Solid Earth Dynamics

Bill Menke, Instructor

Lecture 9

Today

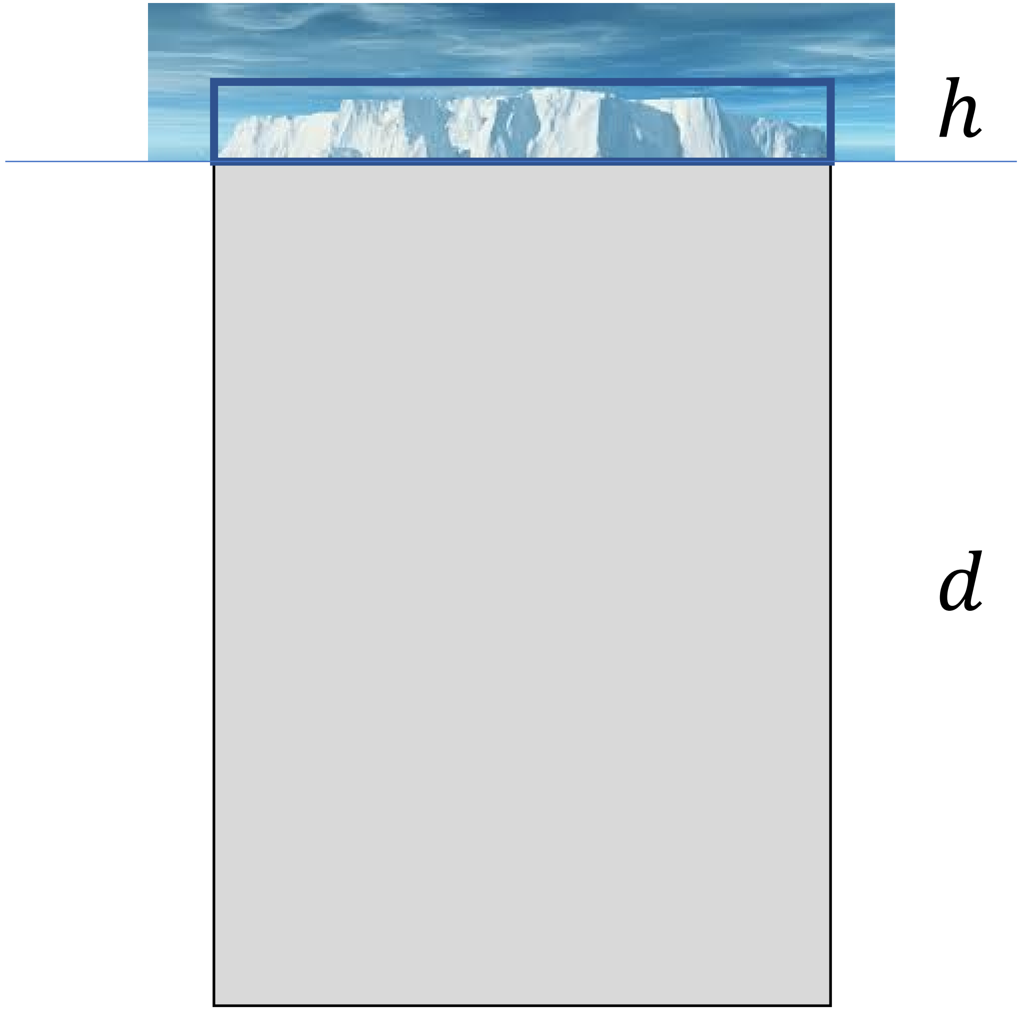
Icebergs, sedimentary
basins and isostasy

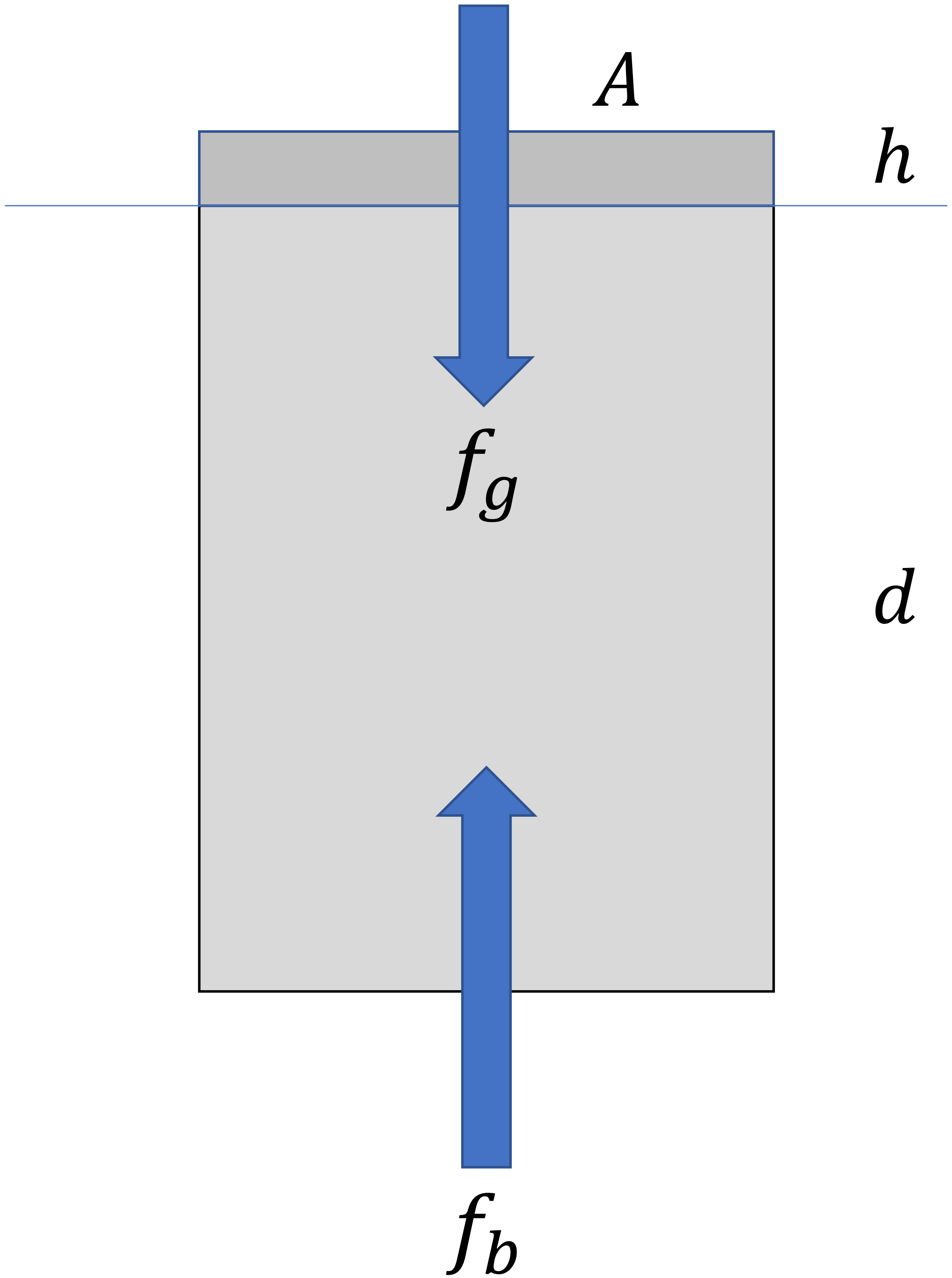
definition of gravity
anomalies

gravity of anomaly of an
isostatically supported mountain

Note

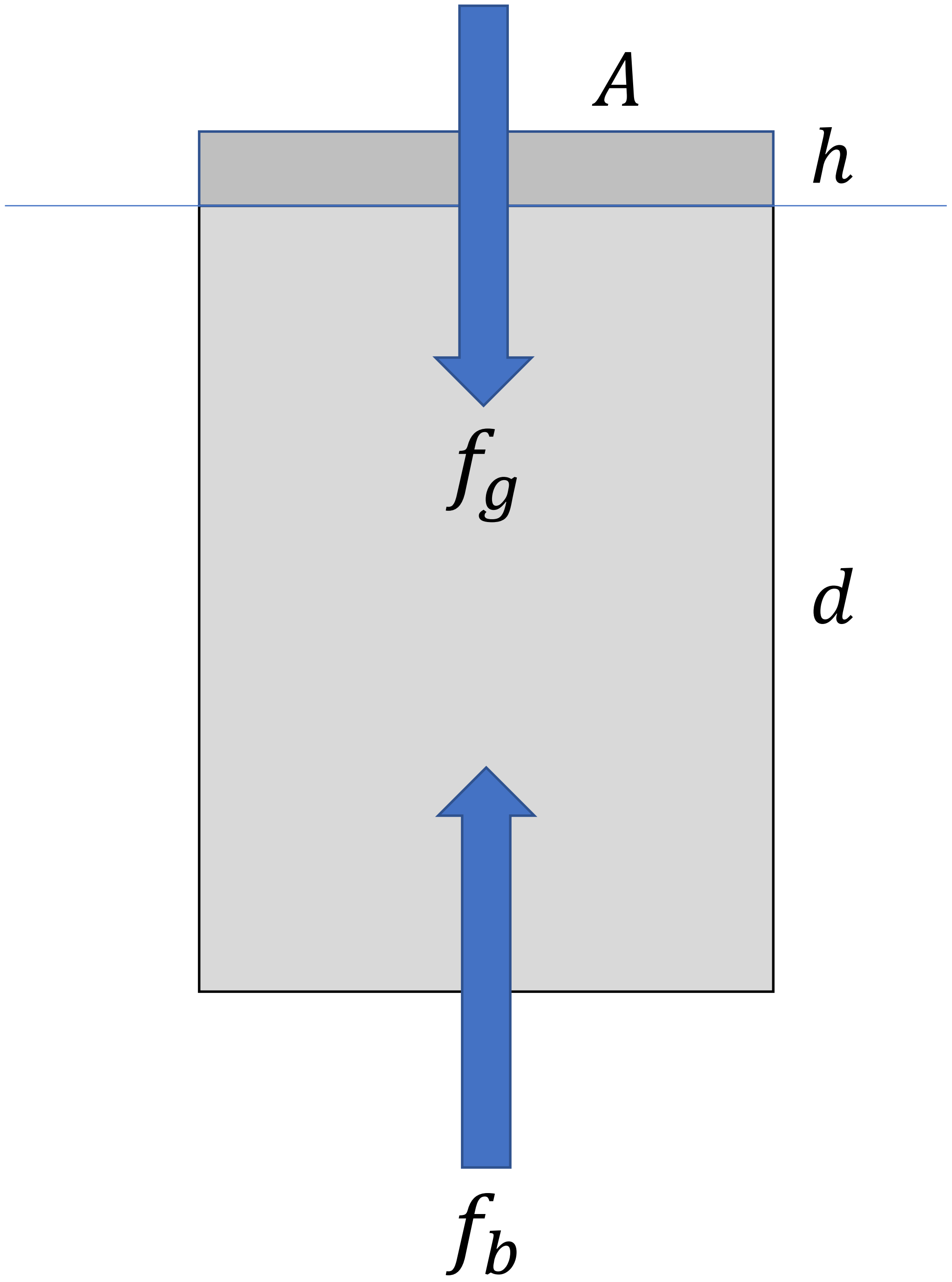
I changed some of the densities in the lecture from what I presented in class to values that I thought more accurate


