EESC UN3201
Solid Earth Dynamics
Spring 2023

Lecture 2

Heat flow:
Sources of heat, modes of heat transport

Harriman SP, New York

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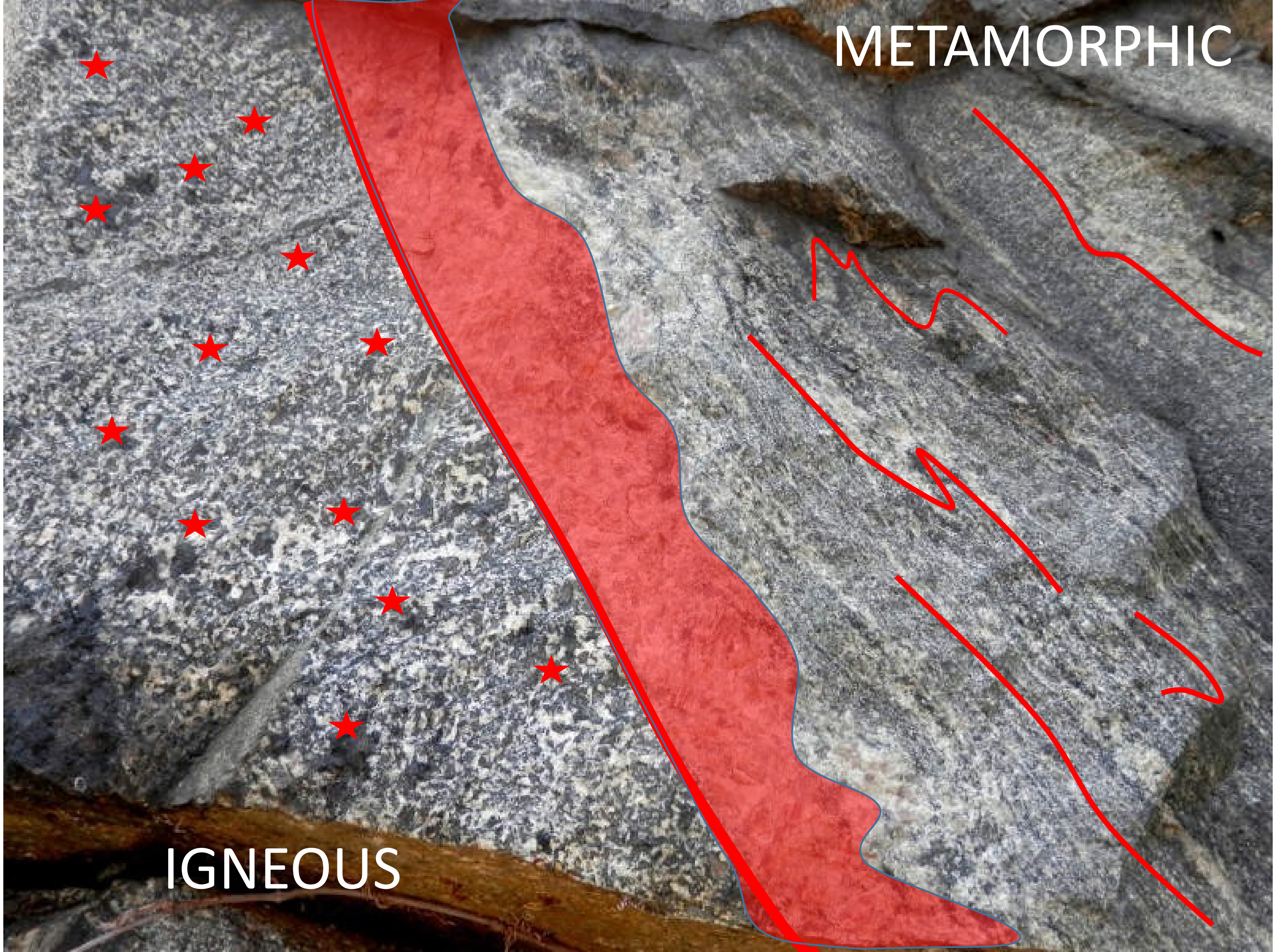
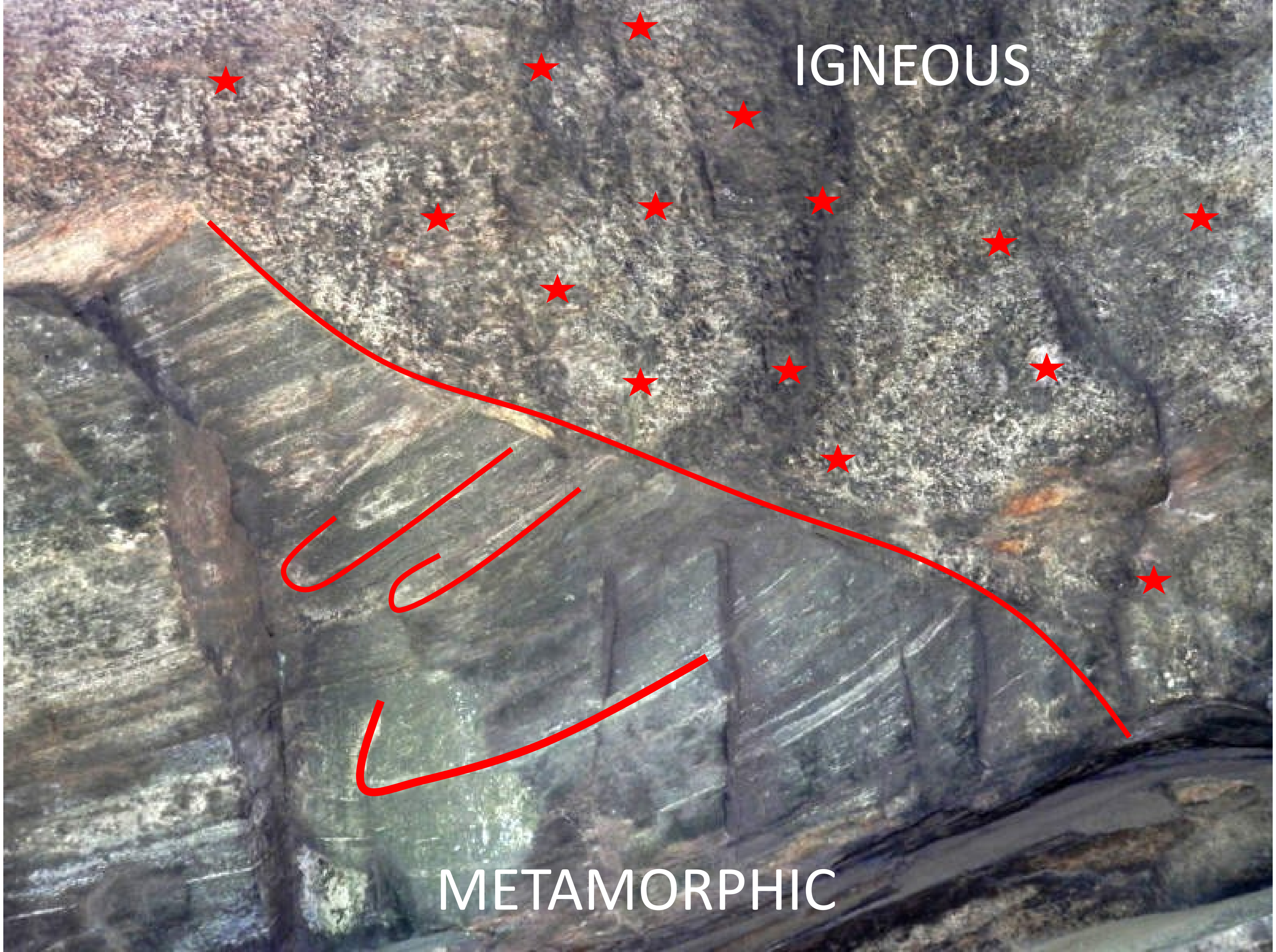


