Solid Earth Dynamics

Bill Menke, Instructor

Lecture 6

Today:

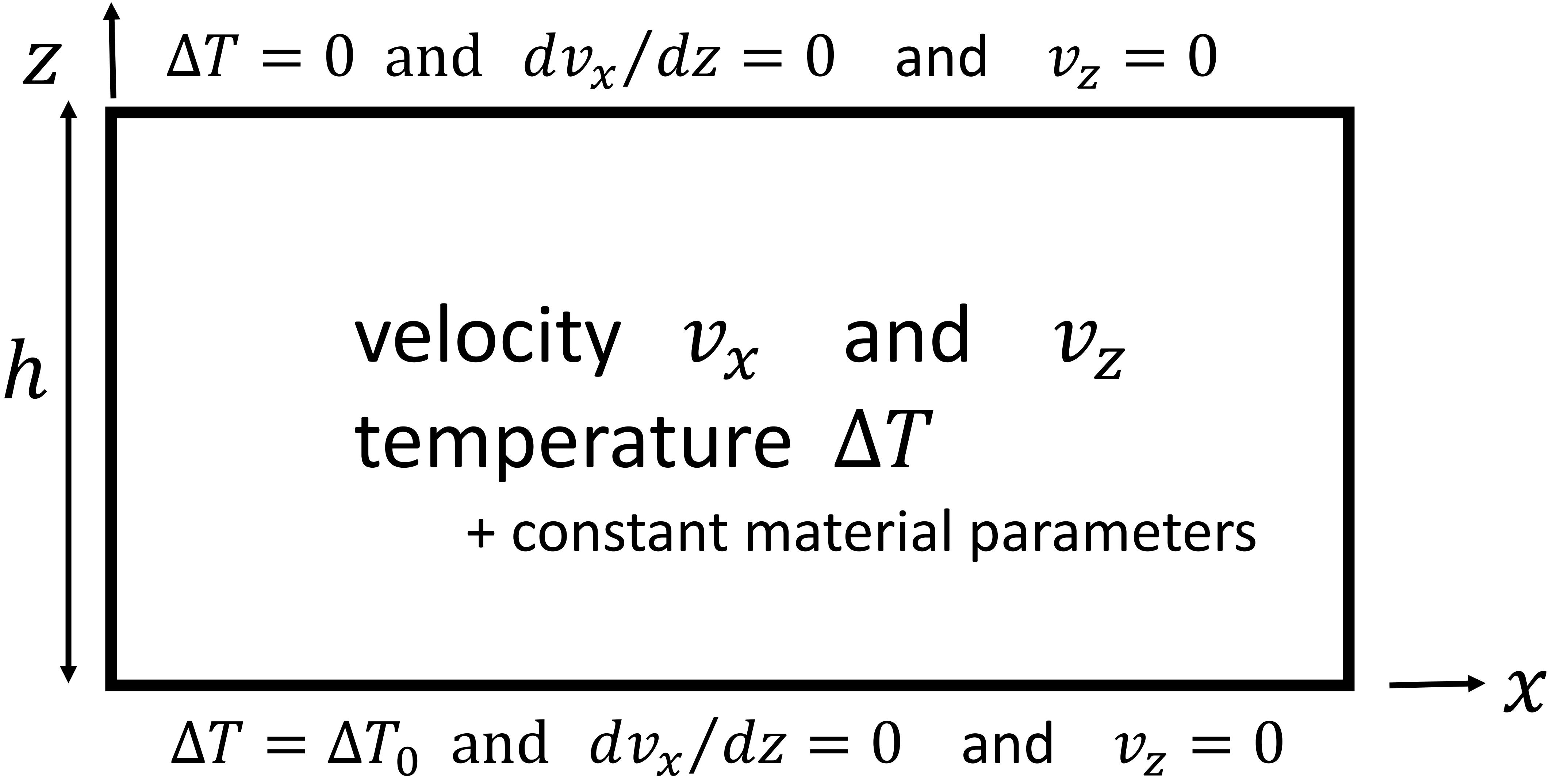
Mantle Convection

Thermal Structure of the Earth

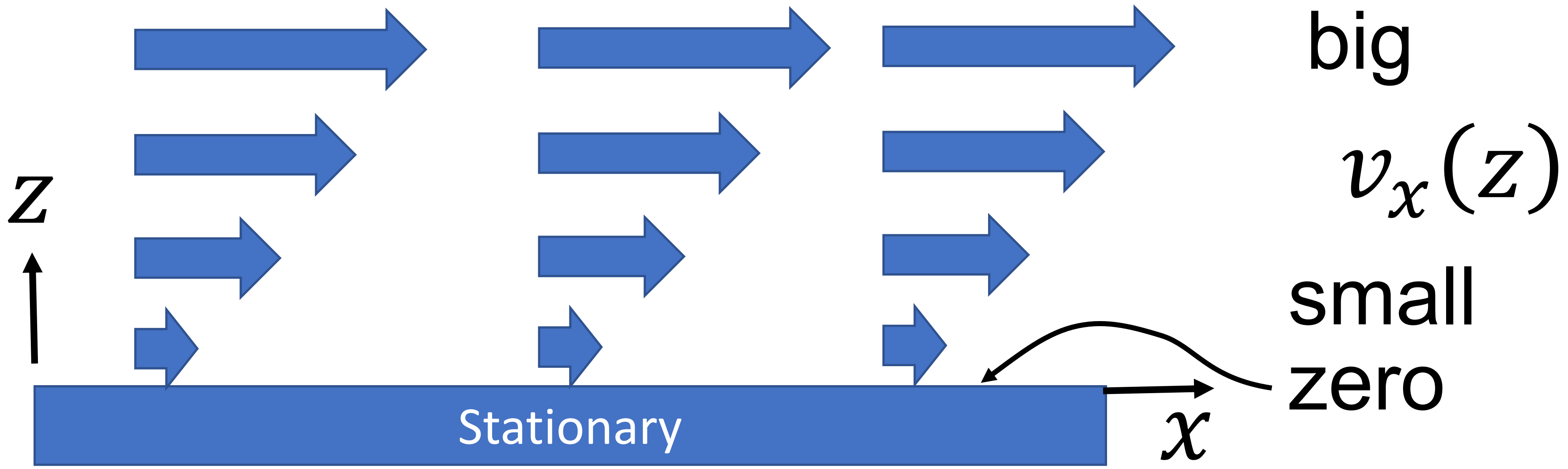
Part 1

Mantle Convection

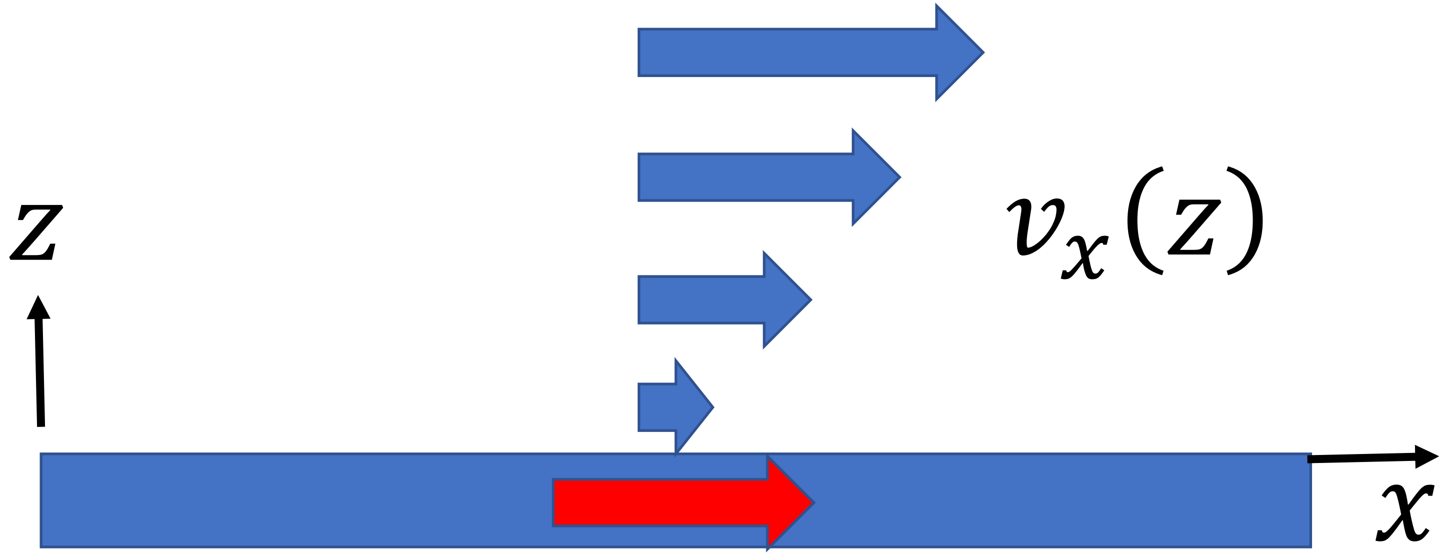
General things we know about Convection in a box



fuid motionless adjacent to a stationary boundary (welded boundary)



Motion of fluid past a stationary object exerts a drag force



$$Traction = force\ per\ unit\ area$$

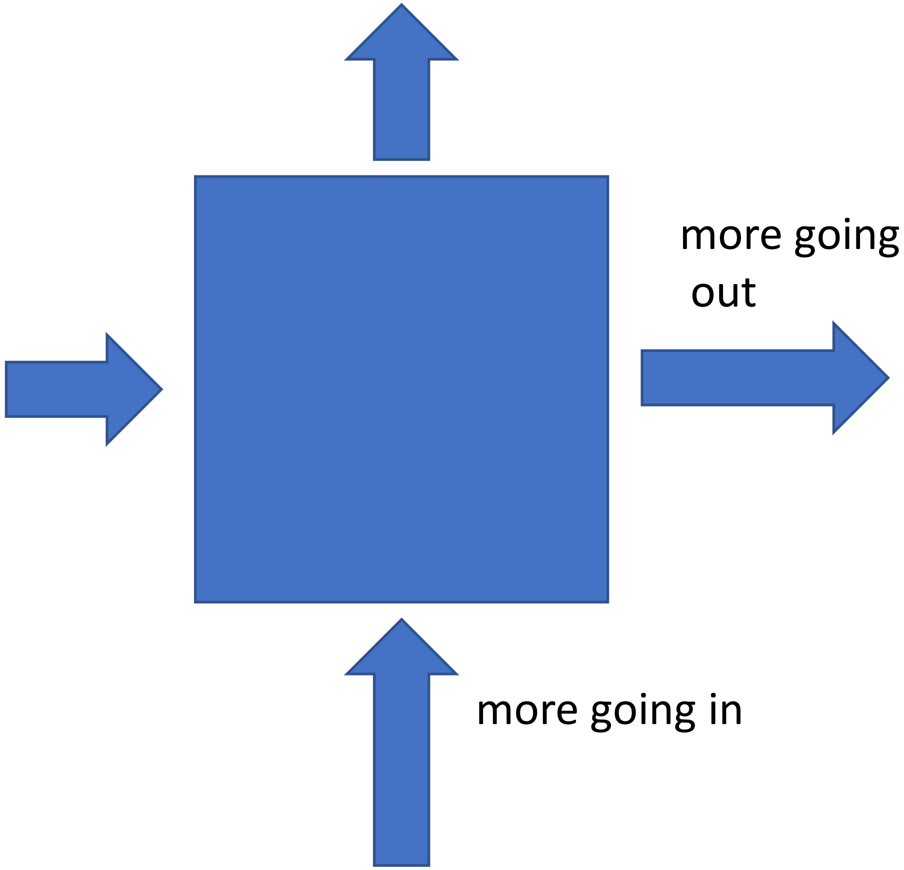
$$Traction\ parallel\ to\ surface = \eta \frac{dv_x}{dz}$$

In two dimensions, and for an incompressible fluid, the velocity has two components