h
d



balance of forces

$$f_b + f_g = 0$$

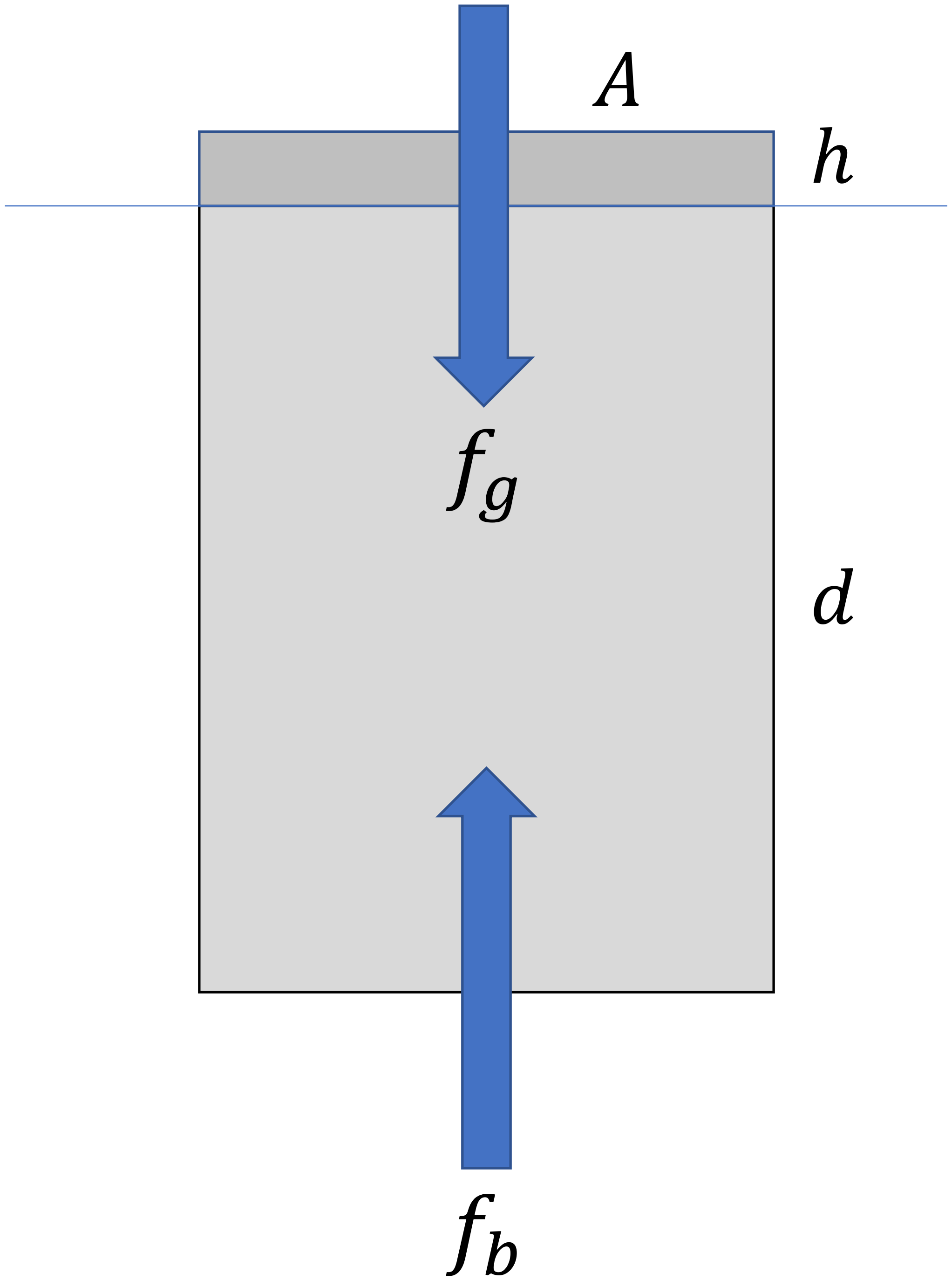


balance of forces

$$f_b + f_g = 0$$

$$f_g = -\rho_{ice} g A h$$

$$f_b = (\rho_{water} - \rho_{ice})\, g A \mathrm{d}$$

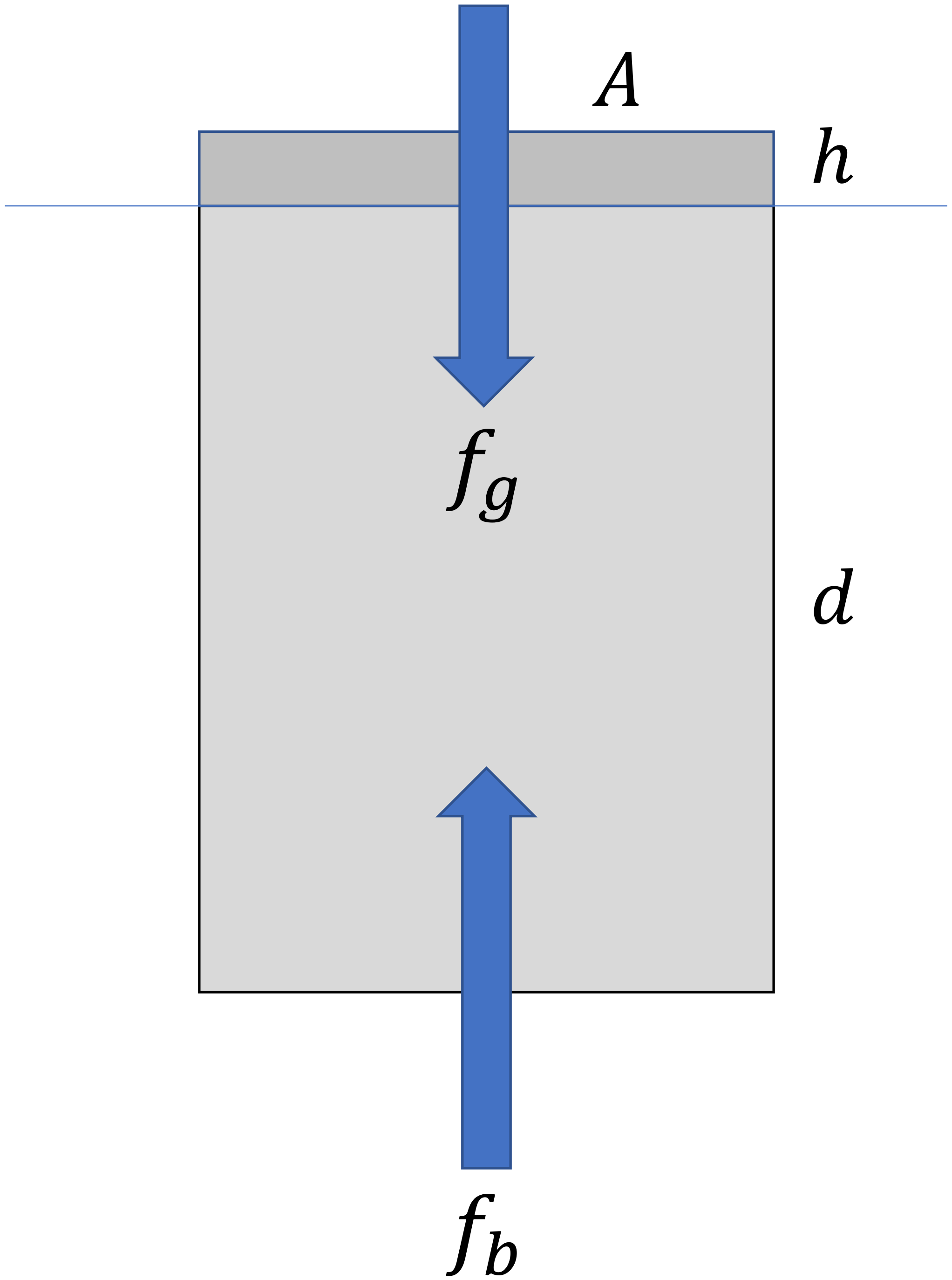


balance of forces

$$f_b + f_g = 0$$

$$f_g = -\rho_{ice} g A h$$

$$f_b = \Delta\rho g A d$$



balance of forces

$$f_b + f_g = 0$$

$$\Delta\rho g A d = \rho_{ice} g A h$$

$$d = \frac{\rho_{ice}}{\Delta\rho} h$$

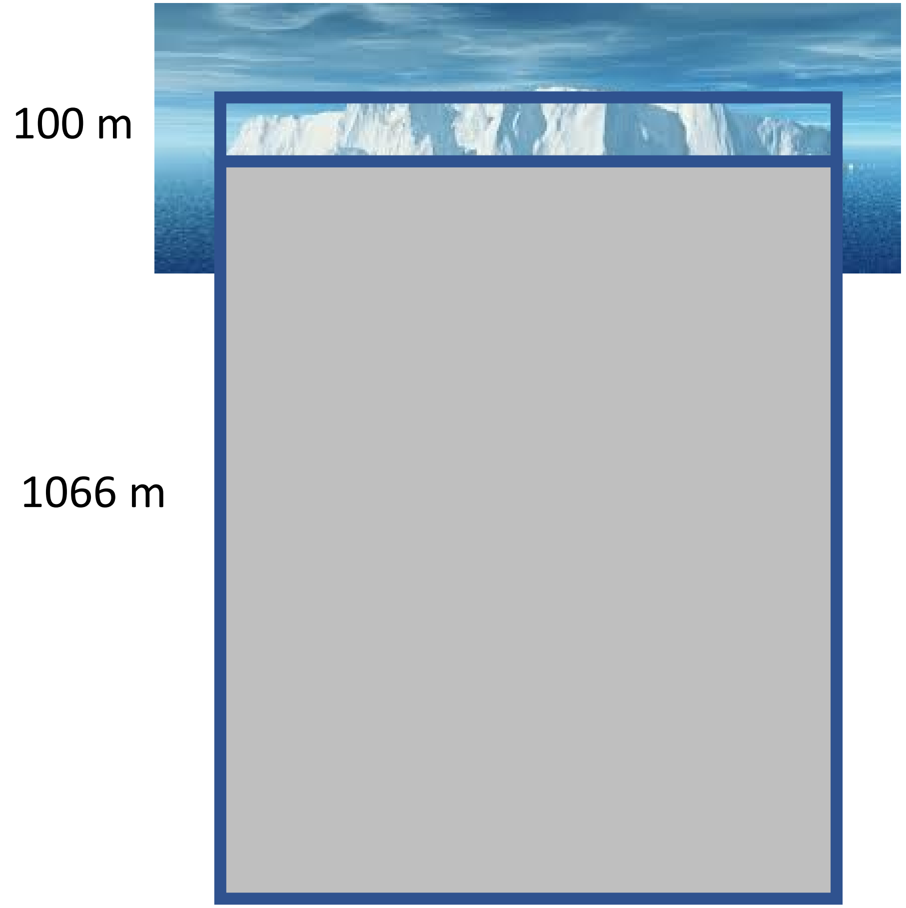
$$d = \frac{\rho_{ice}}{\Delta\rho} h$$

$$\rho_{ice} = 917 \frac{kg}{m^3}$$

$$\rho_{sea\,water} = 1003 \frac{kg}{m^3}$$

$$\Delta\rho = 86 \frac{kg}{m^3}$$

$$\frac{\rho_{ice}}{\Delta\rho} = \frac{917}{86} = 10.66$$



$$d = \frac{\rho_{ice}}{\Delta\rho} h$$

$$\rho_{ice} = 917 \frac{kg}{m^3}$$

$$\rho_{sea\,water} = 1003 \frac{kg}{m^3}$$

$$\Delta\rho = 86 \frac{kg}{m^3}$$

$$d = \frac{\rho_{ice}}{\Delta\rho} h = 1066\, m$$

Floating Board Experiment

$$\rho_{oak} = 800\,\frac{kg}{m^3} \quad \rho_{water} = 1003\,\frac{kg}{m^3} \quad \frac{\Delta\rho}{\rho} = 0.20$$

Analog to Wet Clay Sediment on Granite

$$\rho_{sed} = 2100\,\frac{kg}{m^3} \quad \rho_{grante} = 2500\,\frac{kg}{m^3} \quad \frac{\Delta\rho}{\rho} = 0.20$$

Do Experiment

6 Boards:

Each Board: 4 km of Wet Clay

leaf on top of first board

$$d = \frac{\rho_{sed}}{\Delta\rho} h$$

$$h = \frac{\Delta\rho}{\rho_{sed}} d$$

$$d + h = L$$

$$d = L - h$$

$$d = L - \frac{\Delta\rho}{\rho_{sed}} d$$

$$d + \frac{\Delta\rho}{\rho_{sed}} d = L$$

$$\left(1 + \frac{\Delta\rho}{\rho_{sed}}\right) d = L$$

$$d = L\left(1 + \frac{\Delta\rho}{\rho_{sed}}\right)^{-1}$$

6 Boards:

Each Board: 4 km of Wet Clay

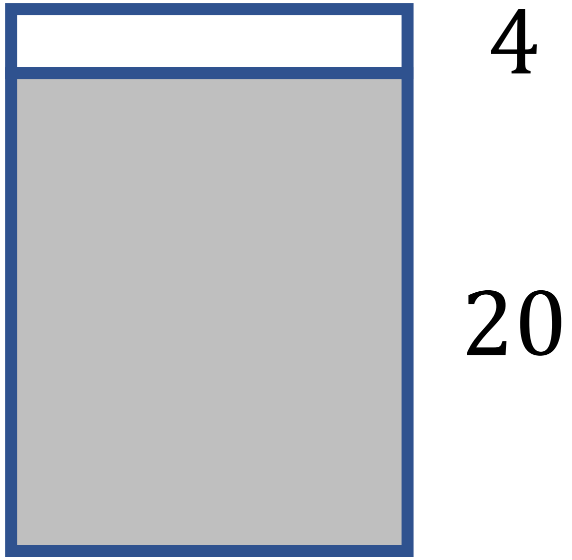
leaf on top of first board

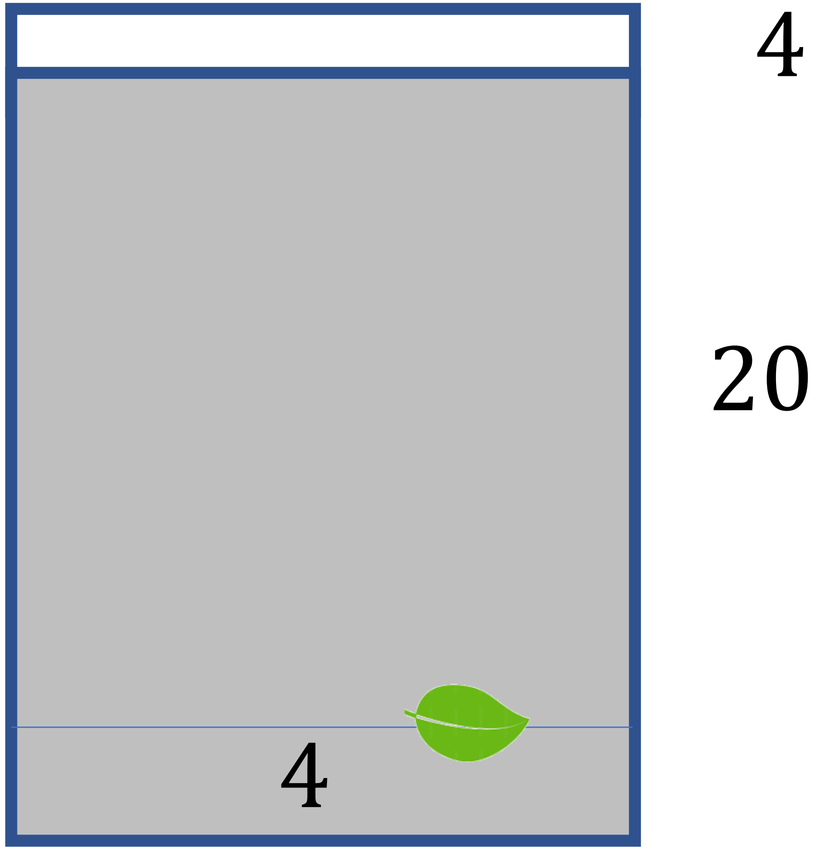
$$d = \frac{\rho_{sed}}{\Delta\rho} h = L\left(1 + \frac{\Delta\rho}{\rho_{sed}}\right)^{-1} = 5.25h = 0.84L$$