Harriman SP, New York

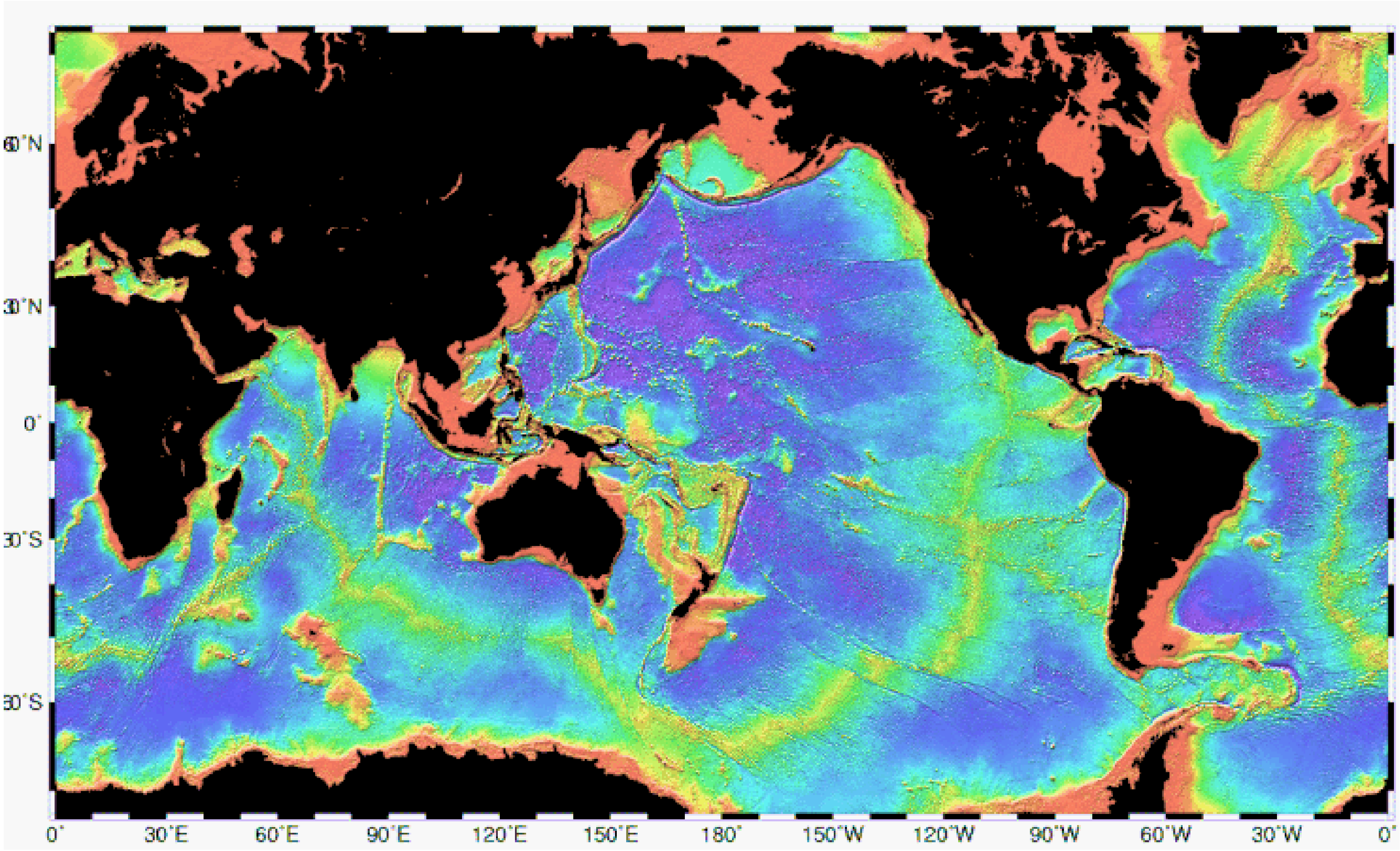


Strokkur geyser, Iceland

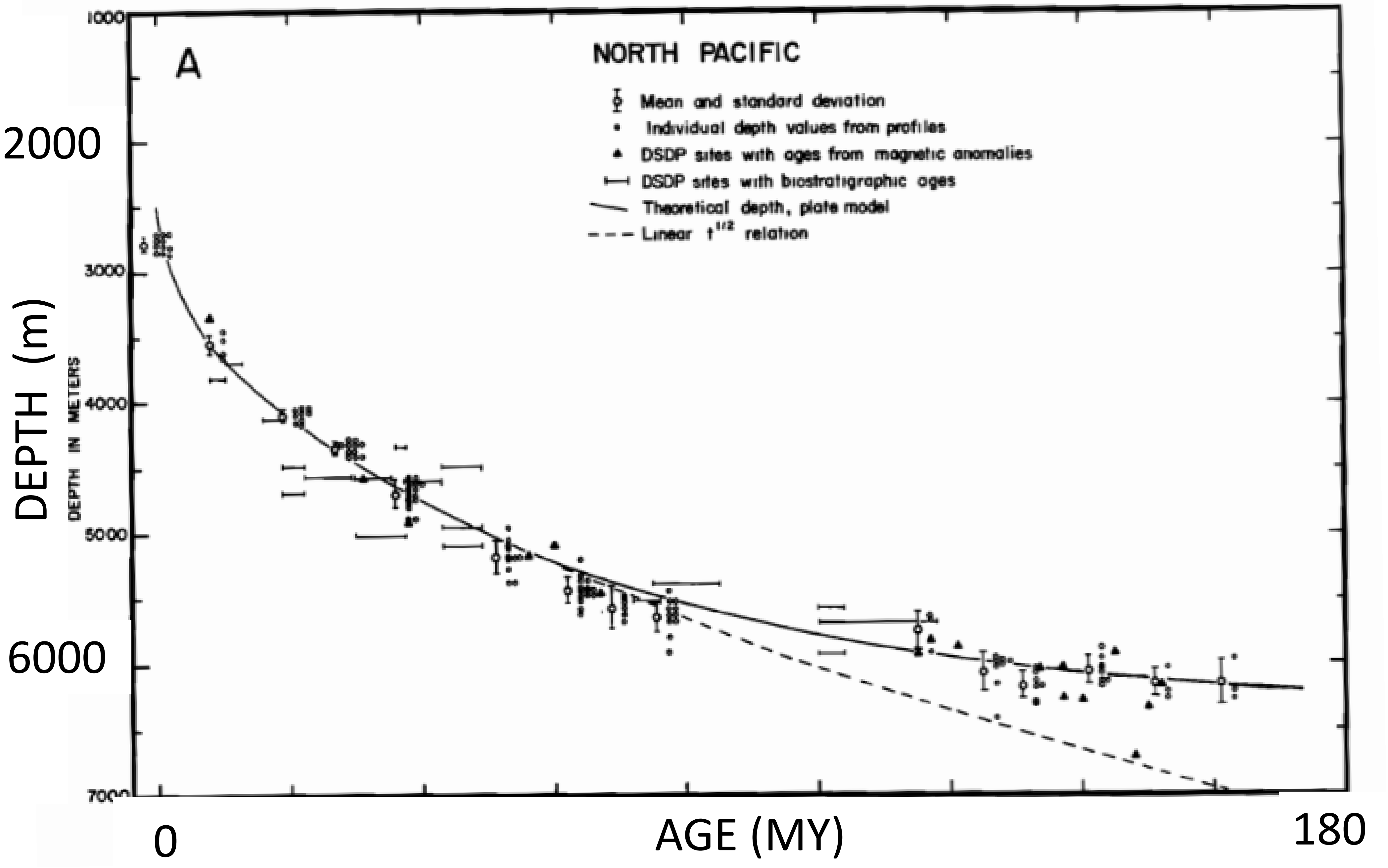

Hellishedi, Iceland

Pacaya, Guatemala

Fjallsjokull Iceland



60˚N
30˚N
0˚
30˚S
60˚S
0˚
30˚E
60˚E
90˚E
120˚E
150˚E
180˚
150˚W
120˚W
90˚W
60˚W
30˚W
0˚



A
NORTH PACIFIC
Mean and standard deviation
Individual depth values from profiles
DSDP sites with ages from magnetic anomalies
DSDP sites with biostratigraphic ages
Theoretical depth, plate model
Linear t^1/2 relation
DEPTH (m)
DEPTH IN METERS
1000
2000
3000
4000
5000
6000
7000
AGE (MY)
0
180

Question?

How much
Heat Energy
is In the Rock?

Question?

Hard and
not-very-useful
question
because it involves
thinking about
absolute zero
temperature

Better Question?

How much Heat Energy is released from the rock as it cools from a daytime temperature (say 25 C) to a nighttime temperature (say 15 C)

Relative temperature:

$$\Delta T$$

Temperature measured with respect to some "normal reference temperature" not with respect to absolute zero

We'll use 0 °C.

Heat released

Change in temperature

times

mass of rock

times

heat capacity

Heat released ΔQ

Change in temperature ΔT

times

mass of rock $\rho\, V$

times

heat capacity c_p

$\rho = \text{density}$
$V = \text{volume}$

Heat released

$$\Delta Q = \rho c_p V \Delta T$$

$$\rho = 2500 \ \text{kg/m}^3$$

c_p = 790 (J/kg °C)

V = 1 (m^3)

$$\Delta T = 10$$

$$\Delta Q = \rho c_p V \Delta T = 2 \times 10^7 \text{ J}$$

Heat released

$$\Delta Q = \rho c_p V \Delta T$$

1 Joule
= 0.000239 kCal

$\rho = 2500 \ \ \mathrm{kg/m^3}$

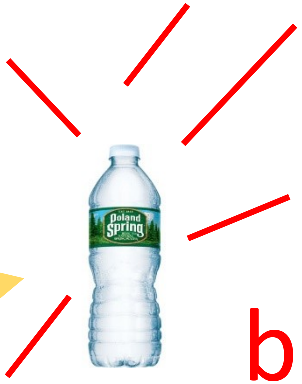
c_p = 790 (J/kg °C)

V = 1 (m^3)

$\Delta T = 10$

$$\Delta Q = \rho c_p V \Delta T = 2 \times 10^7 \ \mathrm{J} \qquad = 4700 \ \ \mathrm{kCal}$$

kCal – heat energy needed to raise temperature of 1 liter of water by 1 deg C

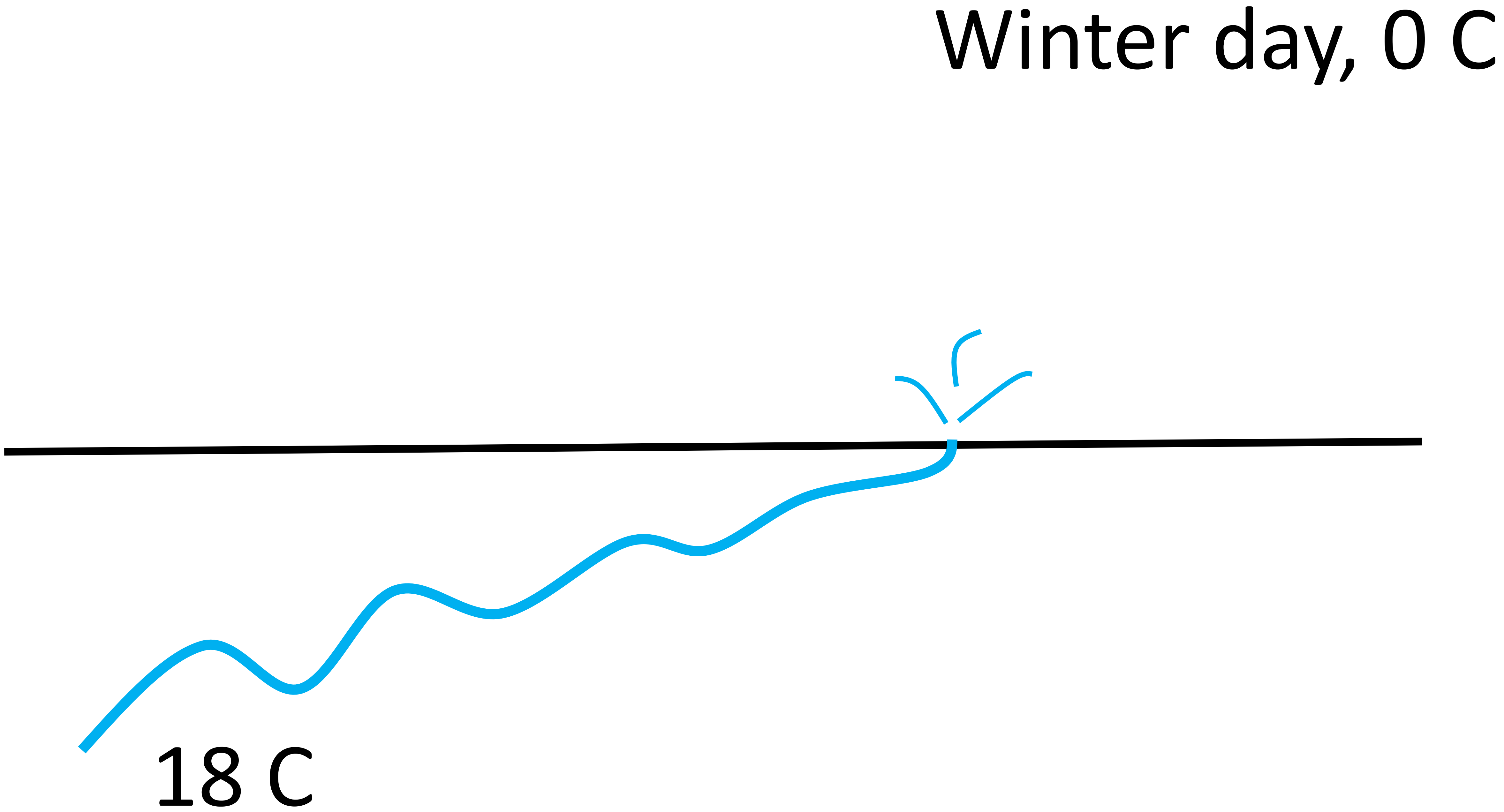


bang!

