left than on the right