$$d + h = 24$$

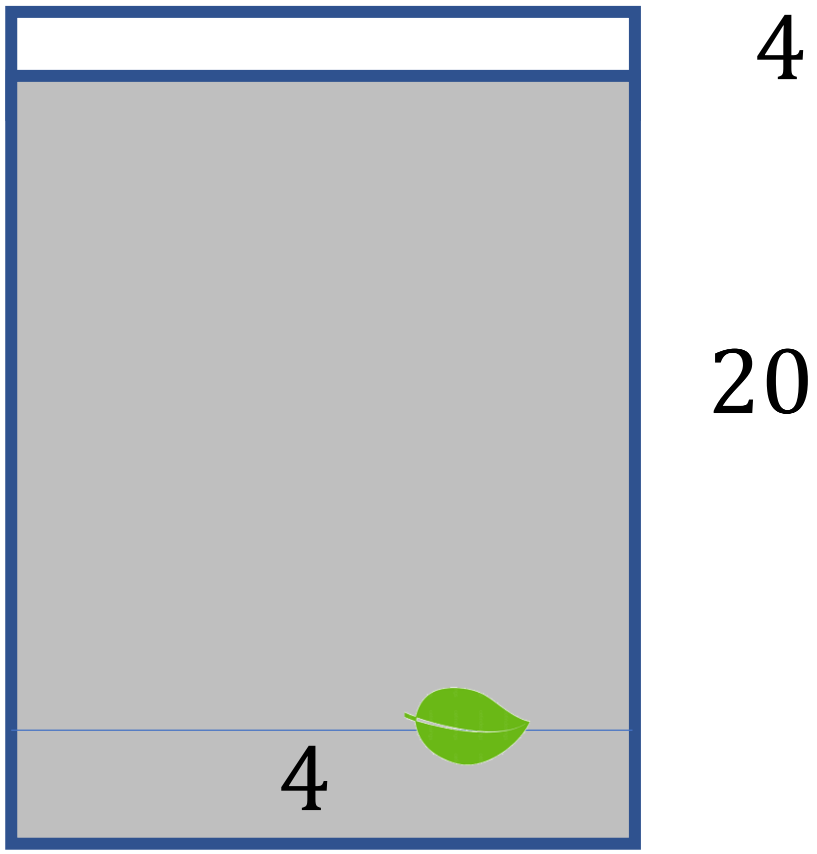
$$d = 20$$

$$h = 4$$



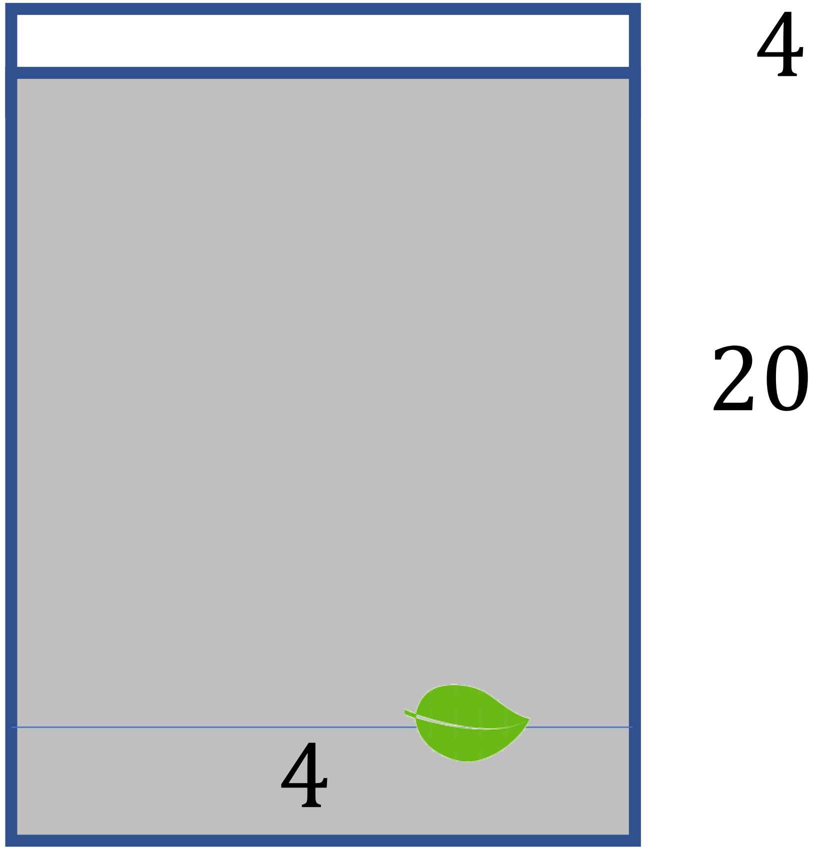


How deep did the leaf get?



How deep did the leaf get?

$$20 - 4 = 16 \text{ km}$$

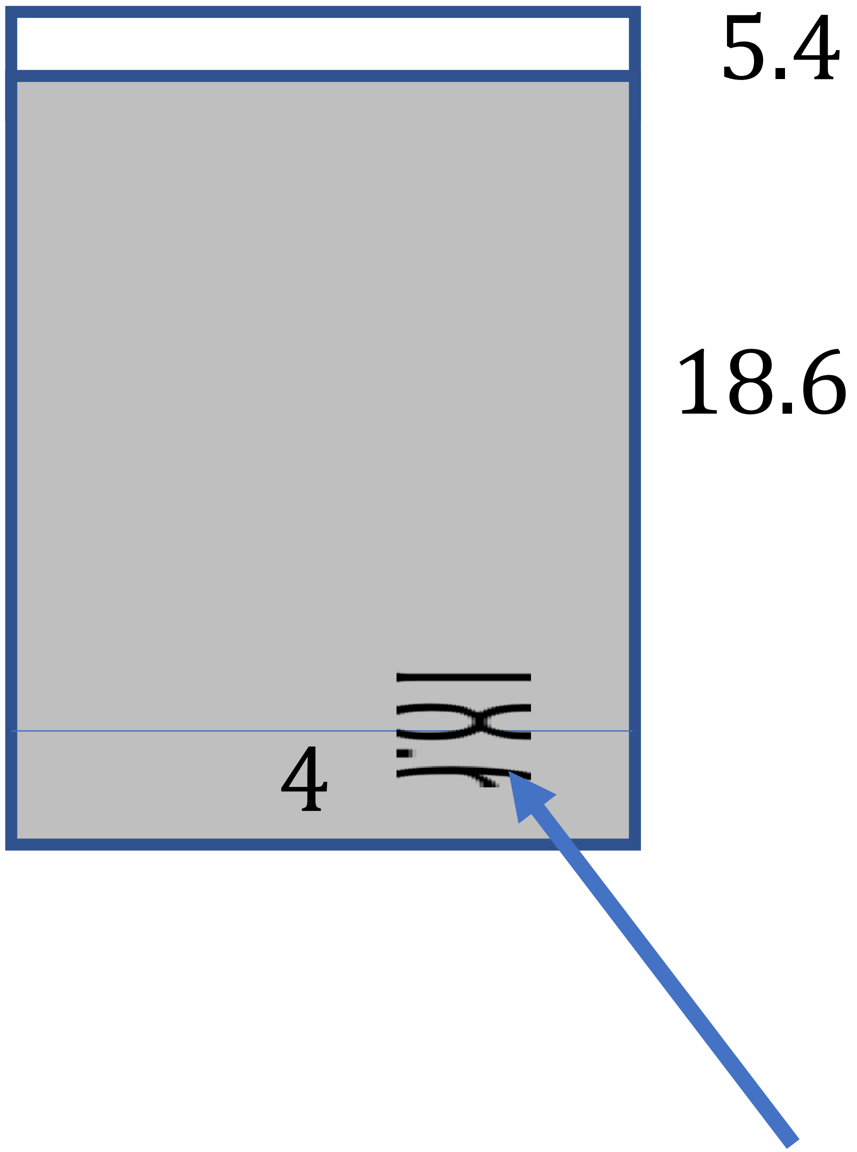


How deep did the leaf get?

$$20 - 4 = 16\ km$$

How hot was it there?

geothermal gradient 20 degC/km



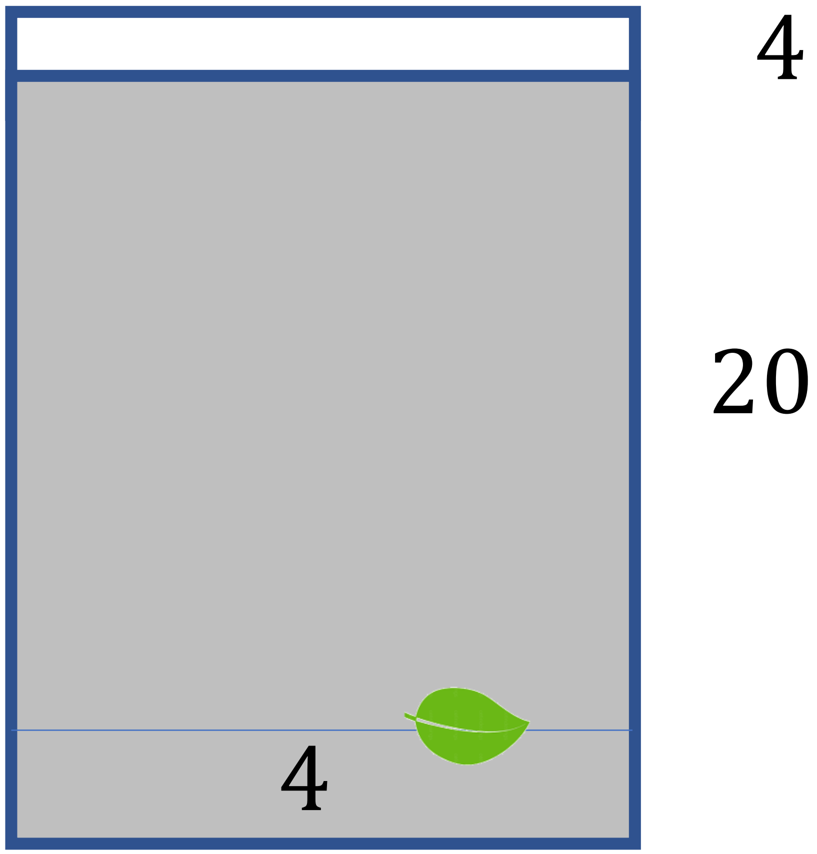
think coal

How deep did the leaf get?

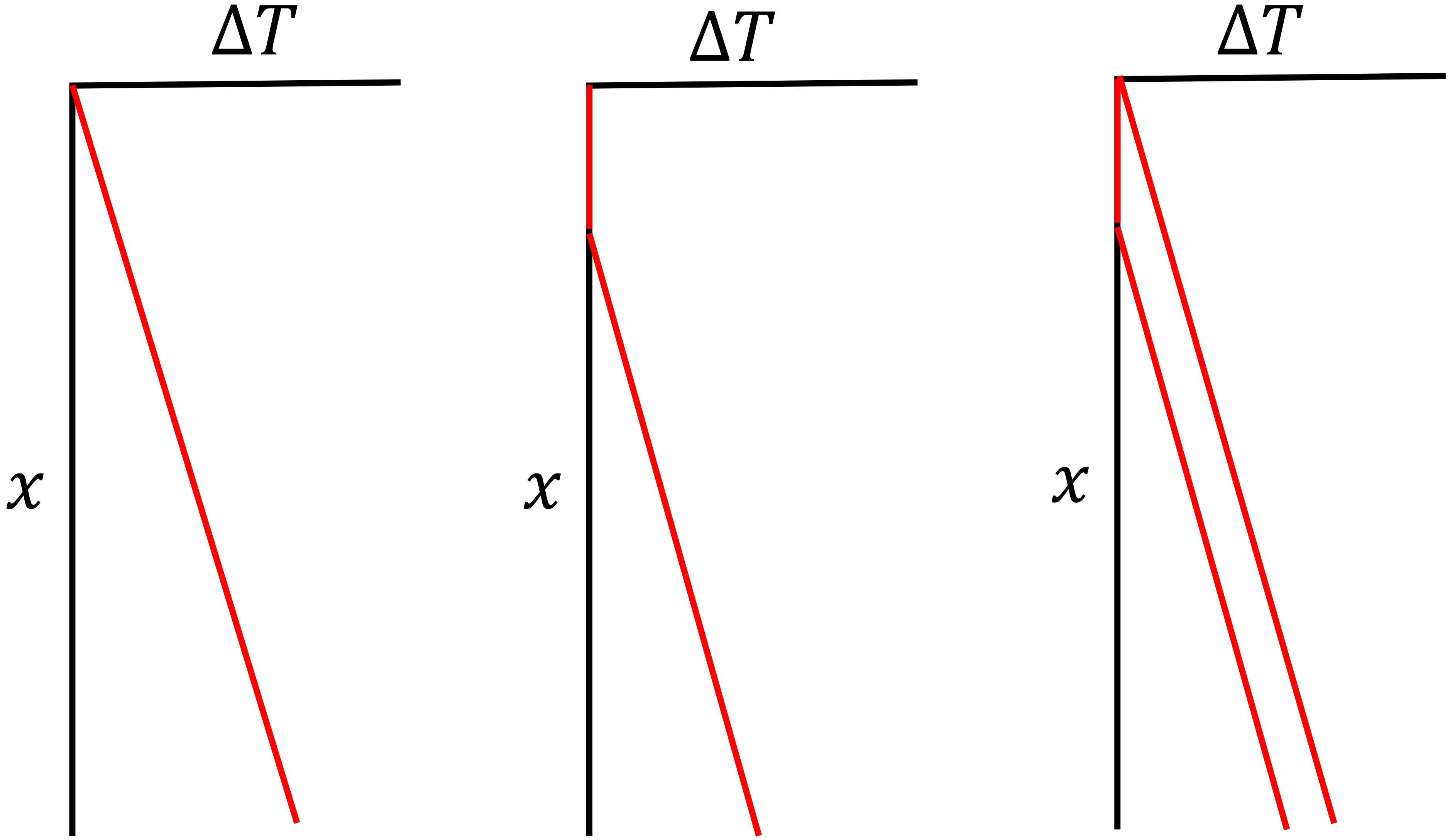
$$18.6 - 4 = 14.4\ km$$

How hot was it there?

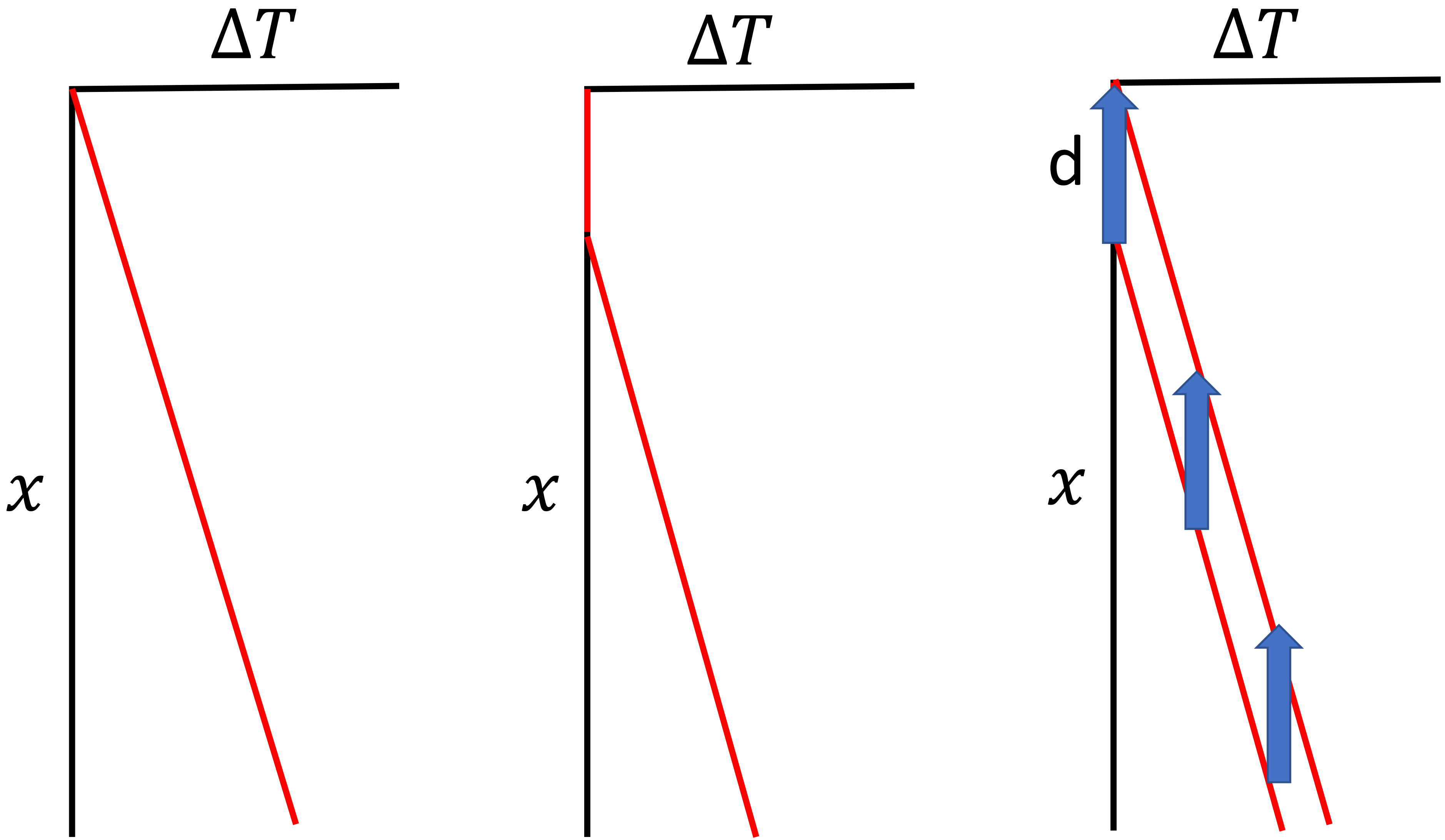
$$16\ km\ \times 20\frac{°C}{km} = 320°C$$



How long does it take to warm up?

