Solid Earth Dynamics

Bill Menke, Instructor

Lecture 11

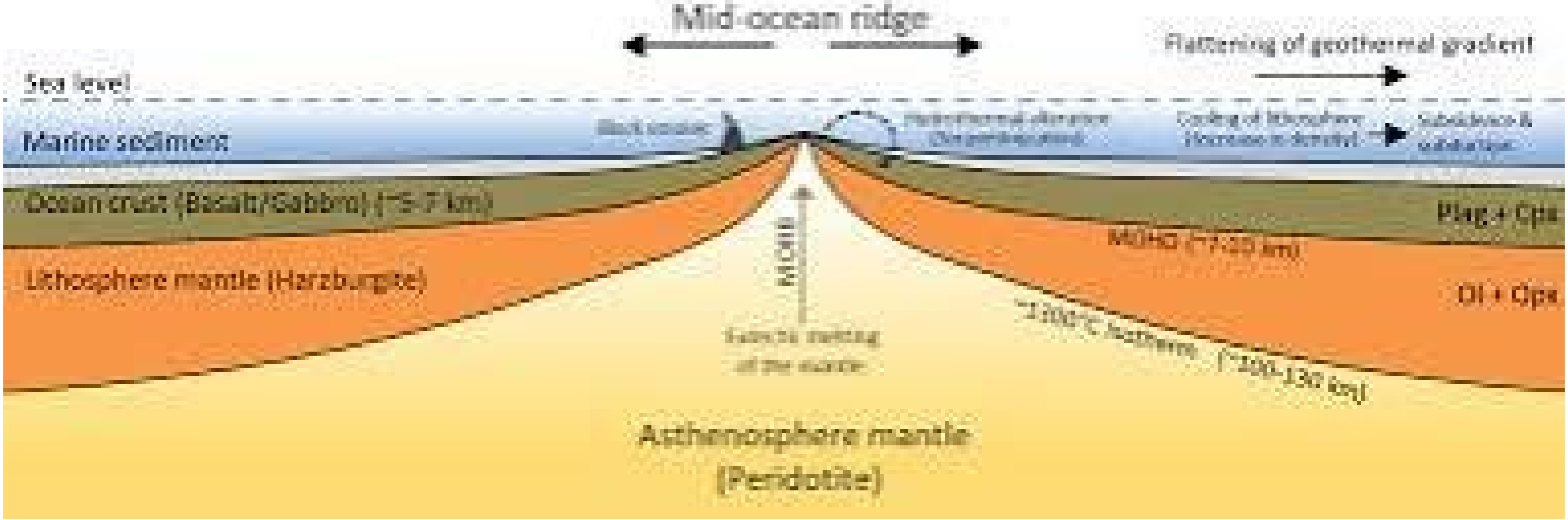
Today:

Effect of isostacy on depth-age

Effect of isostacy on sedimentary basins

Glacial isostatic rebound

Effect of isostacy on depth-age

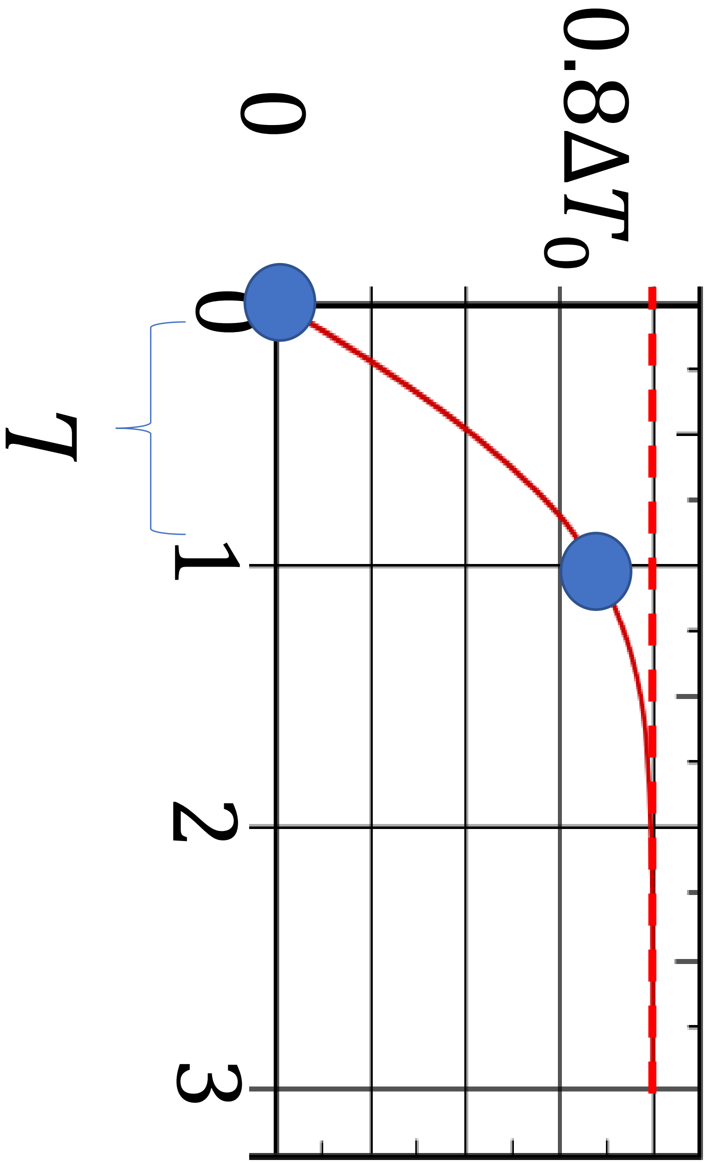


How much cooled?

$$L \approx \sqrt{\frac{4kt}{\rho c_p}}$$

$$t = 100 \text{ my}$$

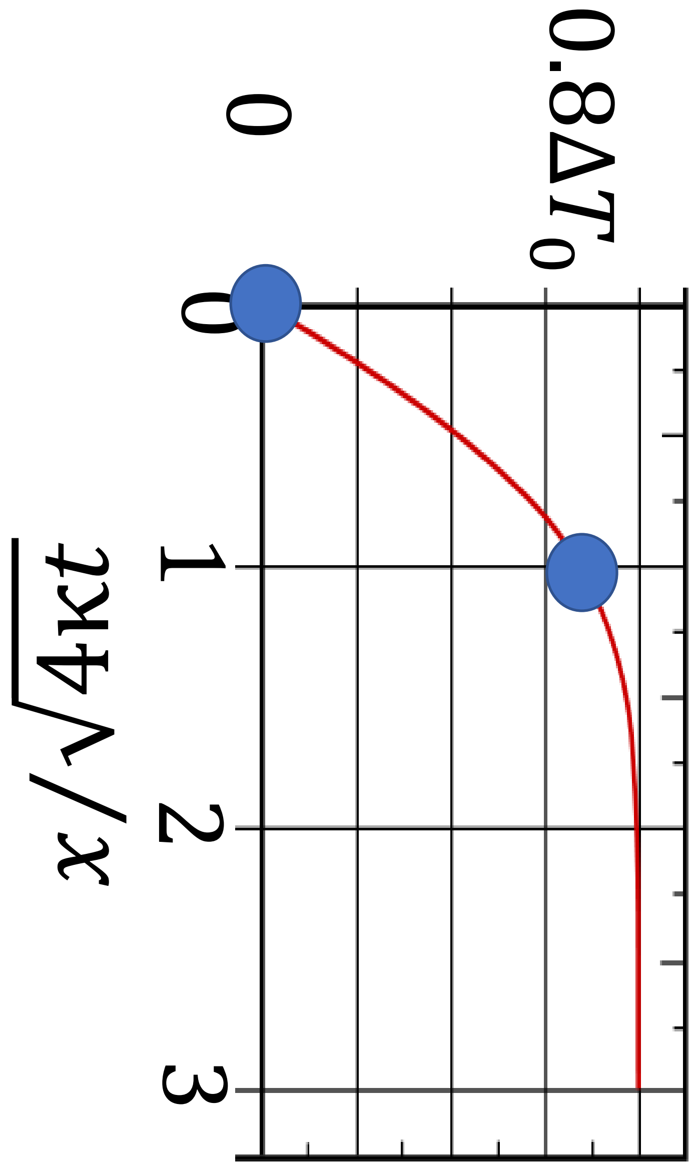
$$x = 150 \text{ km}$$



How much did it cool?

cooled $0.6\Delta T_0$

$$0.6\Delta T_0 = 0.6 \times 1350 = 810°\mathrm{C}$$



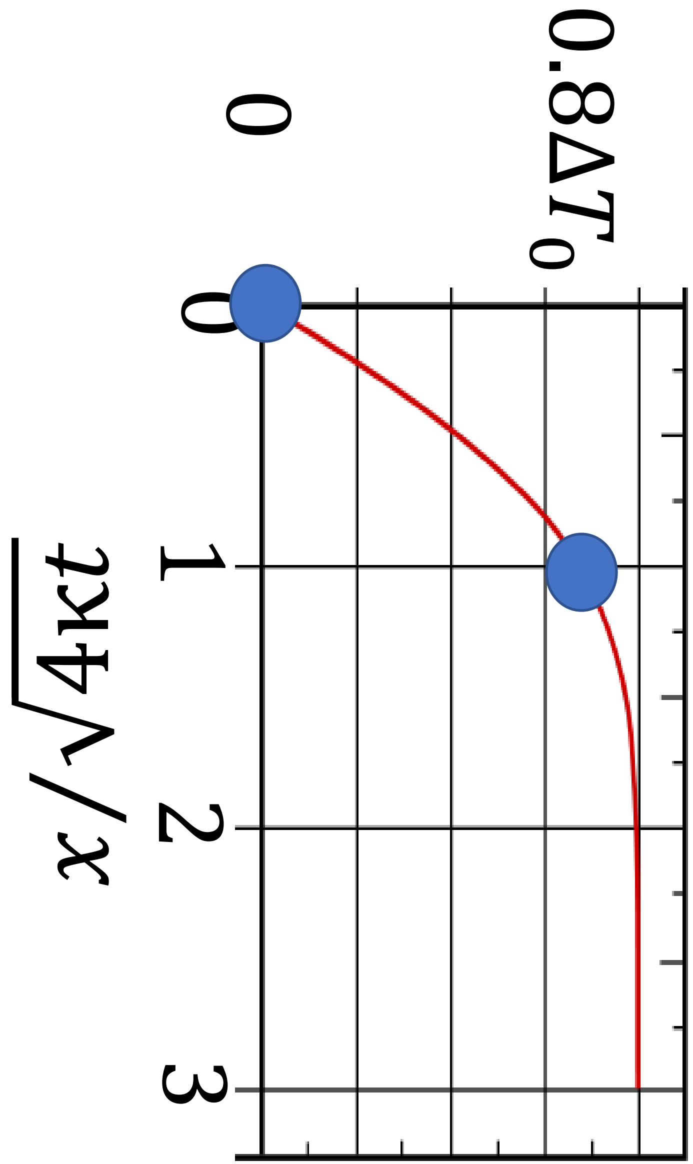
How much did it shrink?

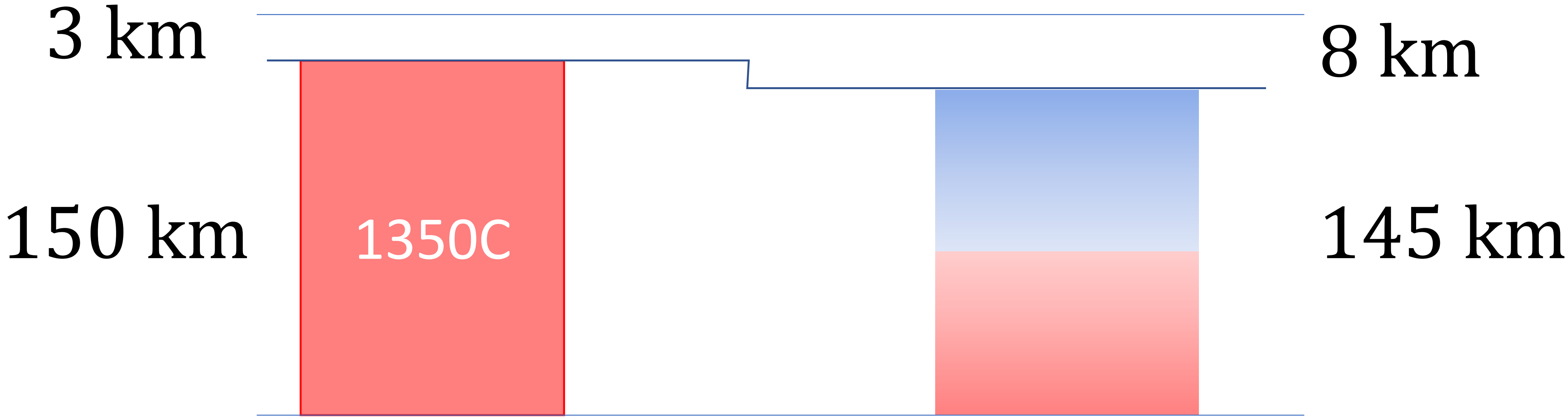
$$\frac{\Delta L}{L} = \alpha \Delta T$$