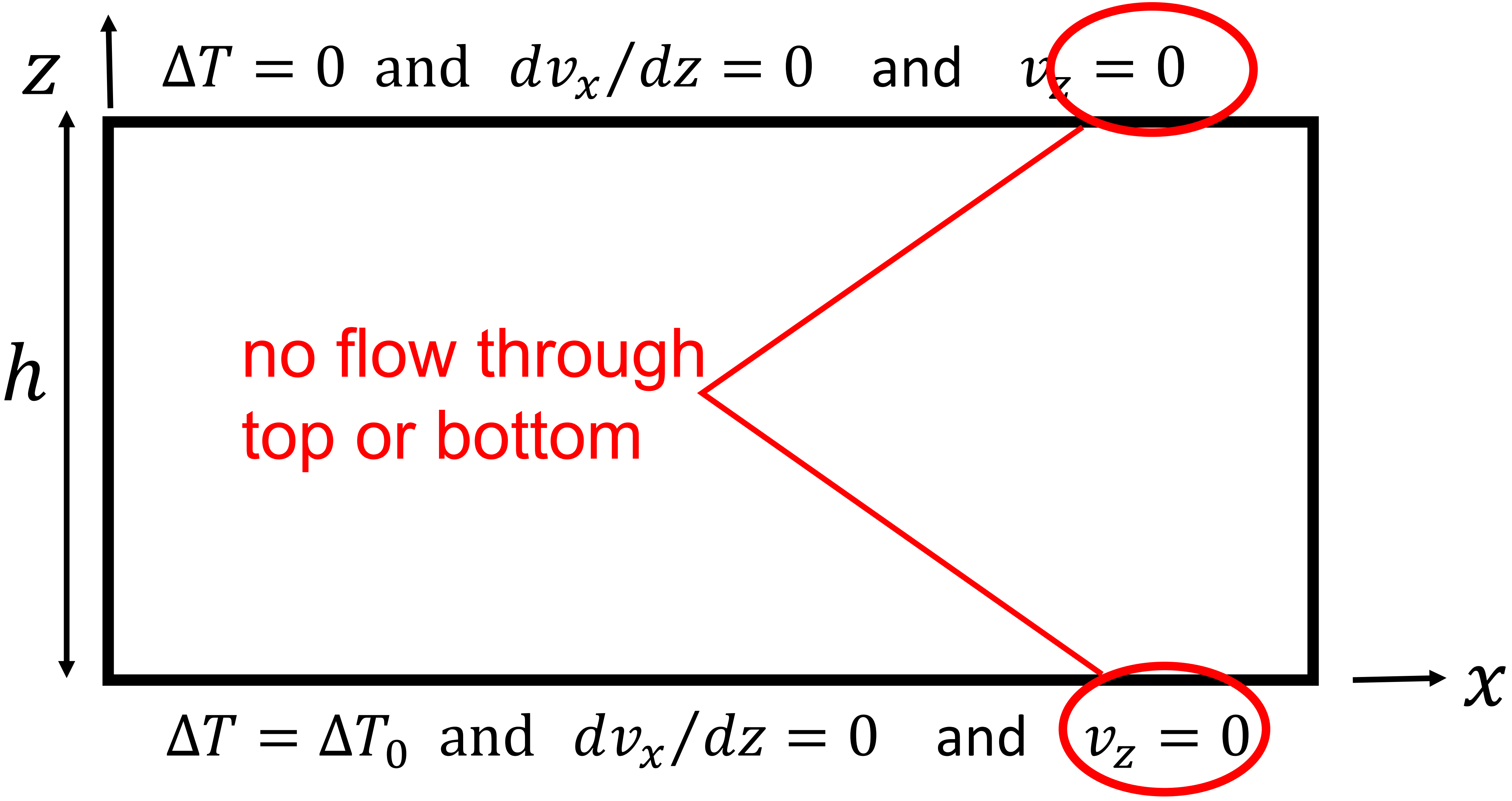
v_x and v_z

that obey

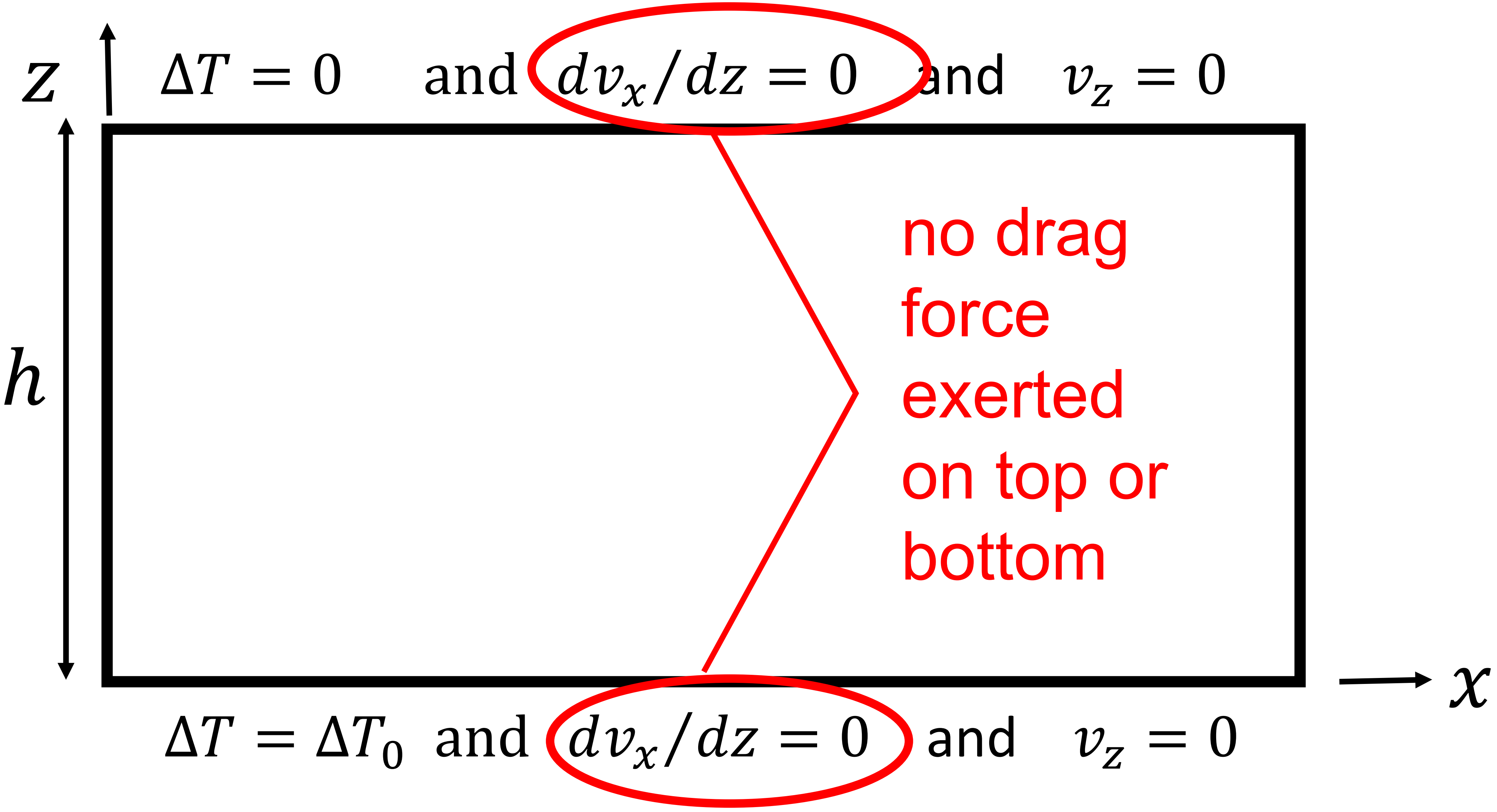
$$dv_x/dx = -dv_z/dz$$



General things we know about Convection in a box



General things we know about Convection in a box



Thing We Know #1: If you work in scaled variables

x/h
z/h
position scaled by height of box

$t\kappa/h^2$ time by rate at which heat conducts

$(\Delta T_0 - ½\Delta T_0)/\Delta T_0$ temperature as fractional deviation from the mean

$v_x \frac{1}{h}\frac{h^2}{\kappa}$
$v_z \frac{1}{h}\frac{h^2}{\kappa}$
velocity by combining position and time scaling

(also scaled pressure which I'm not showing you)

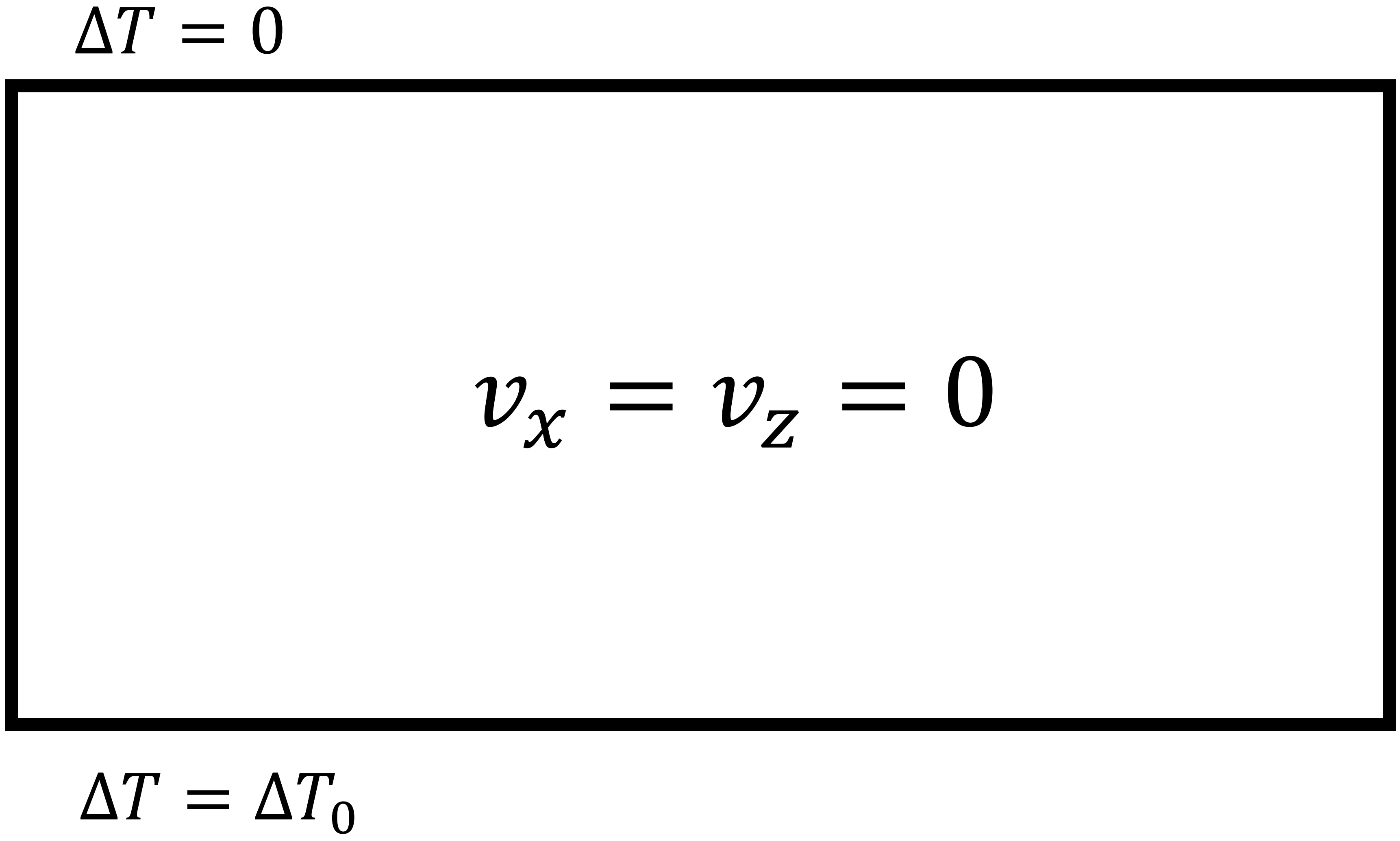
If you work in scaled variables ...

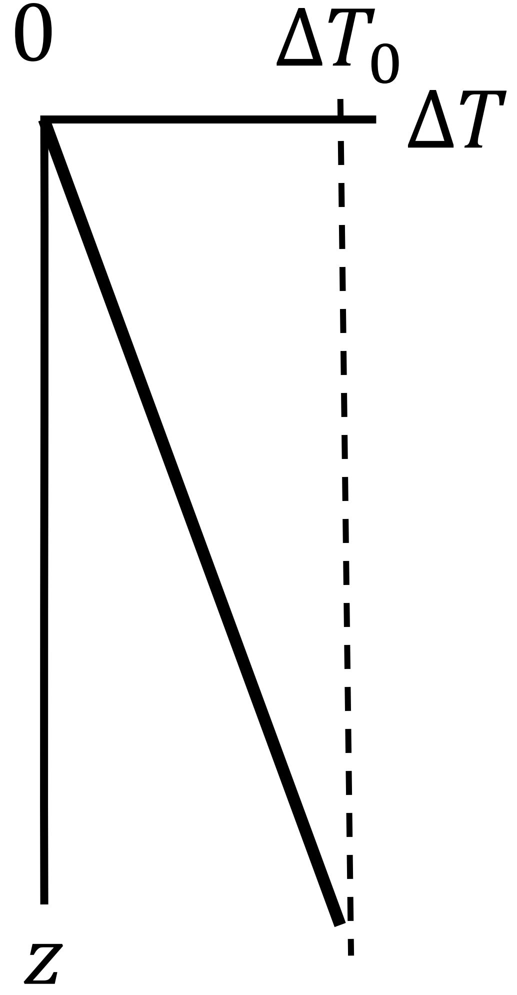
then the ONLY material constant is the Rayleigh Number

$$R_a = \frac{h^3 \rho_0 \alpha \Delta T_0 g}{\mu \kappa}$$

and only this combination of material parameters affects the pattern of convection