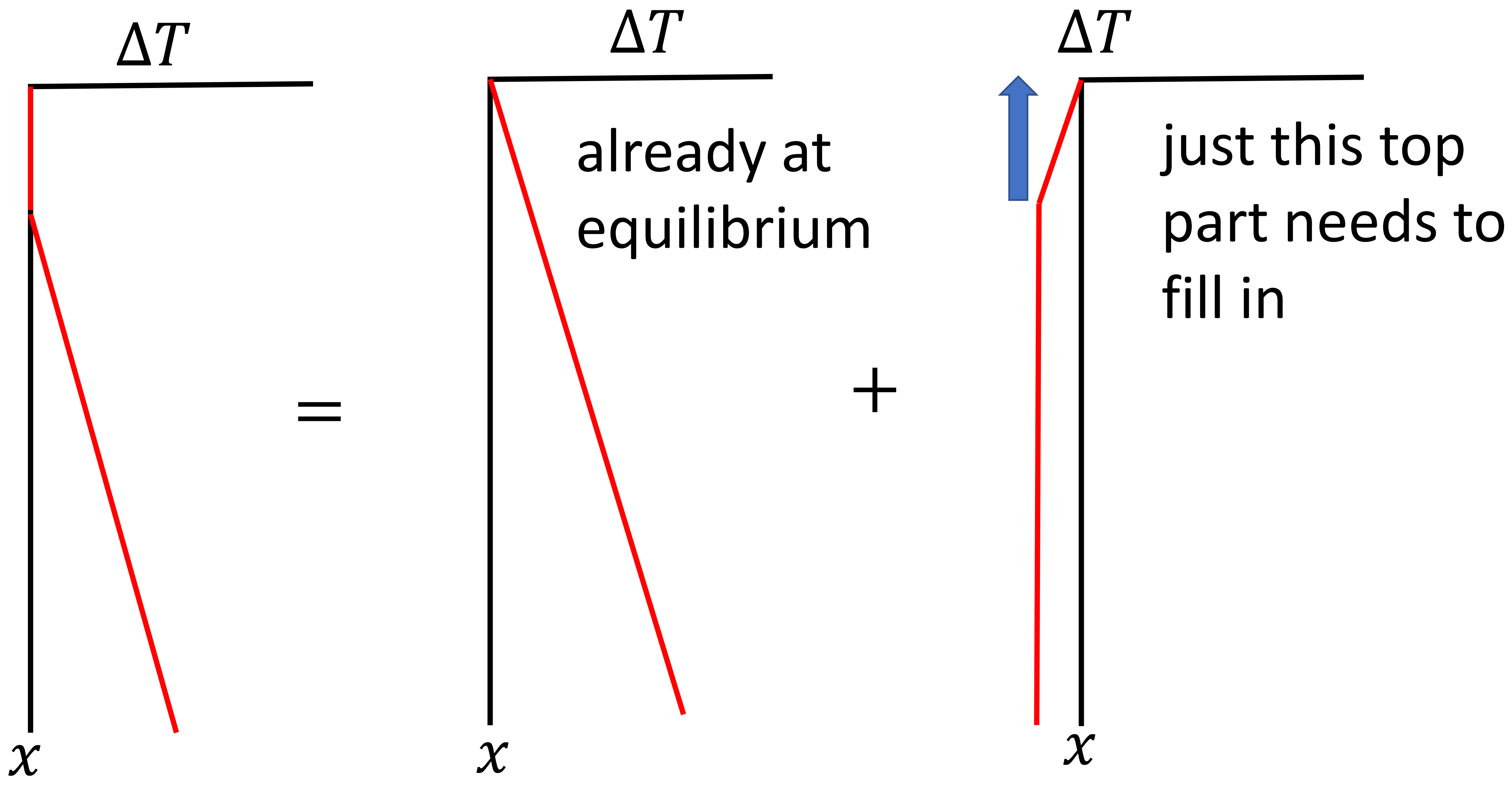


ΔT
x
ΔT
x
ΔT
x



heat has to move a distance d

apply principle of superposition



$$d = \sqrt{4\kappa t}$$

$$t = \frac{d^2}{4\kappa}$$

$$d = 18000\ m$$

$$\kappa = 1.6 \times 10^{-6}\ m^2 s^{-1}$$ (granite)

$$t = 1.74\ my$$

"geologically short time"

A	B
kappa	1.60E-06
d	18600
t s	5.41E+13
t yr	1.74E+06
y my	1.74E+00

Gravity anomaly

gravity minus a reference amount

$$\Delta g = g - g_{ref}$$

Gravity anomaly

often measured in milligals

1 gal = 1 cm/s^2 = 0.01 m/s^2

unfortunately, not an SI unit

Gravity anomaly

gravity minus a reference amount

g_{ref} $\left\{ \begin{array}{l} \text{acceleration at sea level} \\ \text{corrected for latitude, } \varphi \\ \text{corrected for altitude, } h \end{array} \right.$

$$g_{ref} = g_0(\varphi) + f(h, \varphi)$$

for latitude, φ

$$g_0(\varphi) = 9.780327 \text{ m} \cdot \text{s}^{-2} \left(1 + 0.0053024 \sin^2 \phi - 0.0000058 \sin^2 2\phi\right)$$

1967 Geodetic Reference System Formula

just an empirical formula

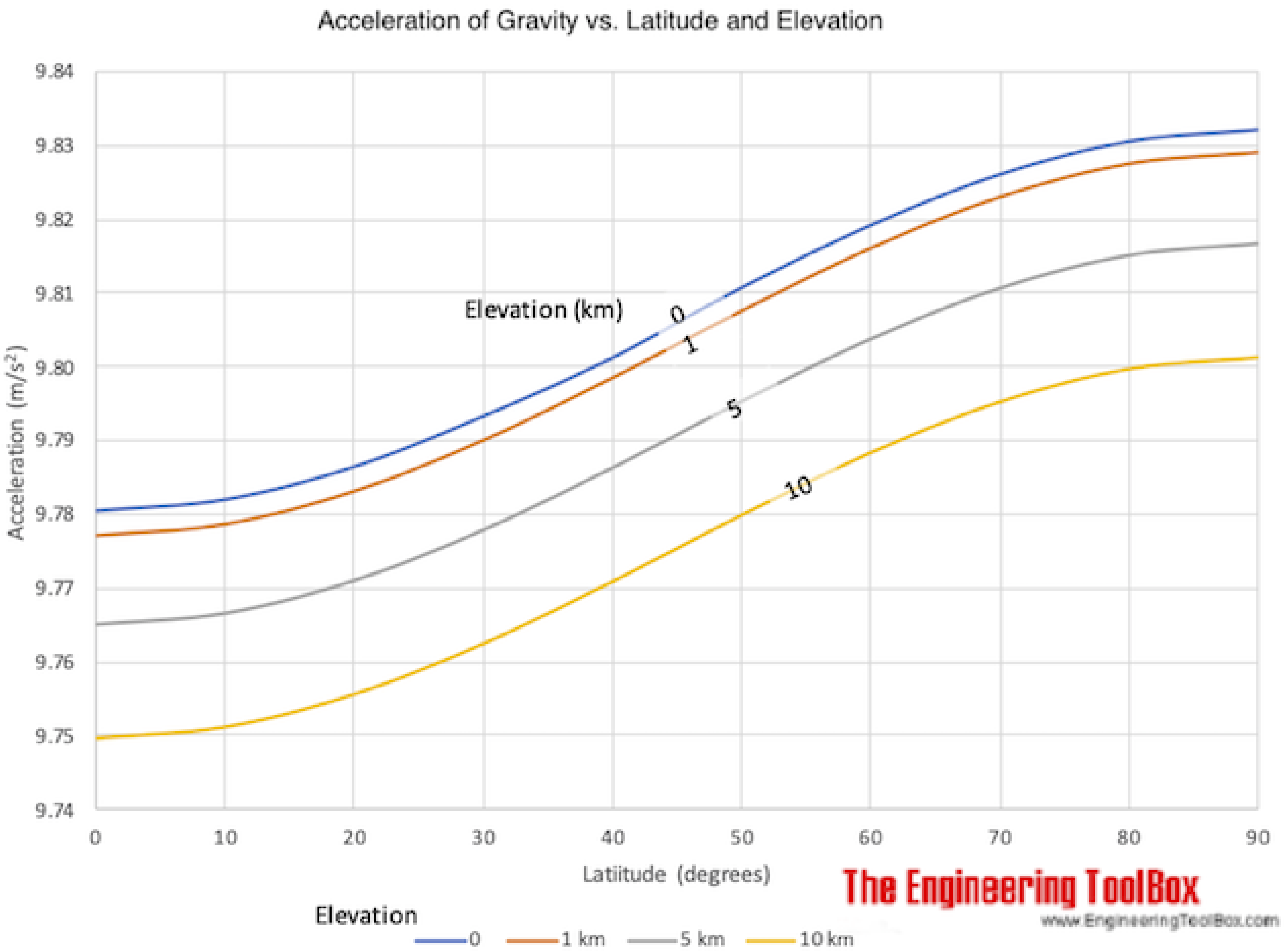
for altitude, h $\varphi = latitude$

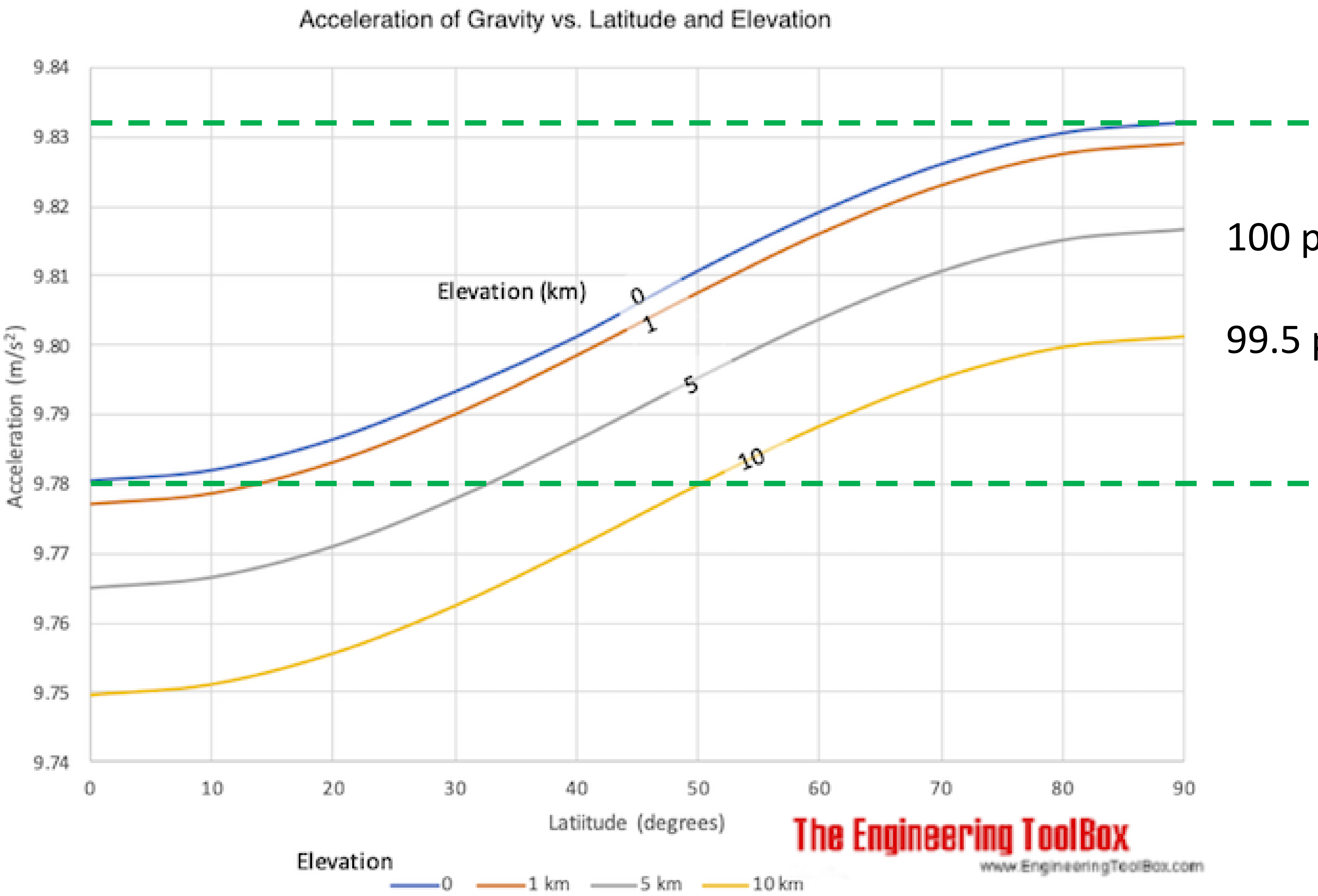
$$g = -\gamma M \frac{1}{(R_o + h)^2} + \omega^2 R_o \cos\varphi \left(1 + \frac{h}{R_o}\right)$$

$$= \frac{\gamma M}{R_0^2}\left(1 + \frac{h}{R_o}\right)^{-2} + \omega^2 R_o \cos\varphi \left(1 + \frac{h}{R_o}\right)$$

$$\approx C - 2\left(\frac{\gamma M}{R_0^2}\right)\left(\frac{h}{R_o}\right) + (\omega^2 R_o \cos\varphi)\left(\frac{h}{R_o}\right)$$