for granite $\alpha = 4 \times 10^{-5}\ \frac{1}{°\mathrm{C}}$

$$\Delta L = \alpha \Delta T L$$

$$\Delta L = 5\ \mathrm{km}$$





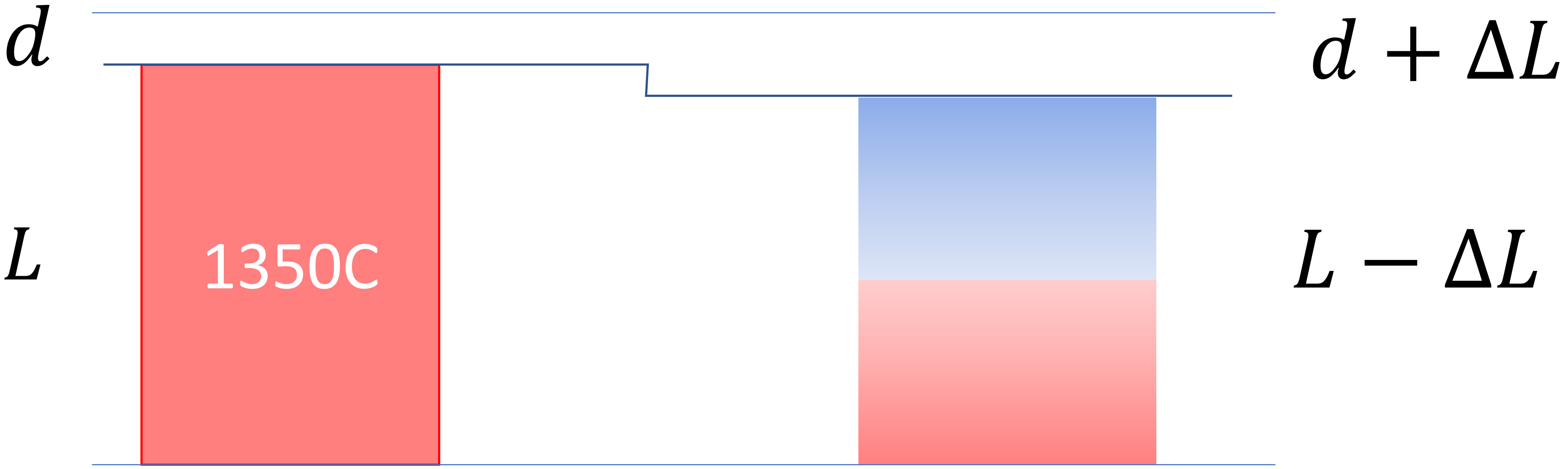
3 km

150 km

1350C

8 km

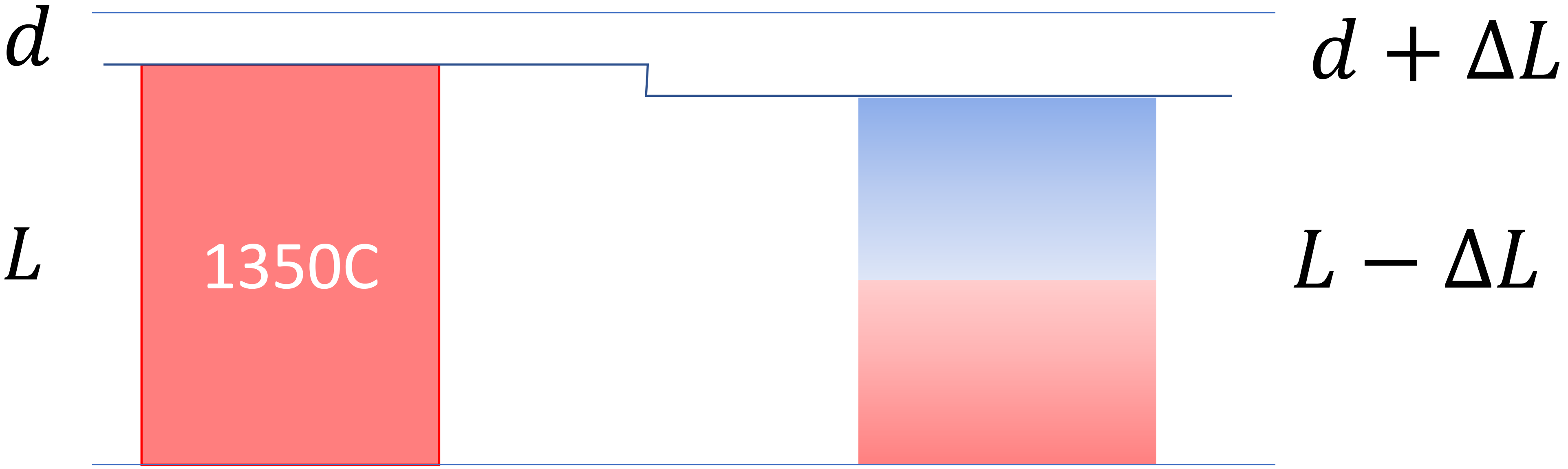
145 km



change in average density of rock

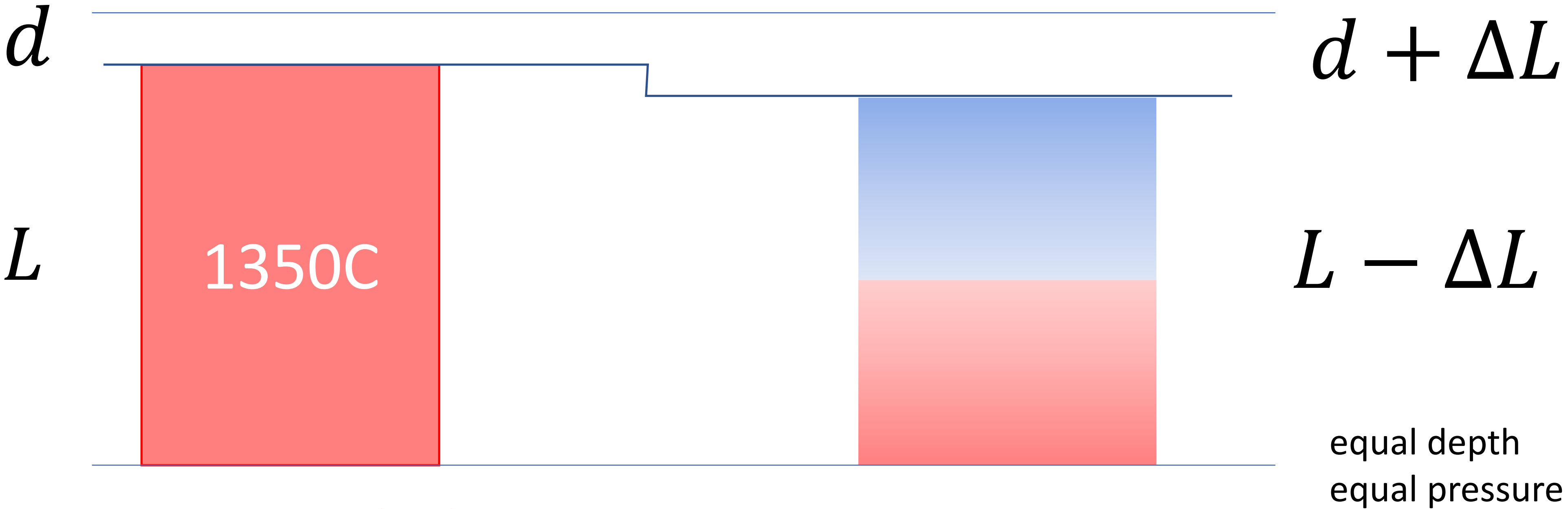
$$\rho_R A L = \quad M \quad = \rho_{CR} A (L - \Delta L)$$

$$\rho_{CR} = \rho_R \frac{L}{(L - \Delta L)} = \frac{\rho_R}{(1 - \Delta L / L)}$$



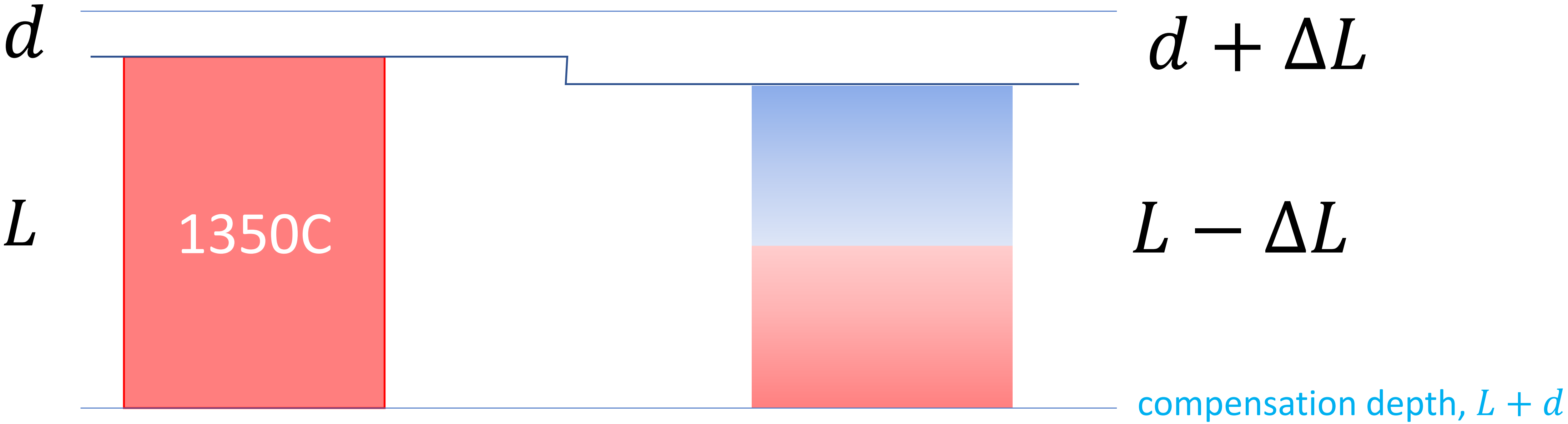
total mass

$\rho_R A L + \rho_W A d$ $\quad$ $\rho_{CR} A (L - \Delta L) + \rho_W A (d + \Delta L)$



are these in isostatic balance?

Pratt Balance: Equal mass column
(equal pressure)
at (deep) equal depth



d

L

1350C

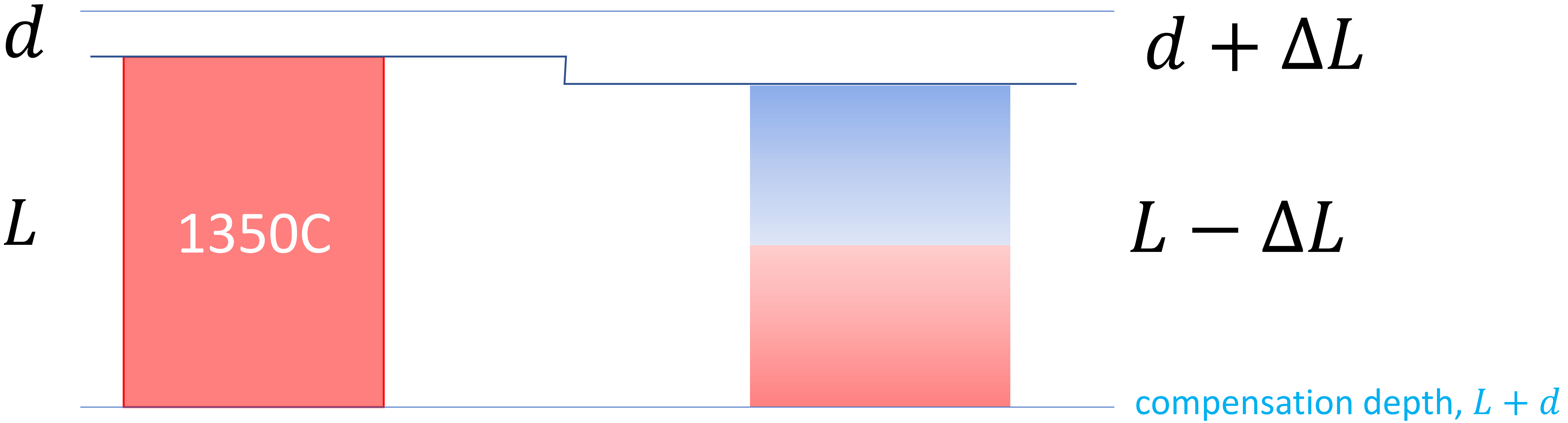
$d + \Delta L$

$L - \Delta L$

compensation depth, $L + d$

are these in isostatic balance?

Pratt Balance: Equal mass column
(equal pressure)
at (deep) equal depth
“depth of compensation”



d

L

1350C

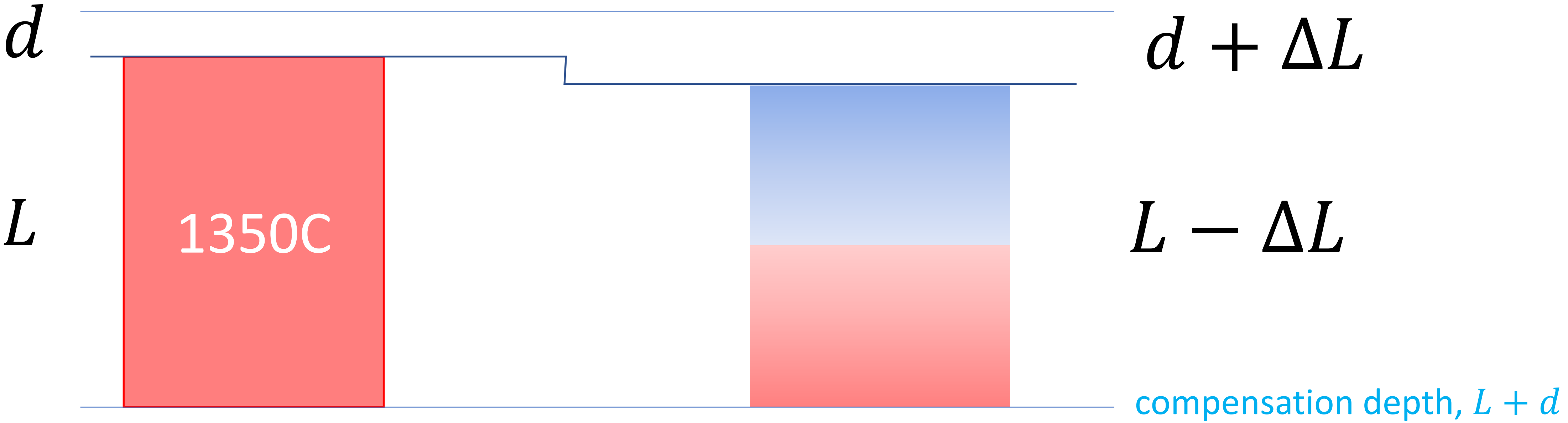
$d + \Delta L$

$L - \Delta L$

compensation depth, $L + d$

are these in isostatic balance?

$$\rho_R A L + \rho_W A d \qquad \rho_{CR} A (L - \Delta L) + \rho_W A (d + \Delta L)$$



d

L

1350C

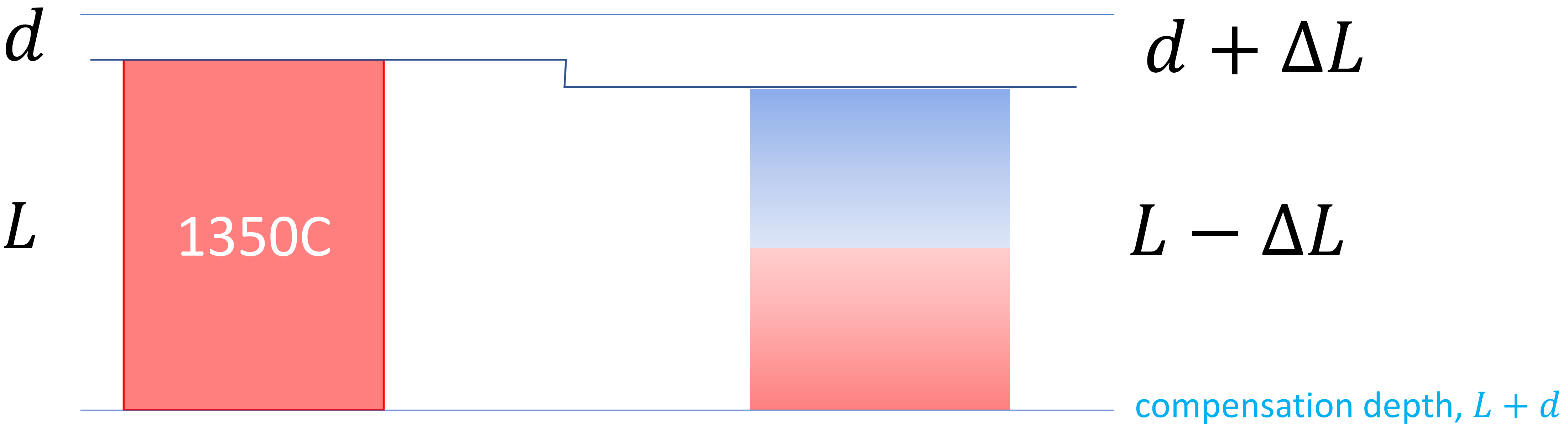
$d + \Delta L$

$L - \Delta L$

compensation depth, $L + d$

are these in isostatic balance?

$$\rho_R L + \rho_W d \qquad \frac{\rho_R}{(1 - \Delta L/L)}(L - \Delta L) + \rho_W(d + \Delta L)$$