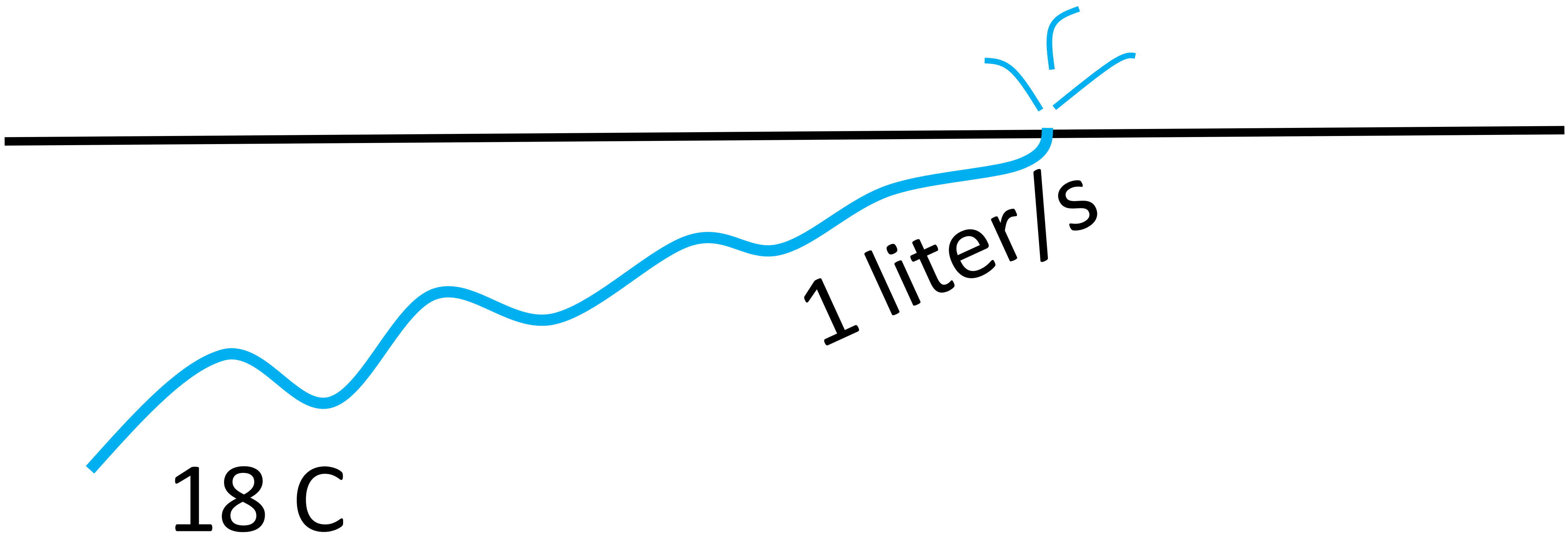
Tempid Spring

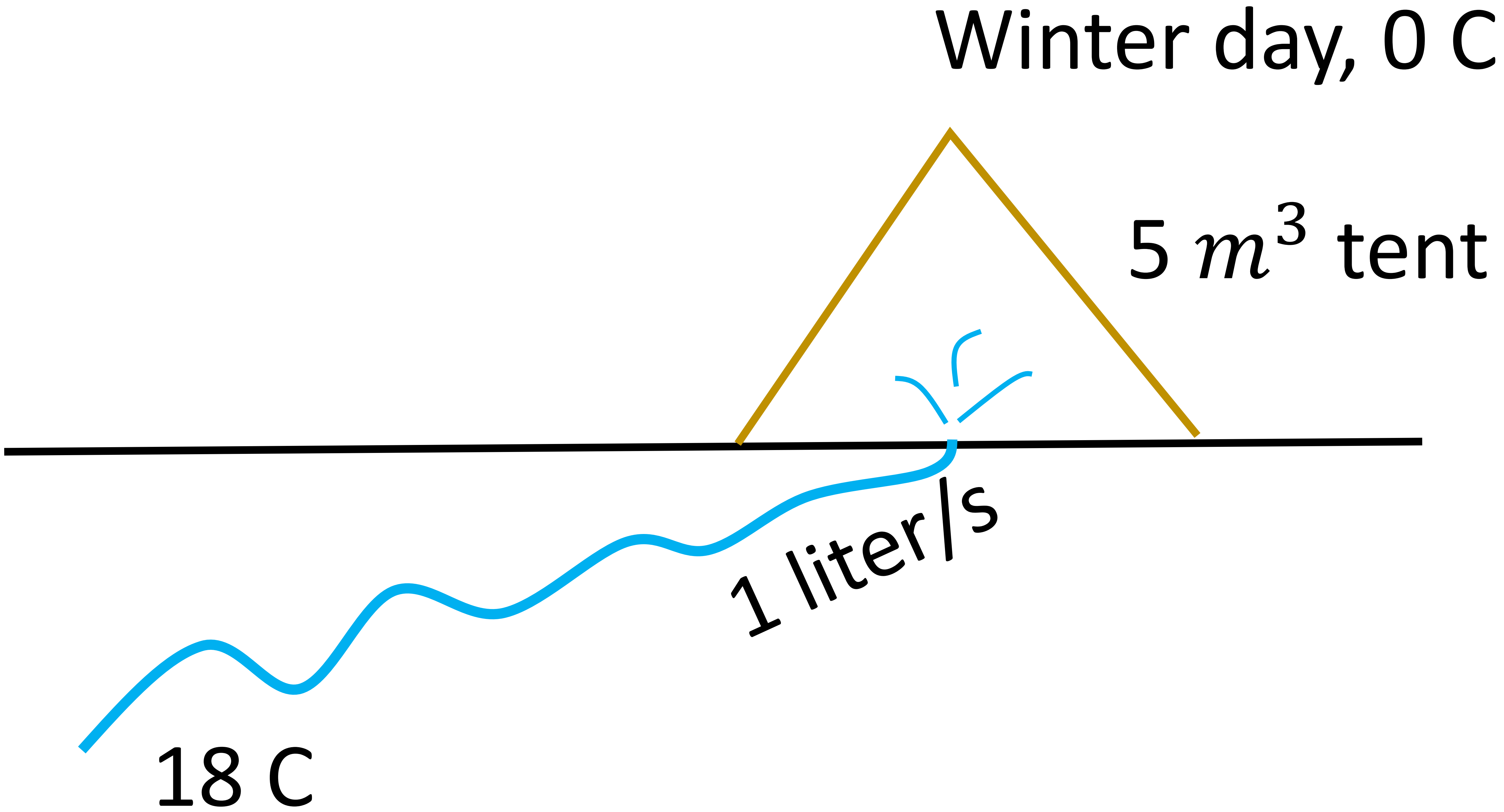
Williamstown, Massachosetts

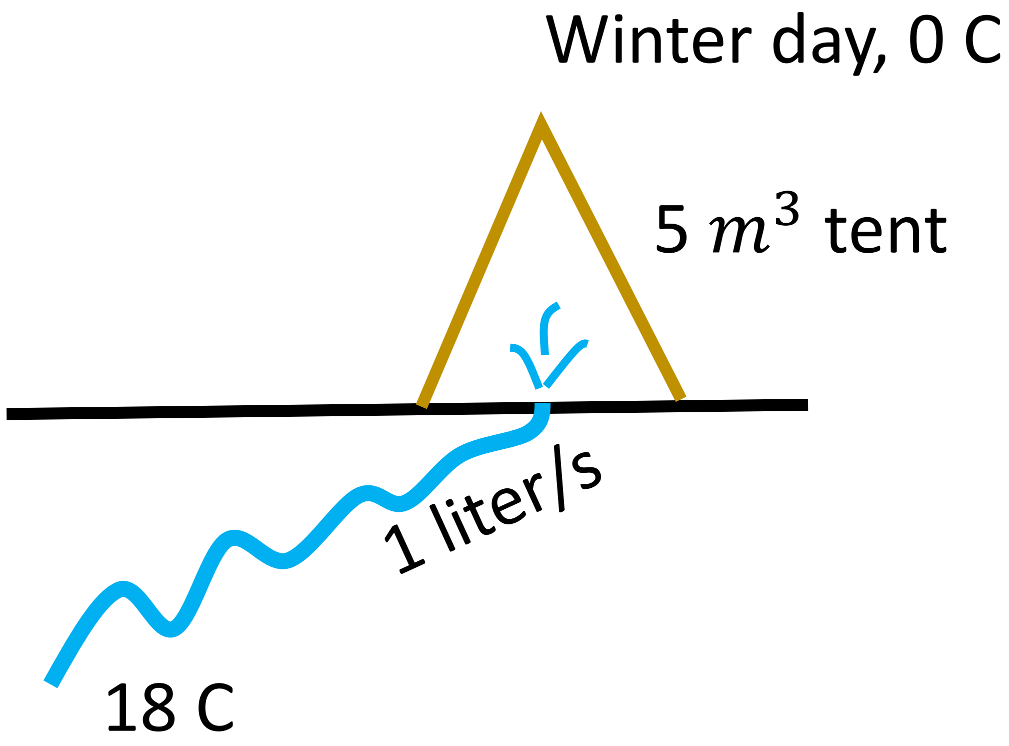
18C


Winter day, 0 C
18 C

Winter day, 0 C




Winter day, 0 C
5 m^3 tent
1 liter/s
18 C



How long does it take for the spring to warm up the tent by 10 C ?

air:

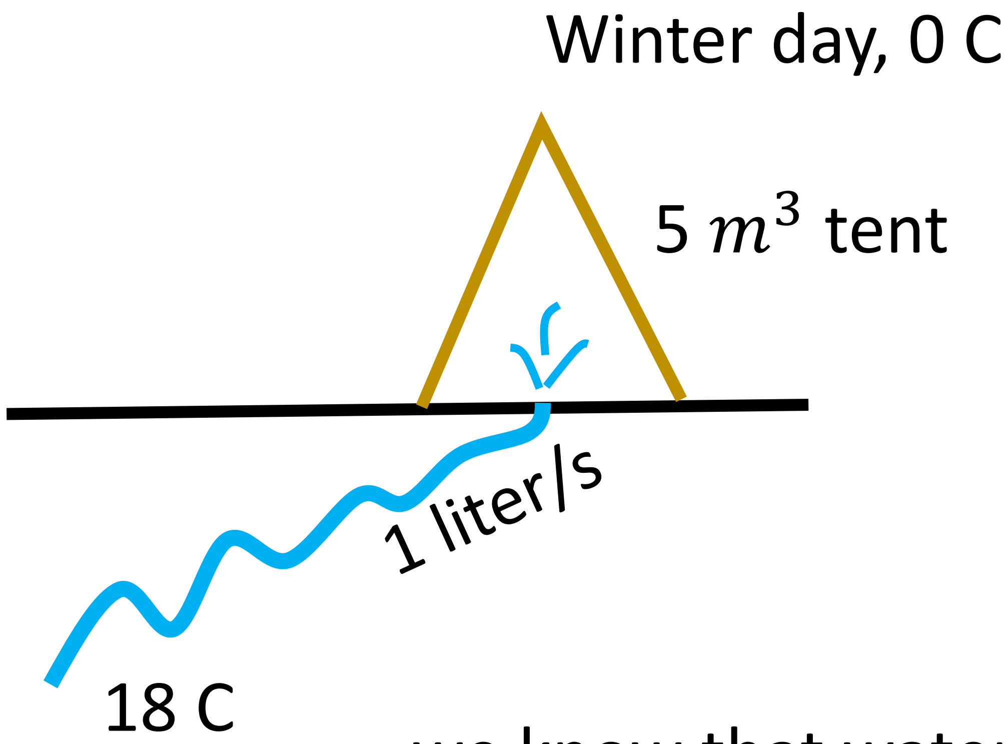
c_p 700 J kg/C

ρ 1.3 kg/m^3

$V = 5\text{m}^3$

$\Delta T = 10\ C$

$$\Delta Q = \rho c_p V \Delta T = 45500\ J$$
$$= 10\ \text{kCal}$$



How long does it take for the spring to warm up the tent by 10 C ?