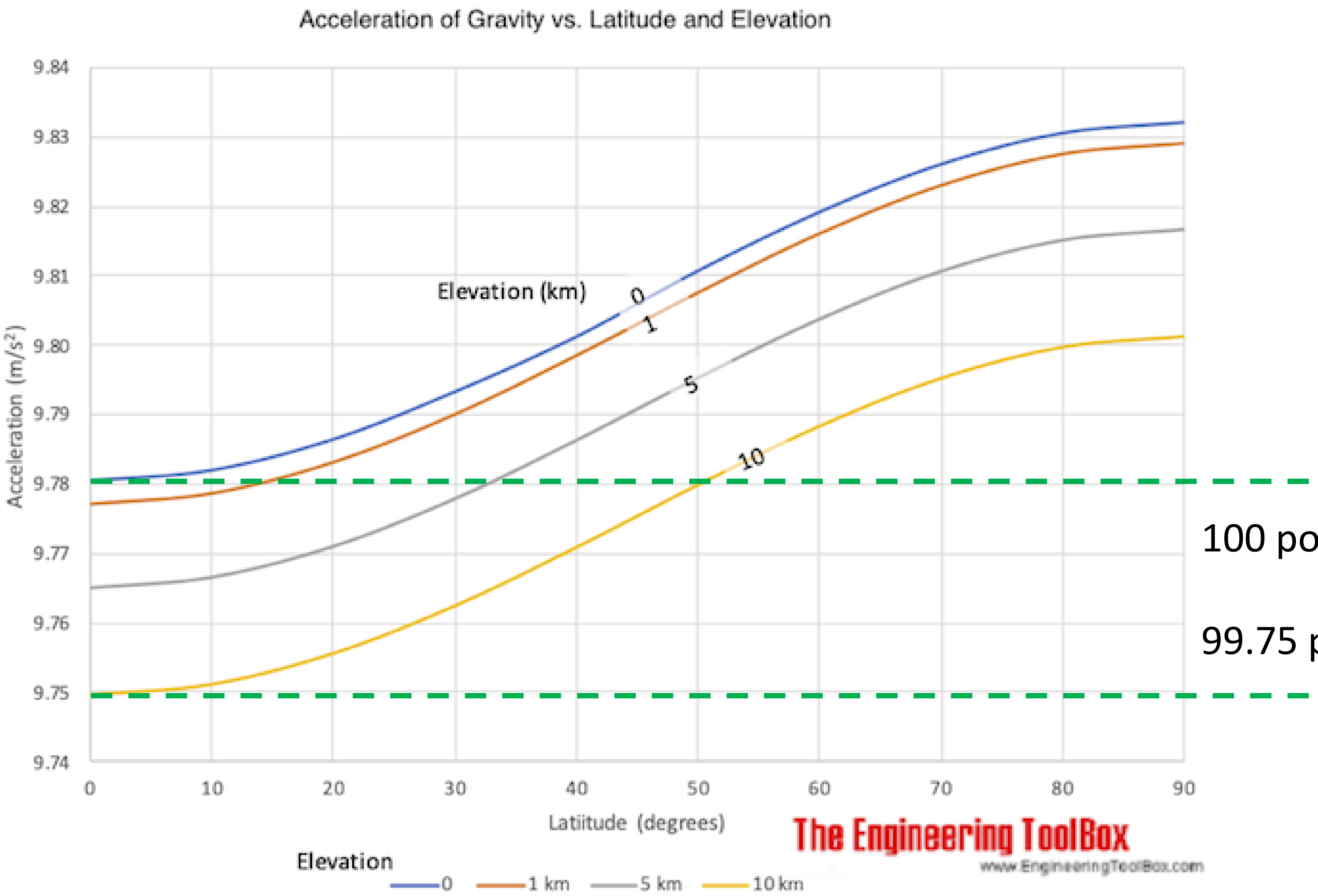
$$\approx C + \left(-2\frac{\gamma M}{R_0^2} + \omega^2 R_o \cos\varphi\right)\left(\frac{h}{R_o}\right) \quad f(h, \varphi)$$


Acceleration of Gravity vs. Latitude and Elevation
9.84
9.83
9.82
9.81
9.80
9.79
9.78
9.77
9.76
9.75
9.74
Acceleration (m/s²)
Elevation (km)
0
1
5
10
0
10
20
30
40
50
60
70
80
90
Latiitude (degrees)
The Engineering ToolBox
www.EngineeringToolBox.com
Elevation
0
1 km
5 km
10 km



100 pounds at pole

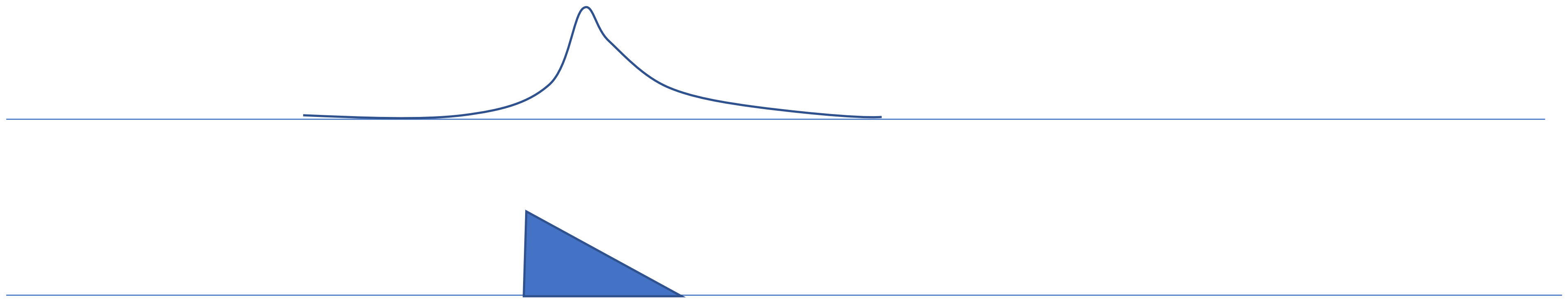
99.5 pounds at equator

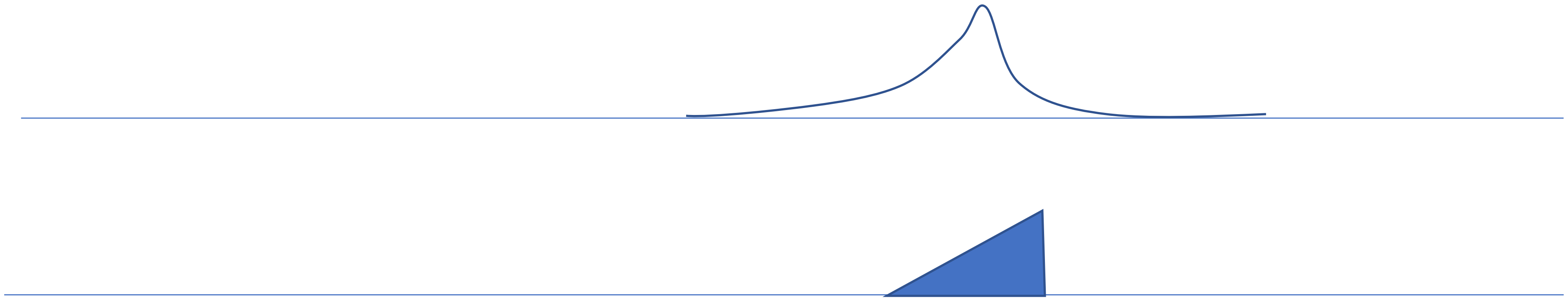


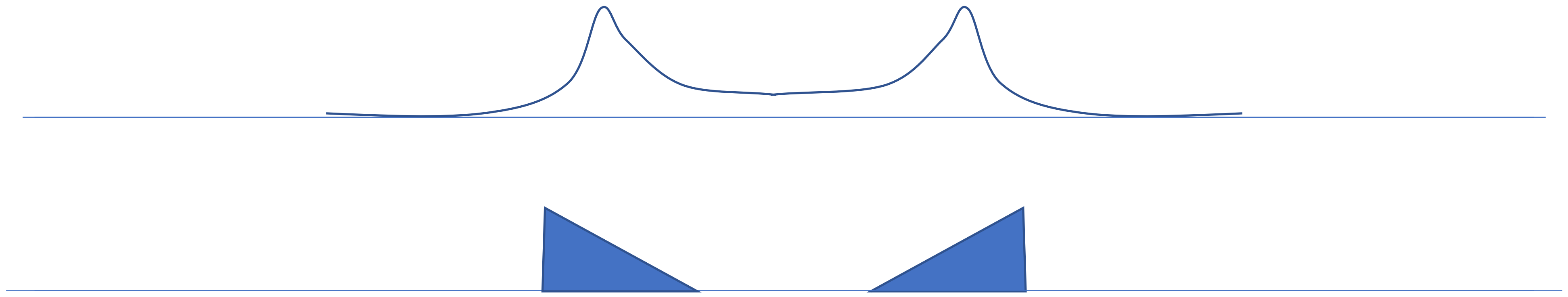
100 pounds sea level

99.75 pounds 10 km

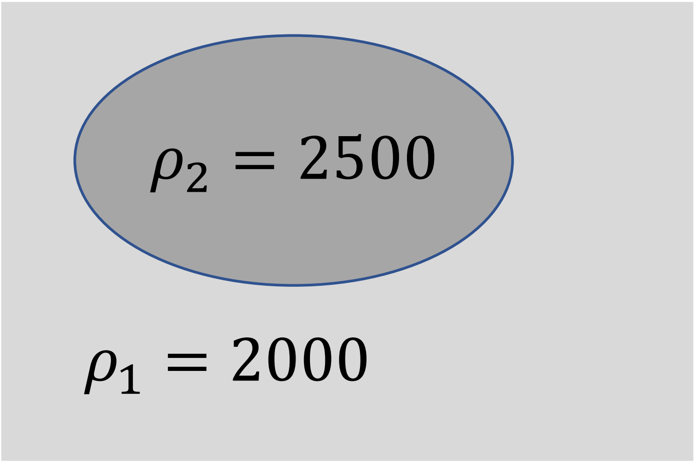
additivity of gravity and gravity anomalies







this is the same as



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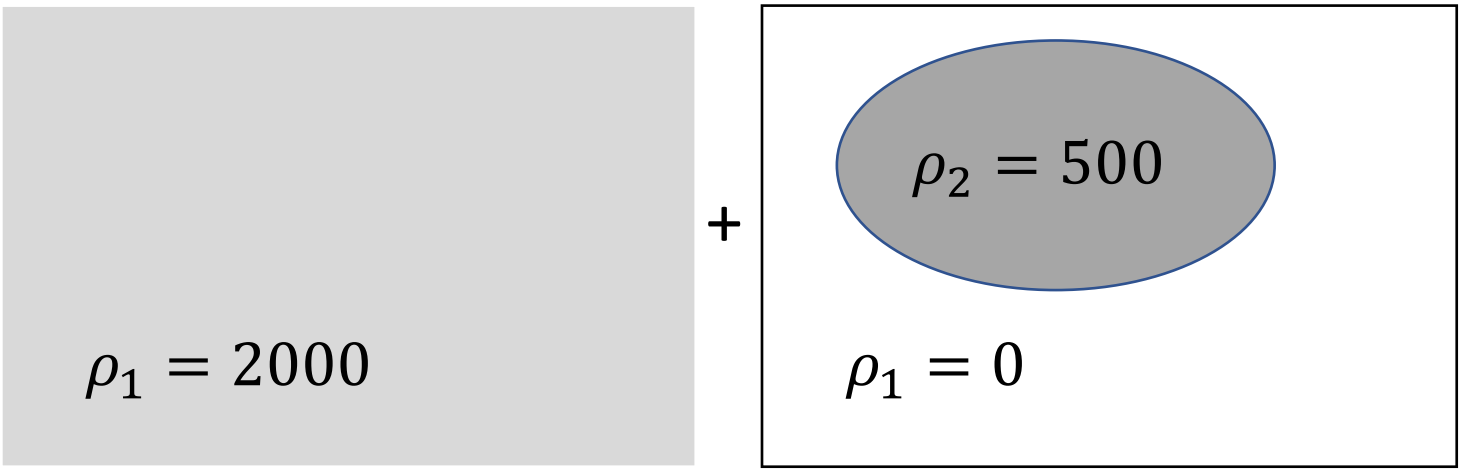
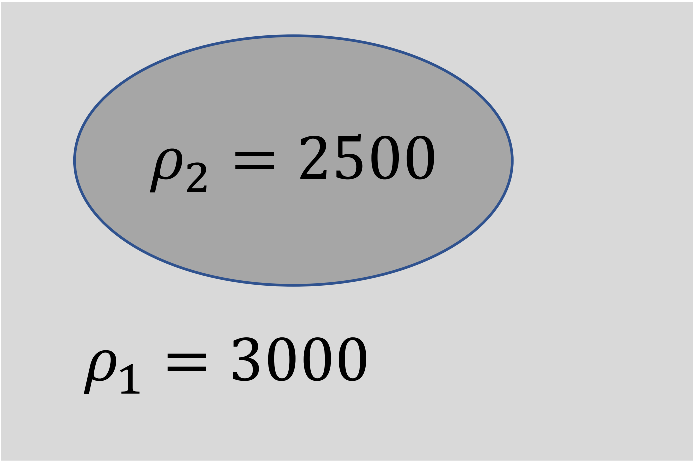
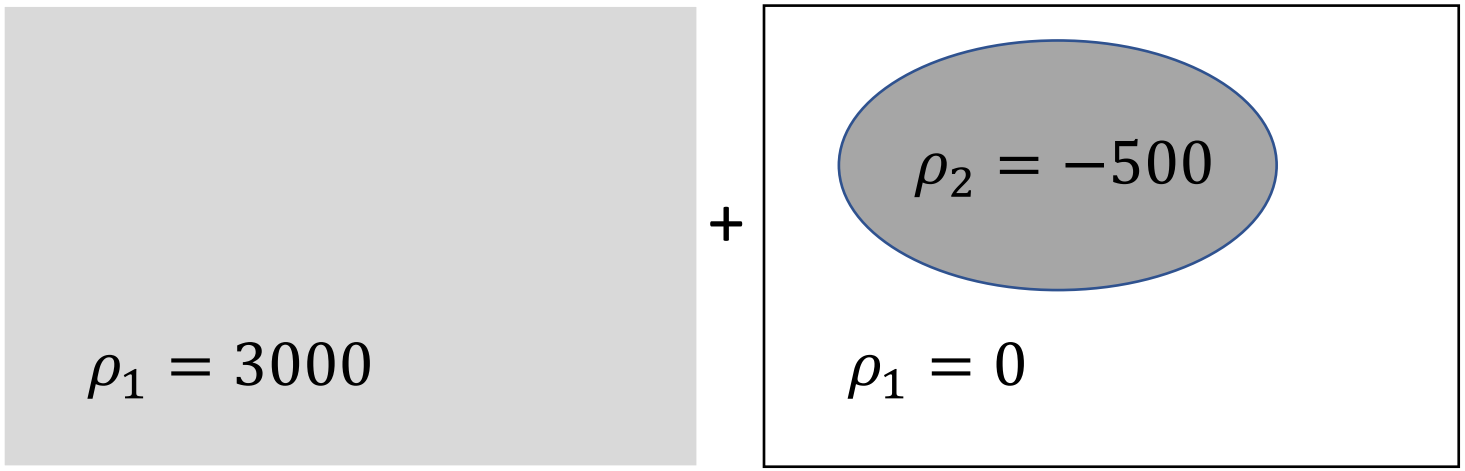