Thing We Know #2: Sometimes "no convection" is possible

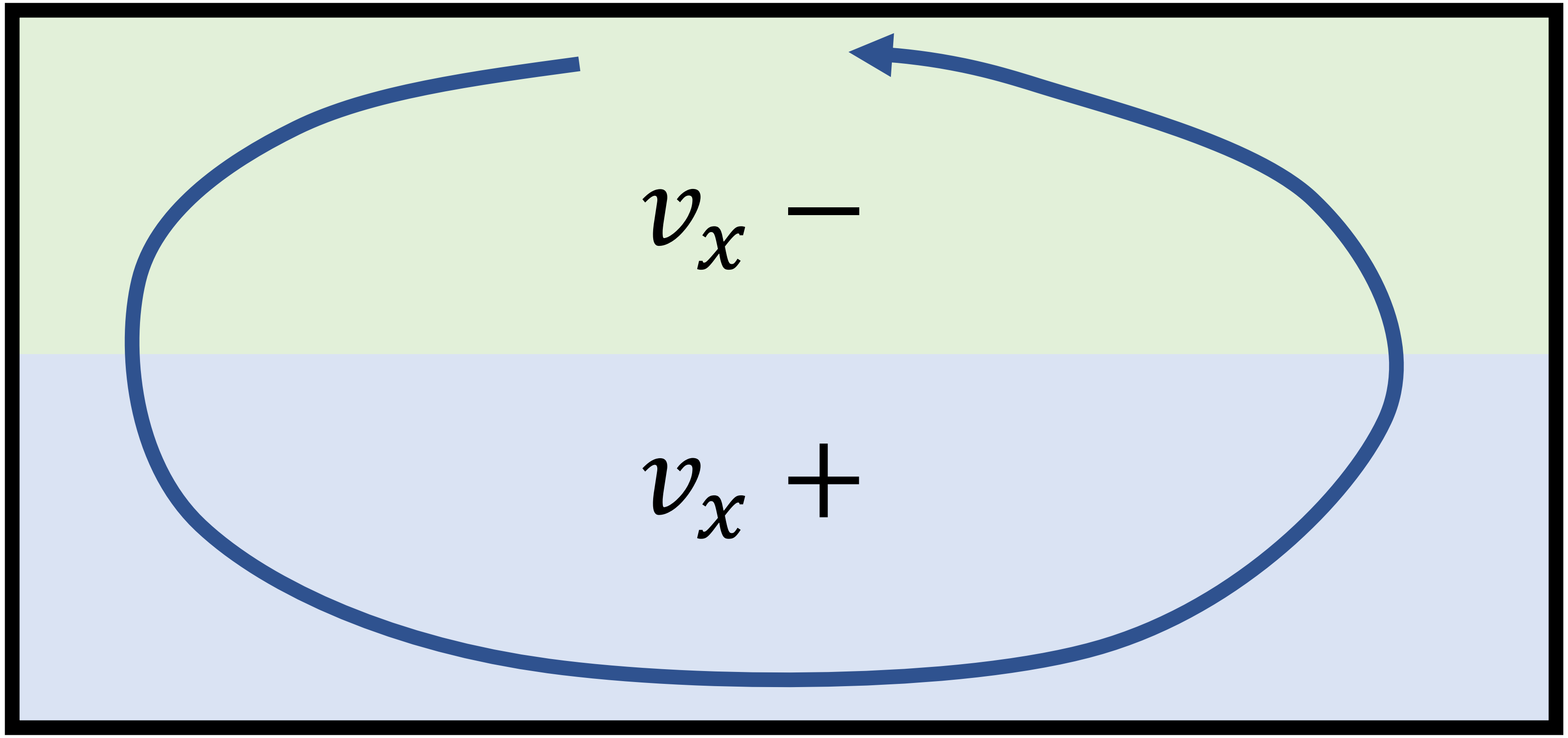


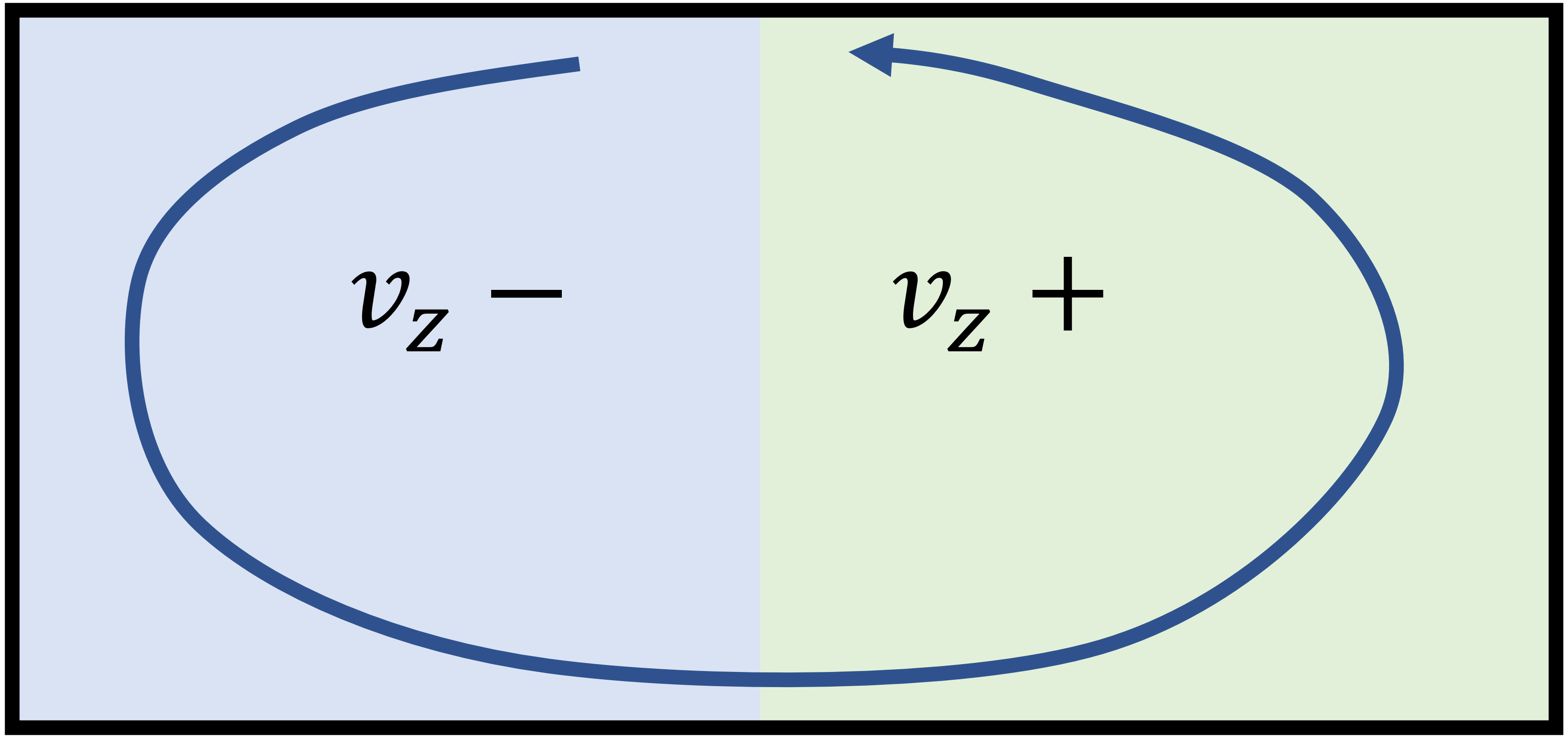


but this solution is not always stable

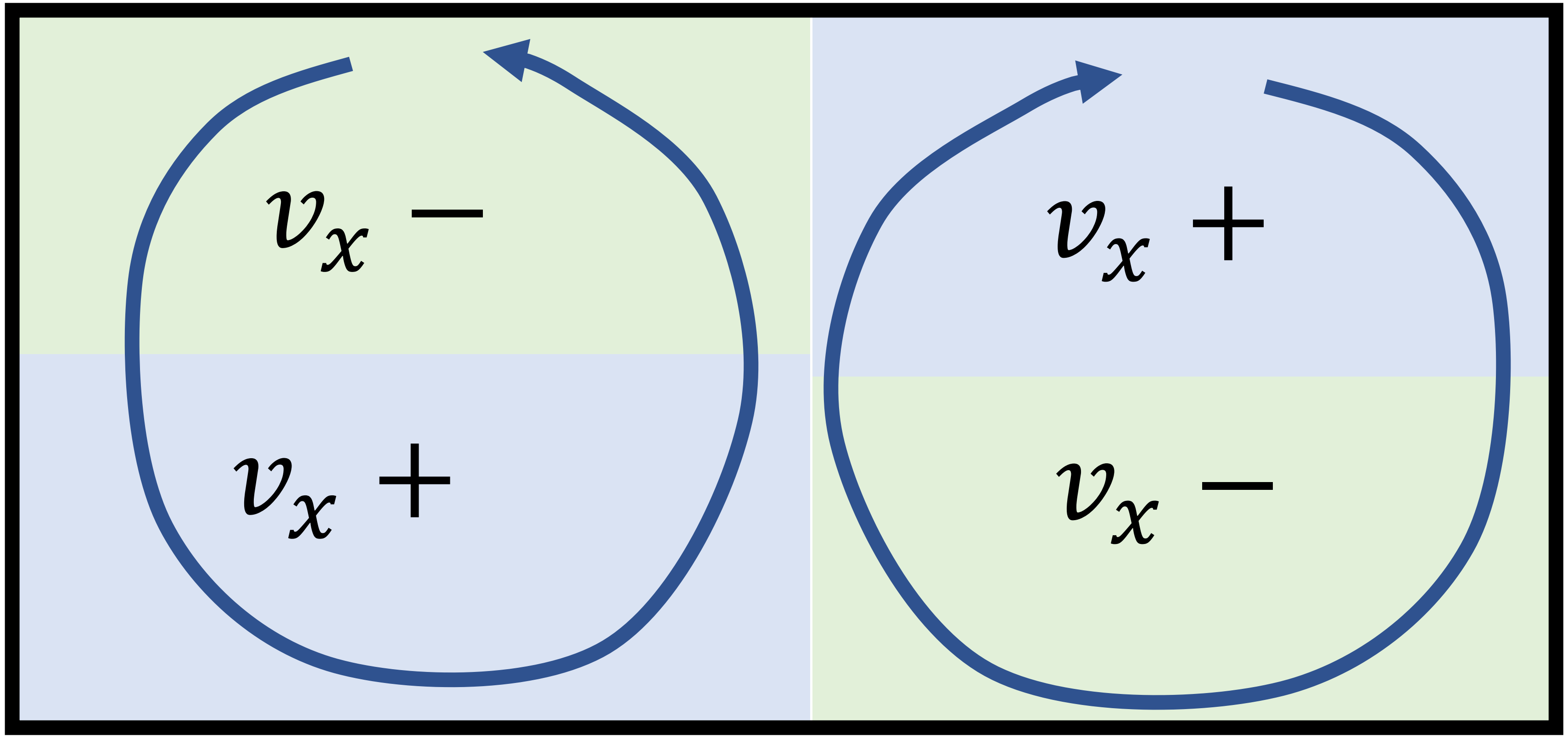
Many possible convection patterns

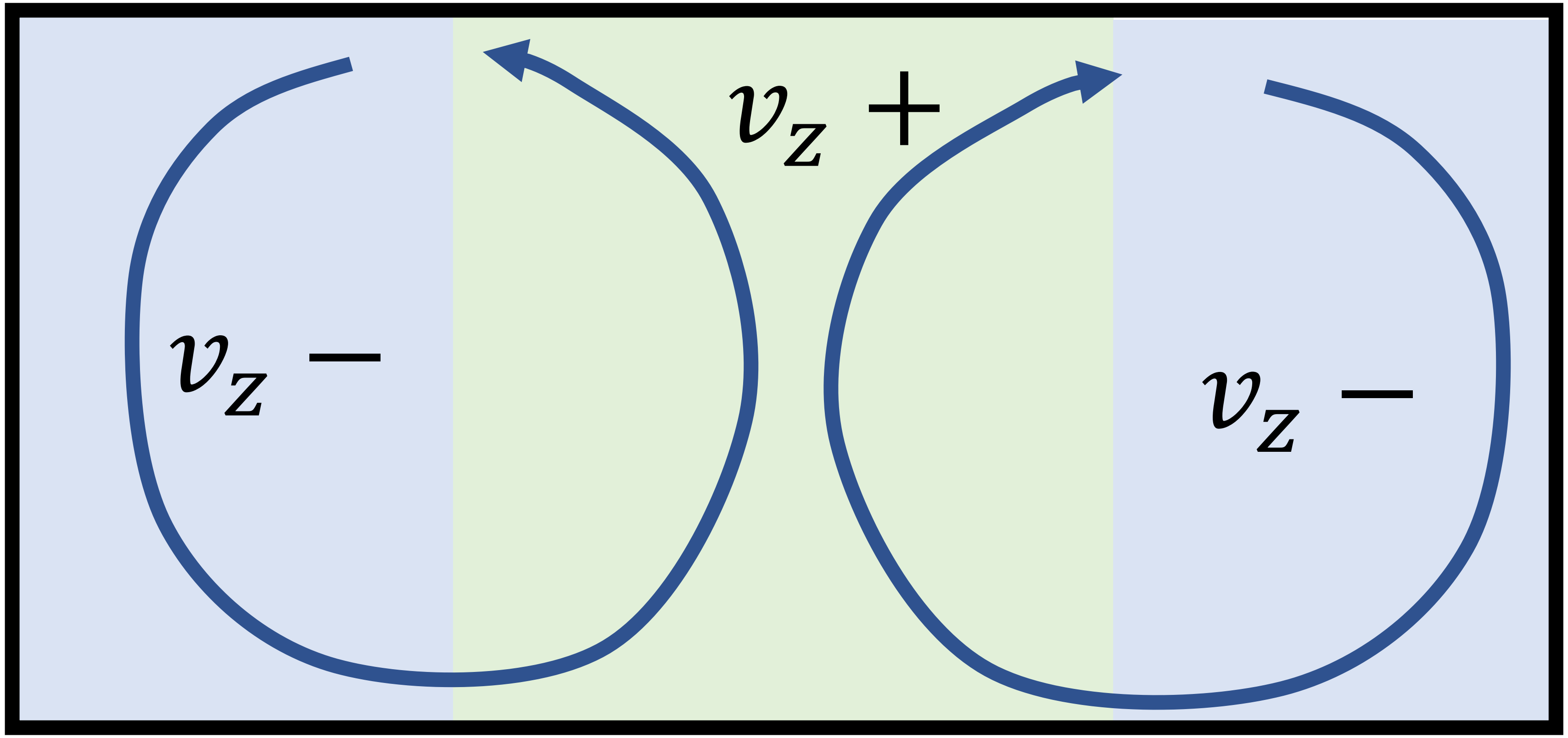
One convective roll




$v_z -$
$v_z +$

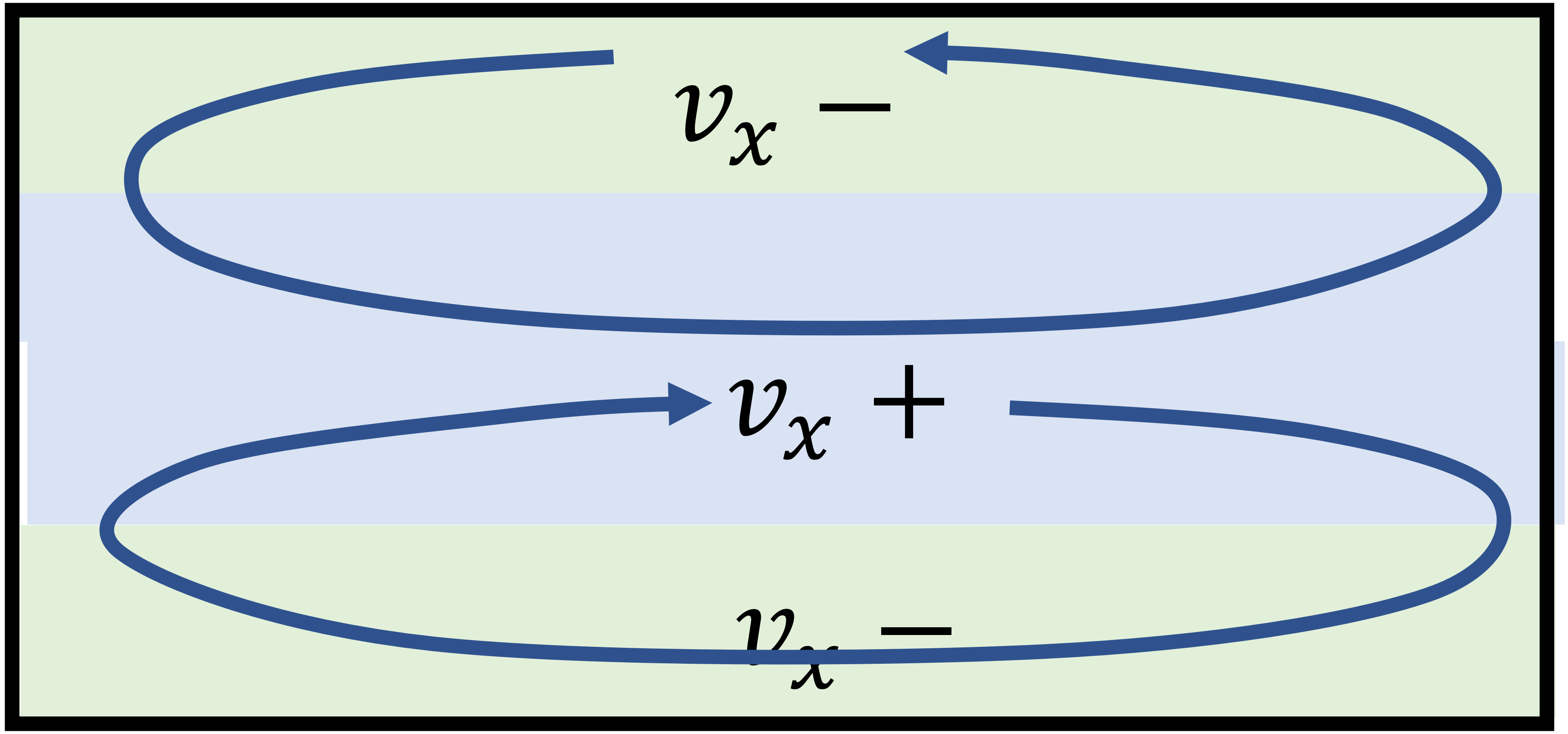
Two convective rolls, side-by-side



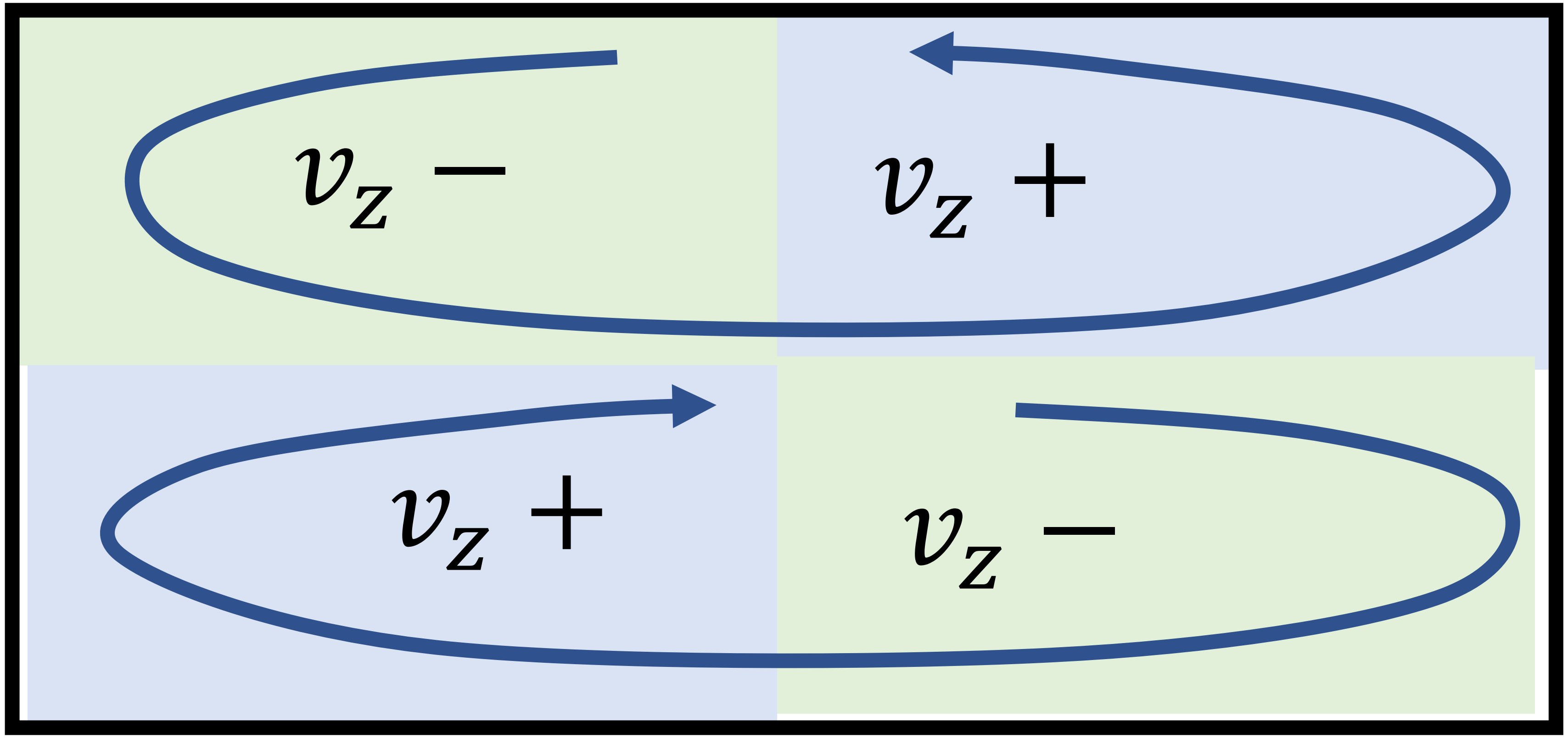


v_z +
v_z −
v_z −

Two convective rolls, one atop the other



Two convective rolls, one atop the other



Suppose that we start with zero velocity and a linear temperature profile

and add a tiny perturbation

v

$$v_z = A \sin\left(m\pi \frac{x}{h}\right) \sin\left(n\pi \frac{z}{h}\right)$$

Suppose that we start with zero velocity and a linear temperature profile

and add a tiny perturbation

v

$$v_z = A \sin\left(m\pi \frac{x}{h}\right) \sin\left(n\pi \frac{z}{h}\right)$$

get v_x from incompressibility equation

Suppose that we start with zero velocity and a linear temperature profile

and add a tiny perturbation

v

rate of roles horizontally

number of roles vertically

$$v_z = A \sin\left(m\pi \frac{x}{h}\right) \sin\left(n\pi \frac{z}{h}\right)$$

Suppose that we start with zero velocity and a linear temperature profile

and add a tiny perturbation

v

does the pattern initially amplify or dissipate with time

$$\Delta T = A \sin\left(m\pi \frac{x}{h}\right) \sin\left(n\pi \frac{z}{h}\right) \exp(\mathrm{st}')$$

t' is scaled time, $t\kappa/h^2$

Suppose that we start with zero velocity and a linear temperature profile

and add a tiny perturbation

v

is s positive or negative?