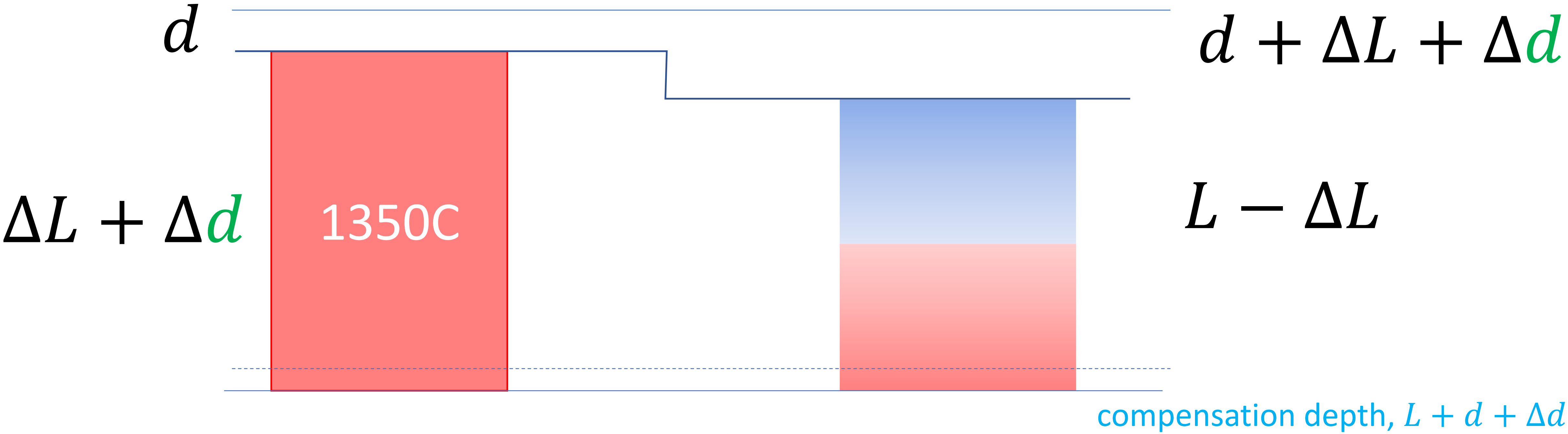


are these in isostatic balance?

$$\rho_R L + \rho_W d \qquad \frac{\rho_R L}{\cancel{(1 - \Delta L/L)}}\cancel{(1 - \Delta L/L)} + \rho_W (d + \Delta L)$$

$$\rho_R L + \rho_W d + \boxed{\rho_W \Delta L}$$

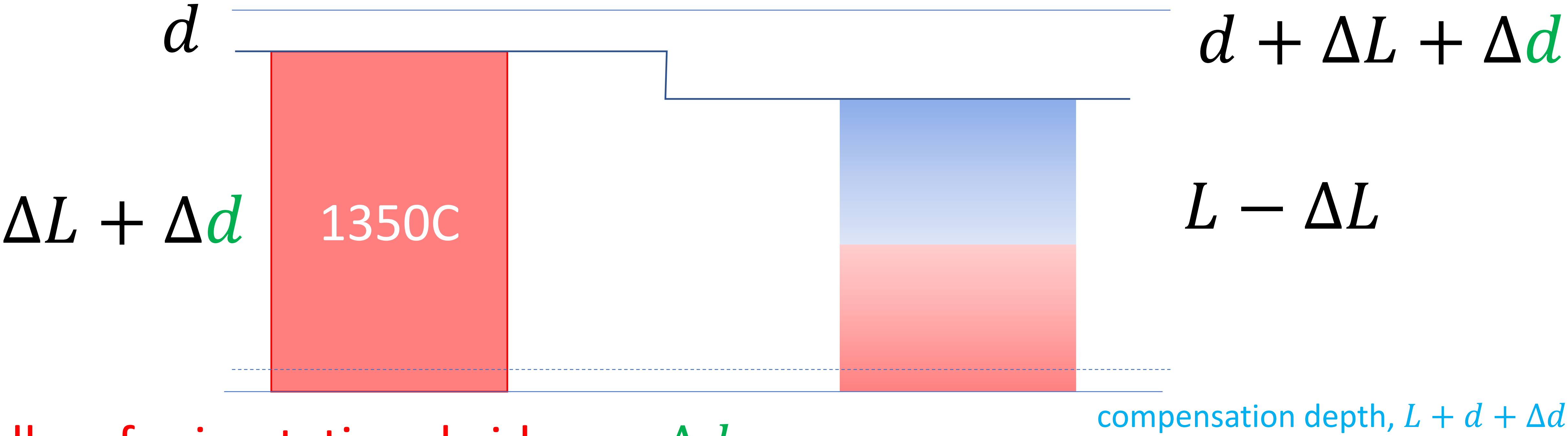
no.



allow for isostatic subsidence, Δd

$\rho_R AL + \underline{\rho_R} A\Delta d + \rho_W dA$

$\rho_{CR}(L - \Delta L)A + \rho_W A(d + \Delta L \underline{+ \Delta} d)$



d

$d + \Delta L + \Delta d$

$\Delta L + \Delta d$

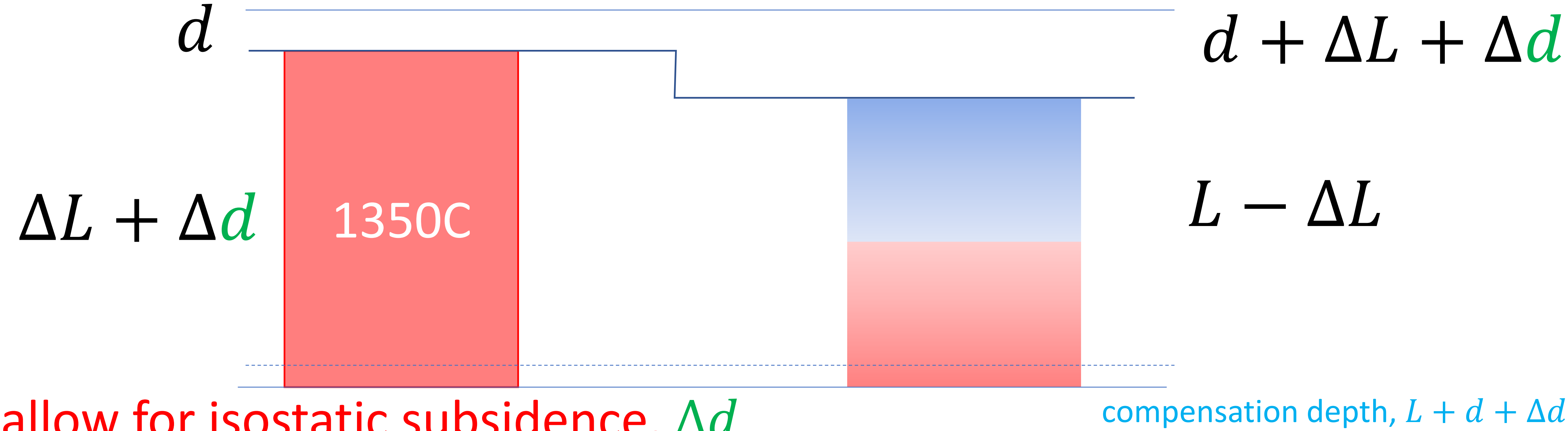
1350C

$L - \Delta L$

compensation depth, $L + d + \Delta d$

allow for isostatic subsidence, Δd

$$\rho_R L + \rho_R \Delta d + \rho_W d \quad = \quad \rho_R L + \rho_W d + \rho_W \Delta L + \rho_W \Delta d$$



d

$d + \Delta L + \Delta d$

$\Delta L + \Delta d$

1350C

$L - \Delta L$

compensation depth, $L + d + \Delta d$

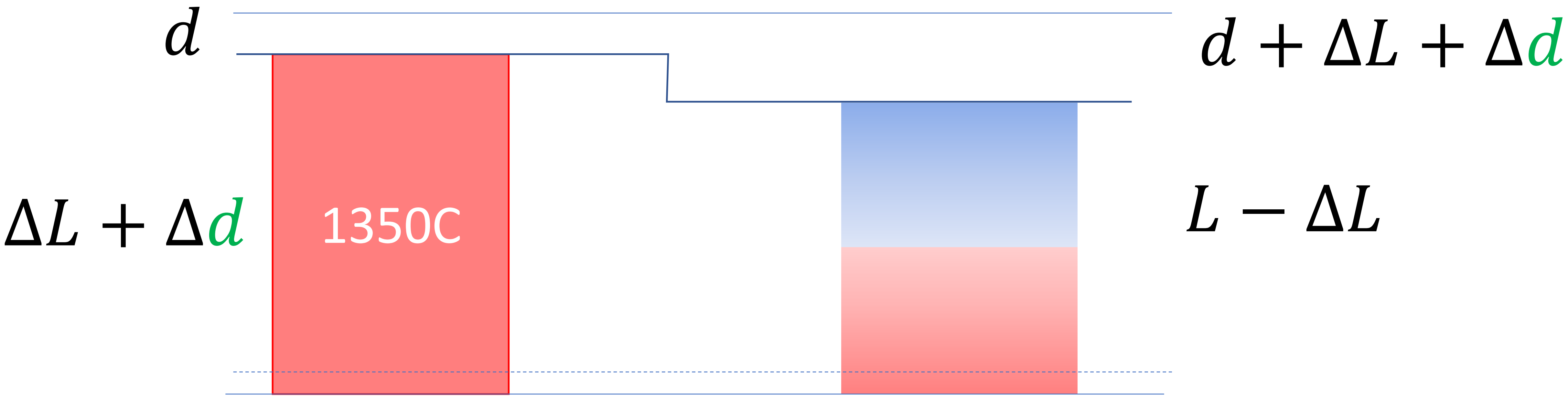
allow for isostatic subsidence, Δd

$$\rho_R L + \rho_R \Delta d + \rho_W d = \rho_R L + \rho_W d + \rho_W \Delta L + \rho_W \Delta d$$

$$\rho_R \Delta d = \rho_W \Delta L + \rho_W \Delta d$$

$$(\rho_R - \rho_W)\Delta d = \rho_W \Delta L$$

$$\Delta d = \frac{\rho_W}{(\rho_R - \rho_W)} \Delta L$$



allow for isostatic subsidence, Δd

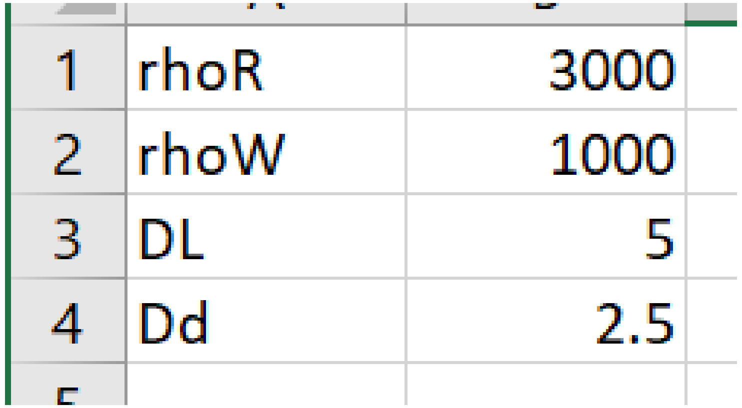
compensation depth, $L + d + \Delta d$

$$\rho_R L + \rho_R \Delta d + \rho_W d = \rho_R L + \rho_W d + \rho_W \Delta L + \rho_W \Delta d$$

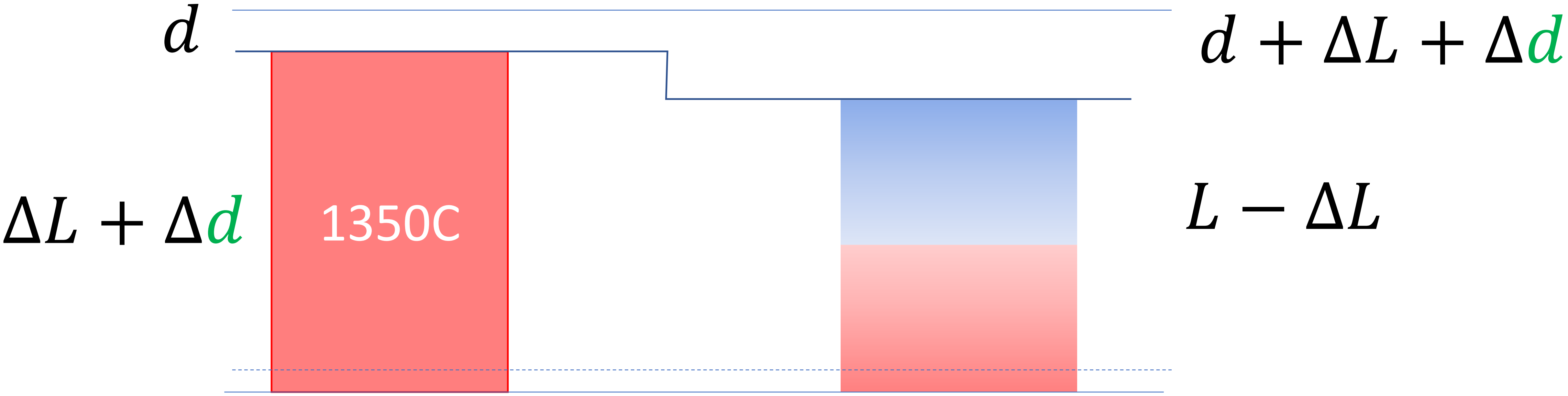
$$\rho_R \Delta d = \rho_W \Delta L + \rho_W \Delta d$$

$$(\rho_R - \rho_W)\Delta d = \rho_W \Delta L$$

$$\Delta d = \frac{\rho_W}{(\rho_R - \rho_W)} \Delta L$$



1	rhoR	3000
2	rhoW	1000
3	DL	5
4	Dd	2.5



allow for isostatic subsidence, Δd

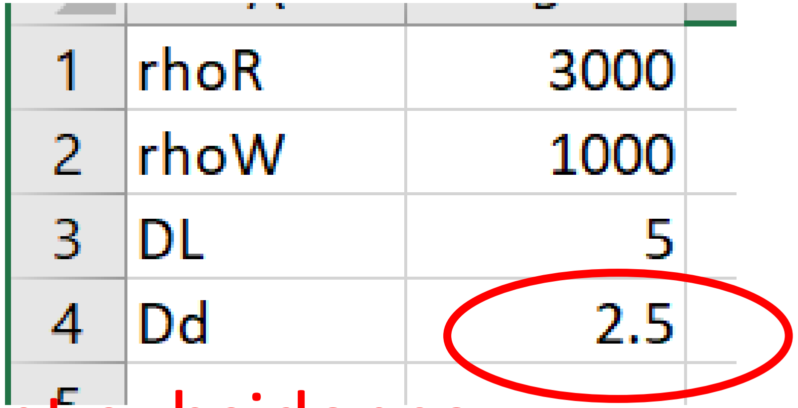
compensation depth, $L + d + \Delta d$

$$\rho_R L + \rho_R \Delta d + \rho_W d = \rho_R L + \rho_W d + \rho_W \Delta L + \rho_W \Delta d$$