negative density anomalies allowed, too

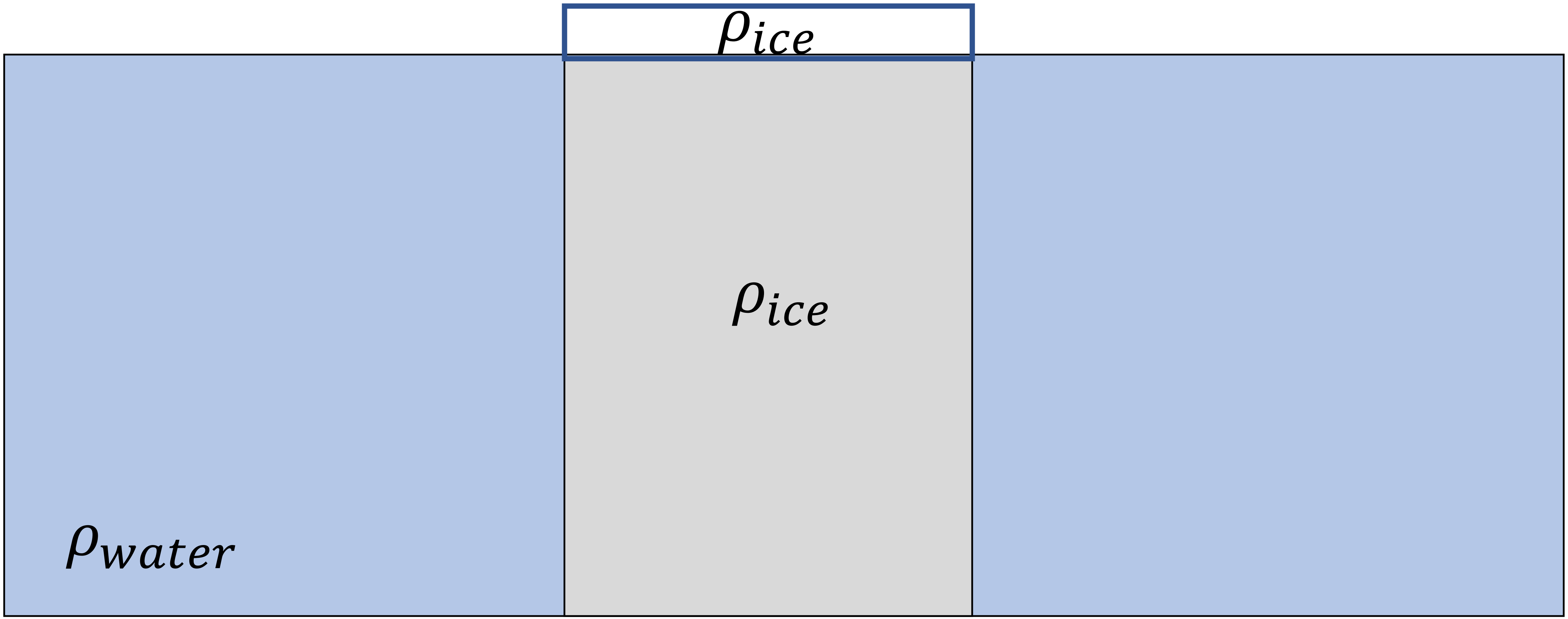
this is the same as



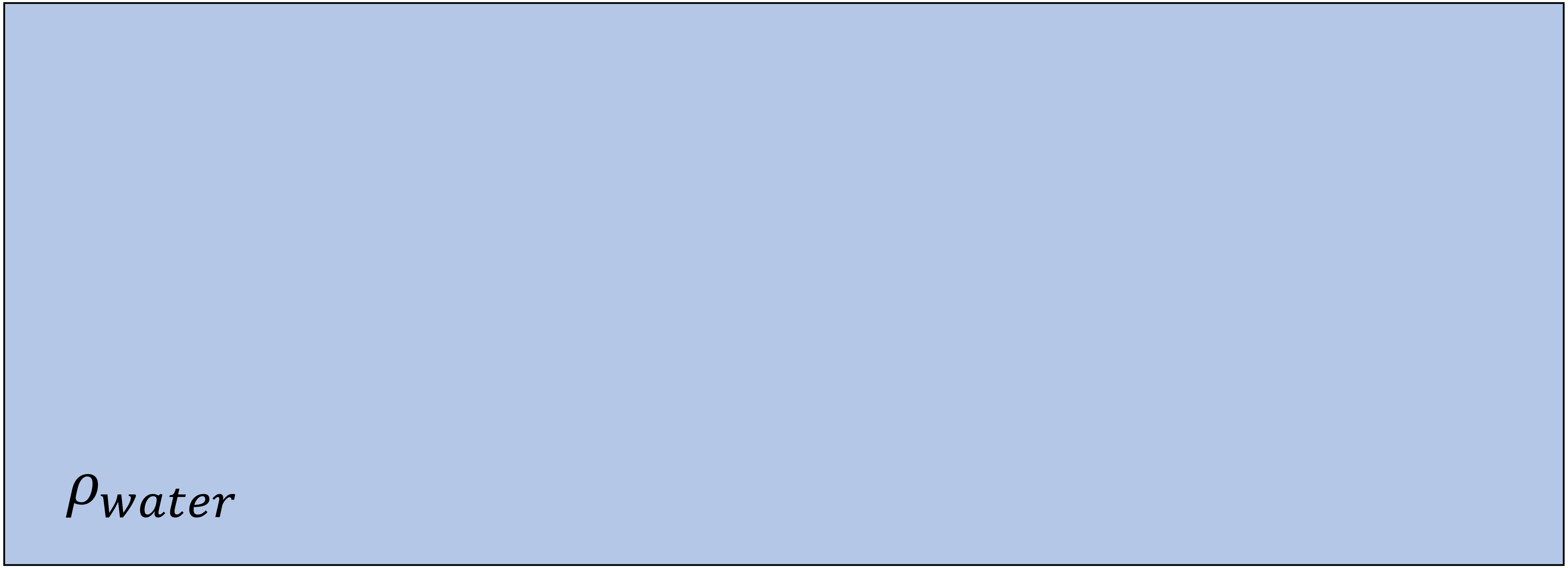
this is the same as



what is the gravity anomaly over the iceberg?

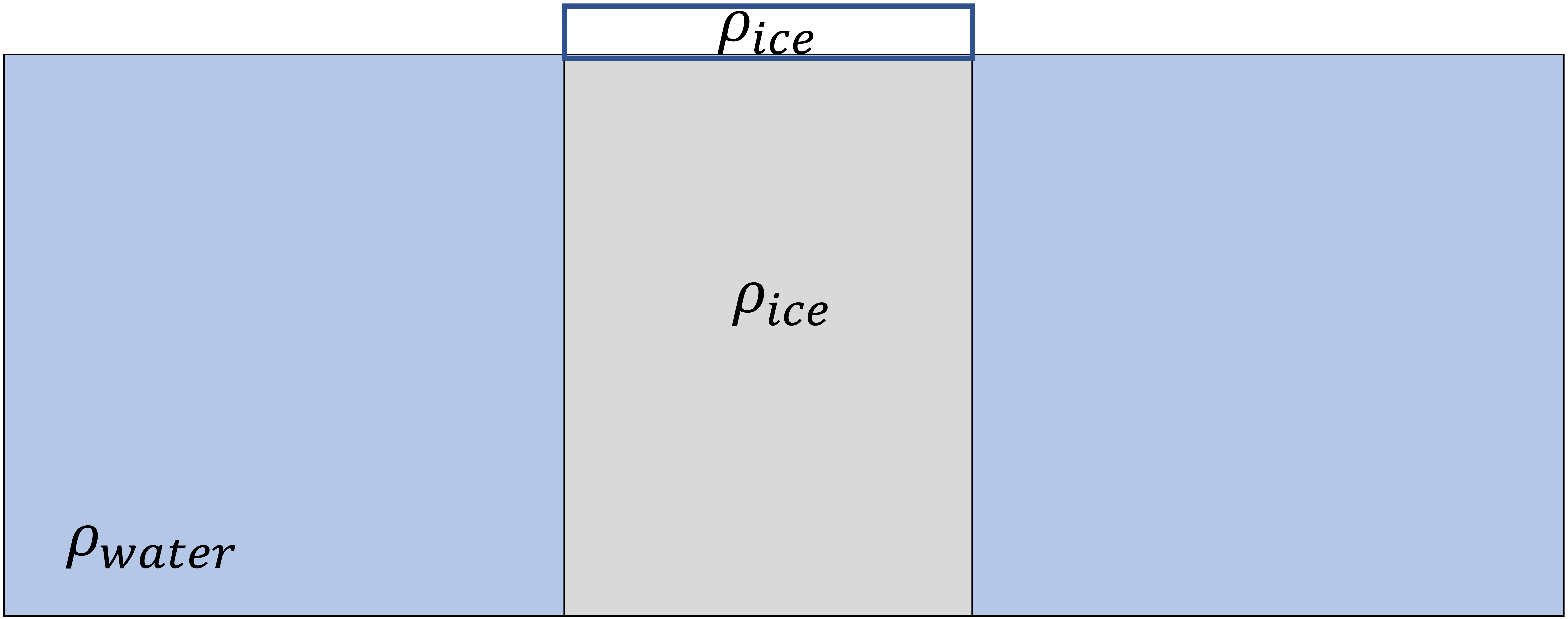


ρ_{ice}
ρ_{ice}
ρ_{water}


ρ_{water}

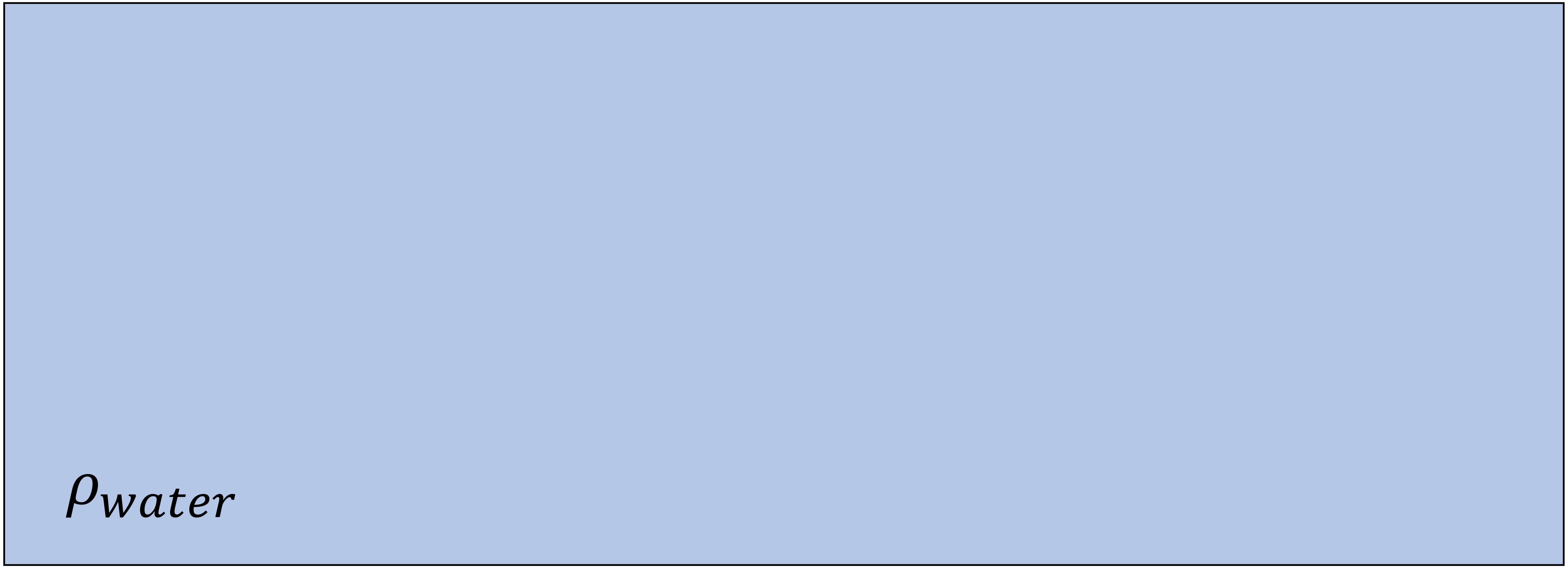
ρ_{ice}

$$\rho_{ice} - \rho_{water}$$

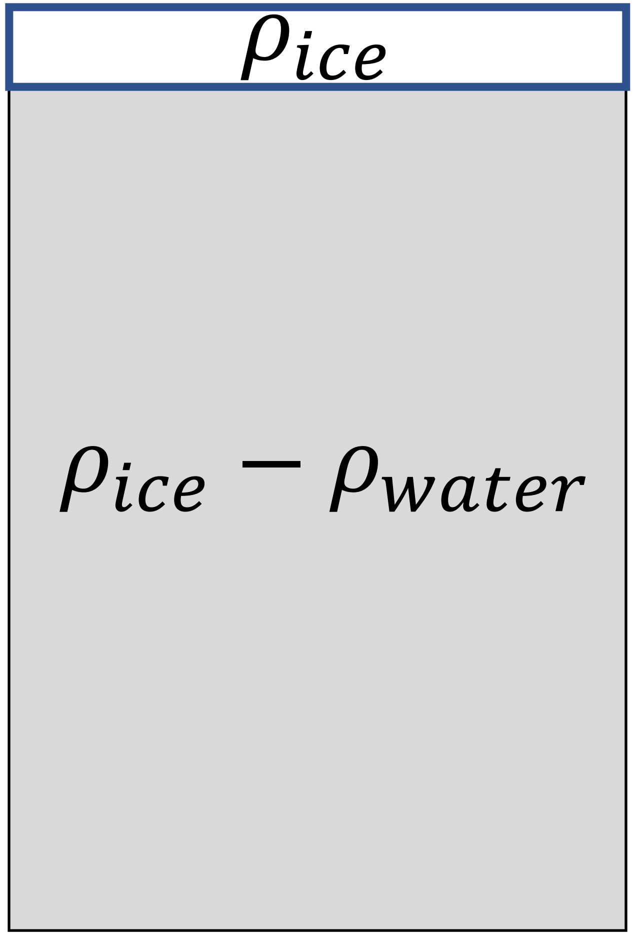


ρ_{ice}
ρ_{ice}
ρ_{water}

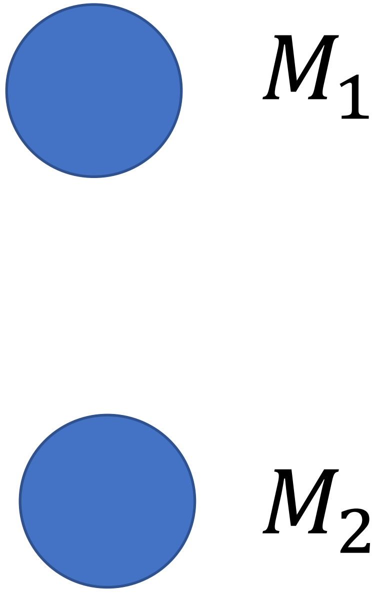
Ths one produces the reference field, g_{ref}