$$\Delta T = A \sin\left(m\pi \frac{x}{h}\right) \sin\left(n\pi \frac{z}{h}\right) \exp(\mathrm{s}t')$$

t' is scaled time, $t\kappa/h^2$

$$s = \frac{m^2 R_a}{(m^2 + n^2\pi^2)^2} - (m^2 + n^2\pi^2)$$

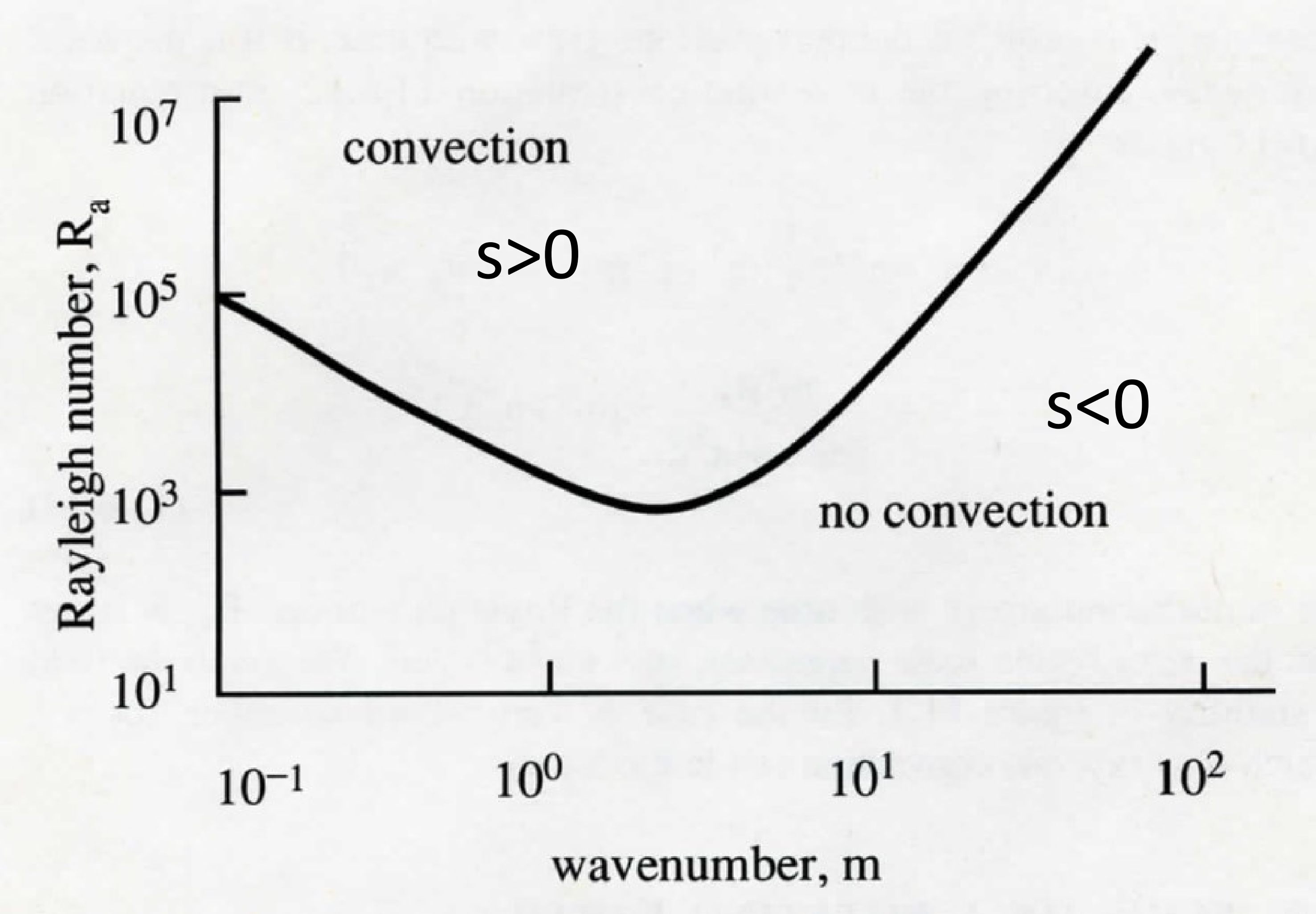
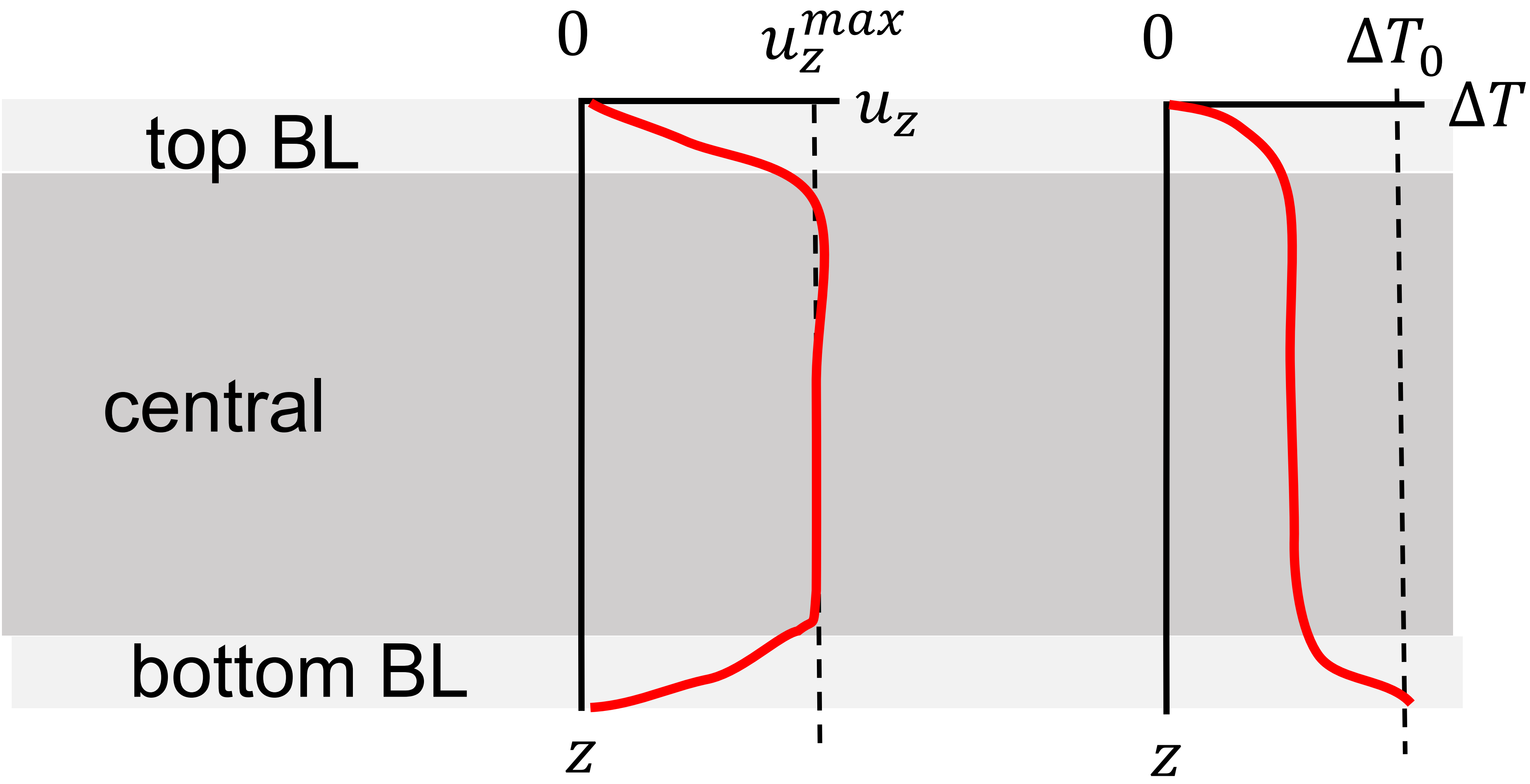


Diagram for n=1

(one roll vertically)

Thing We Know #3: averaged over time, the solution is consists of a central uniform region and boundary layers



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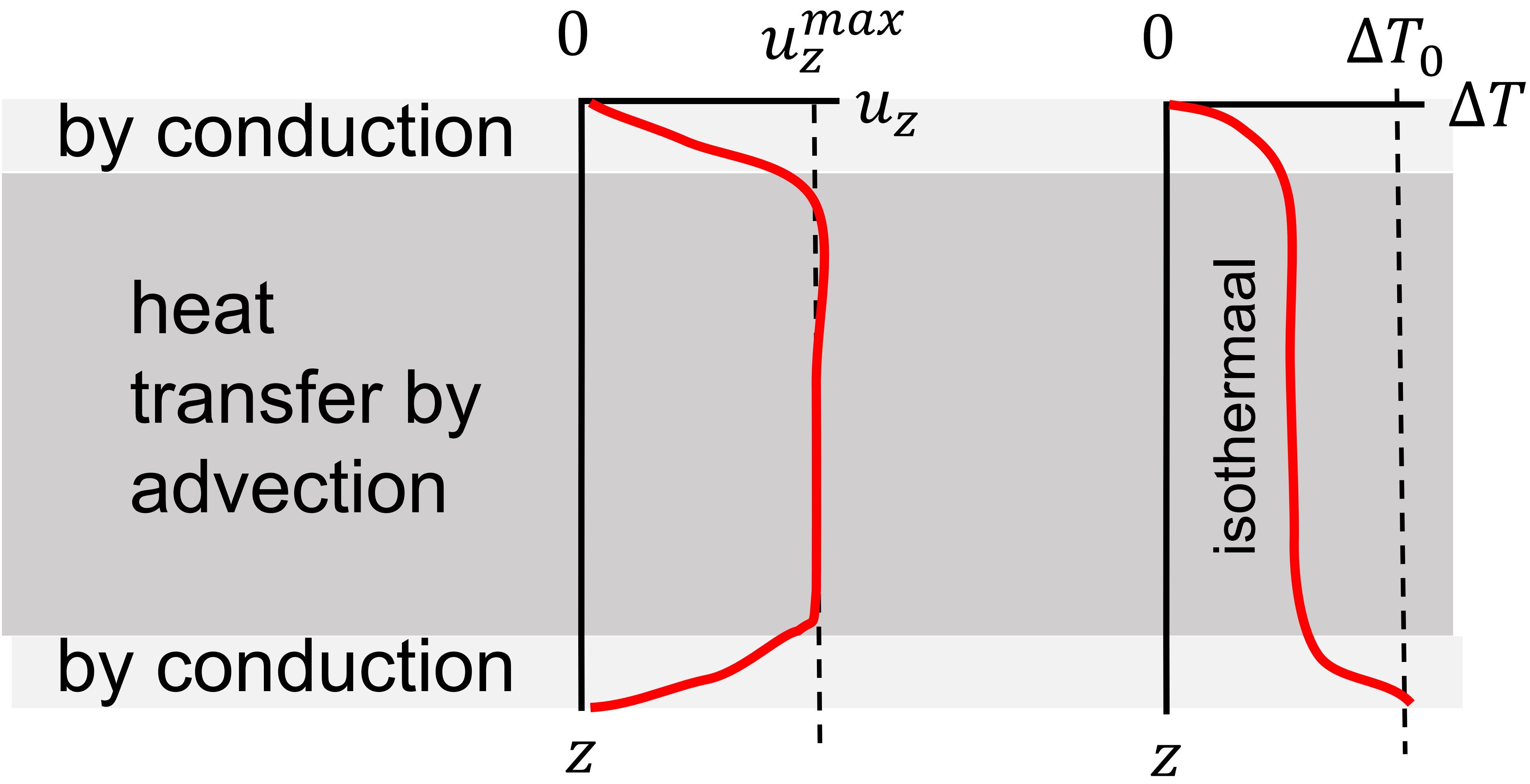
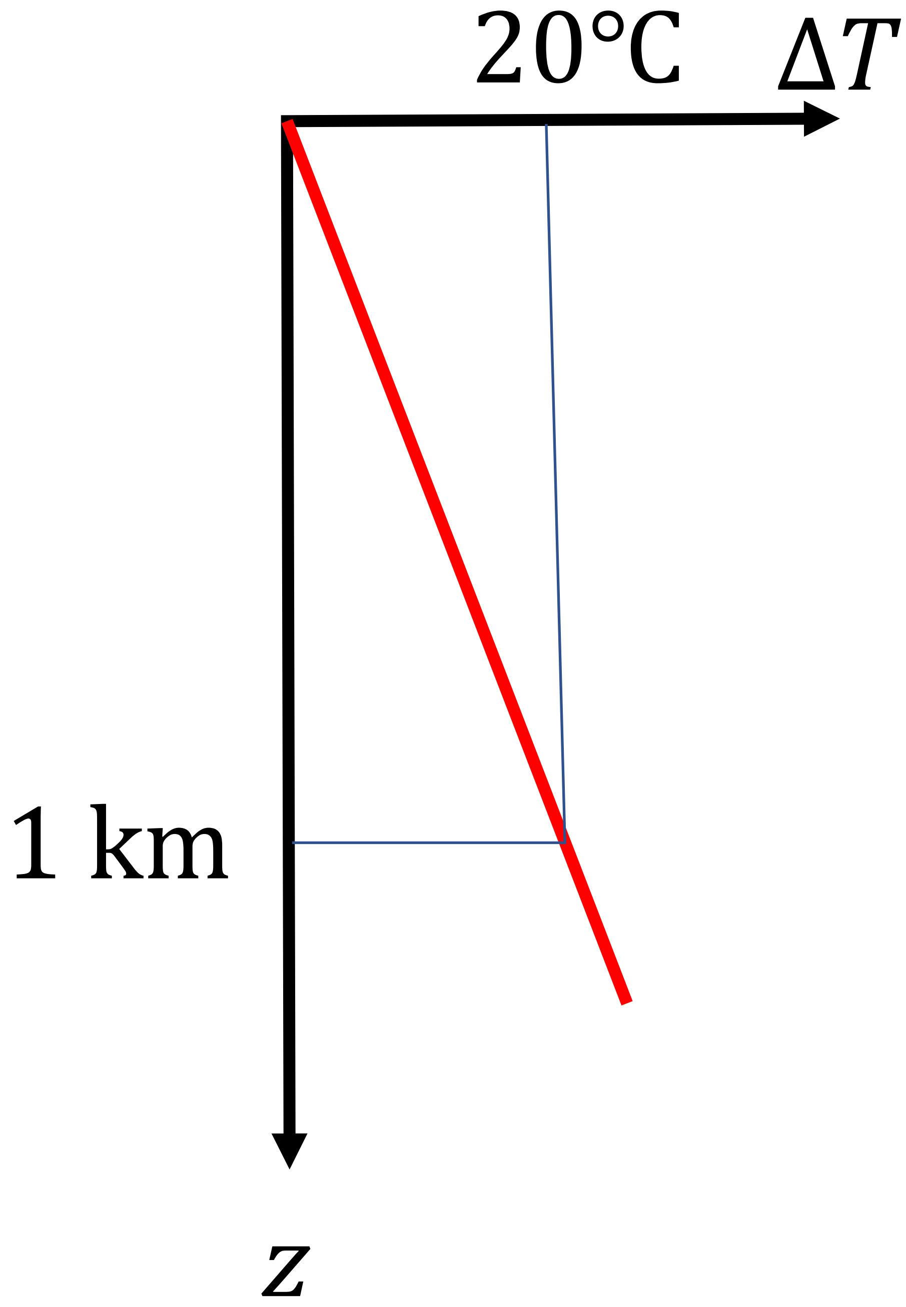