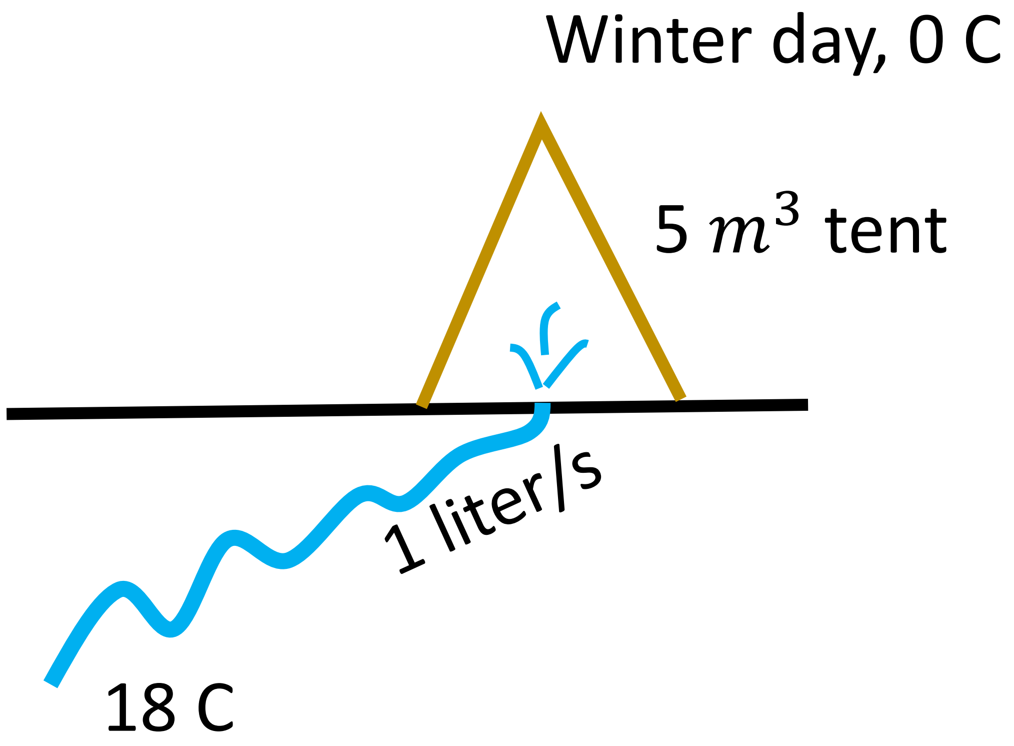
we know that water stores 1 kCal of heat per liter per degree C

spring:

$\Delta T = 8\ °C$

flux of heat

q = ΔQ per second = $8\ \mathrm{kCal}/s$



How long does it take for the spring to warm up the tent by 10 C ?

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$$\Delta Q = \rho c_p V \Delta T = 45500\ J$$
$$= 10\ \text{kCal}$$

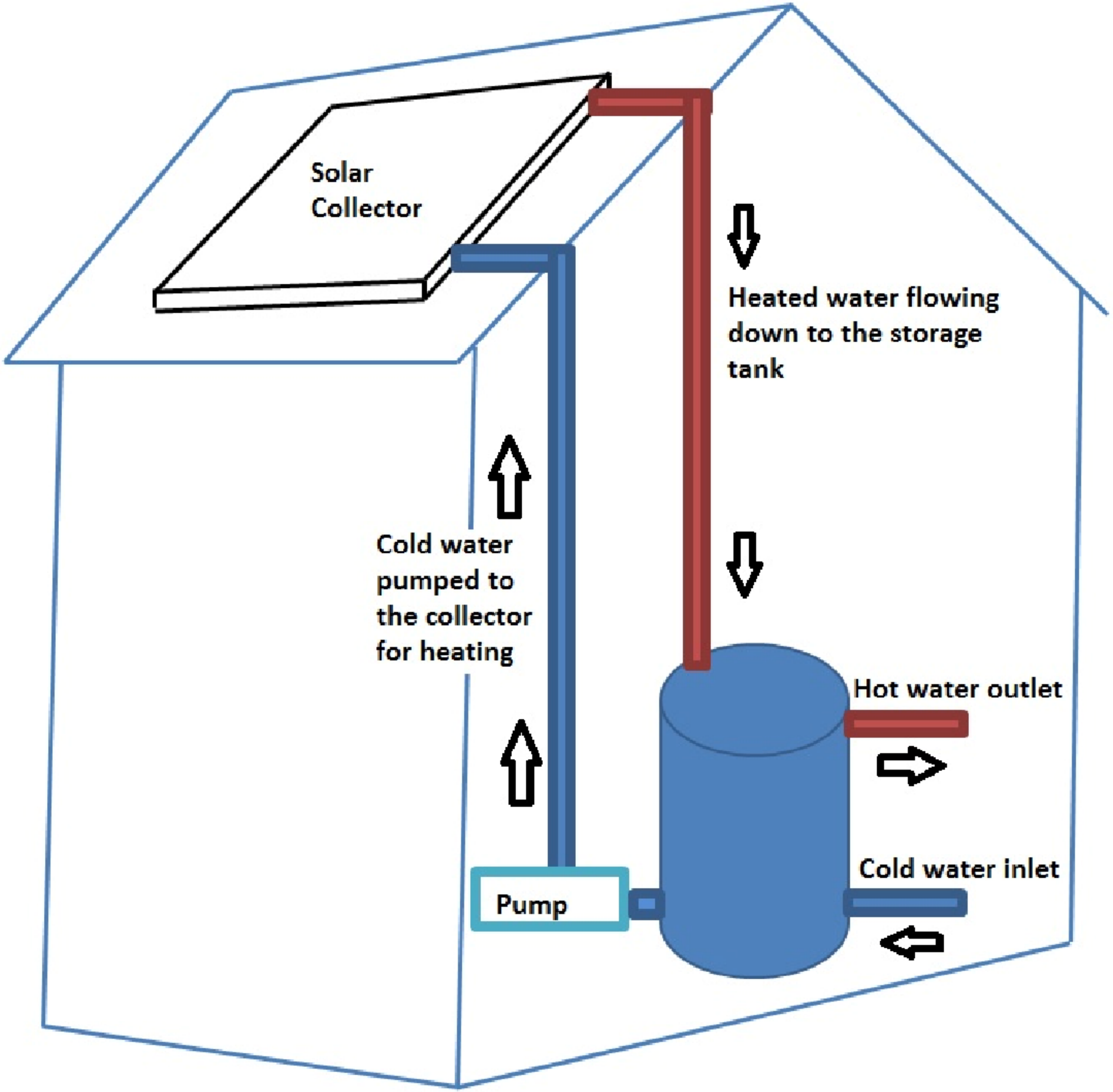
spring:

$\Delta T = 8\ ^\circ\text{C}$

flux of heat

q = ΔQ per second = 8 kCal/s

$$t = \Delta Q / q = 1.25\ \text{s}$$



you can see why water is frequently used to transfer heat “advection”

Moving heat energy by moving hot material characteristic time

Quantity divided by flux

$$t = \Delta Q / q$$

“conduction”

heat flow from ______________ to ___________

(without the material moving)

“conduction”

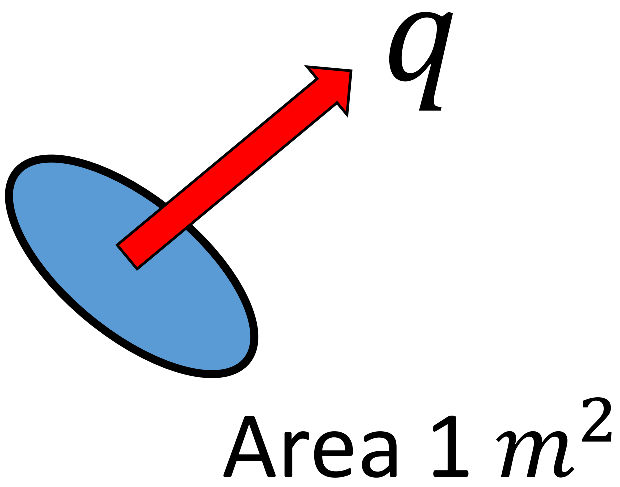
heat flow from __Hot__ to __Cold___

(without the material moving)

heat flux, q, in a solid:

heat energy crossing a surface with unit are per second

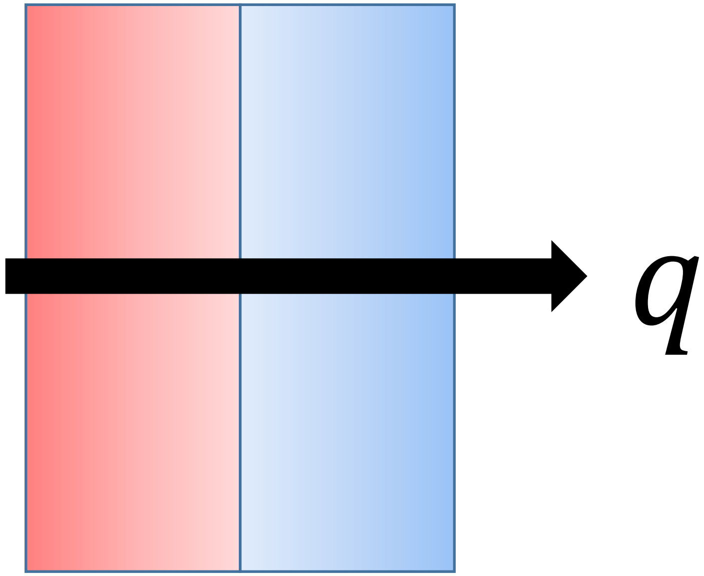
$$q\colon\ \frac{J}{m^2\ s}$$



heat flux, q, in a solid:

heat flows from cold to hot

$$q = -k \frac{dT}{dx}$$

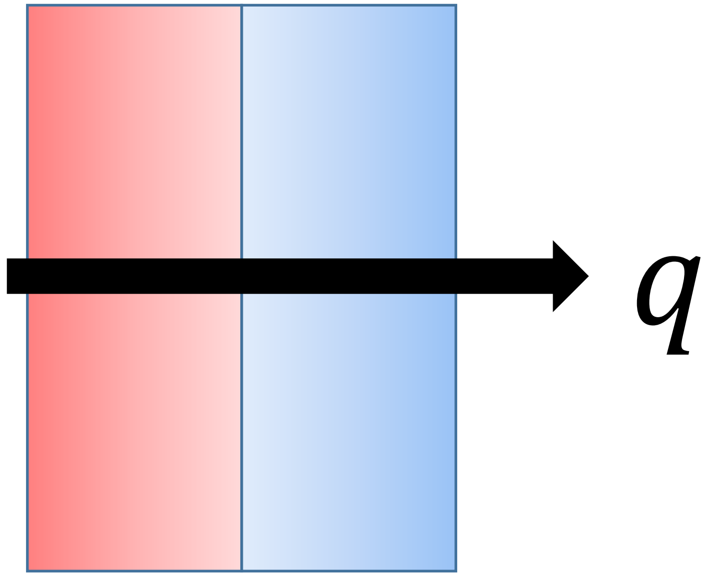


k: thermal conductivity

$$q = -k \frac{dT}{dx}$$

$$\frac{J}{m^2 s} \qquad \frac{J}{ms°C} \frac{°C}{m}$$

$$\frac{W}{m°C}$$

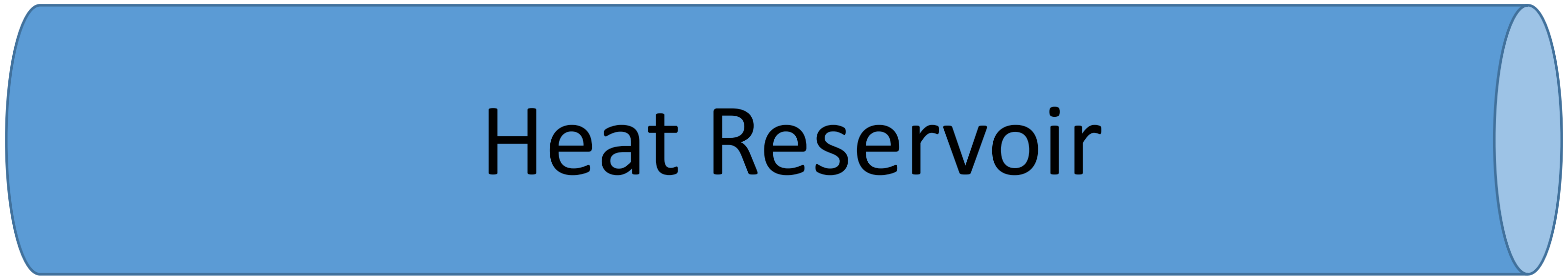


thermal conductivity rock $k = 3 \frac{W}{m°C}$

Solid rod with insulated surface

insulated = no heat flux

Solid rod with insulated surface



Approximation:

Temperature varies only along length of rod

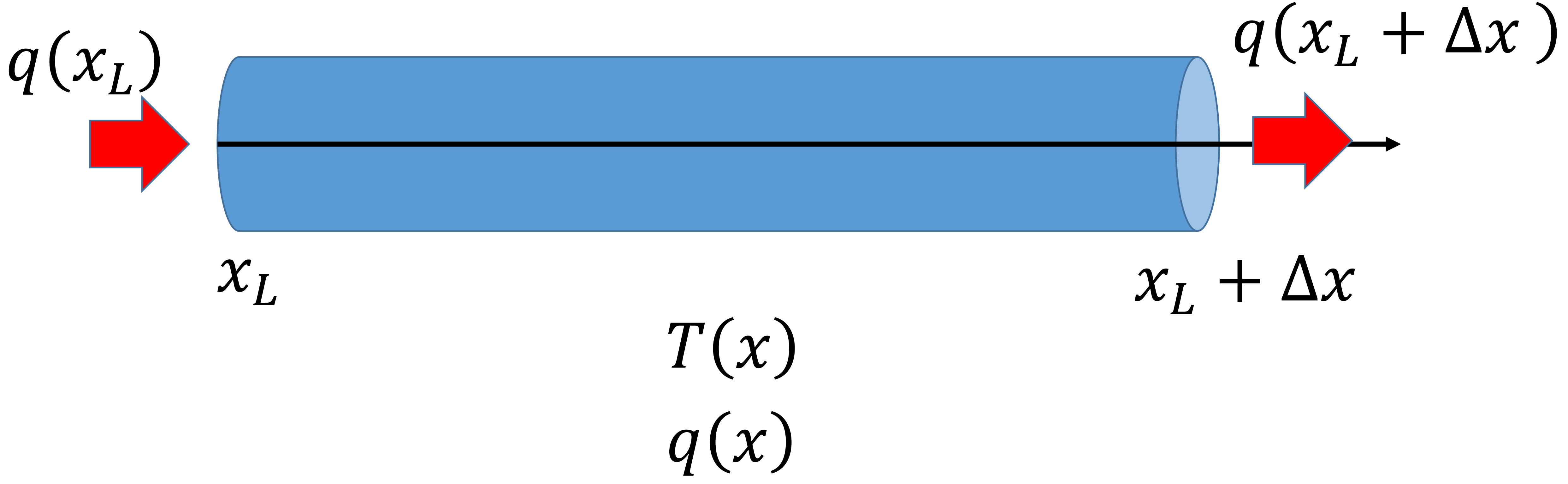
Heat flux is along length of rod

$T(x)$

$q(x)$

Conservation of heat energy

$$\frac{d\Delta Q}{dt} = q(x_L) - q(x_L + \Delta x)$$



Conservation of heat energy

$$\frac{d\Delta Q}{dt} = Aq(x_L) - Aq(x_L + \Delta x\ )$$

$$\rho c_p A\Delta x \frac{d\Delta T}{dt} = Aq(x_L) - Aq(x_L + \Delta x\ )$$

$$\rho c_p \frac{d\Delta T}{dt} = -\frac{dq}{dx}$$

Conservation of heat energy

$$\frac{d\Delta Q}{dt} = Aq(x_L) - Aq(x_L + \Delta x\ )$$

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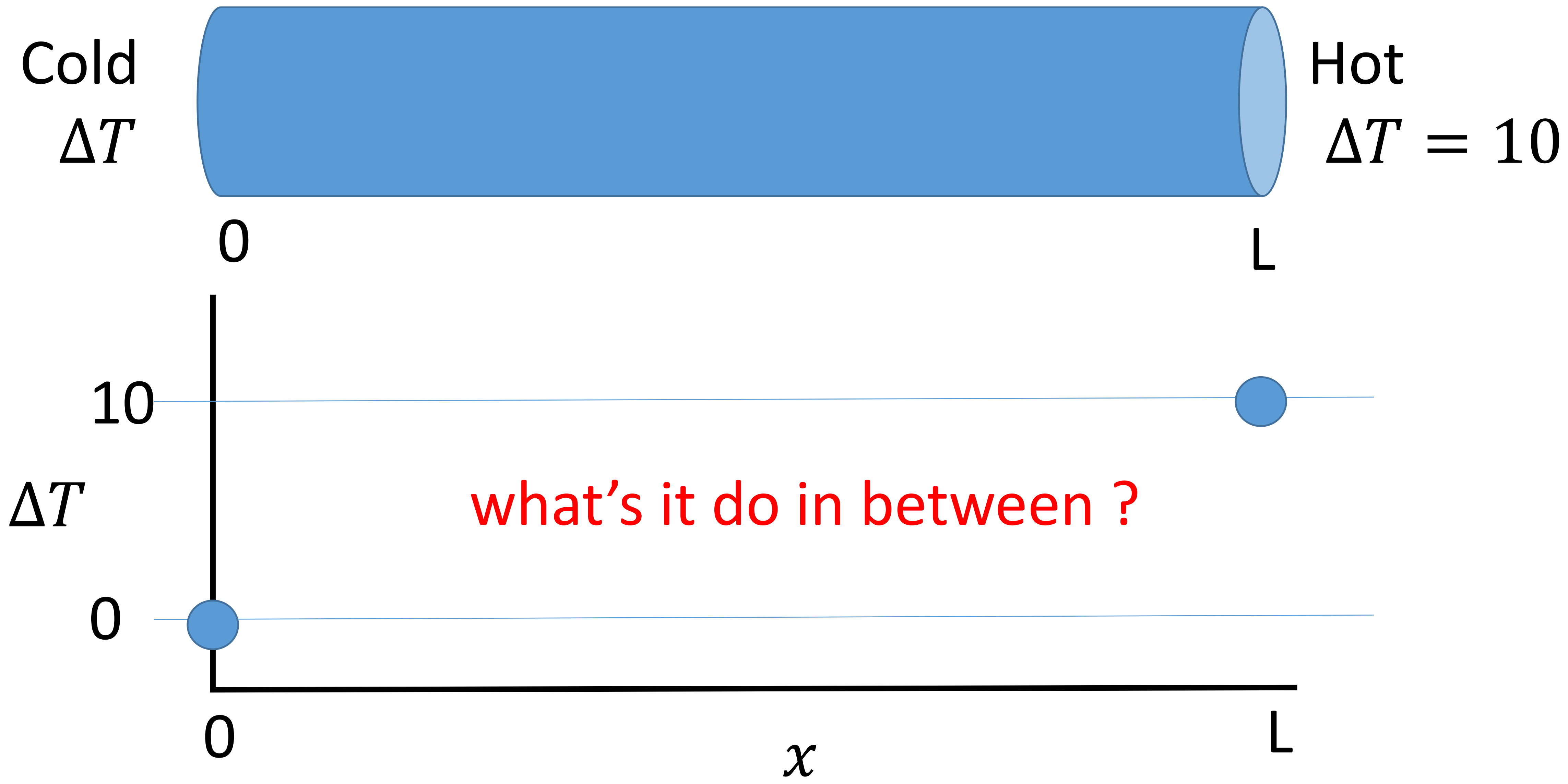
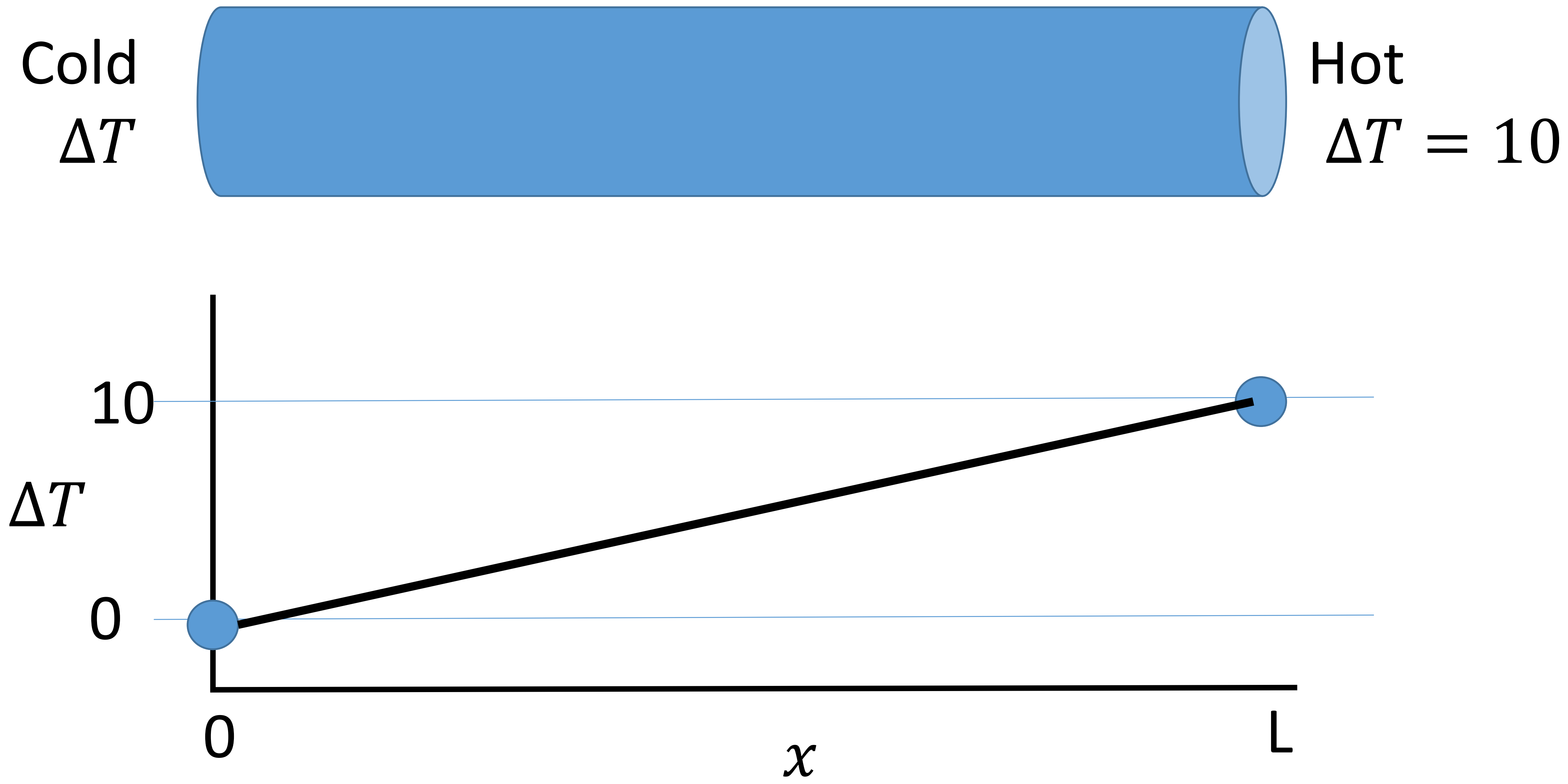
$$\rho c_p \frac{d\Delta T}{dt} = -\frac{dq}{dx}$$