These two produce the anomaly, Δg



approximate as point masses



but what are the right masses?

approximate as point masses

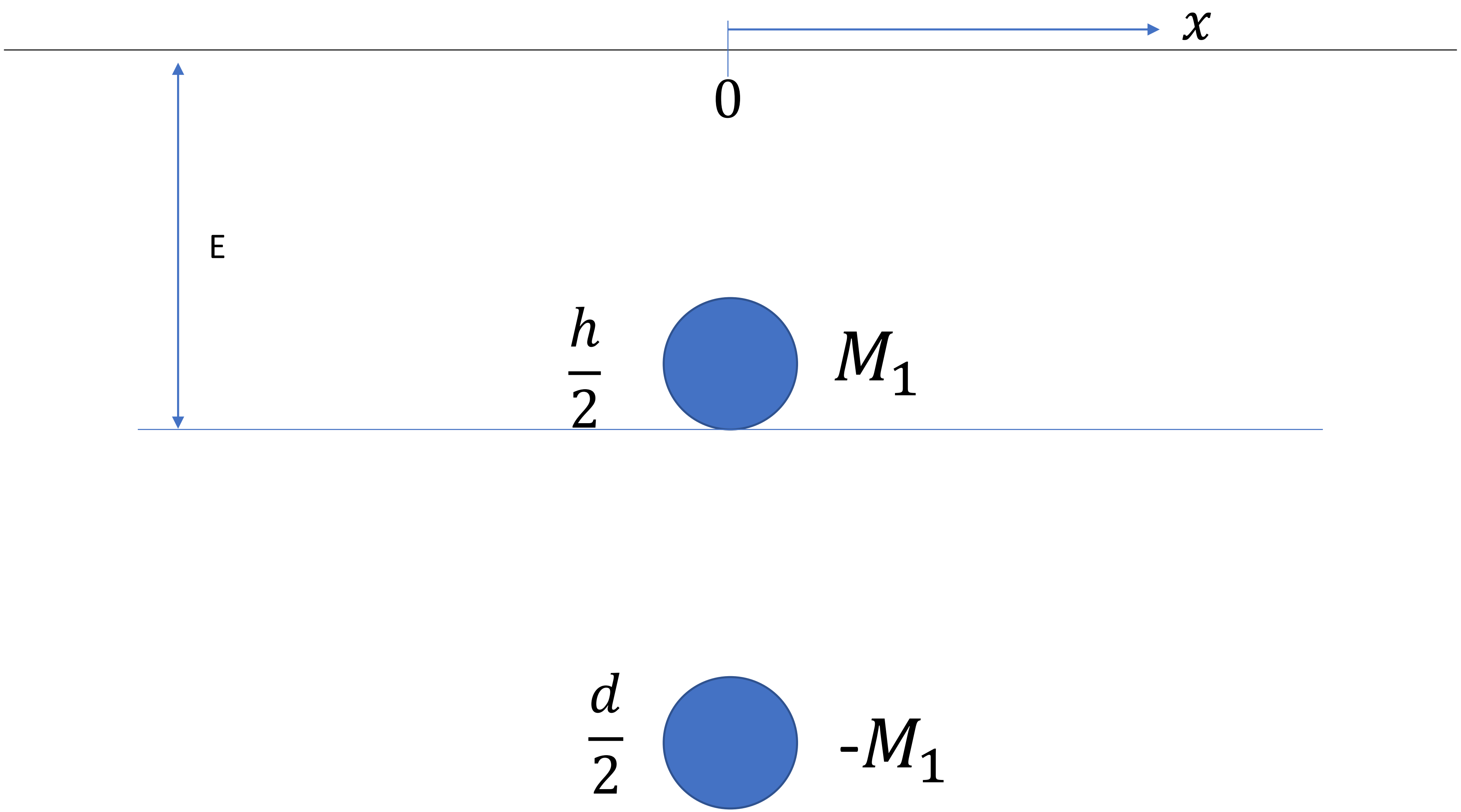
$$M_1 = \rho_{ice} A h$$

$$M_2 = -\Delta\rho A \mathrm{d}$$

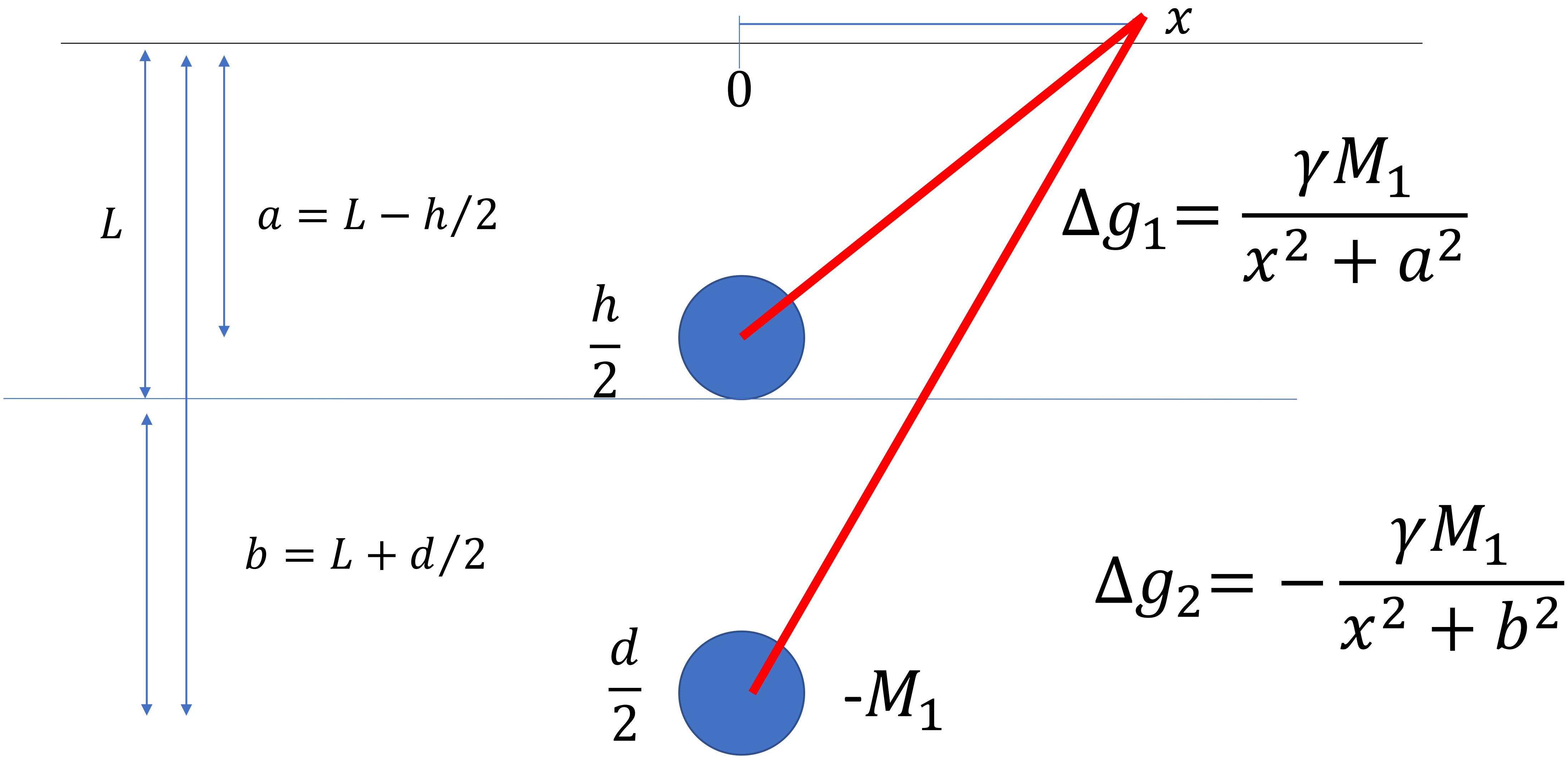
$$= -\Delta\rho A \frac{\rho_{ice}}{\Delta\rho} h$$

$$= -\rho_{ice} A h$$

$$= -\rho_{ice} M_1$$



x
0
E
$\frac{h}{2}$
M_1
$\frac{d}{2}$
$-M_1$

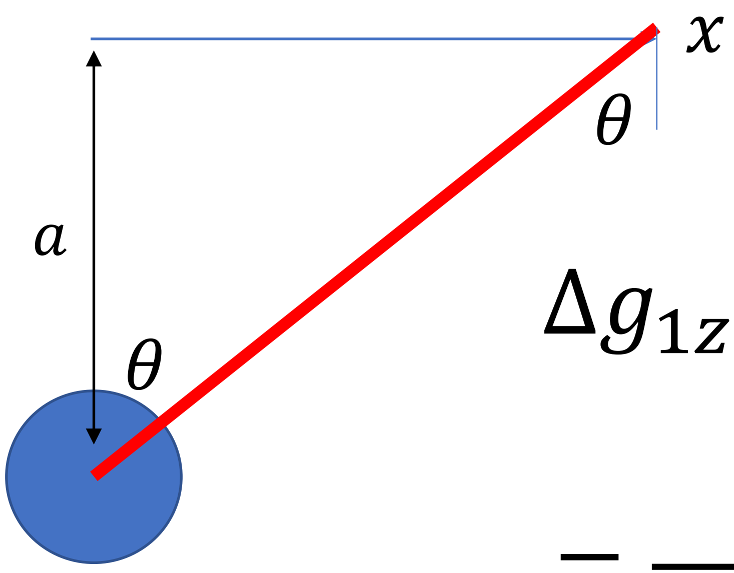


x
0
L
$a = L - h/2$
$\frac{h}{2}$
$\Delta g_1 = \frac{\gamma M_1}{x^2 + a^2}$
$b = L + d/2$
$\Delta g_2 = -\frac{\gamma M_1}{x^2 + b^2}$
$\frac{d}{2}$
$-M_1$

but the two forces are not in the same direction

solution

use vertical component, only

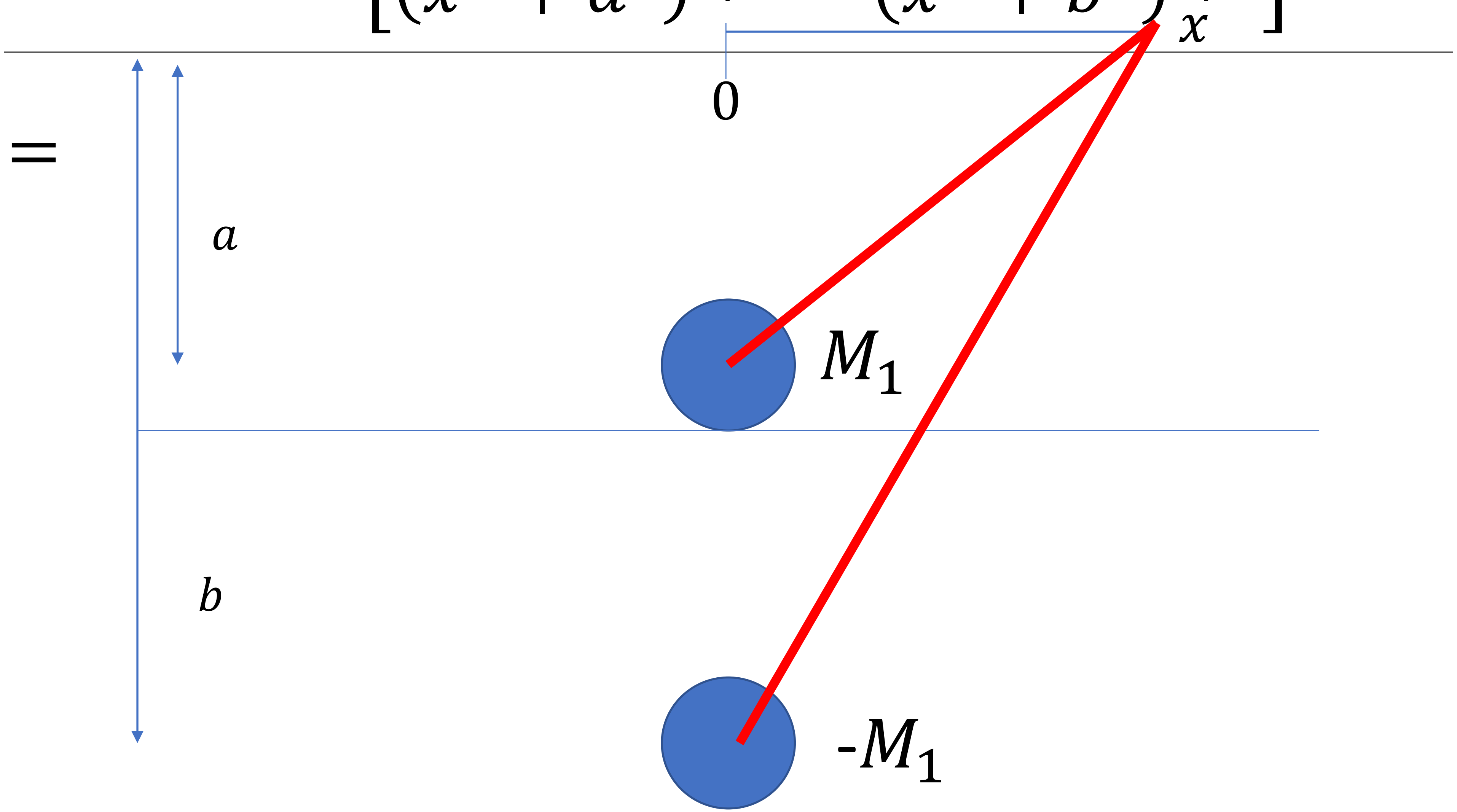


$$\Delta g_{1z} = \frac{\gamma M_1}{x^2 + a^2} \cos\theta$$

$$= \frac{\gamma M_1 a}{(x^2 + a^2)^{3/2}}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{\sqrt{x^2 + a^2}}$$

$$\Delta g_z = \gamma M_1 \left[\frac{a}{(x^2 + a^2)^{3/2}} - \frac{b}{(x^2 + b^2)^{3/2}} \right]$$



$$\Delta g_z = \gamma M_1 \left[\frac{a}{(x^2 + a^2)^{3/2}} - \frac{b}{(x^2 + b^2)^{3/2}} \right]$$

$$x = 0$$

$$\Delta g_z = \gamma M_1 \left[\frac{1}{a^2} - \frac{1}{b^2} \right] > 0 \qquad \text{as } b > a$$

$$\Delta g_z = \gamma M_1 \left[\frac{a}{(x^2 + a^2)^{3/2}} - \frac{b}{(x^2 + b^2)^{3/2}} \right]$$