Part 2

Thermal Structure of the Earth

What we know

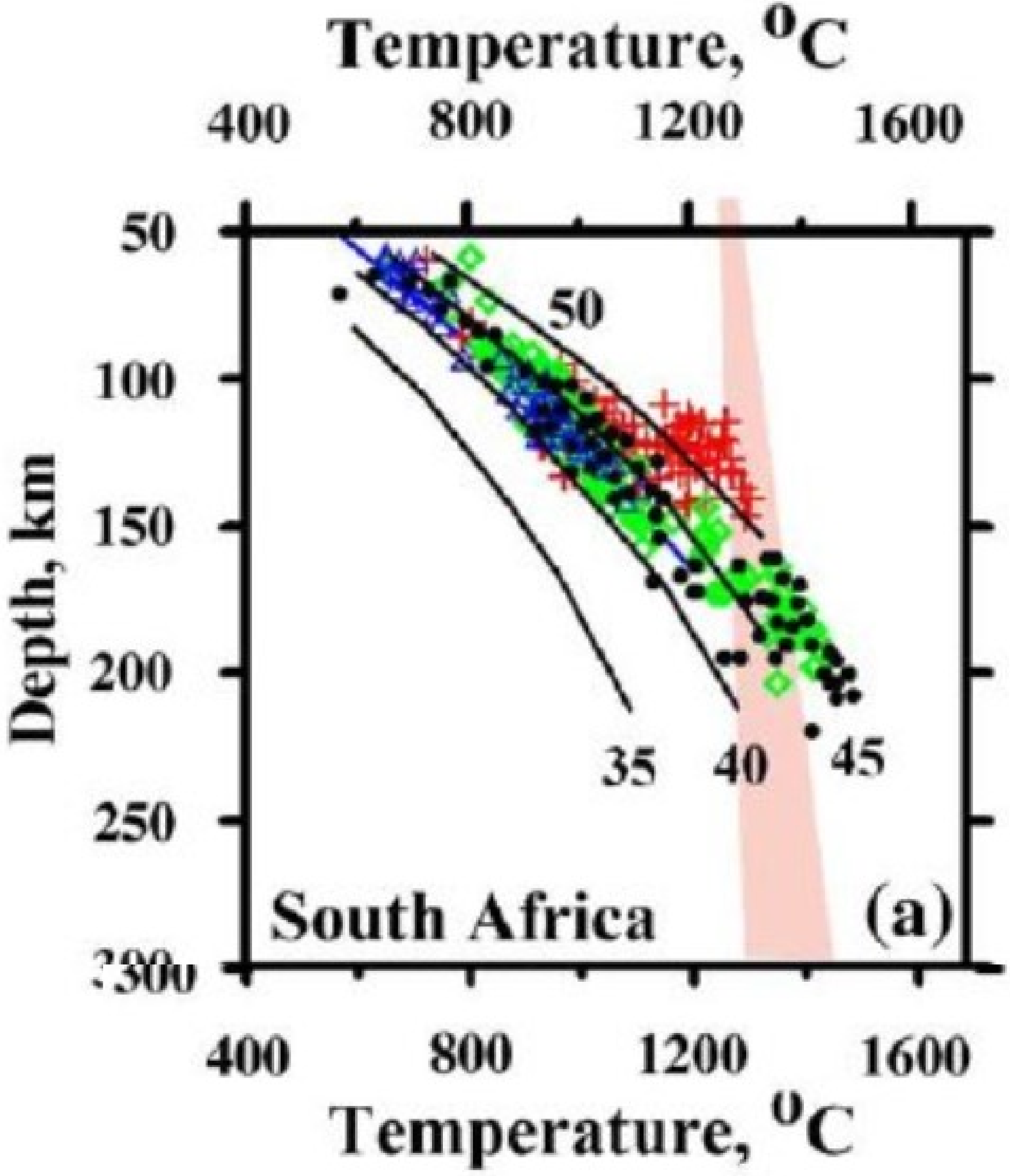
or can reasonably infer



(A) surface heat flow of q=0.06 W/m^2 implies hotter temperatures at depth, increasing by about 20 degC / km

$$\text{for } k = 3.1 \, \frac{W}{m°\text{C}}$$

$$\frac{d\Delta T}{dz} = q/k = 20\ °\text{C}/km$$



(B) geotherms constructed by P-T estimates on peridotite xenoliths indicate near-melting temperatures at 200 km depth. Yet they must roll over before completely melting the mantle, because volcanism is pretty rare.

$$L = 100 \text{ km}, \Delta T_0 = 100 \text{ °C}$$

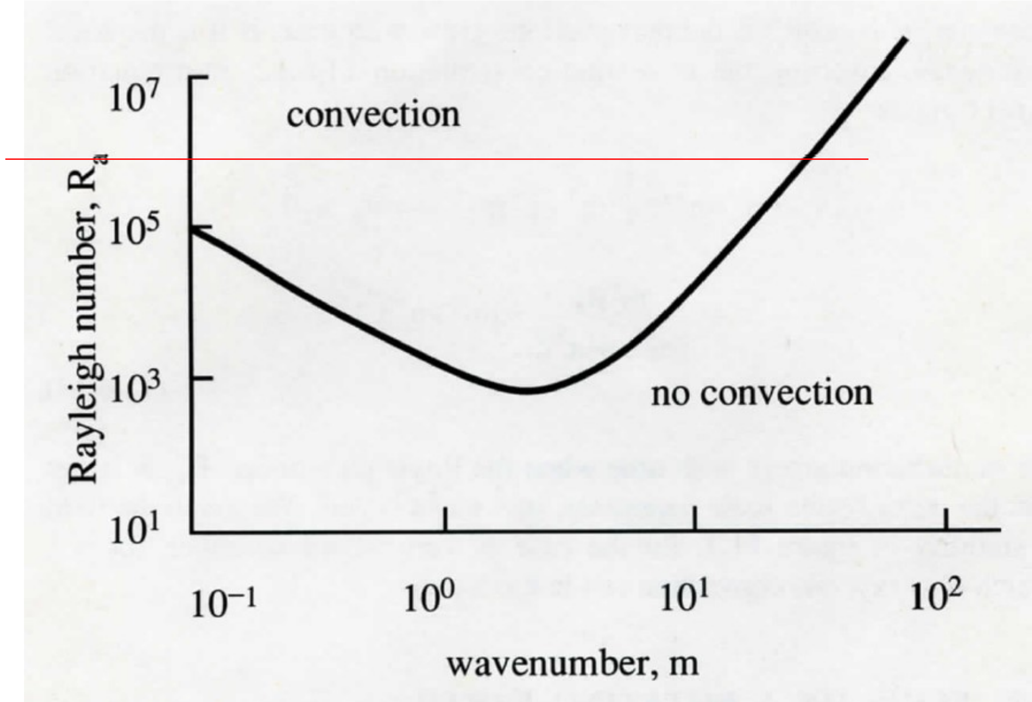
$$R_a = \frac{L^3 \rho_0 \alpha \Delta T_0 g}{\mu\kappa} = 170$$

$$L = 1000 \text{ km}, \Delta T_0 = 1000 \text{ °C}$$

$$R_a = \frac{L^3 \rho_0 \alpha \Delta T_0 g}{\mu\kappa} = 1{,}700{,}000$$

so interior of mantle likely tend to isothermal (actually adiabatic)

(C) Rayleigh number of mantle suggestive of convection



Detour

Adiabatic: material gets hotter as you compress it, cooler as you decompress it

air 8 deg C/km

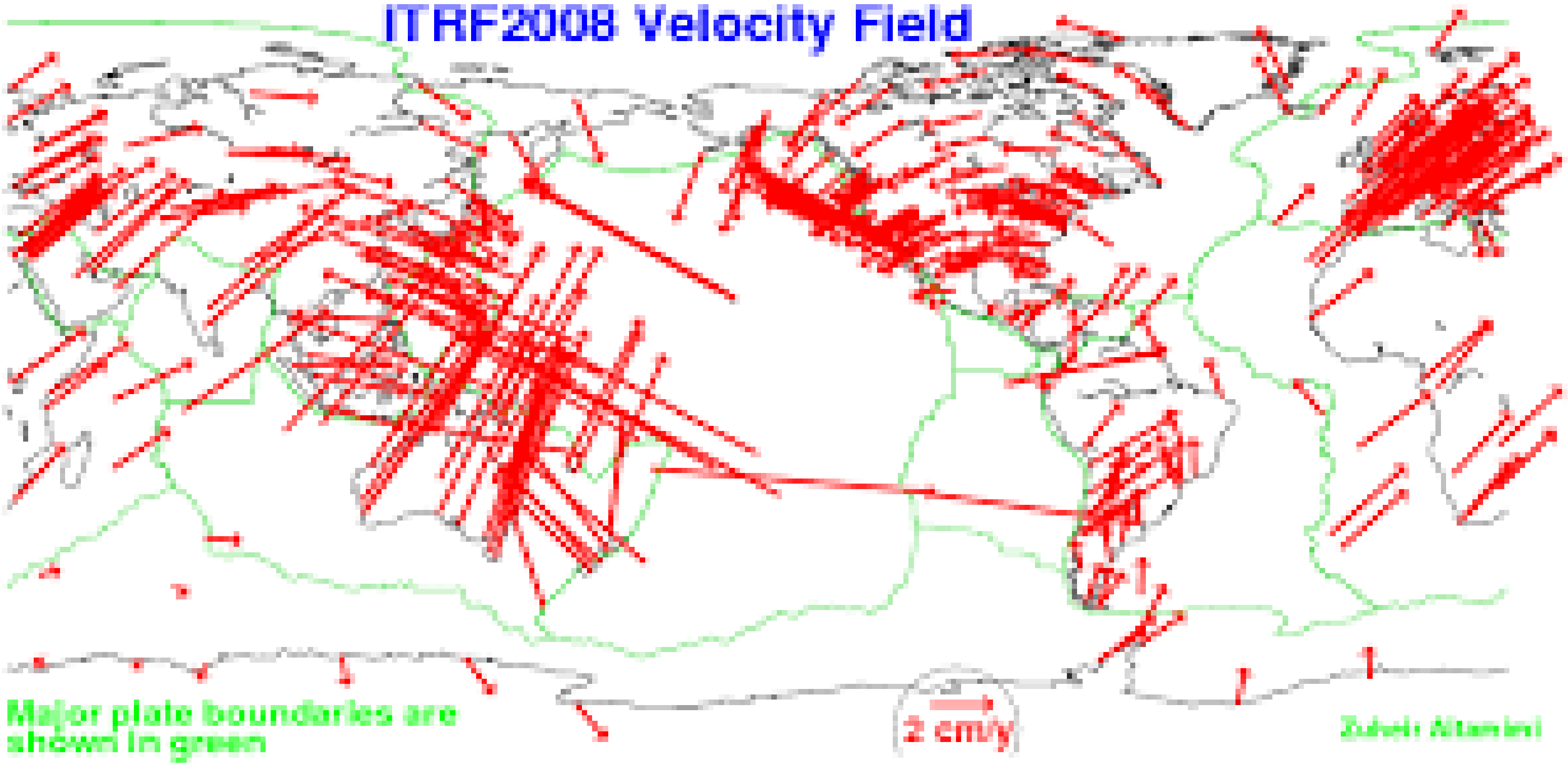
mantle 0.5 degC/km

4000 m
32 deg C

Temperature: what you measure with a thermometer placed at the depth of measurement

Potential temperature: what you would measure if you instantaneously brought the rock up to the Earth's surface

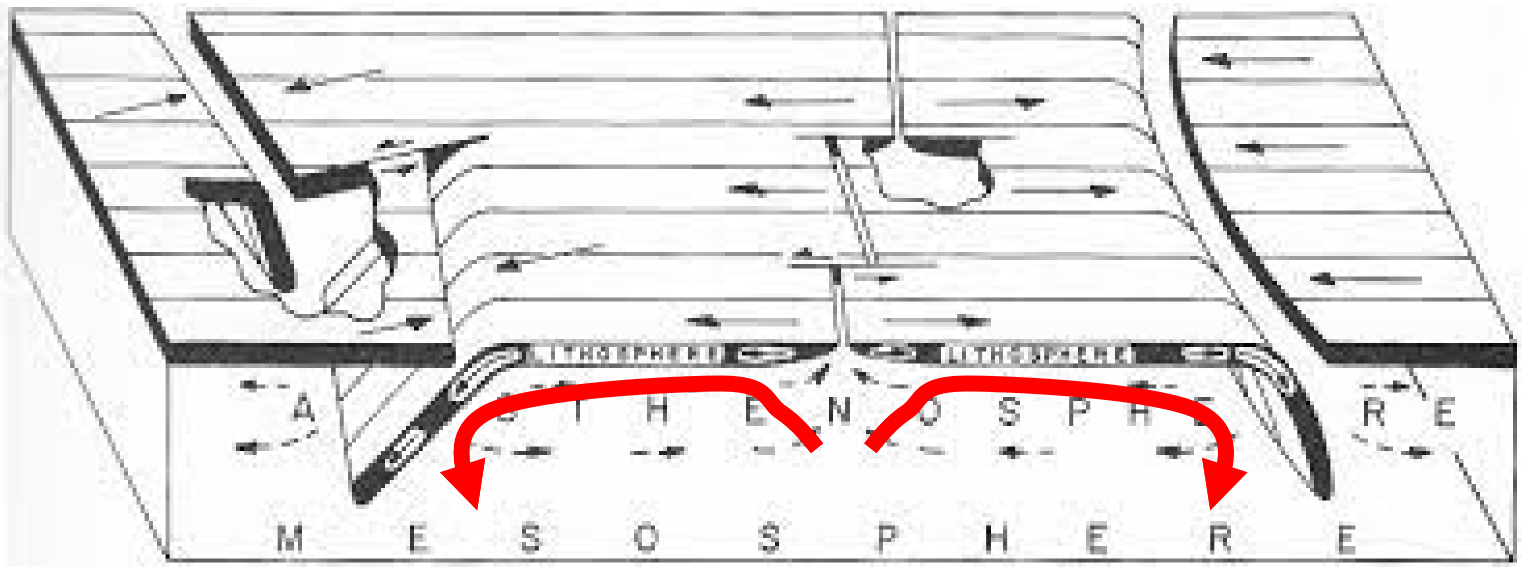
What we know (or can reasonably surmise)



Actual GPS measurements

(D) 5 cm/yr (or so) motion of the tectonic plates likely reflects mantle convection at similar rates

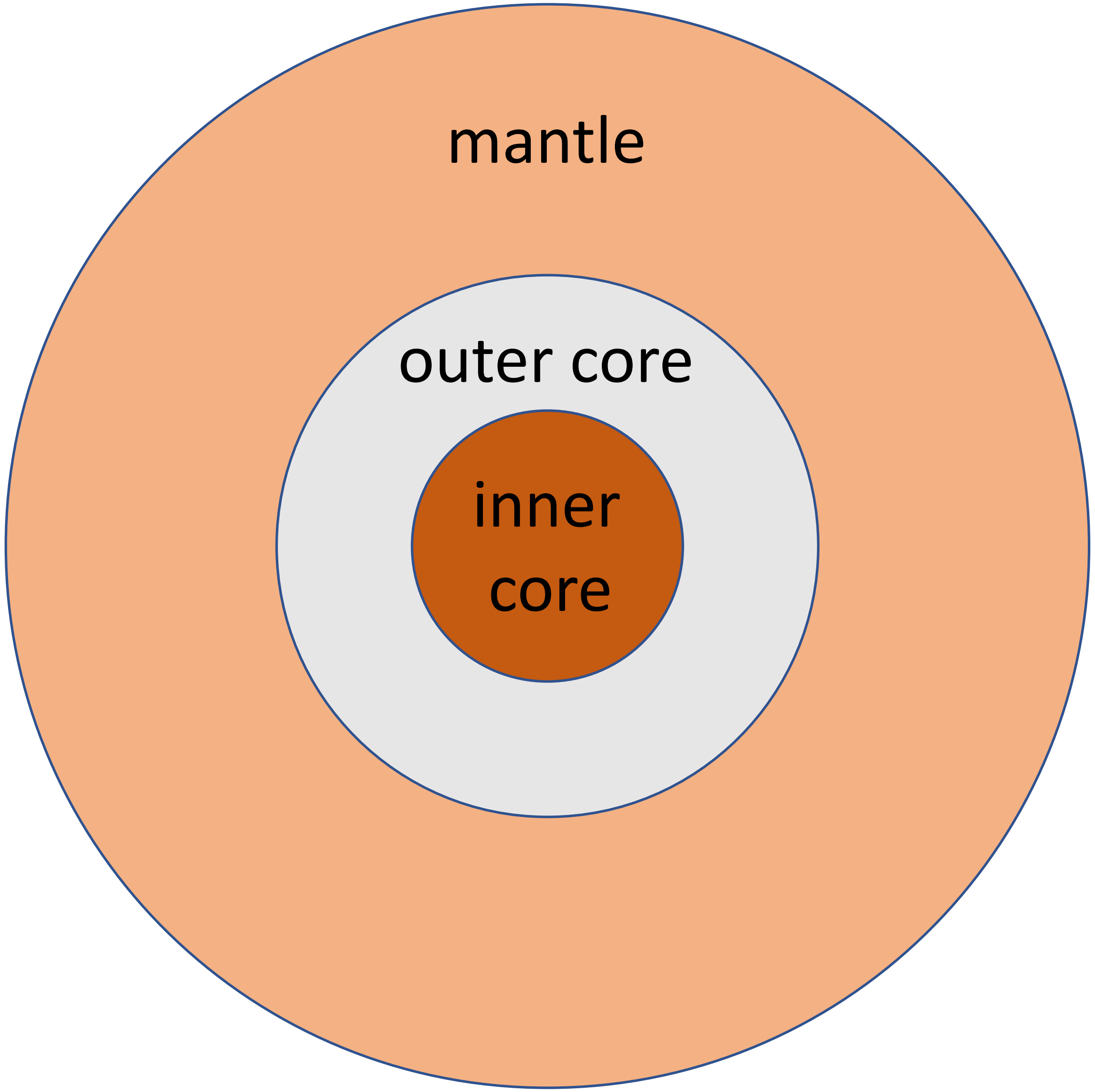
What we know (or can reasonably surmise)