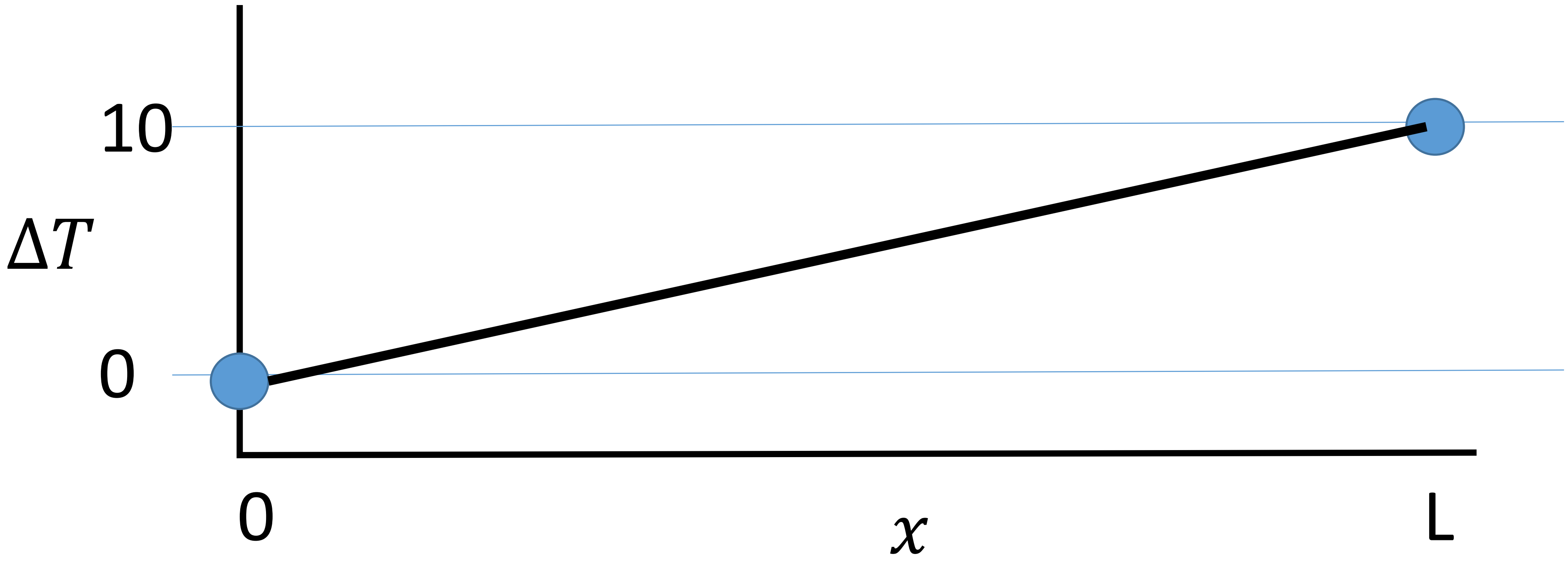
$$\rho c_p \frac{d\Delta T}{dt} = k\frac{d^2\Delta T}{dx^2}$$

Equilibrium temperature $\frac{d\Delta Q}{dt} = 0$

$$\rho c_p \frac{d\Delta T}{dt} = k \frac{d^2 \Delta T}{dx^2} \quad \Rightarrow \quad 0 = \frac{d^2 \Delta T}{dx^2}$$


Cold
ΔT
Hot
$\Delta T = 10$
0
L
10
ΔT
what’s it do in between ?
0
0
x
L

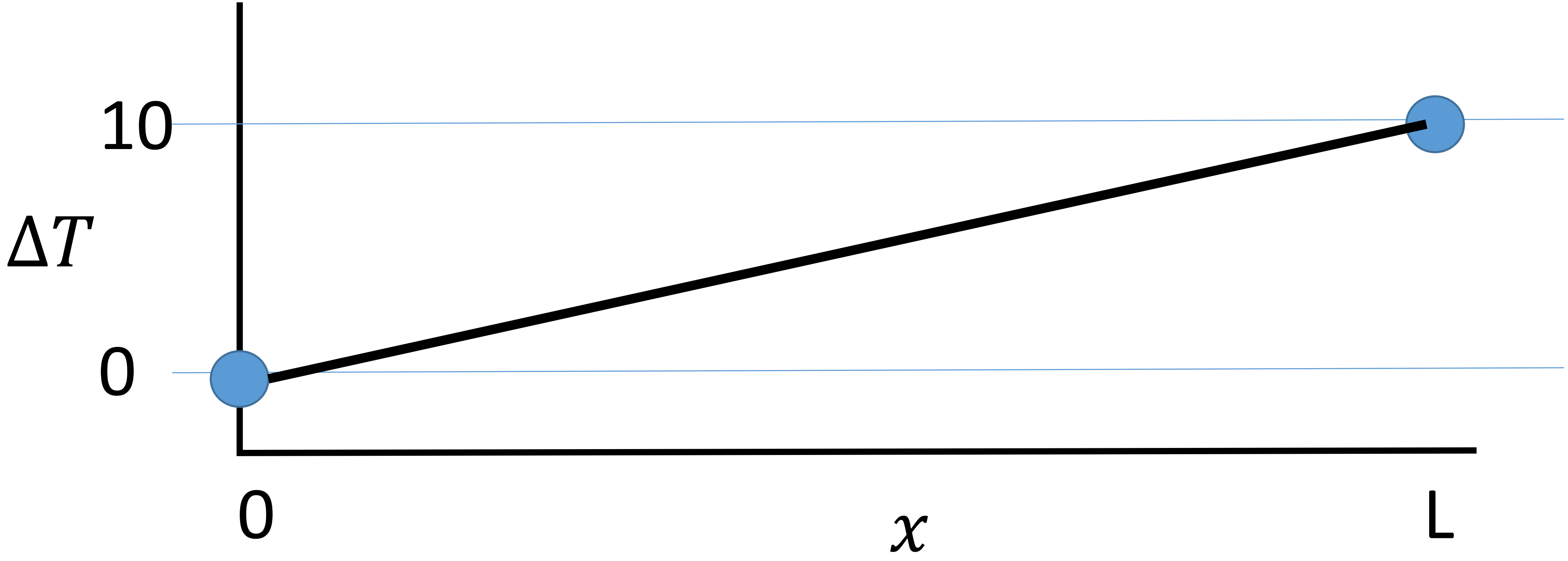

Cold
ΔT
Hot
$\Delta T = 10$
10
ΔT
0
0
x
L



$$\Delta T = \frac{10}{L}x$$

compatible?

$$0 = \frac{d^2 \Delta T}{dx^2}$$



$$\Delta T = \frac{10}{L} x$$

$$0 = \frac{d^2 \Delta T}{dx^2}$$

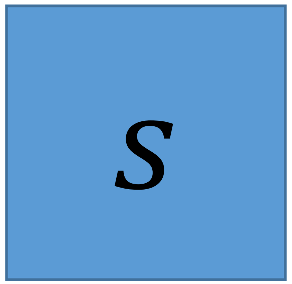
does it depend on the type of rock?