$x \gg b$

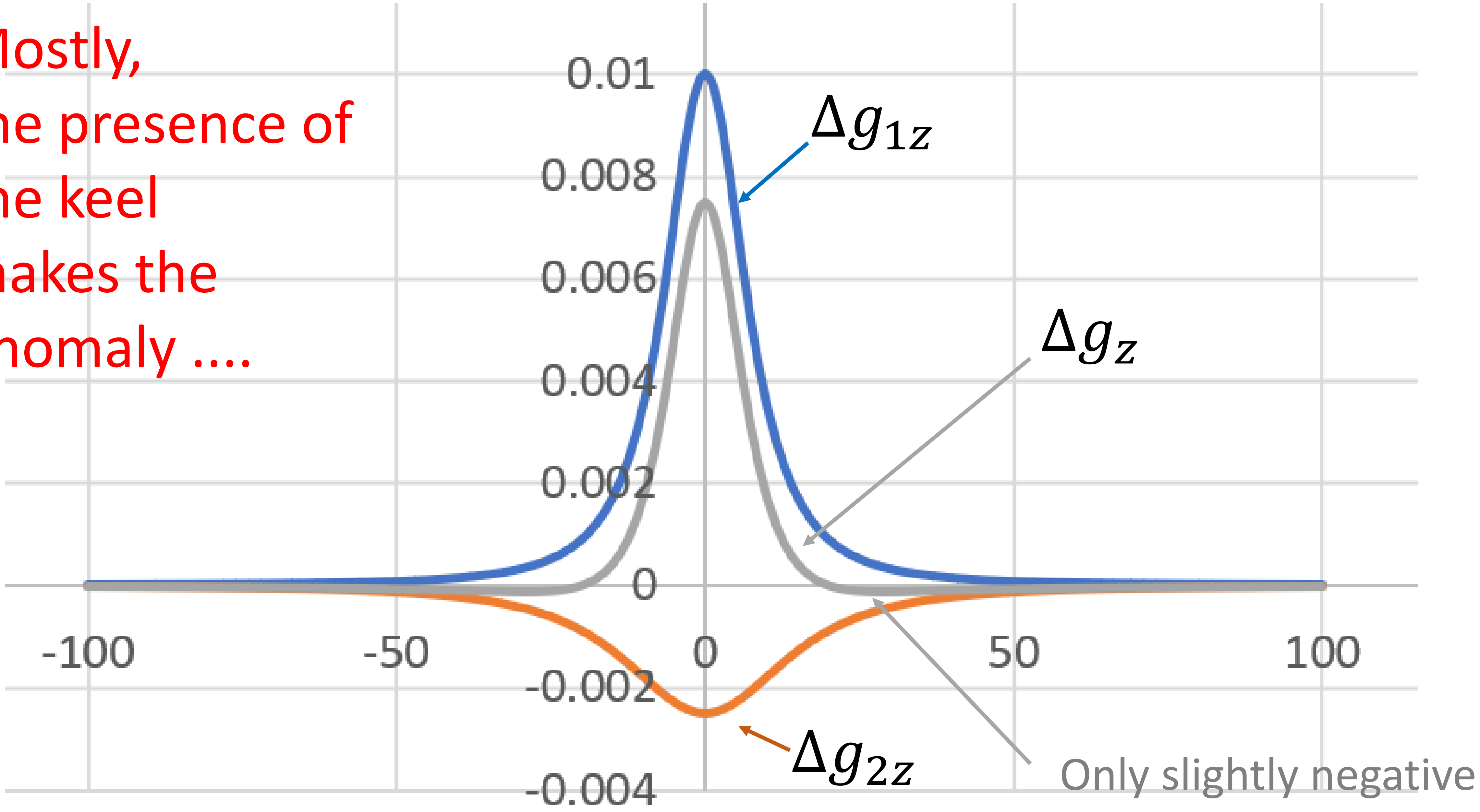
$$\Delta g_z = \frac{\gamma M_1}{x^3} [a - b] < 0 \qquad \text{as } b > a$$

$$\Delta g_z = \gamma M_1 \left[\frac{a}{(x^2 + a^2)^{3/2}} - \frac{b \qquad x}{(x^2 + b^2)^{3/2}} \right]$$

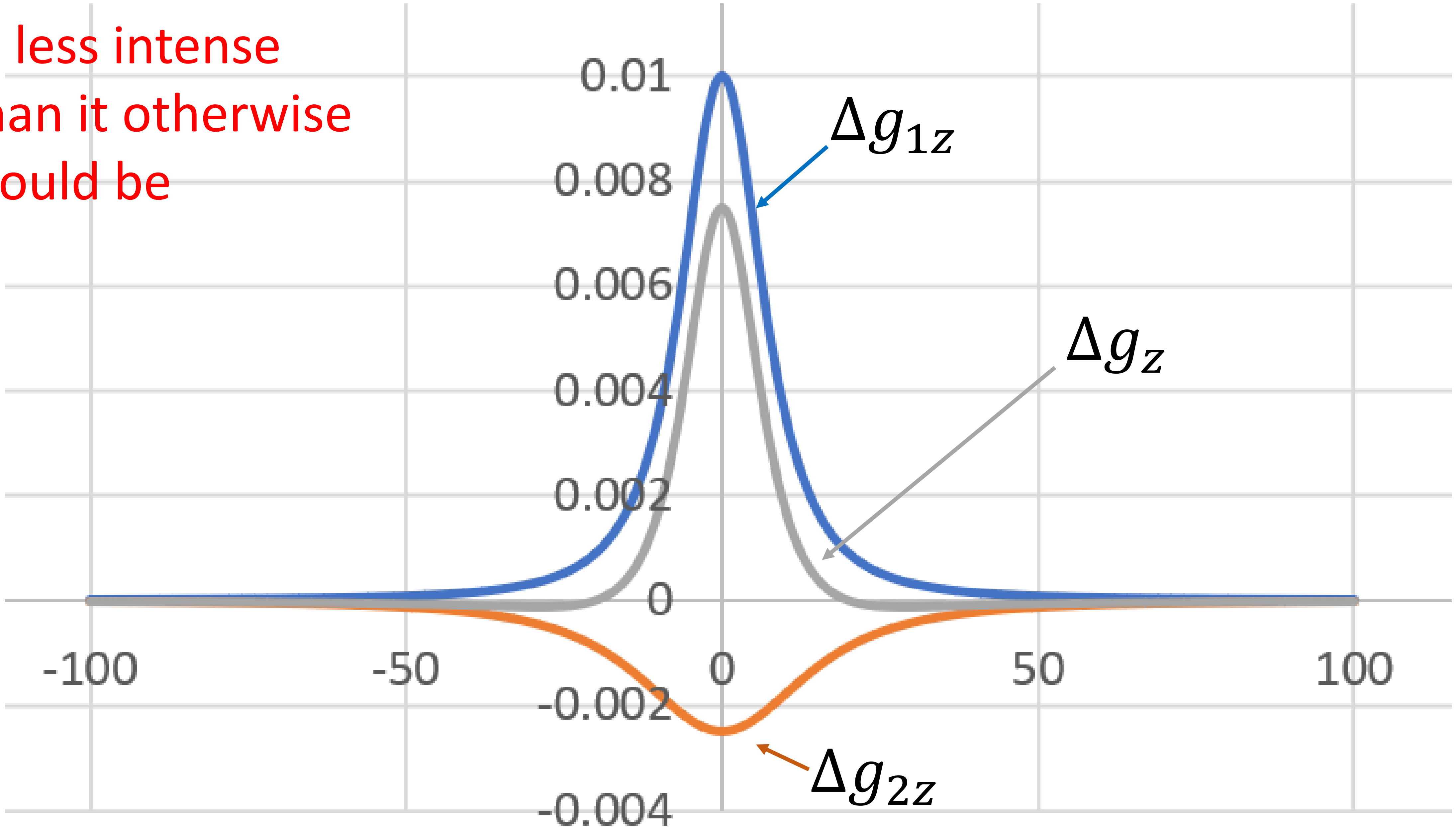
$$x \to \infty$$

$$\Delta g_z = \gamma M_1 [0 - 0] = 0$$

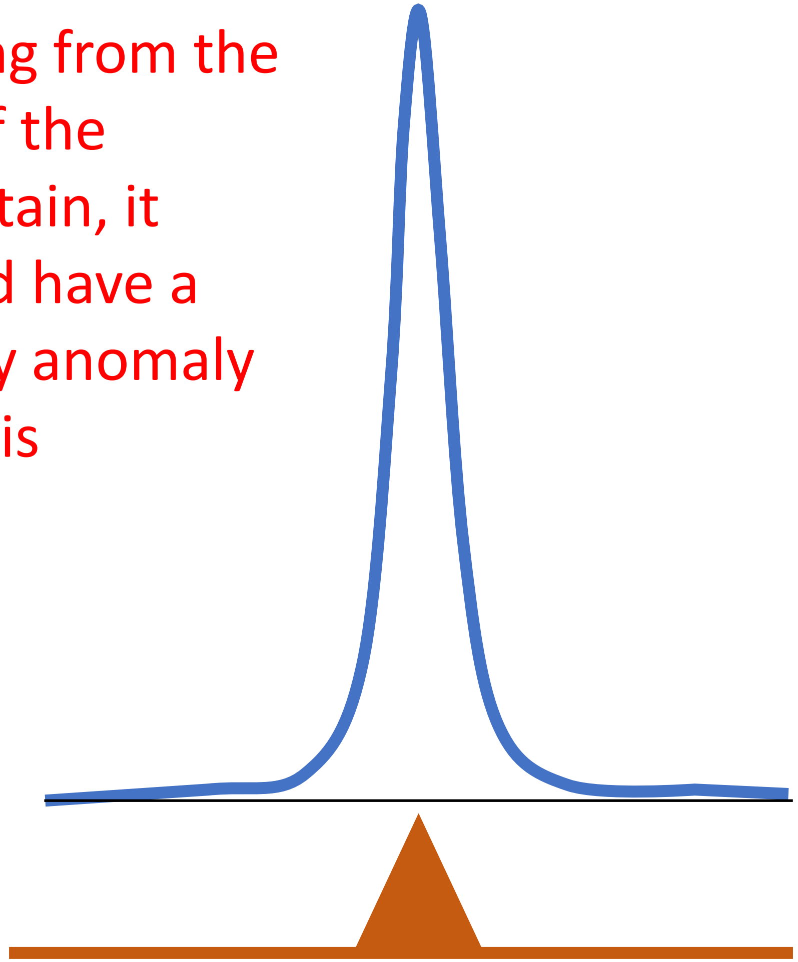
Mostly,
the presence of
the keel
makes the
anomaly ....



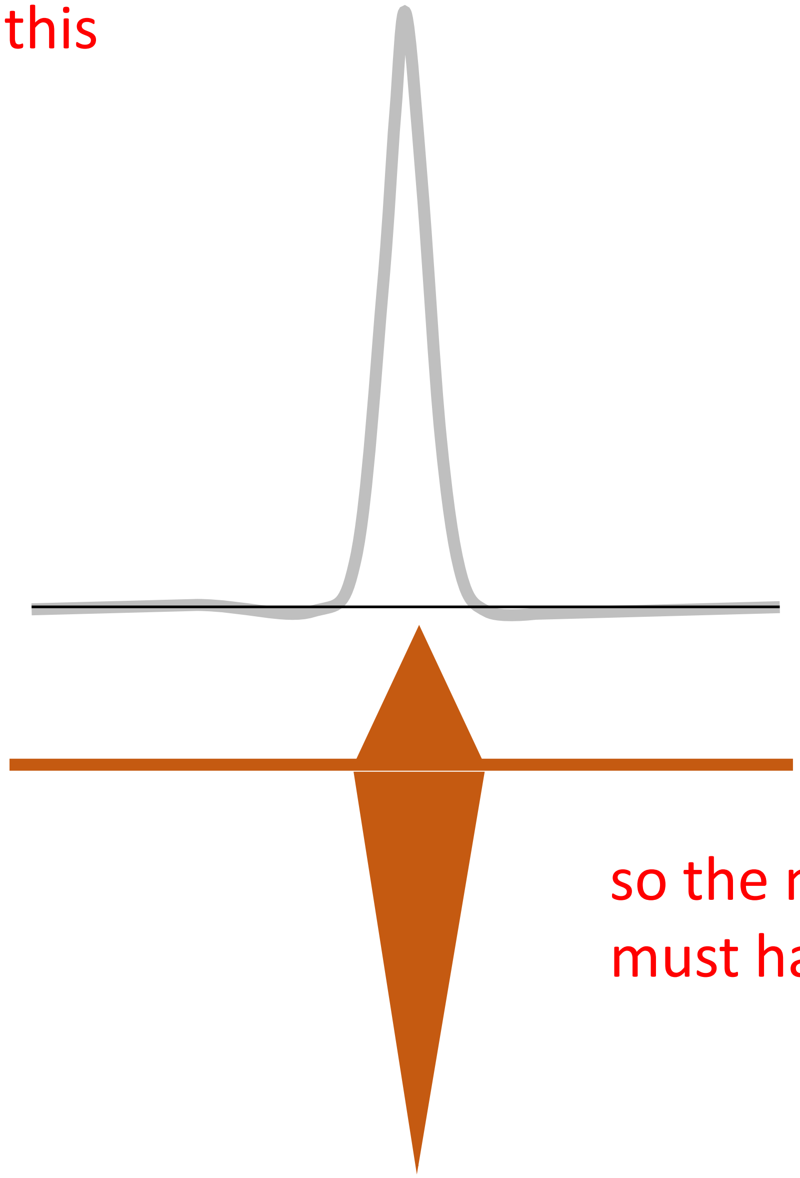
... less intense than it otherwise would be



Judging from the size of the mountain, it should have a gravity anomaly like this



But actually, it has a smaller anomaly like this



so the mountain must have a root