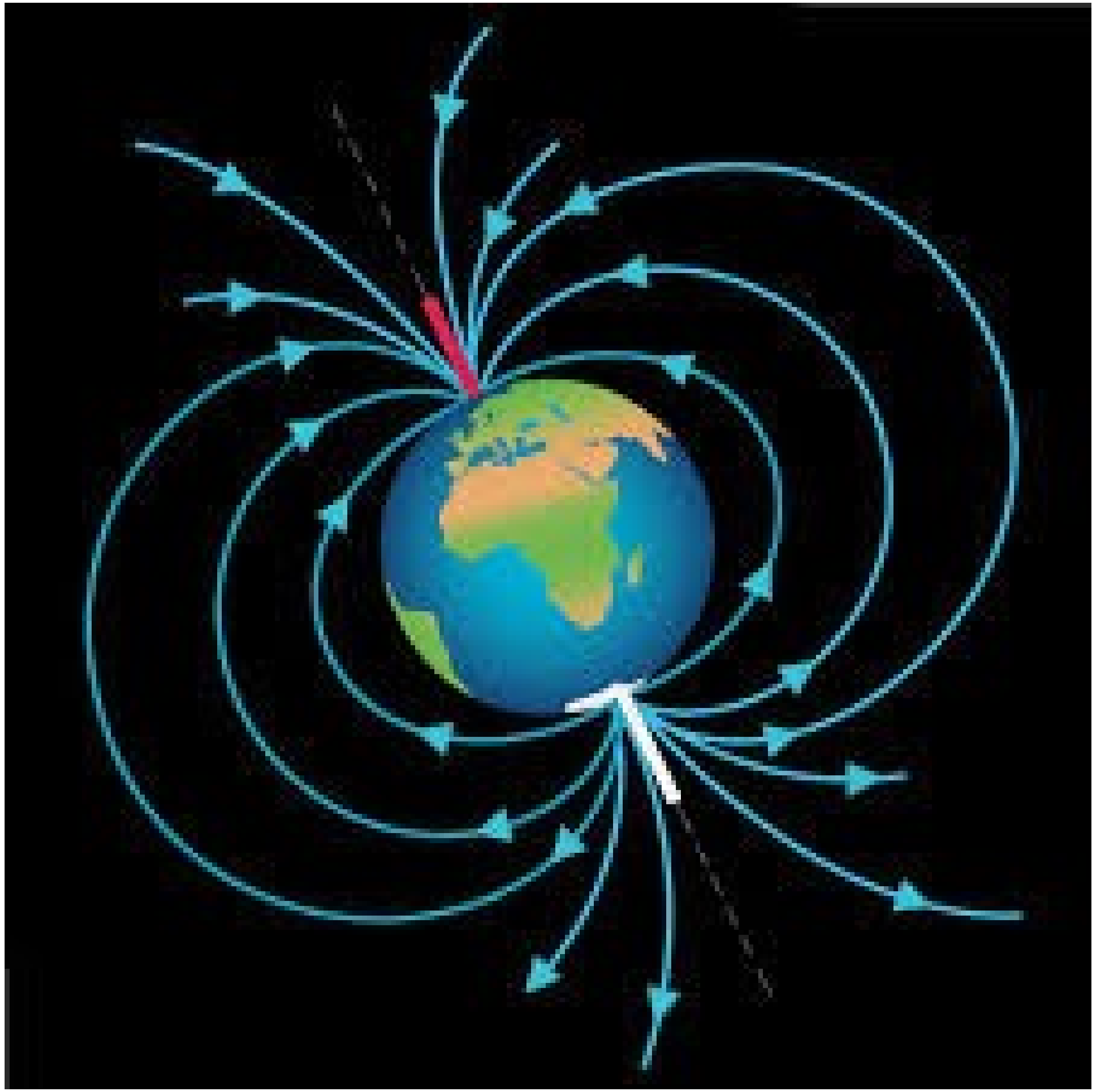
(E) Ridges are likely rising arms of convection cells because of divergent velocities
and volcanism

Subduction zones are likely downwelling zones due to convergent velcities and deep earthquakes (proxy for cold temps)

so at least some convective cells 10,000 km wide

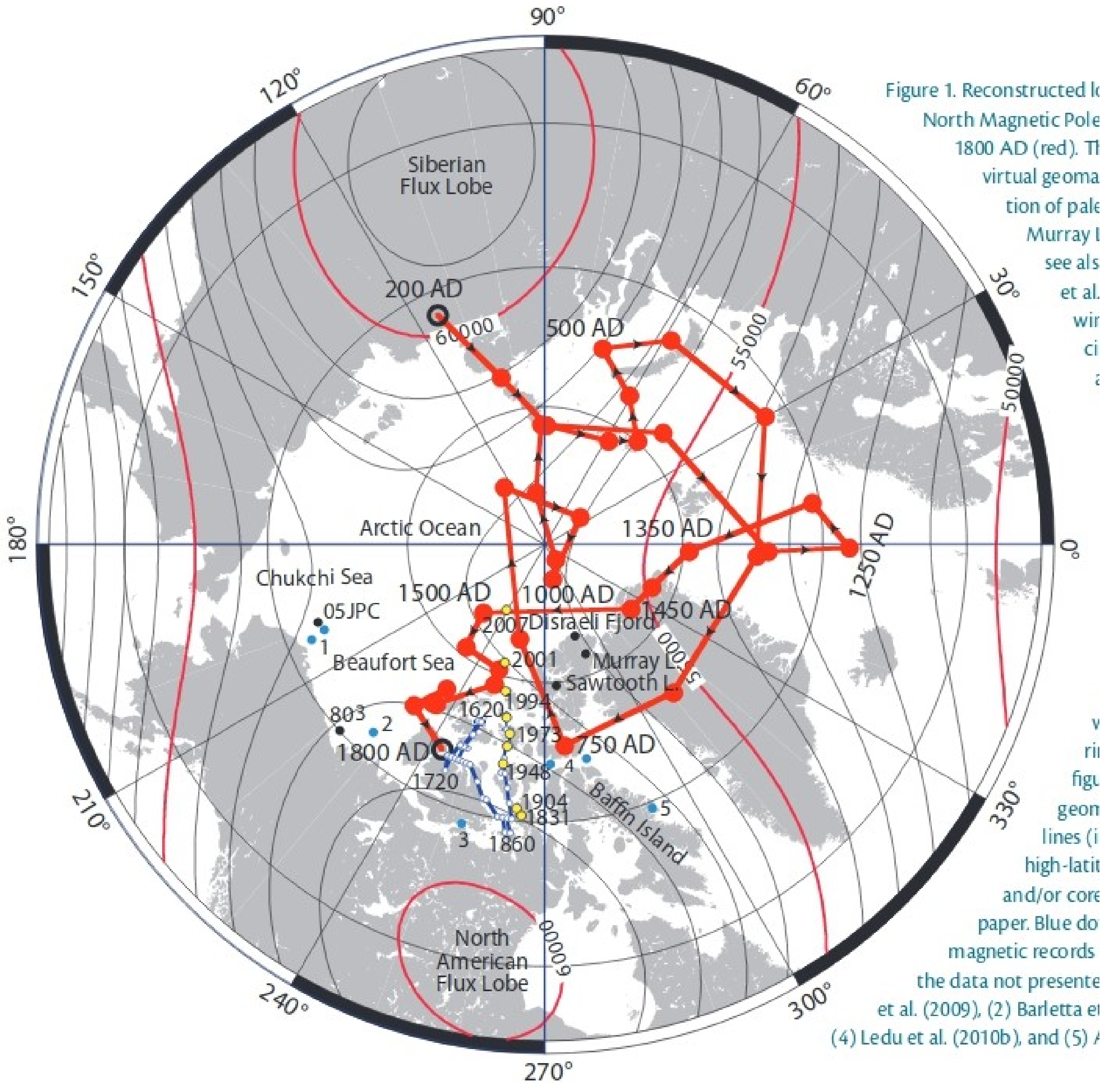


(F) Seismic character of outer core indicates it is fully molten



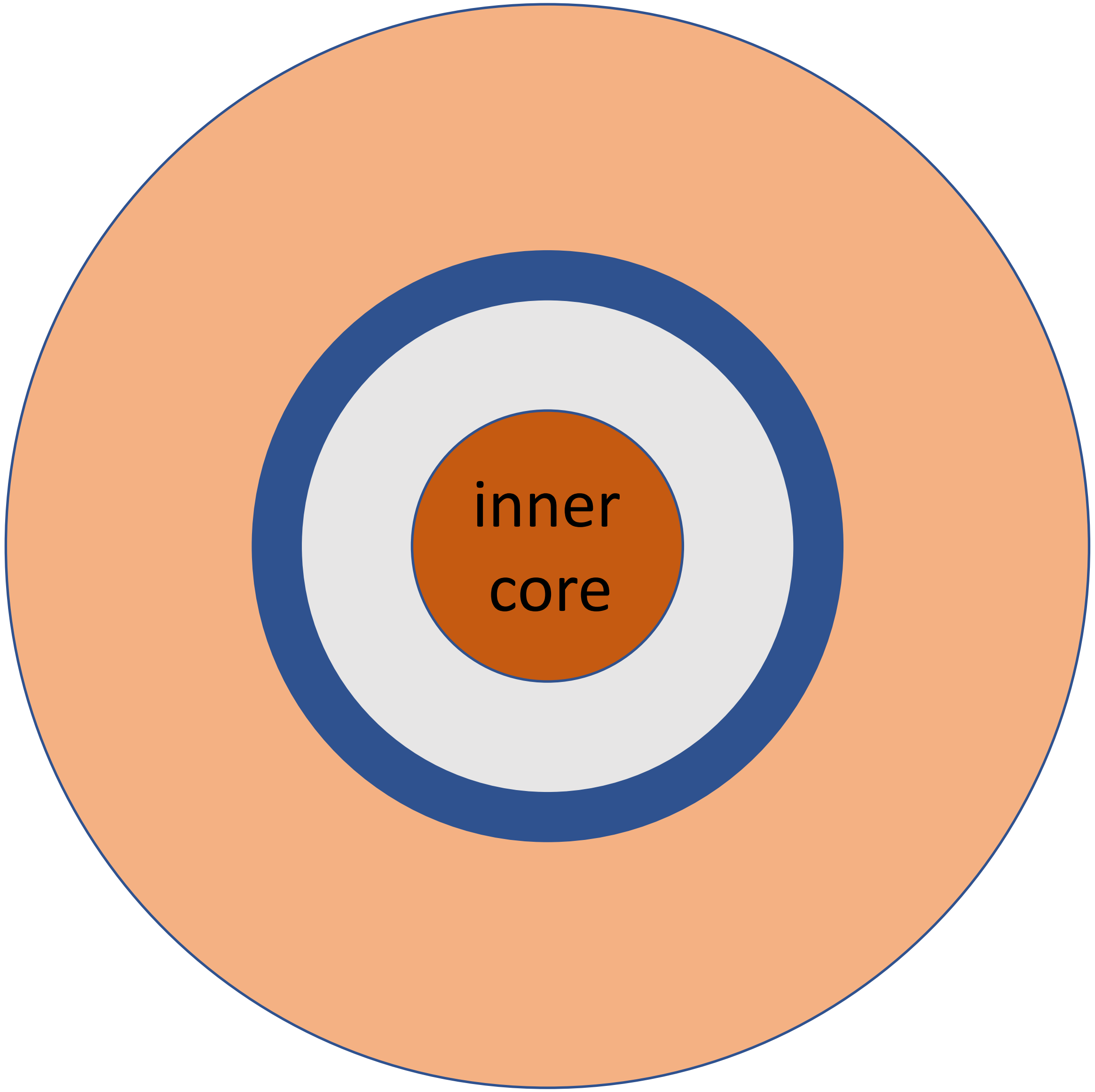
(F) Earth's magnetic field indicates core is electrically conductive

(G) Iron Meteroites indicate core suggests core is probably mostly iron



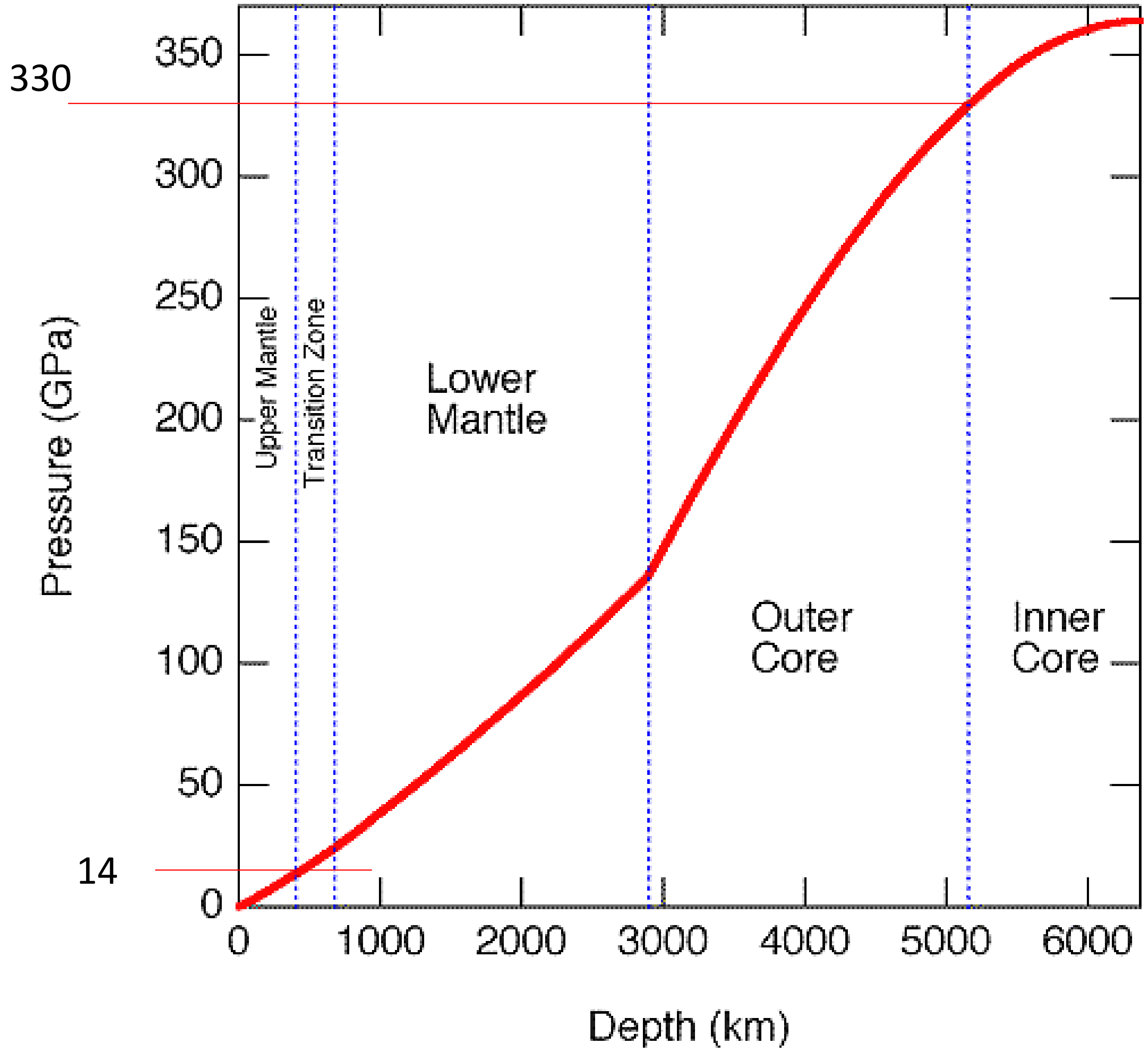
(H) Drift of magnetic pole suggests outer core convecting with velocities of ~10 km/year