$$\rho_R \Delta d = \rho_W \Delta L + \rho_W \Delta d$$

$$(\rho_R - \rho_W)\Delta d = \rho_W \Delta L$$

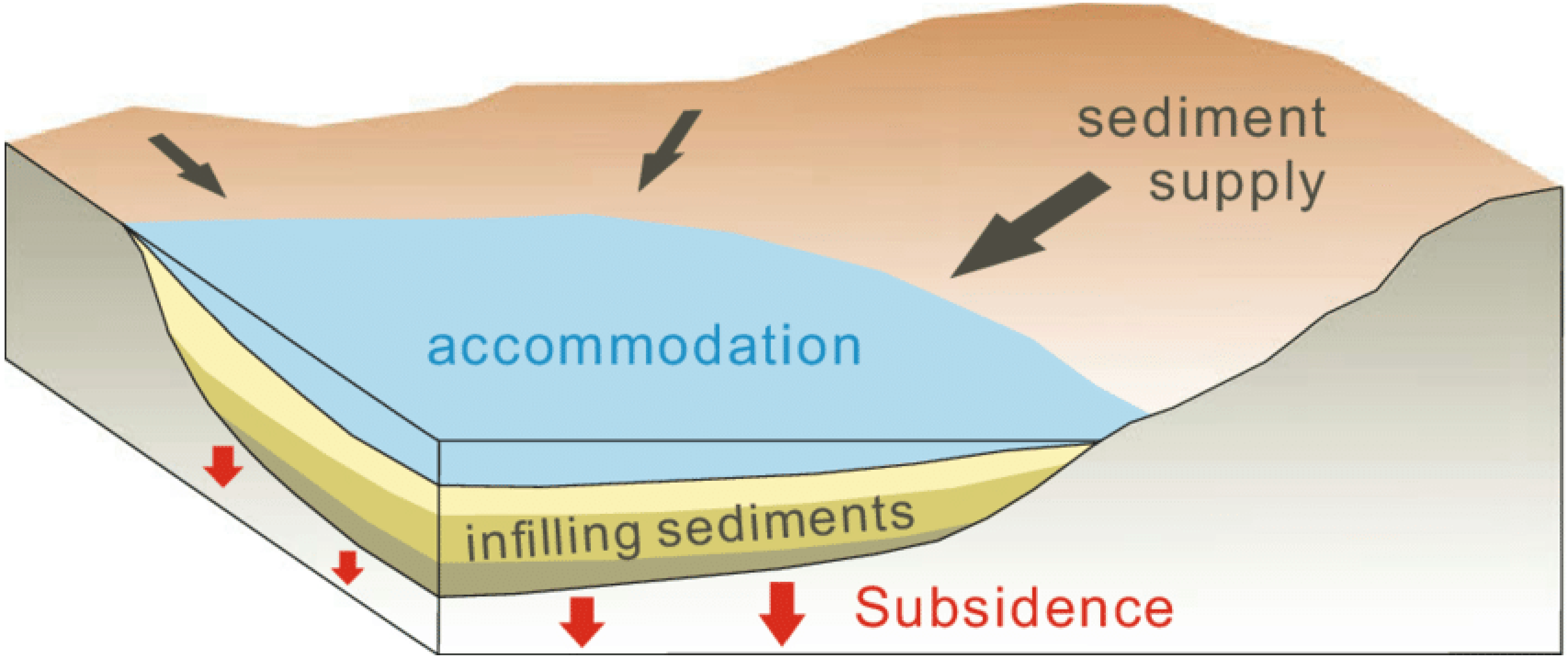
$$\Delta d = \frac{\rho_W}{(\rho_R - \rho_W)} \Delta L$$

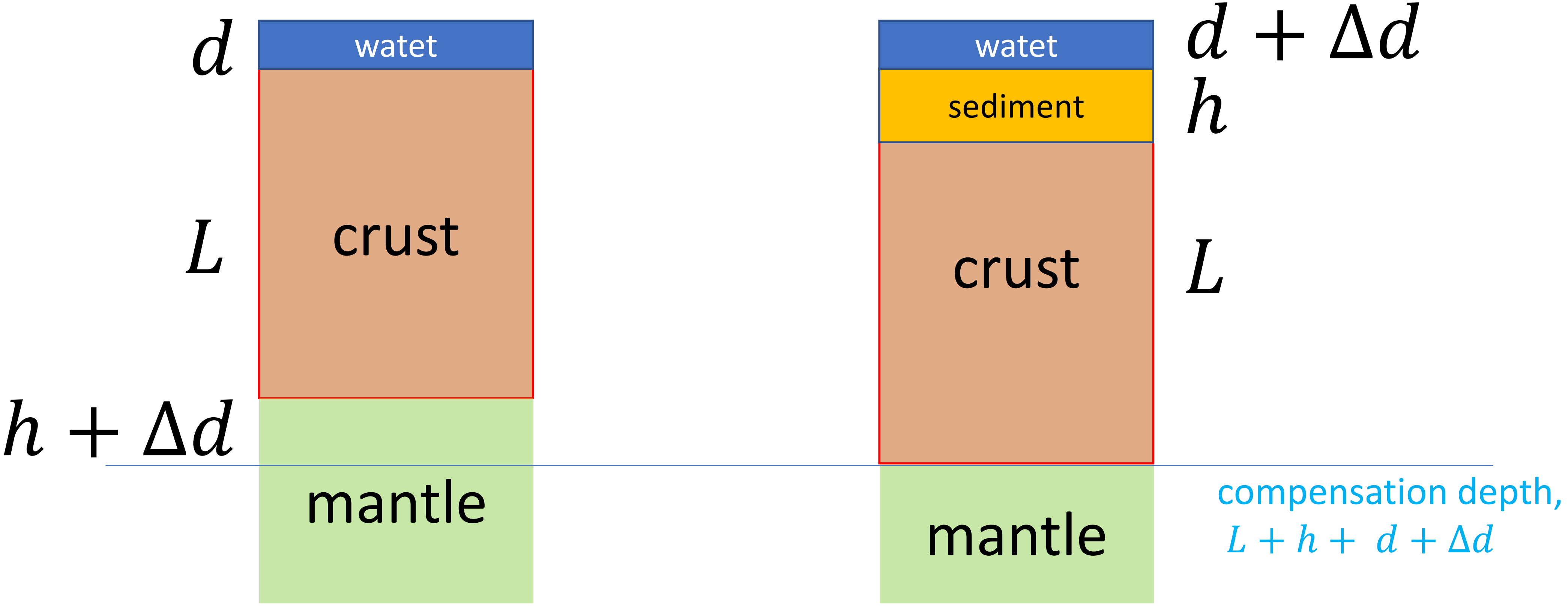


1	rhoR	3000
2	rhoW	1000
3	DL	5
4	Dd	2.5

significant subsidence

Effect of isostacy on sedimentary basins





d

watet

crust

L

$h + \Delta d$

mantle

$d + \Delta d$

watet

sediment

h

crust

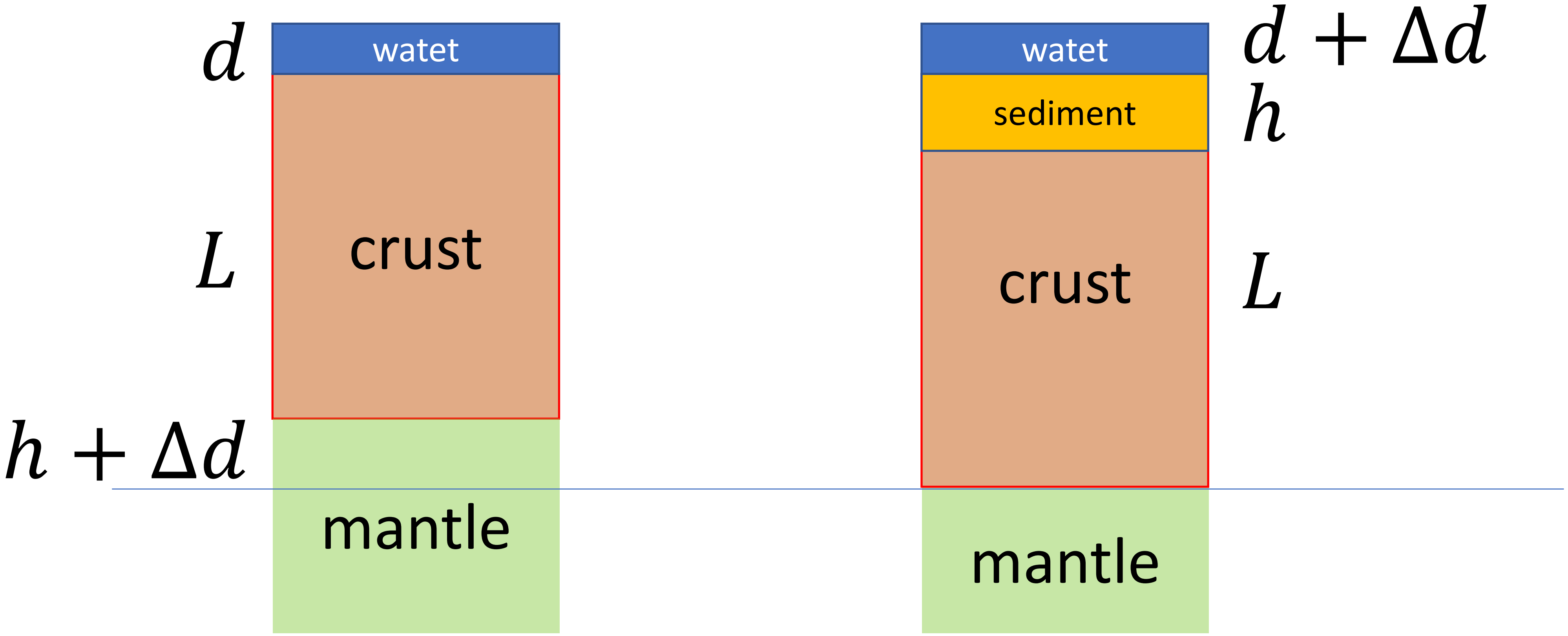
L

mantle

compensation depth,
$L + h + \ d + \Delta d$

$$\rho_W dA + \rho_C LA + \rho_M A(h + \Delta d)$$

$$\rho_W A(d + \Delta d) + \rho_S hA + \rho_C LA$$

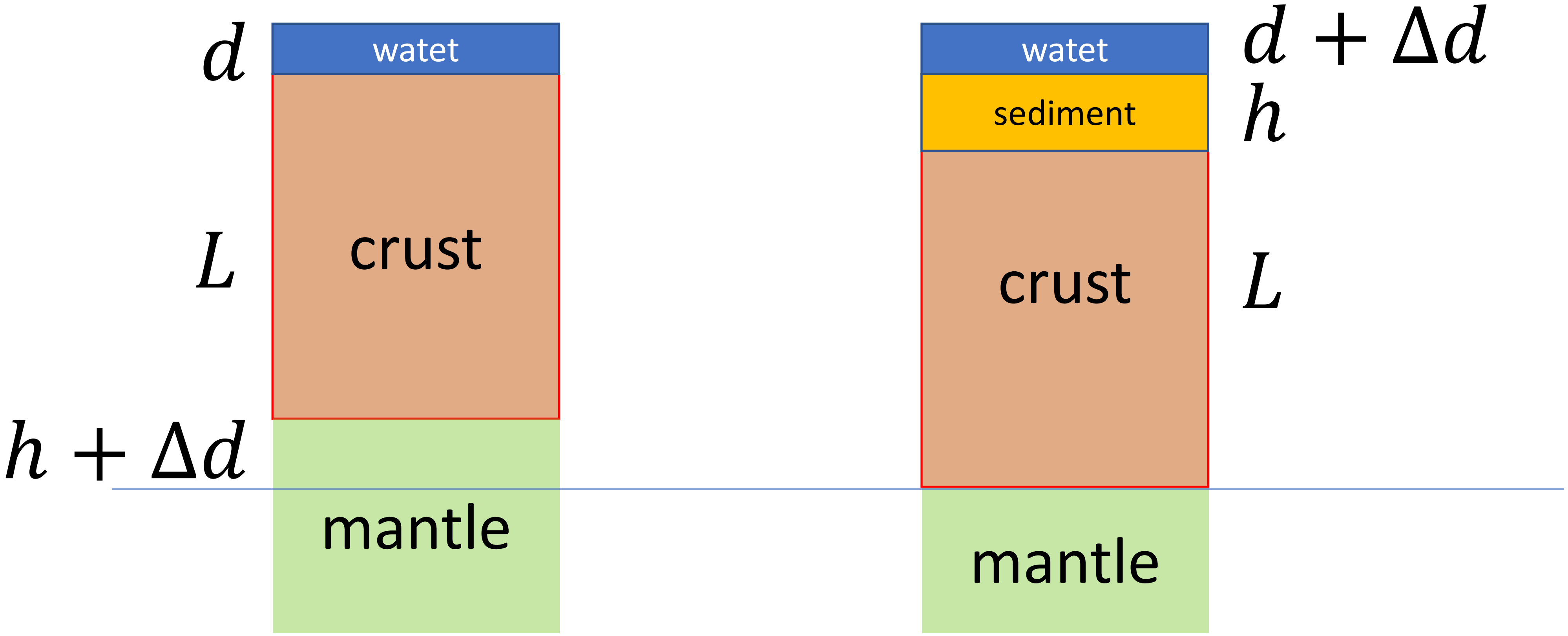


$$\cancel{\rho_W d} + \cancel{\rho_C L} + \rho_M h + \rho_M \Delta d \quad = \quad \cancel{\rho_W d} + \rho_W \Delta d + \rho_S h + \cancel{\rho_C L}$$

$$\rho_M h + \rho_M \Delta d = \rho_W \Delta d + \rho_S h$$

$$(\rho_M - \rho_W)\Delta d = -(\rho_M - \rho_S)h$$

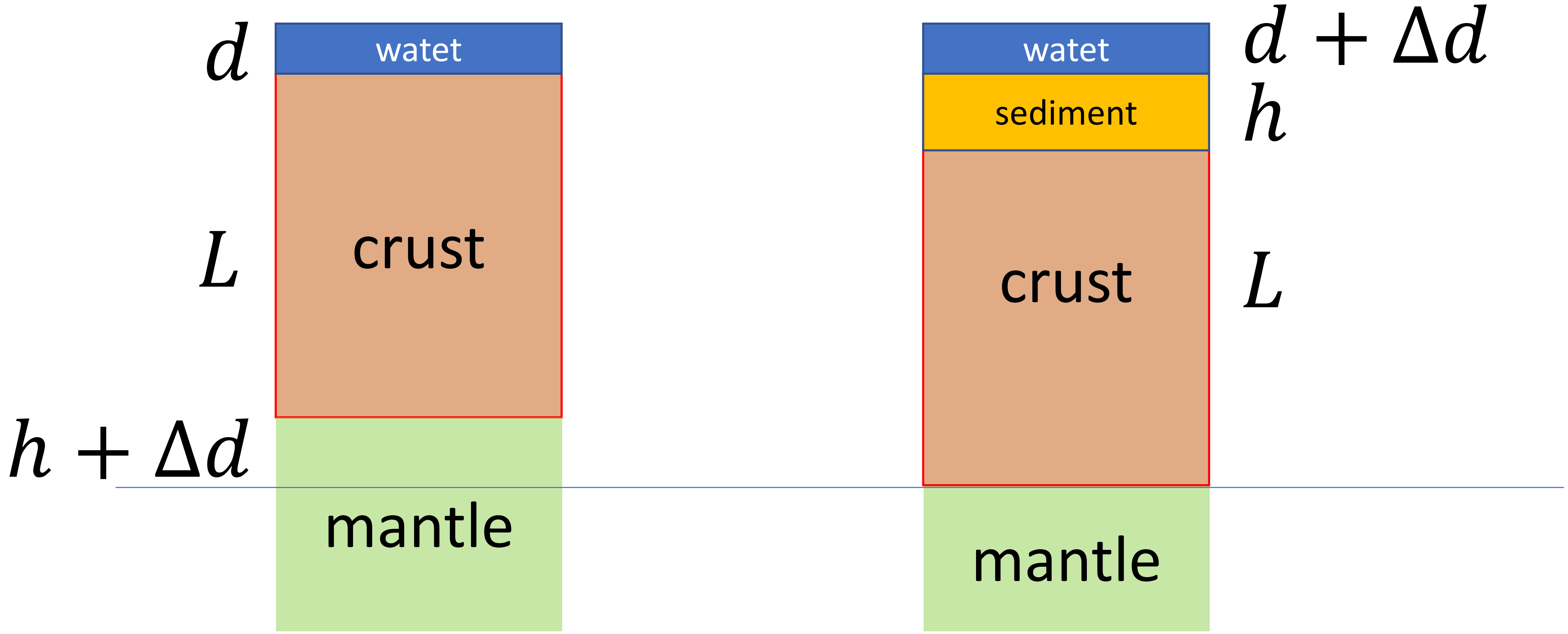
$$\Delta d = -\frac{(\rho_M - \rho_S)}{(\rho_M - \rho_W)}h$$