heat source, s:

heat energy generated per unit volume per second

$$s\text{:}\ \ \frac{J}{m^3\ \ s}$$

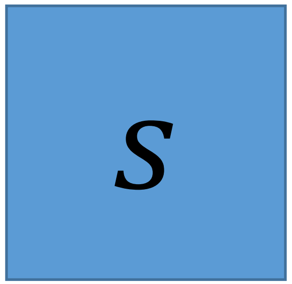
$$\frac{W}{m^3}$$



volume 1 m^2

heat source, s:

heat energy generated per unit volume per second



volume 1 m^2

$$\text{granite } s = 5 \times 10^{-6}\ W/m^3$$

Conservation of heat energy

$$\frac{d\Delta Q}{dt} = Aq(x_L) - Aq(x_L + \Delta x\ ) + A\Delta xs$$

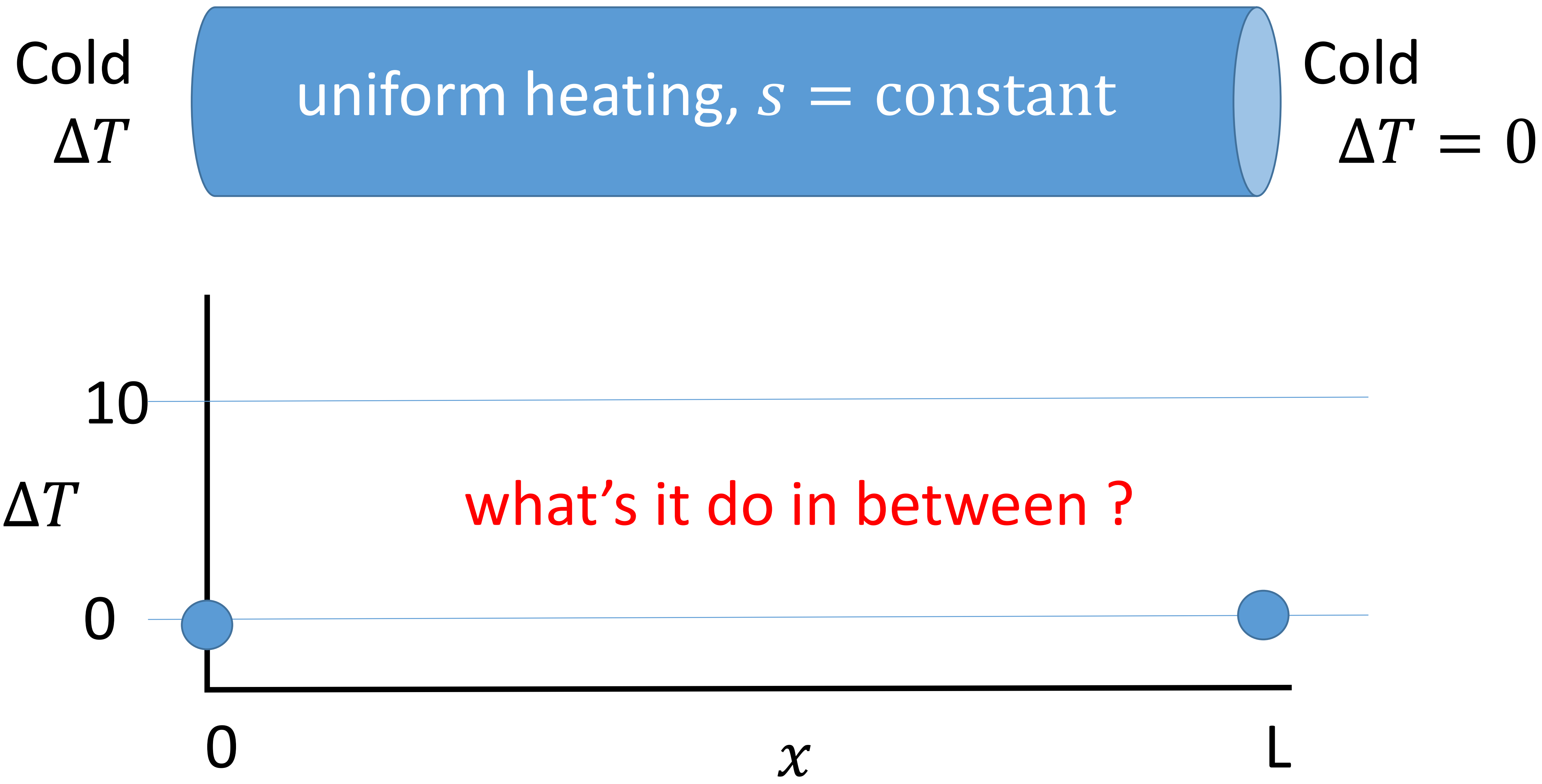
$$\rho c_p A\Delta x \frac{d\Delta T}{dt} = Aq(x_L) - Aq(x_L + \Delta x\ ) + A\Delta xs$$

$$\rho c_p \frac{d\Delta T}{dt} = -\frac{dq}{dx} + s$$

$$\rho c_p \frac{d\Delta T}{dt} = k\frac{d^2\Delta T}{dx^2} + s$$

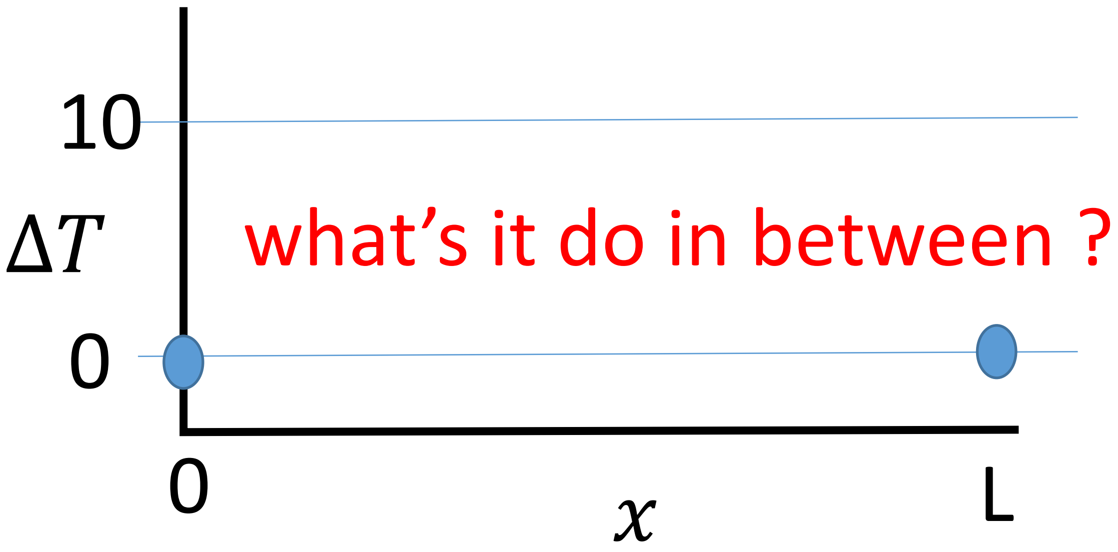
Equilibrium temperature $\frac{d\Delta Q}{dt}=0$

$$\rho c_p \frac{d\Delta T}{dt} = k \frac{d^2 \Delta T}{dx^2} + s \quad \Rightarrow \quad 0 = \frac{d^2 \Delta T}{dx^2} + s/k$$



Cold
ΔT
uniform heating, s = constant
Cold
$\Delta T = 0$
10
ΔT
what's it do in between ?
0
0
x
L

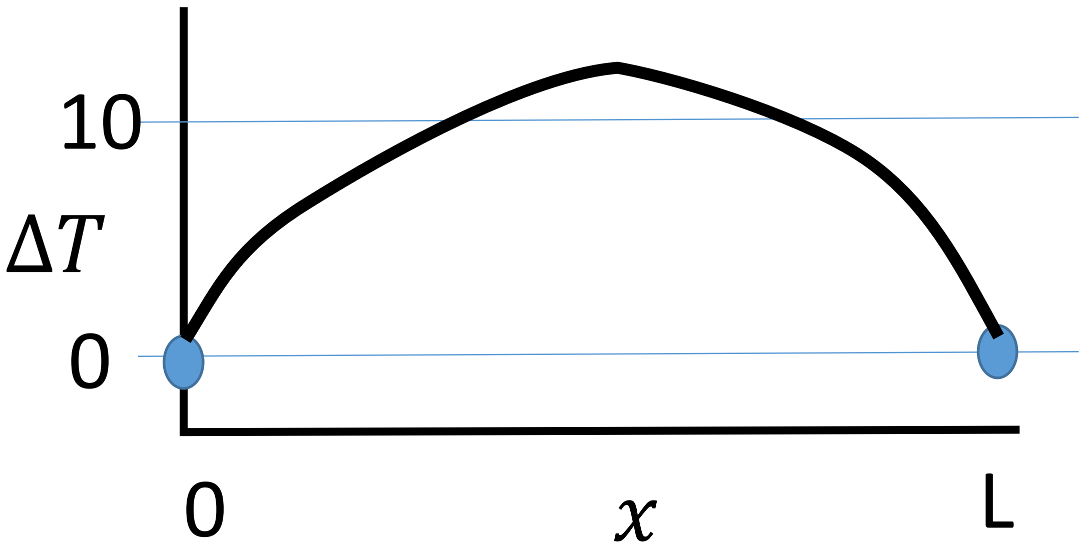
intuitive thinking



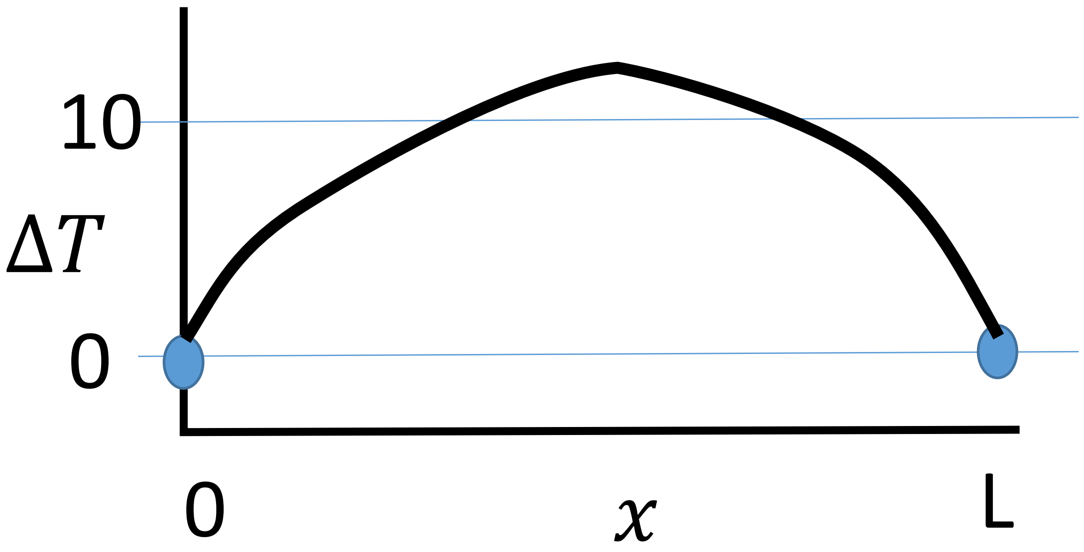
where's it the biggest?

does the shape have any symmetry?