So most of outer core is probably isothermal (really adiabatic) too

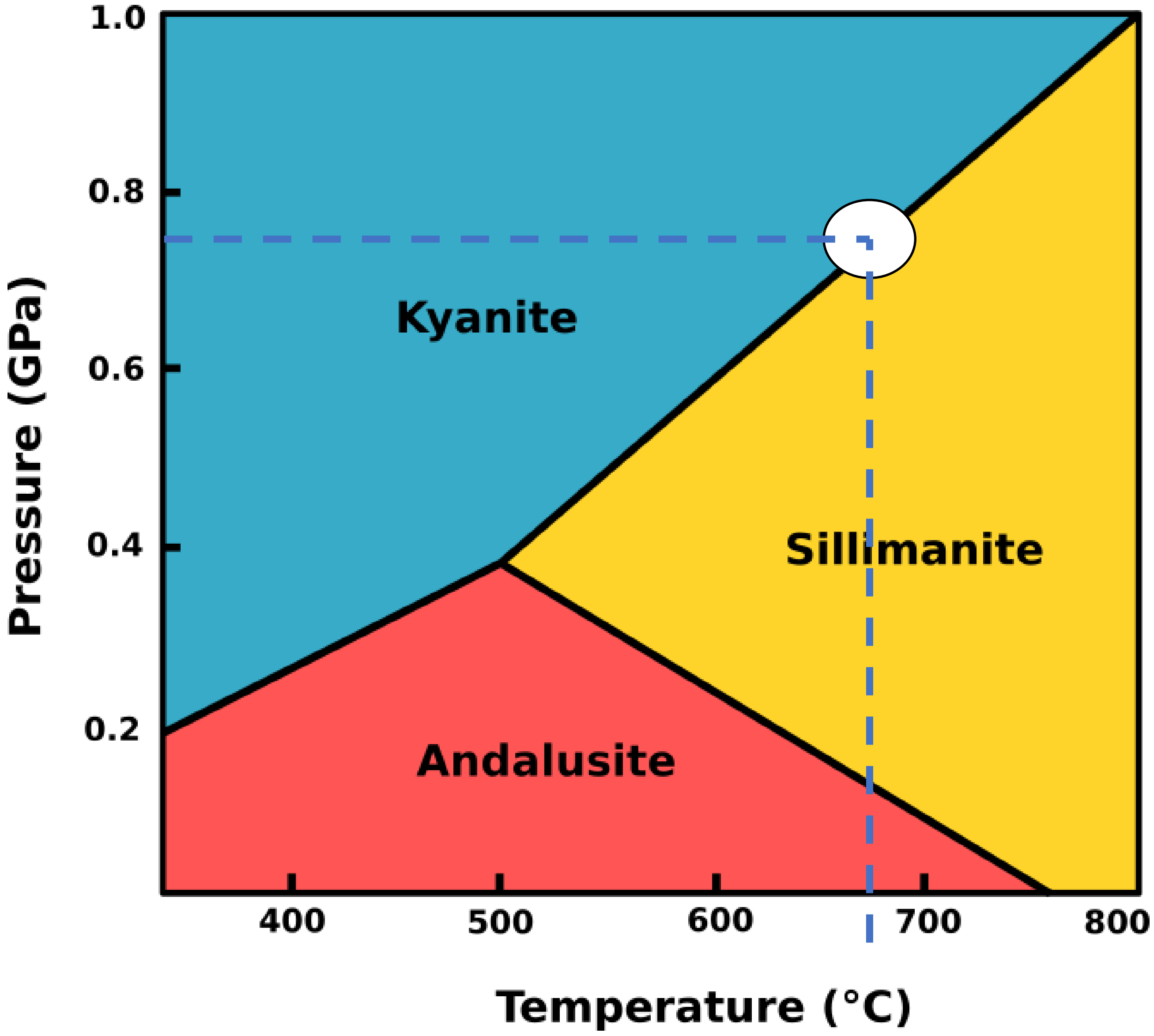


so core-mantle boundary likely has double boundary layer

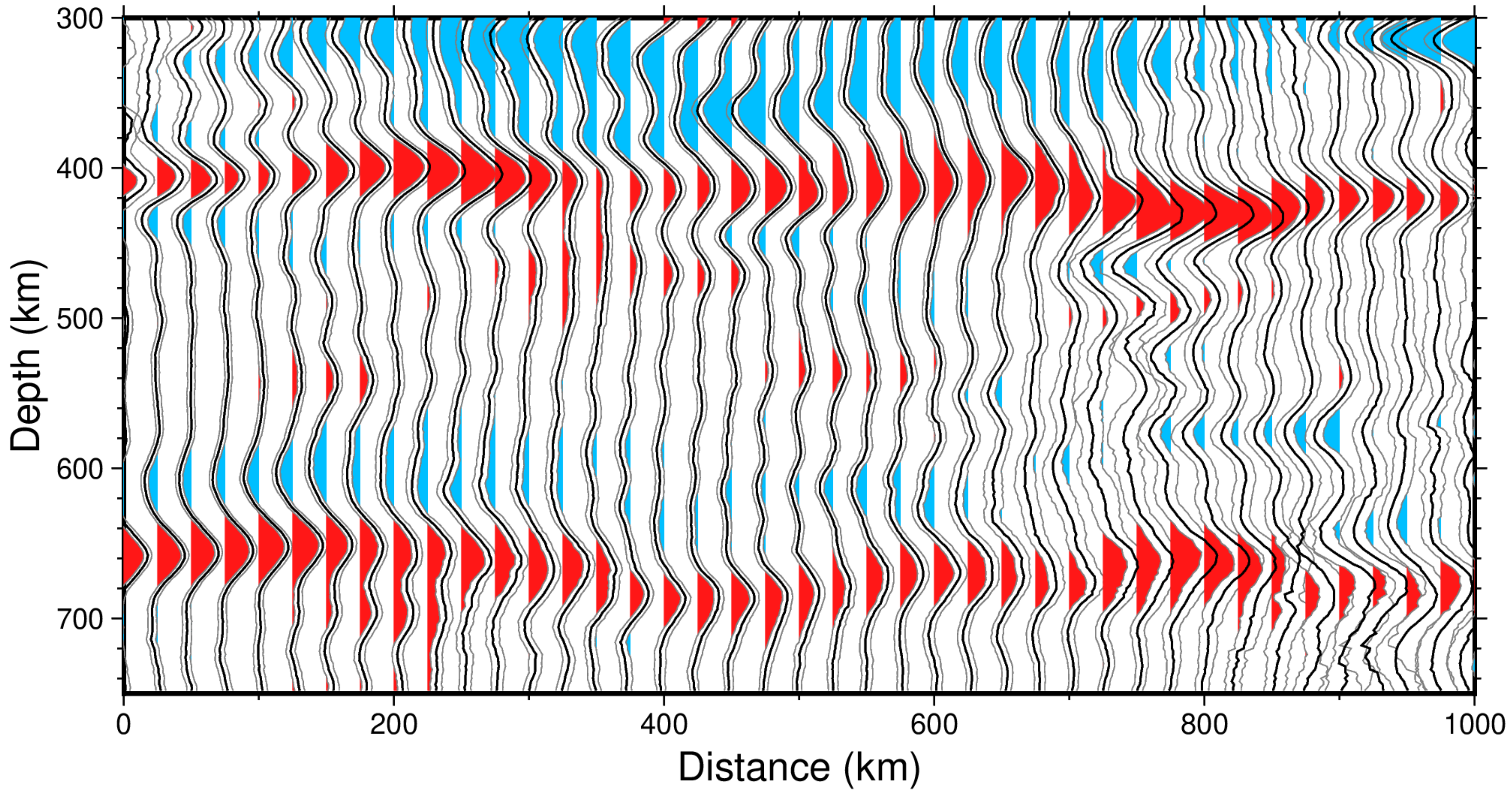
and a very big temperature jump



(I) Pressure vs depth can be very accurately predicted in the Earth, because it depends only on density and gravity

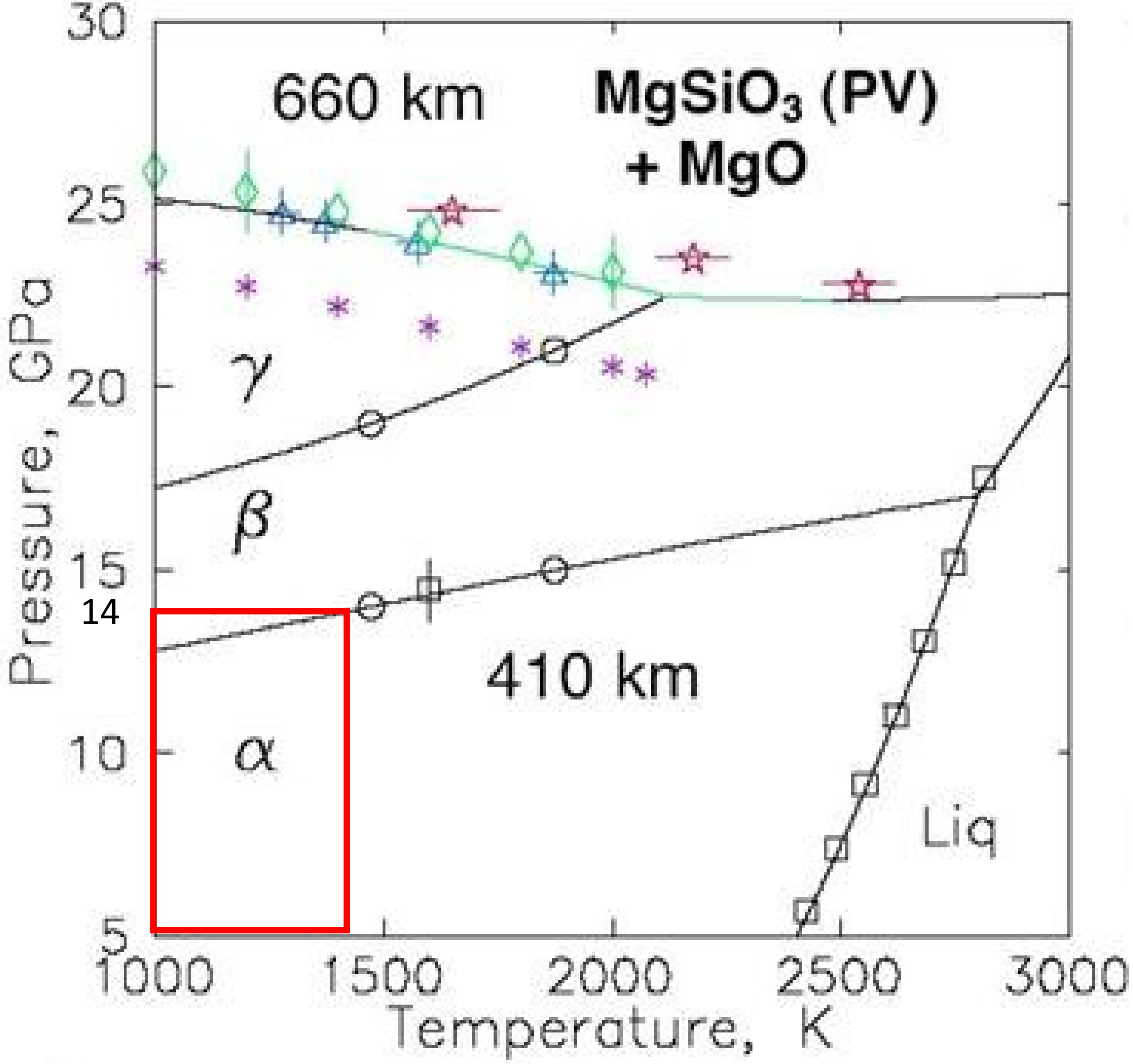


(J) If you can identify the depth of a phase boundary in the Earth, and you known the phase diagram, then you can infer the temperature, for the depth vs pressure curve is well known,