$$\Delta d = -\frac{(\rho_M - \rho_S)}{(\rho_M - \rho_W)} h$$

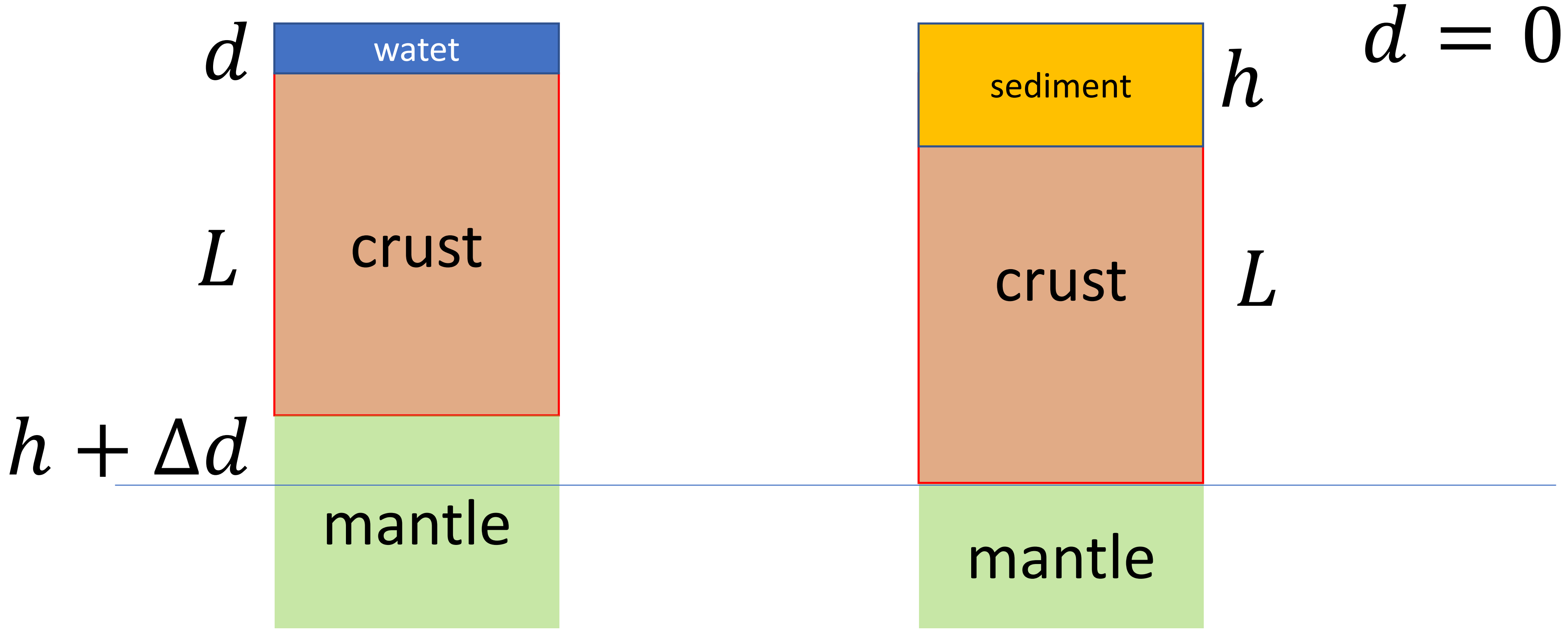
$$\frac{(\rho_M - \rho_S)}{(\rho_M - \rho_W)} = \frac{(3000 - 2500)}{(3000 - 1000)} = \frac{1}{4} = 0.25$$

For every 1 km of sediment put in, water depth deceases by 250 meters



$$\Delta d = -\frac{(\rho_M - \rho_S)}{(\rho_M - \rho_W)} h = -\frac{1}{4} d$$

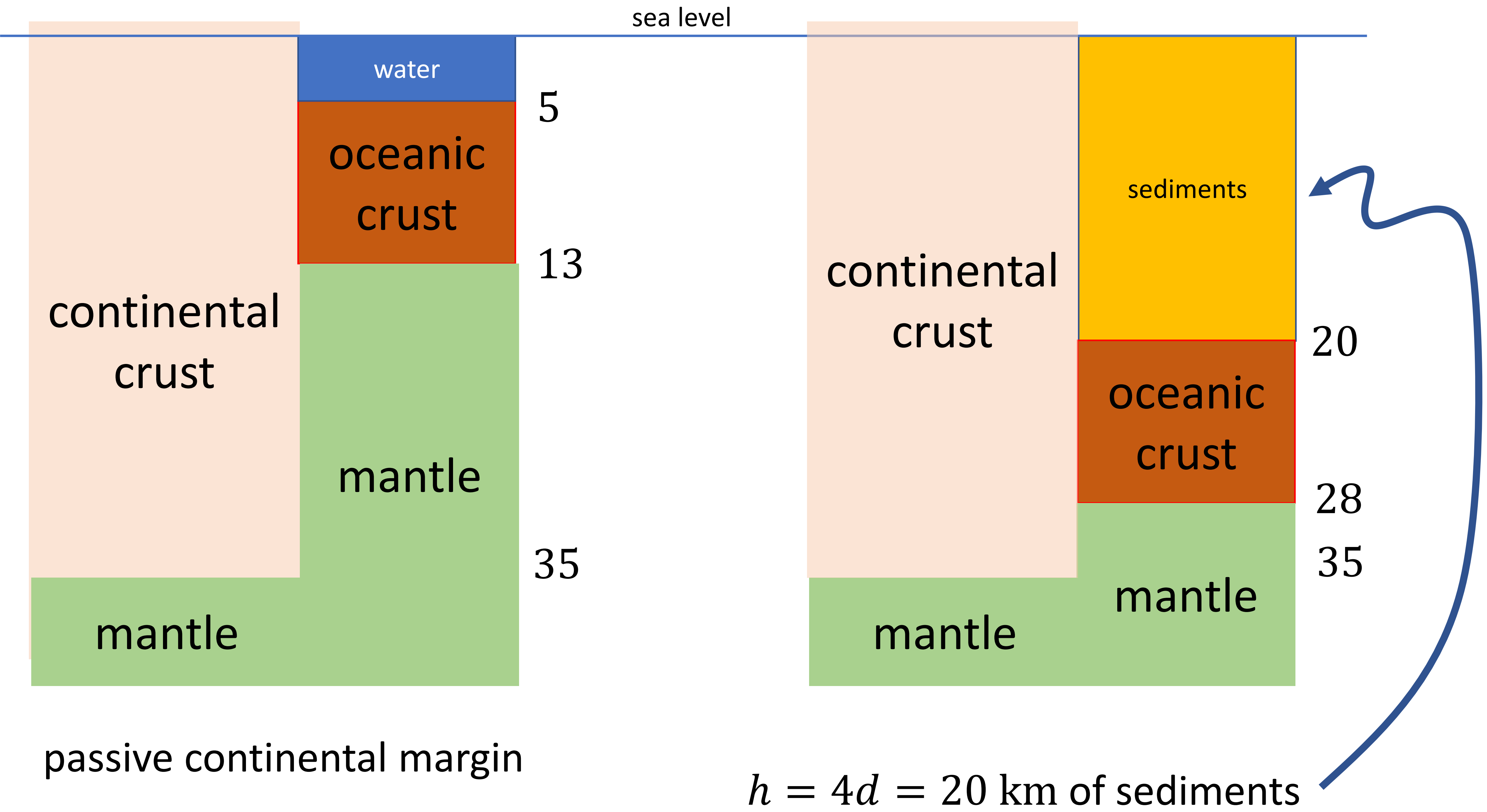
say you started with $d = 5$ km of water and replace it with sediments