intuitive thinking
parabola



$$\Delta T = c\,x(L - x)$$
$$= c\,(Lx - x^2)$$

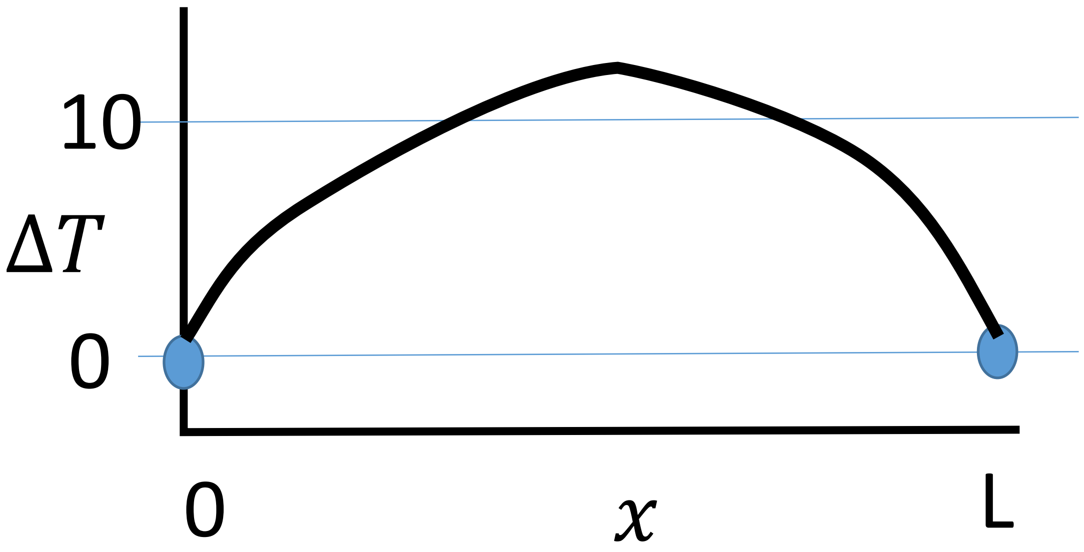


$$\Delta T = c\, x(L - x)$$
$$= c\,(Lx - x^2)$$

compatible?

$$0 = k\frac{d^2\Delta T}{dx^2} + s$$

intuitive thinking



compatible?

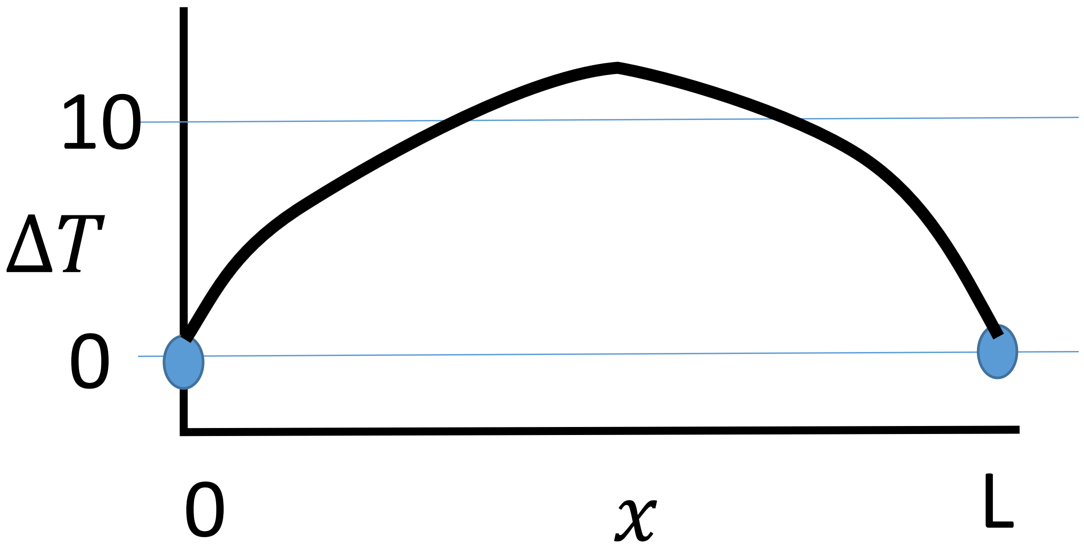
$$0 = \frac{d^2 \Delta T}{dx^2} + s/k$$

$$\Delta T = c\,(Lx - x^2)$$

$$\frac{d\Delta T}{dx} = c\,(L - 2x)$$

$$\frac{d^2 \Delta T}{dx^2} = -2c$$

yes, if $c = s/(2k\,)$

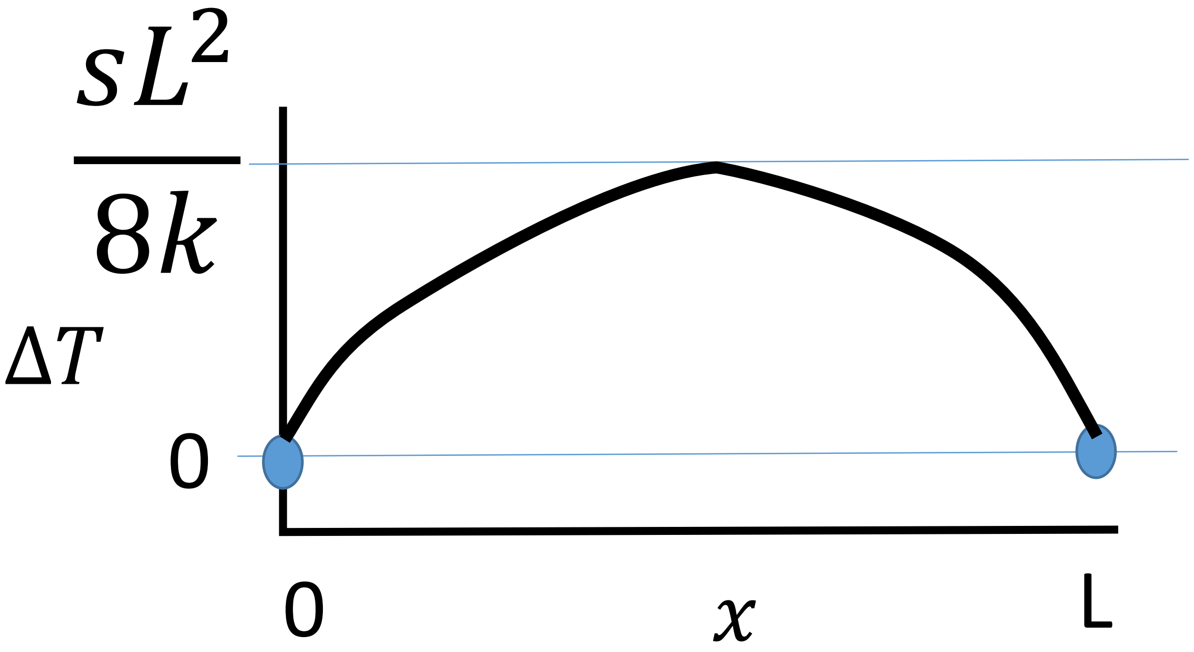


$$\Delta T = \frac{s}{2k} x(L - x)$$

$$\Delta T_{max} = \frac{s}{2k} \frac{L}{2} \left(L - \frac{L}{2} \right)$$

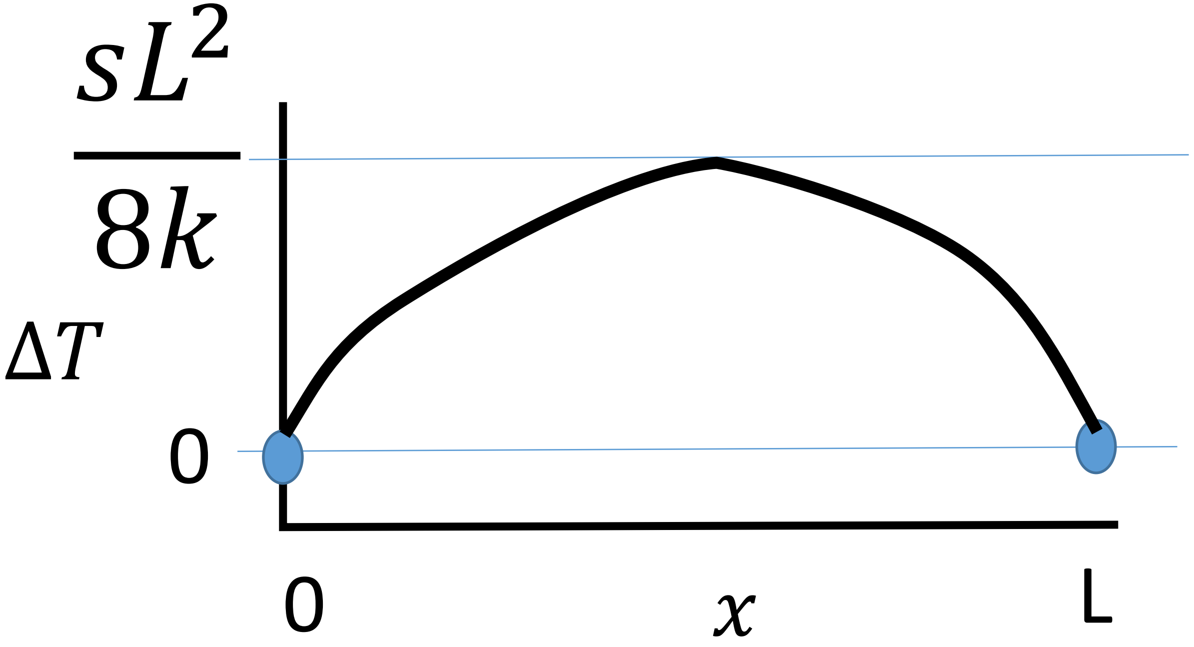
$$= \frac{s}{2k} \frac{L}{2} \frac{L}{2} = \frac{sL^2}{8k}$$

$$\Delta T = \frac{s}{2k}x(L - x)$$

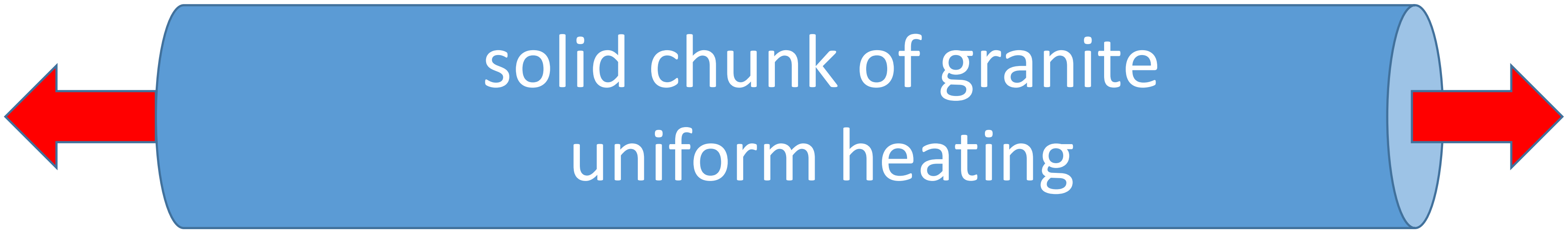


longer bar
hotter/colder
maximum?

$$\Delta T = \frac{s}{2k} x(L - x)$$



if you make it too long, it will melt



To what extent can this serve as a simple mode of the Earth?