(K) two phase boundaries occur in the upper mantle at 410 and 670 km depth

Note: This diagram doesn't look correct to me

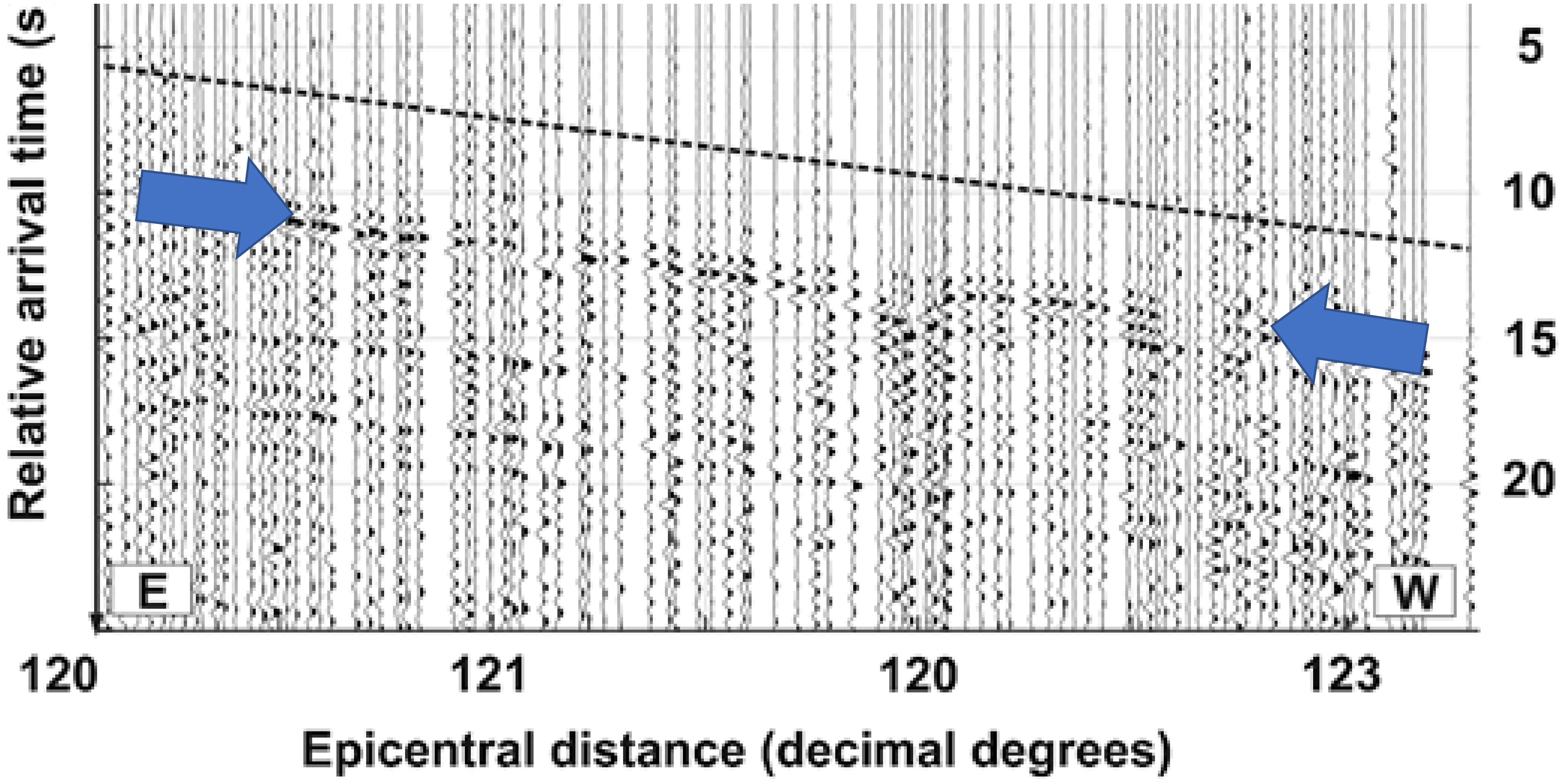


(L) The upper mantle phase diagram is not all that well known

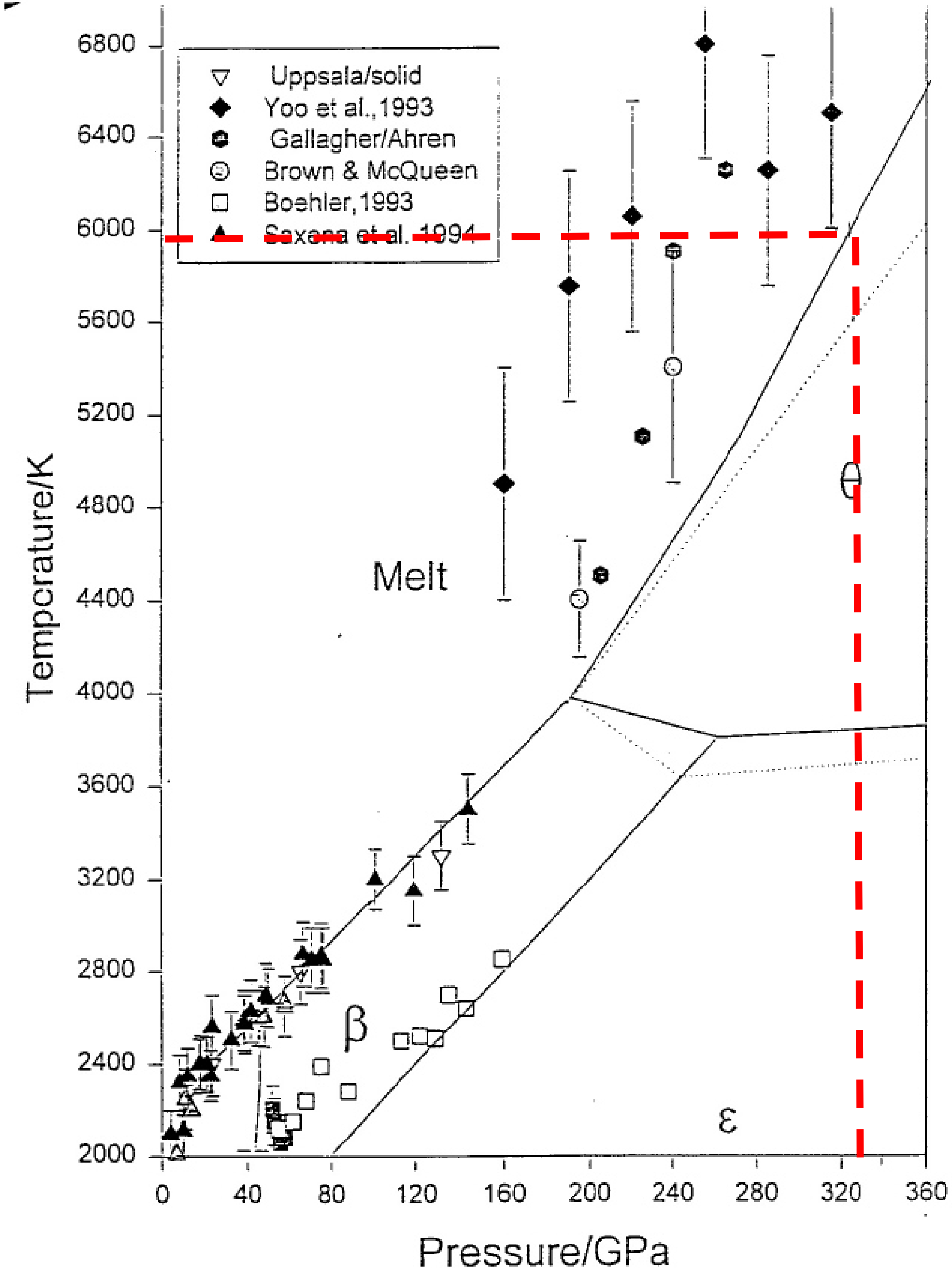
but 410 km implies 14 Gpa implies temp of ~1400 K

however one author calculates 1565C

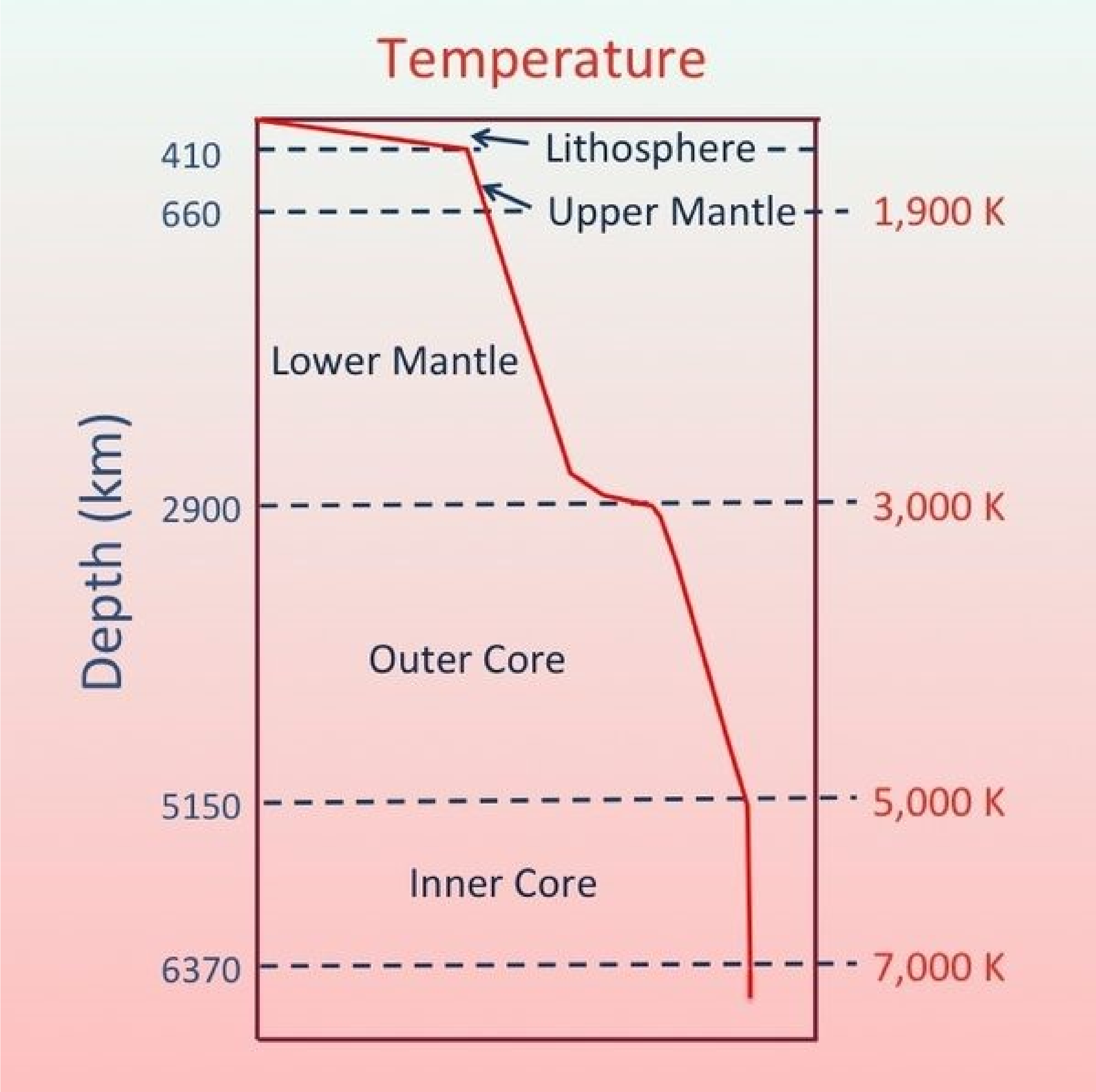
echo from inner core – outer core boundary.



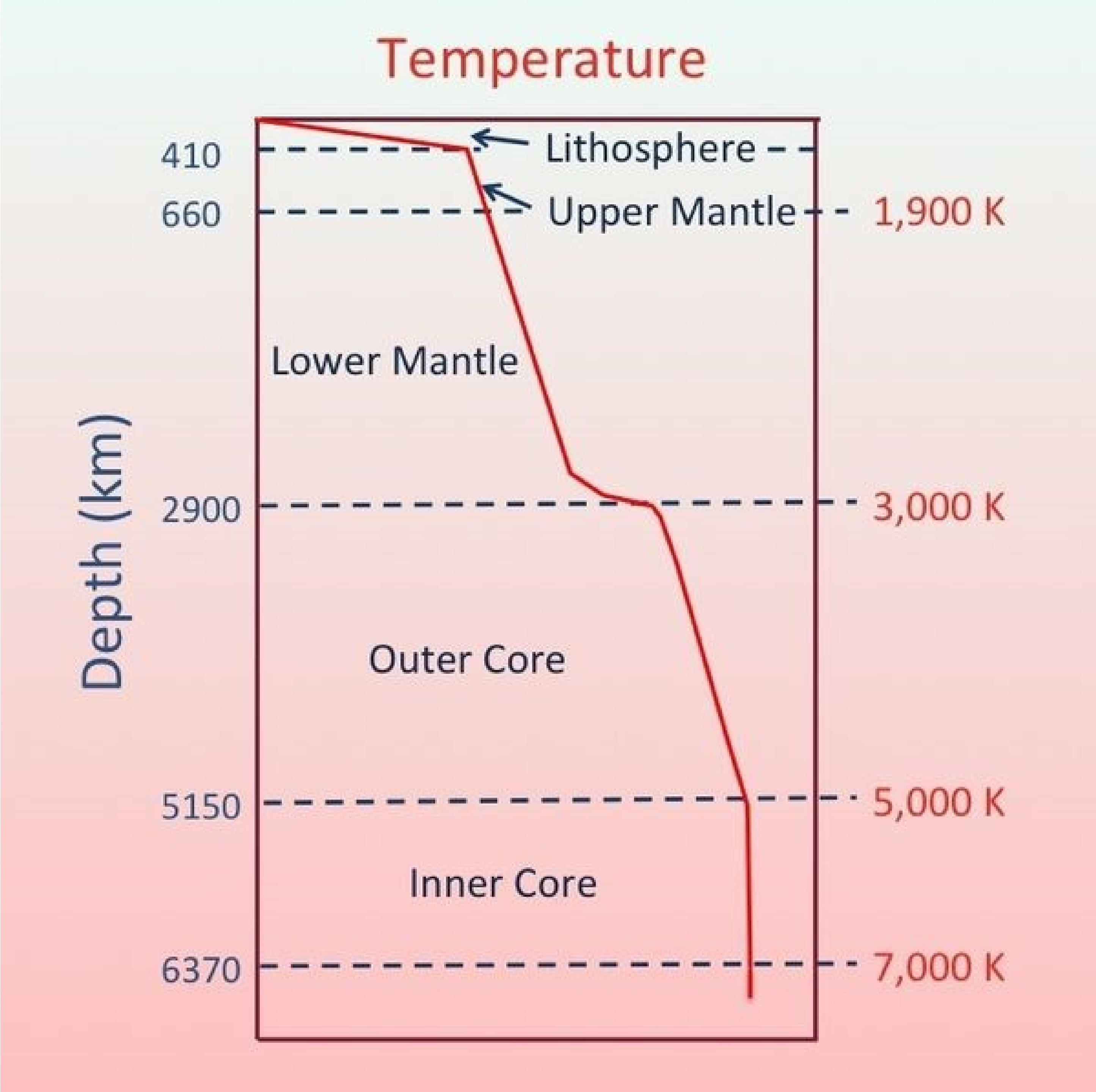
(M) the inner core – outer core boundary is thought to be the iron melt to solid iron phase boundary.



(N) pressure of 330 Gpa implies temperature of 6000 K (5730 C) at inner-core outer core boundary



Putting it all together



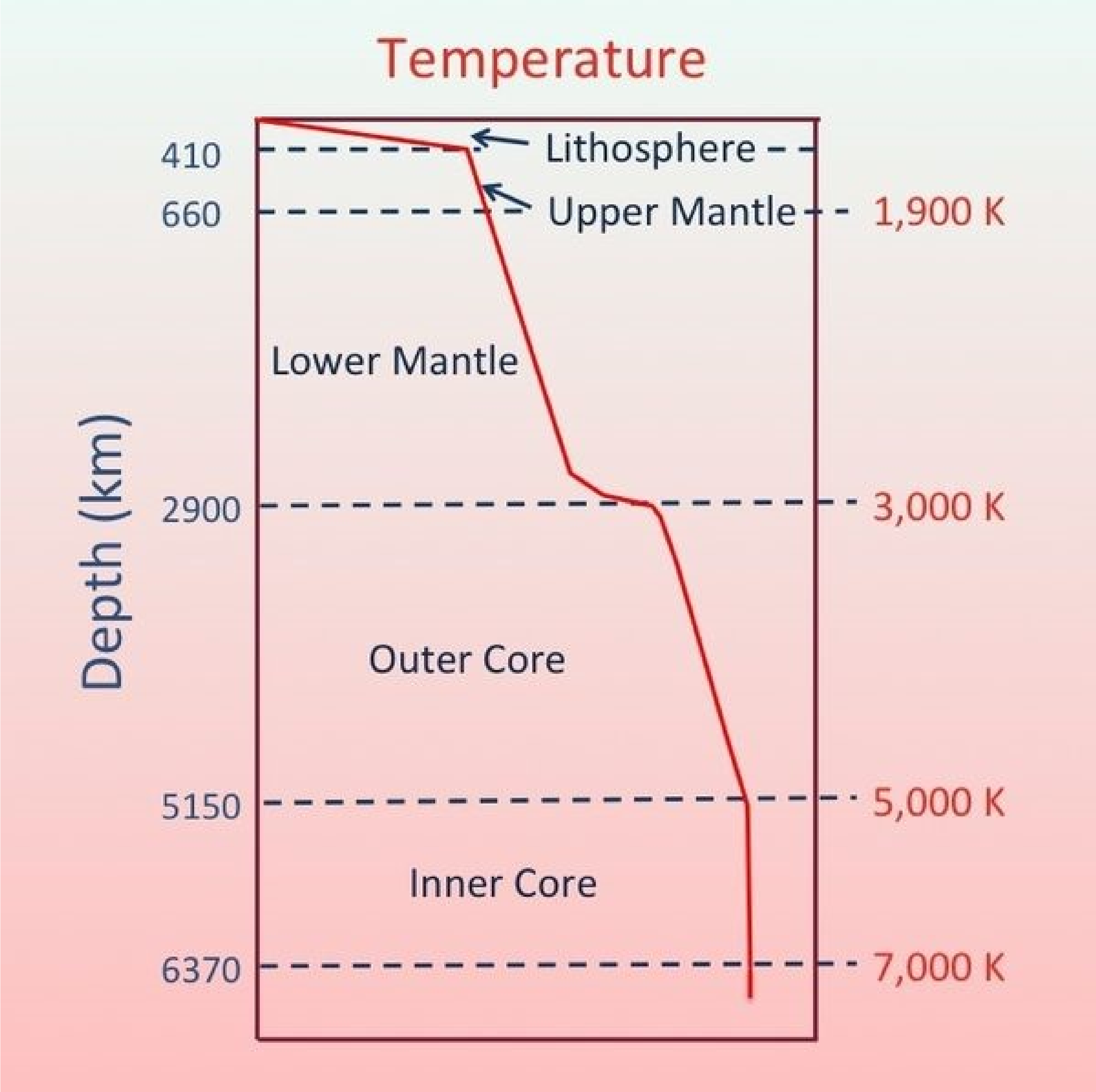
boundary layer

adiabatic

boundary layer

adiabatic layer

freezing front



boundary layer

adiabatic

boundary layer

adiabatic layer

freezing front

as earth cools
inner core grows