$$\Delta d = -\frac{(\rho_M - \rho_S)}{(\rho_M - \rho_W)} h$$

$h = 4d = 20$ km of sediments

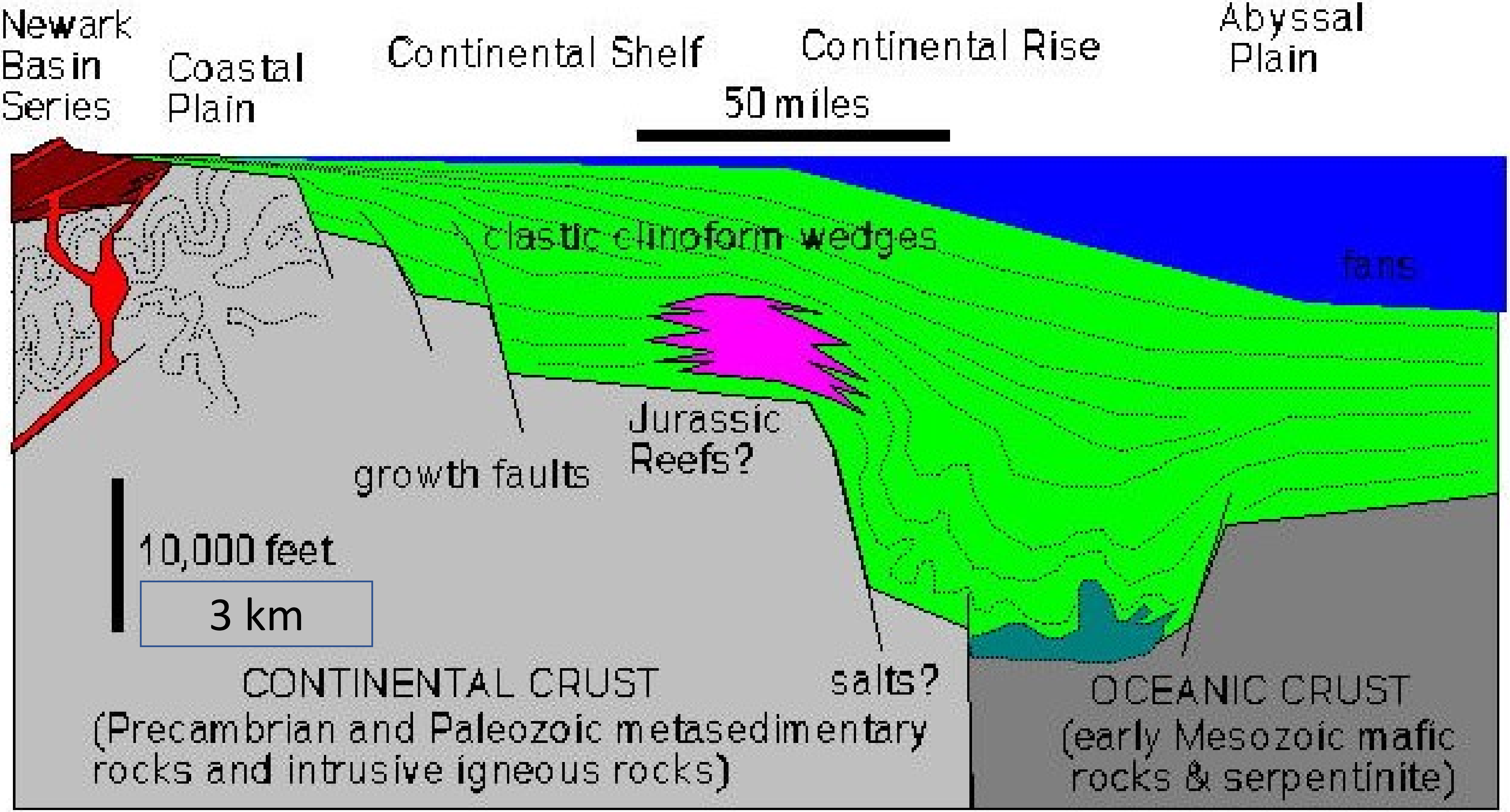
$$\Delta d = -d = -5 \text{ km}$$

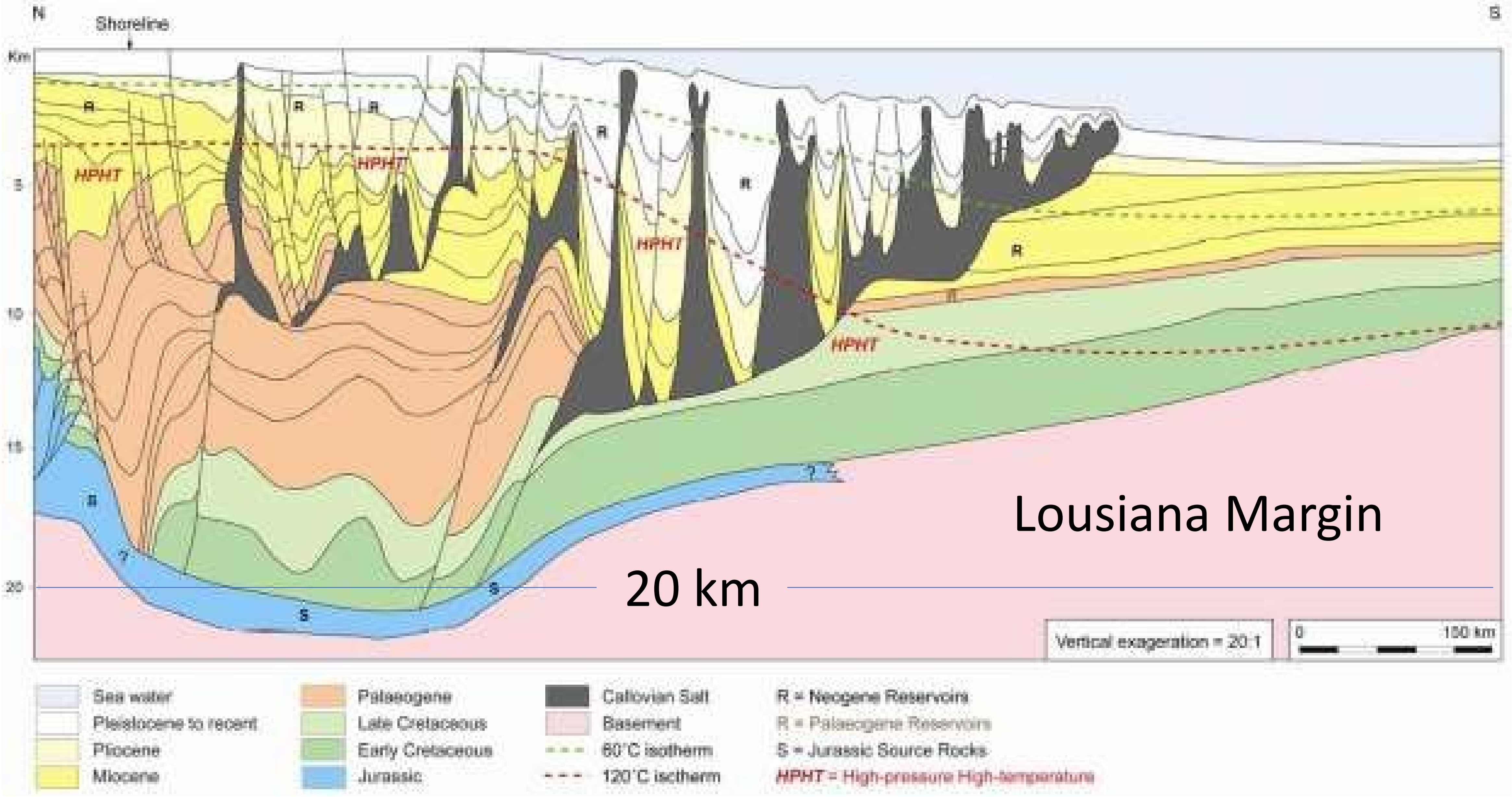


passive continental margin

$h = 4d = 20$ km of sediments

New Jersey Margin

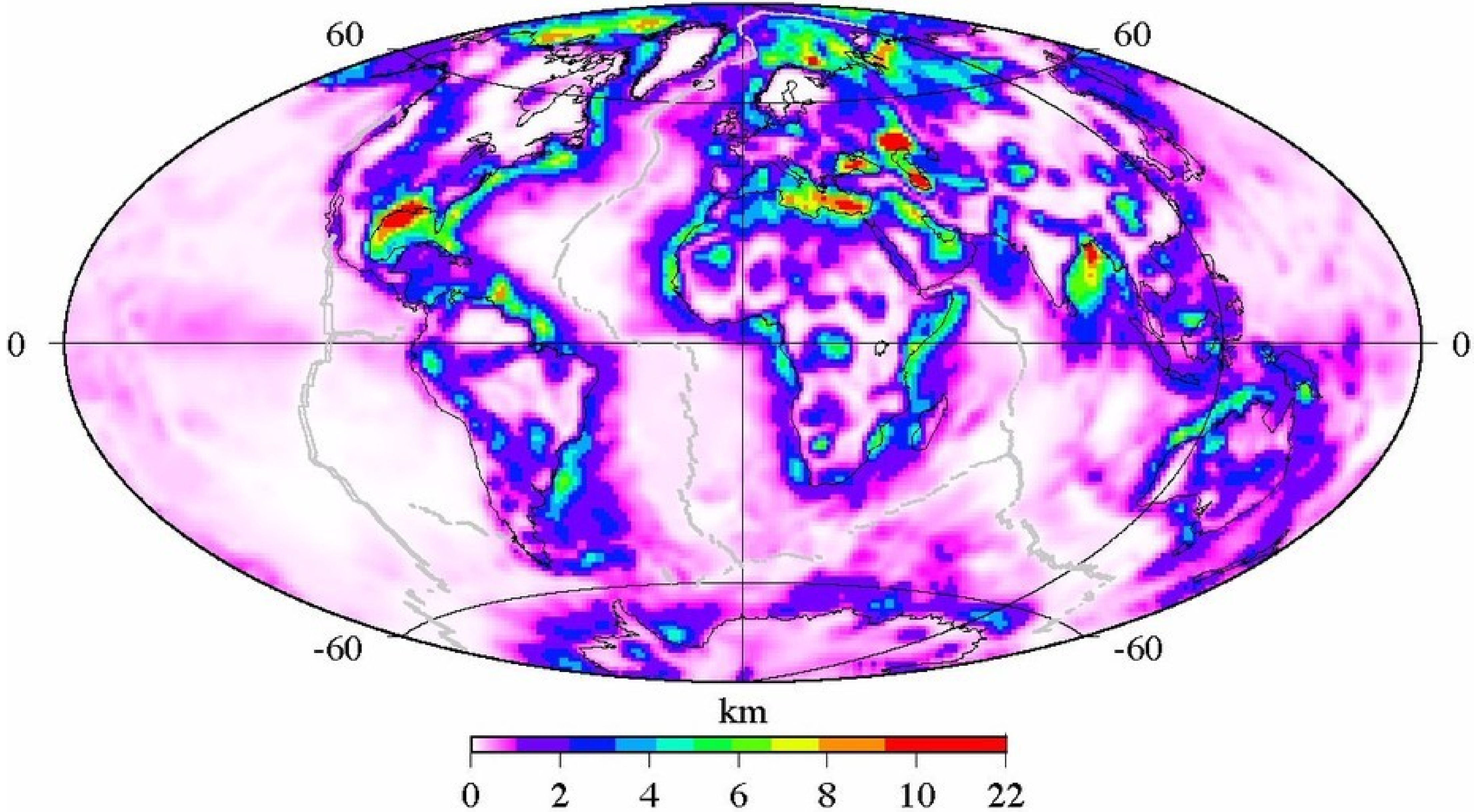




Lousiana Margin

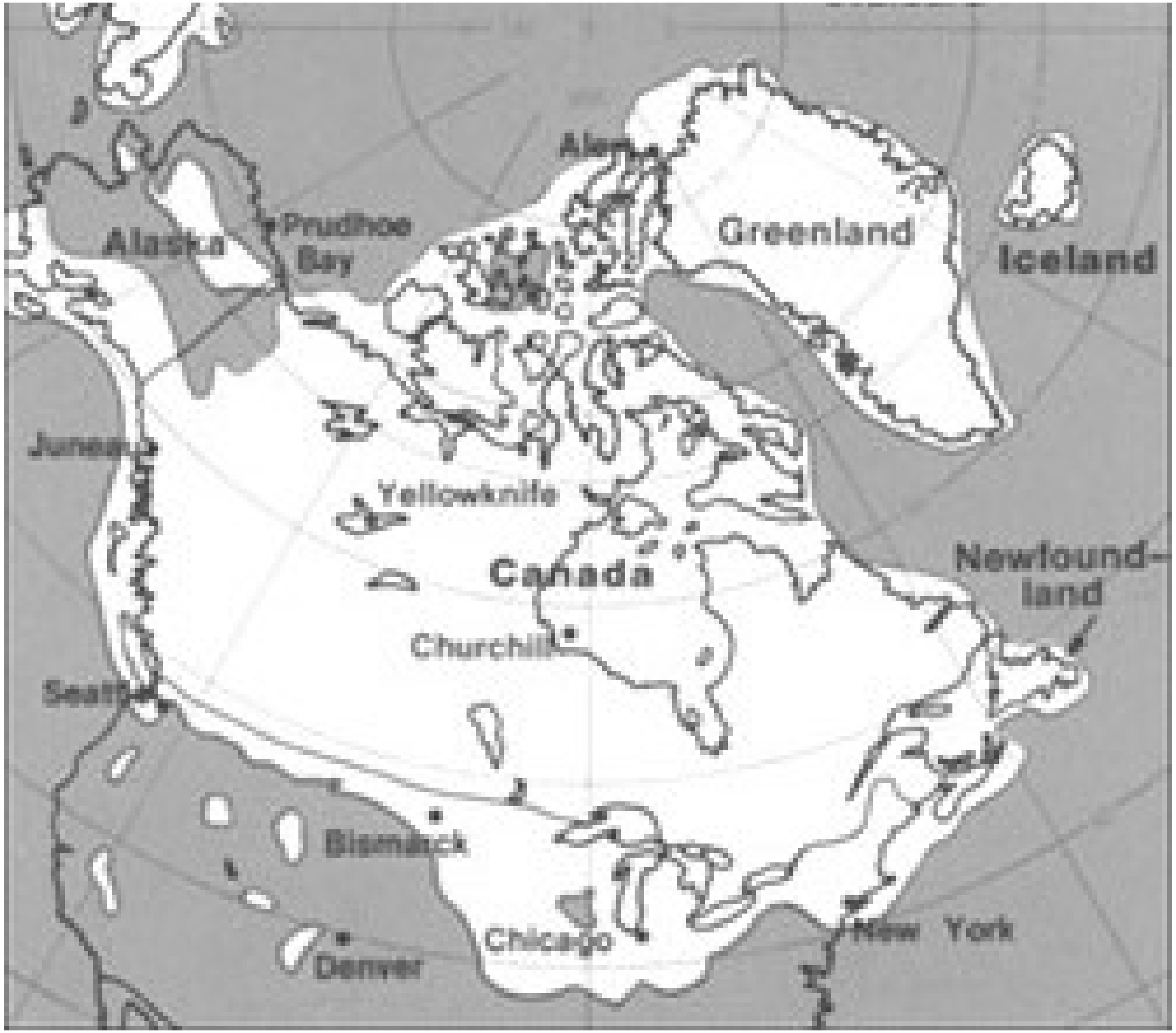
20 km

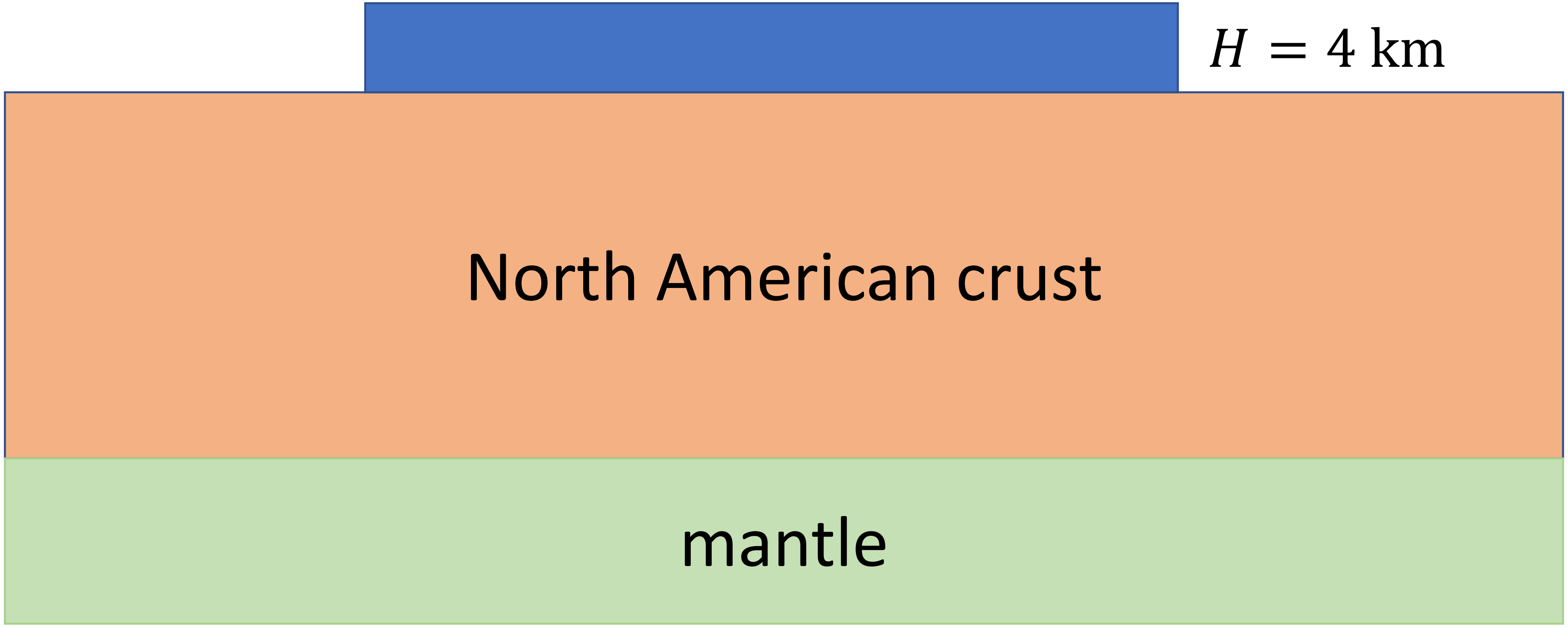
Global sediment thickness



Laske and Masters (1997).

Glacial isostatic rebound

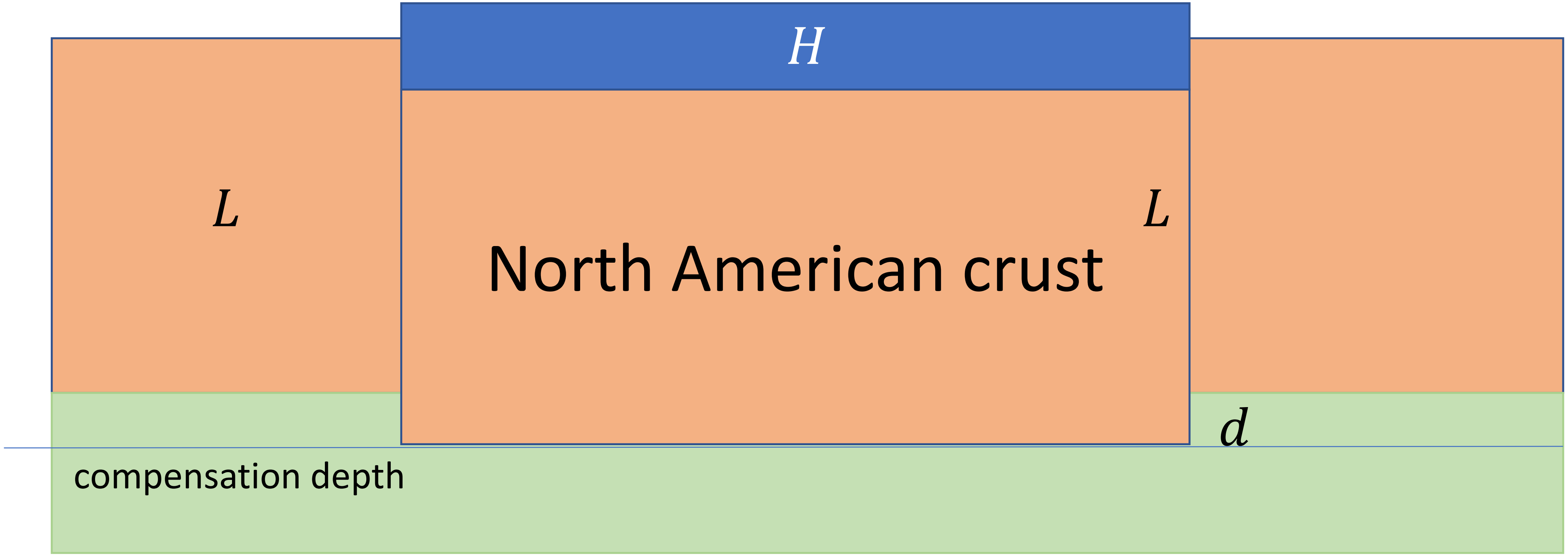




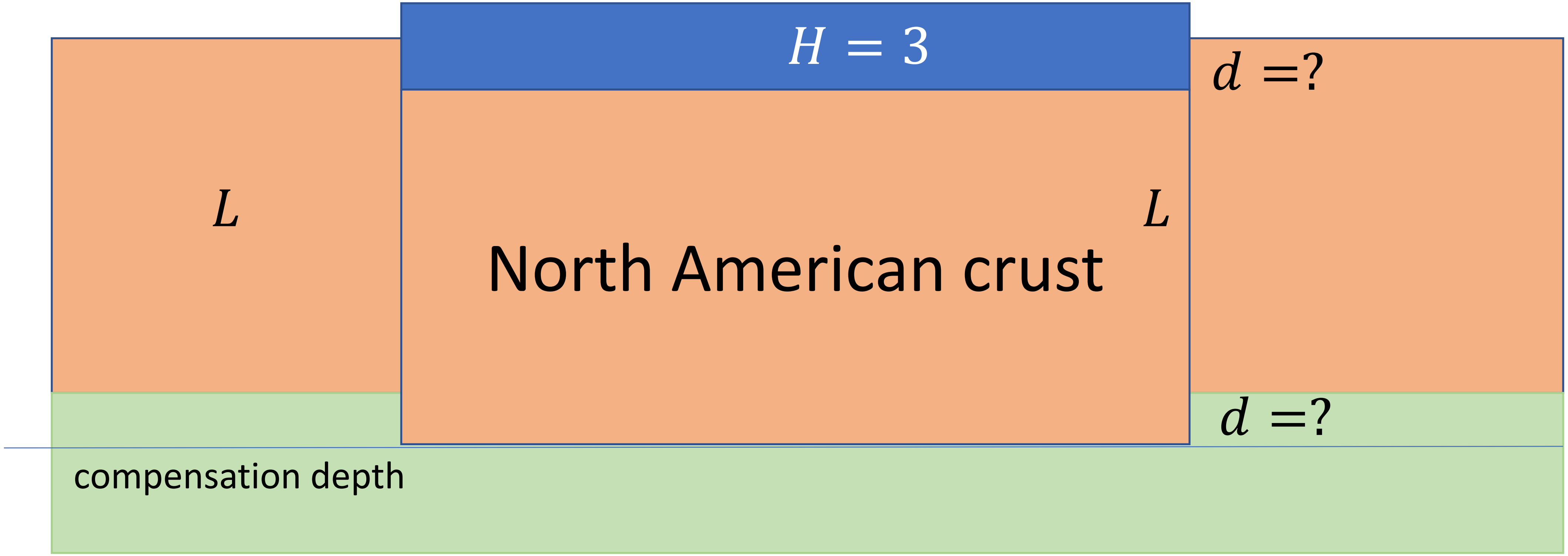
$H = 4$ km

North American crust

mantle



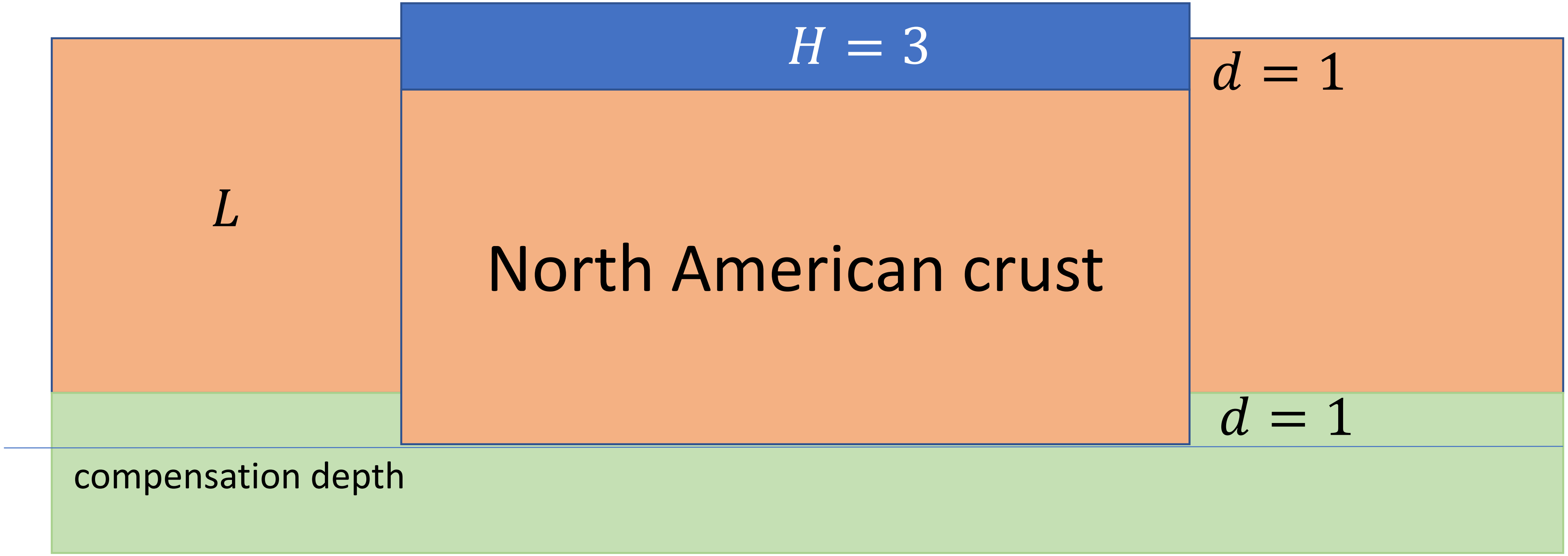
$$\rho_C L + \rho_M d = \rho_I H + \rho_C L$$



$$\rho_C L + \rho_M d = \rho_I H + \rho_C L$$

$$\rho_M d = \rho_I H$$

$$d = \frac{\rho_I}{\rho_M} H$$



$$\rho_C L + \rho_M d = \rho_I H + \rho_C L$$

$$\rho_M d = \rho_I H$$

$$d = \frac{1000}{3000} H = \frac{1}{3} H = 1\,\text{km}$$

How long does it take the hole to dissipate after the ice melts?

$$v = \frac{2r^2\ \Delta\rho\ g}{9\ \mu}$$

stokes law for
settling of sphere
(pretty sleazy)

$$t = \frac{d}{v} = \frac{9\ \mu}{2r\ \Delta\rho\ g}$$

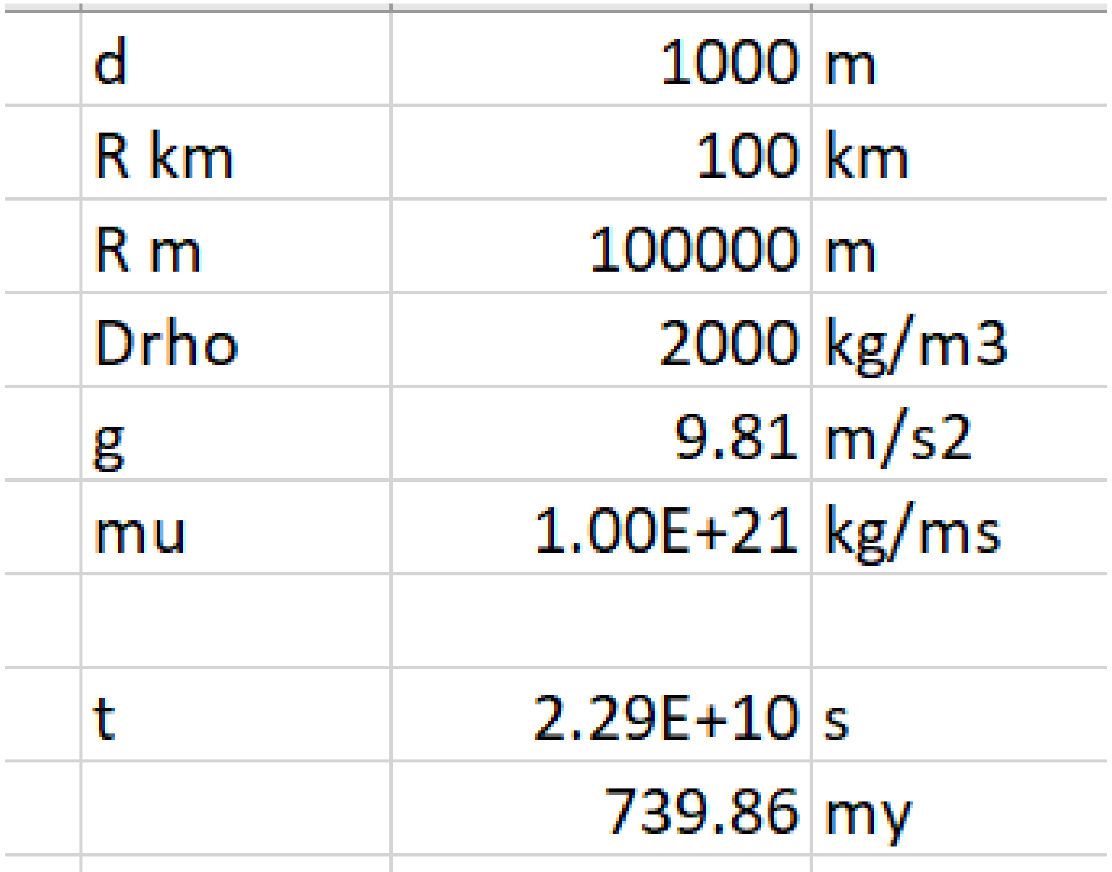
$$t = \frac{r}{v} = \frac{9\ \mu\ d}{2r^2\Delta\rho\ g}$$

How long does it take the hole to dissipate after the ice melts?

$$t = \frac{r}{v} = \frac{9\,\mu\,d}{2r^2\Delta\rho\;g}$$

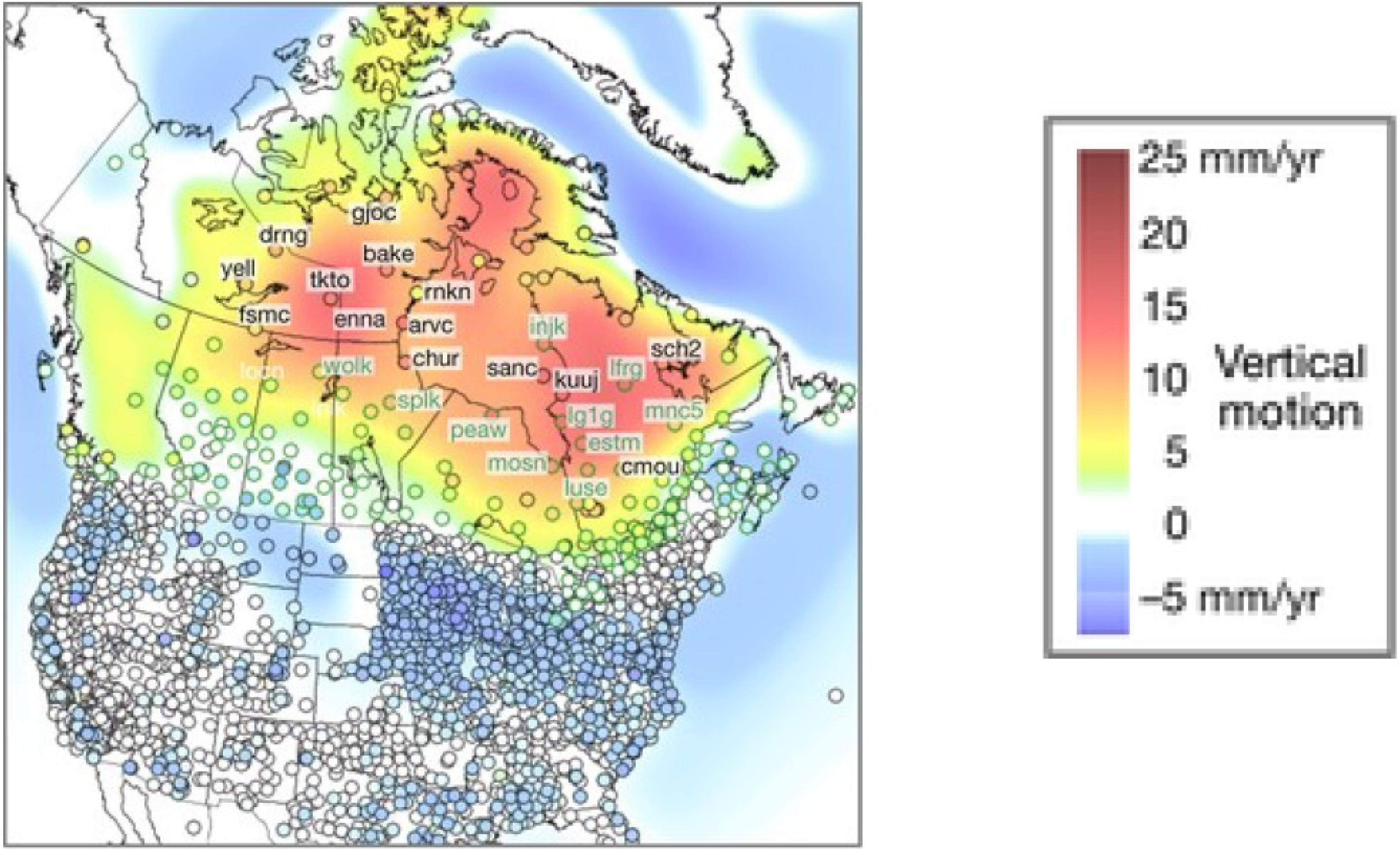
what value for r?

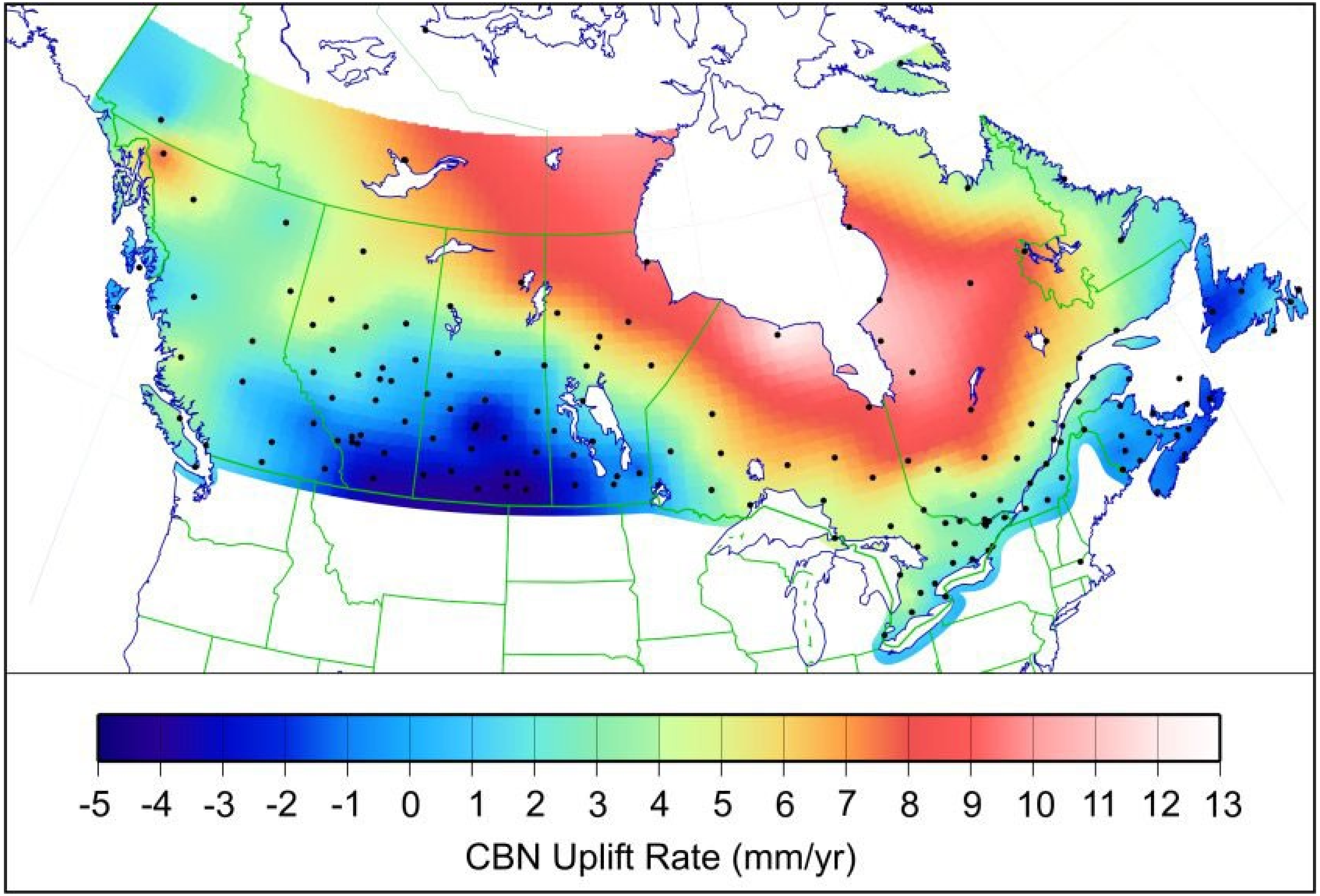
I'll use 100 km …

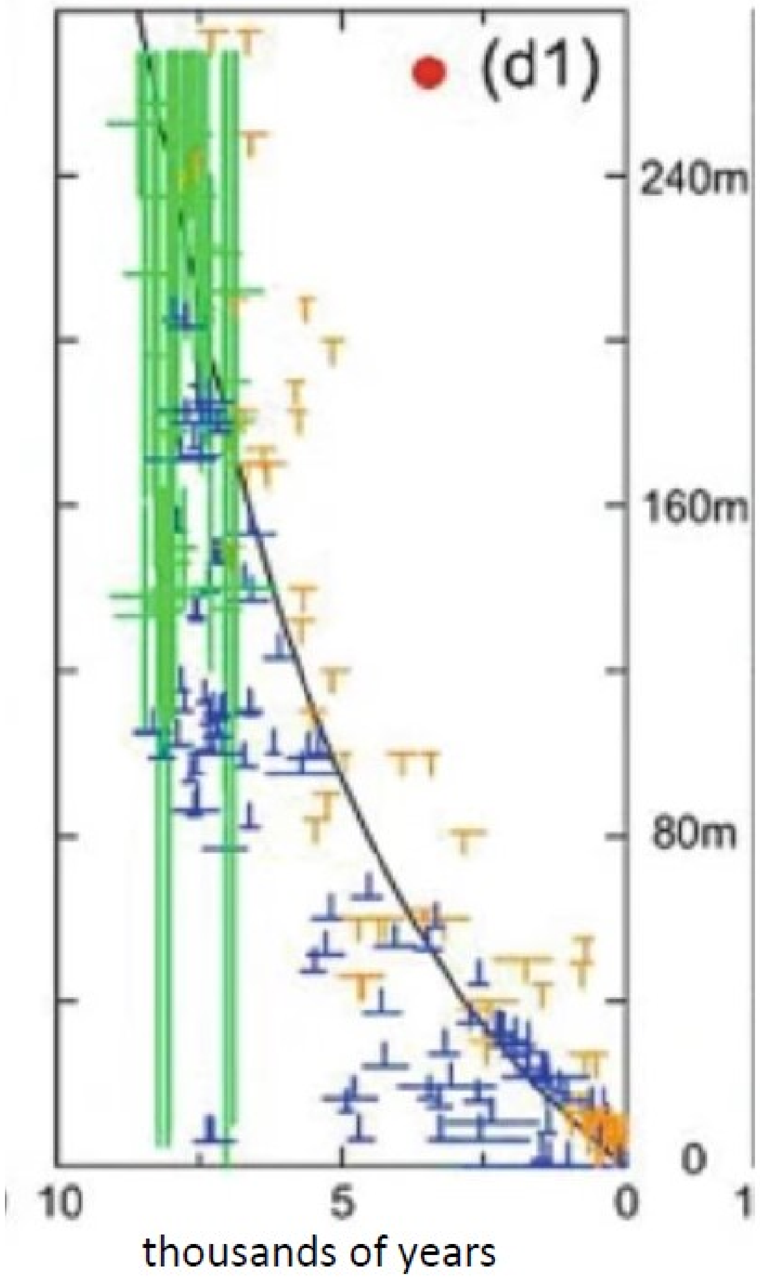


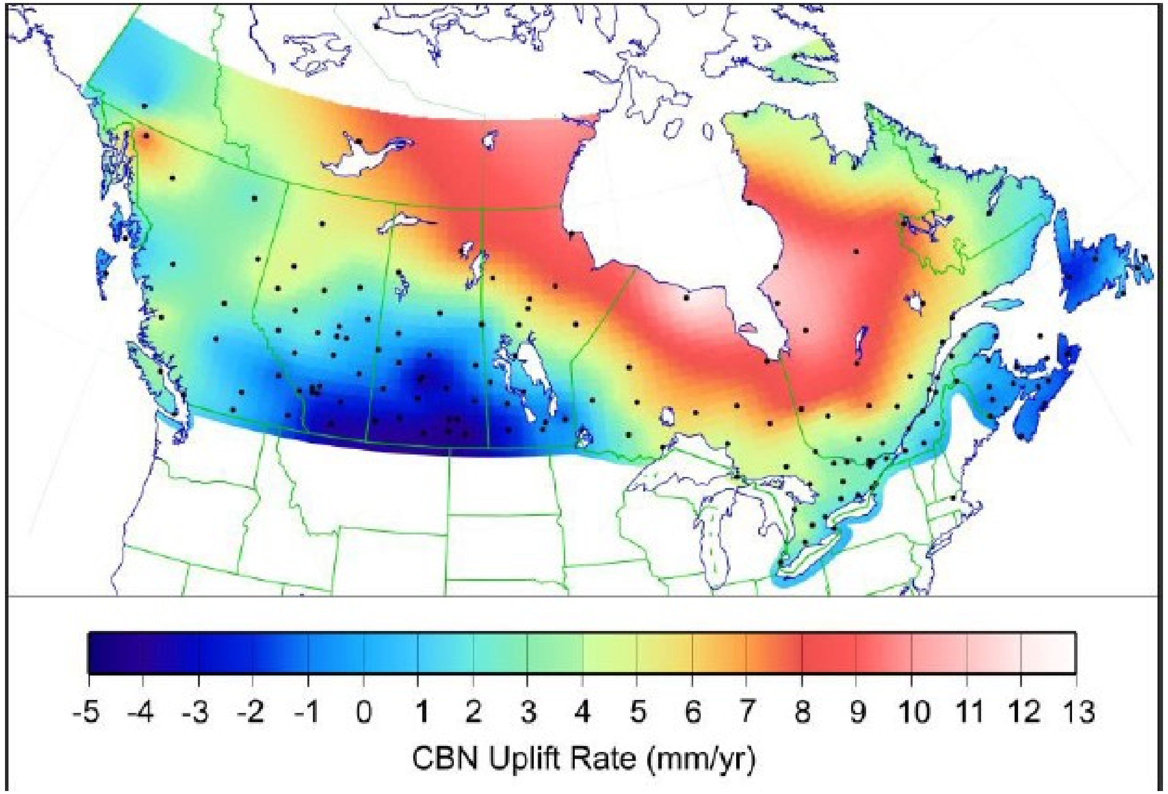
	d	1000	m
	R km	100	km
	R m	100000	m
	Drho	2000	kg/m3
	g	9.81	m/s2
	mu	1.00E+21	kg/ms
	t	2.29E+10	s
		739.86	my

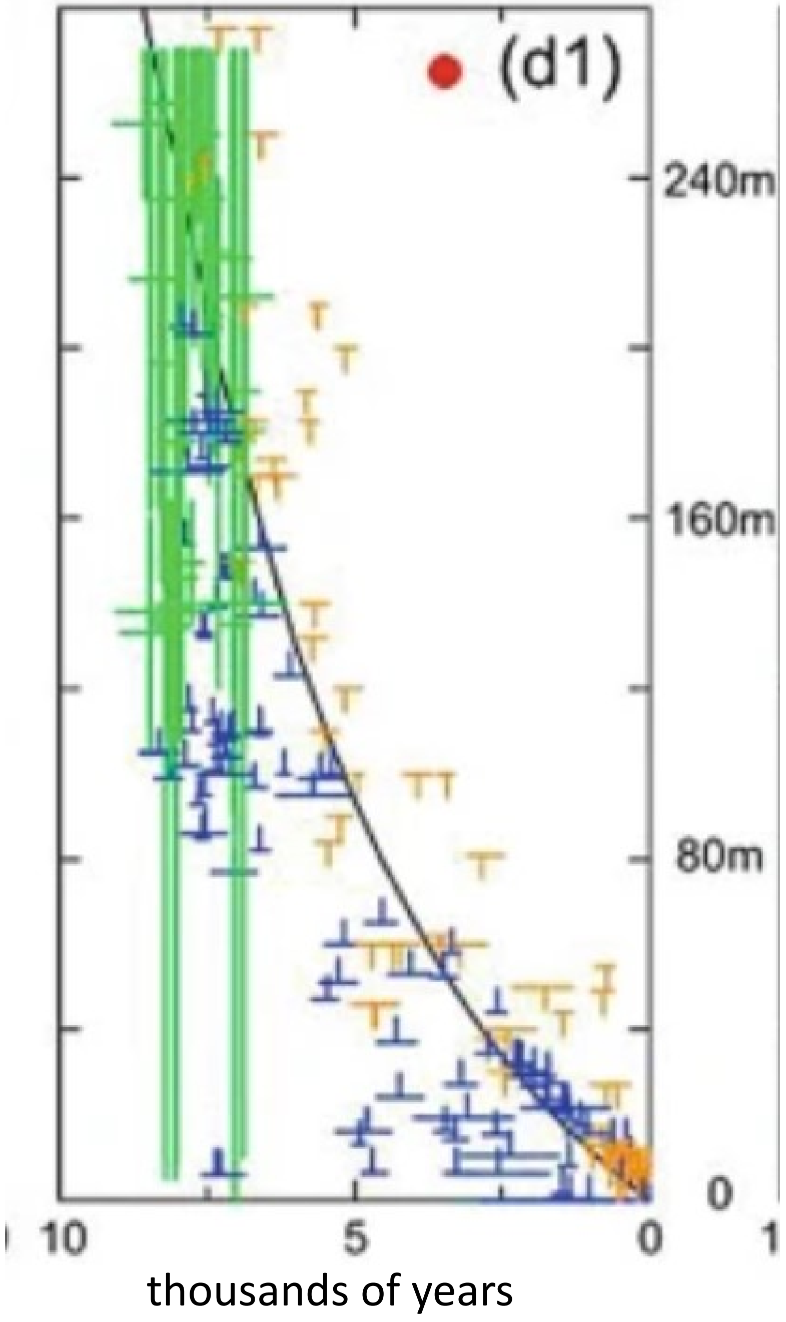
very sensitive to value of r, but probably time is "geologically short"


gjoc
drng
bake
yell
tkto
rnkn
fsmc
enna
arvc
injk
sch2
chur
sanc
kuuj
lfrg
locn
wolk
splk
lg1g
mnc5
peaw
estm
mosn
cmou
luse
25 mm/yr
20
15
10
5
0
−5 mm/yr
Vertical motion


-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
CBN Uplift Rate (mm/yr)


(d1)
240m
160m
80m
0
10
5
0
1
thousands of years


-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
CBN Uplift Rate (mm/yr)

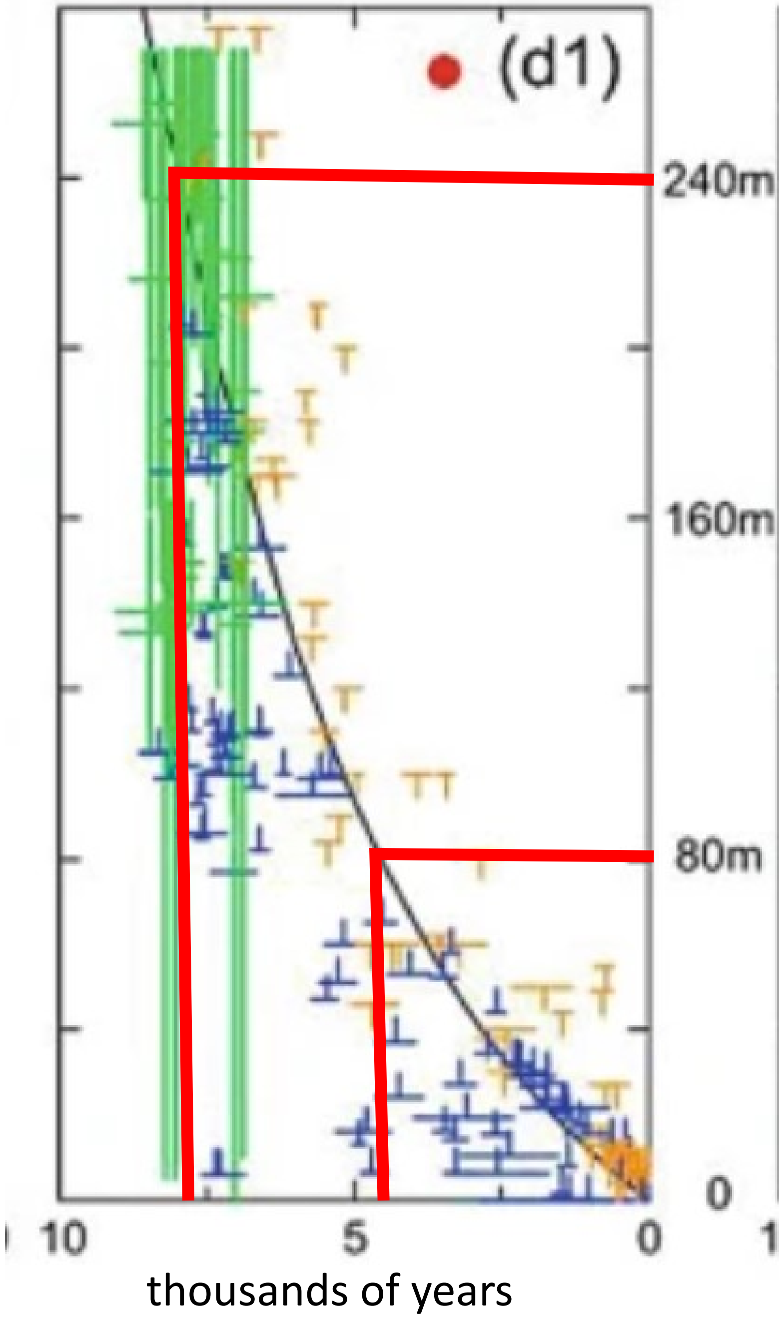


$$H = H_0 \exp\{-t/\tau\}$$

τ characteristic decay time

when $t = \tau$

$$\frac{H}{H} = \exp\{-1\} \approx \frac{1}{3}$$



$$H = H_0 \exp\{-t/\tau\}$$

τ characteristic decay time

when $t = \tau$

$$\frac{H}{H} = \exp\{-1\} \approx \frac{1}{3}$$

$$\tau \approx 3000 \text{ years}$$

oscillatory
functions
on a sphere

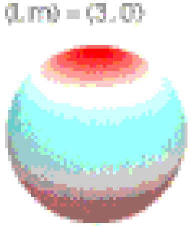
n = number of half-oscillations between north and south pole

n

0


1


2

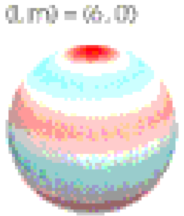

3


4


n

5


6


7


8


9


oscillatory functions on a sphere

n = 4
top about the
right size for the
ice cap

n

0


1


2

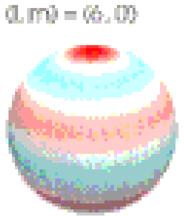

3

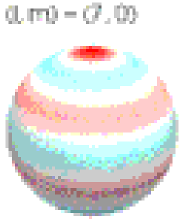

4


n

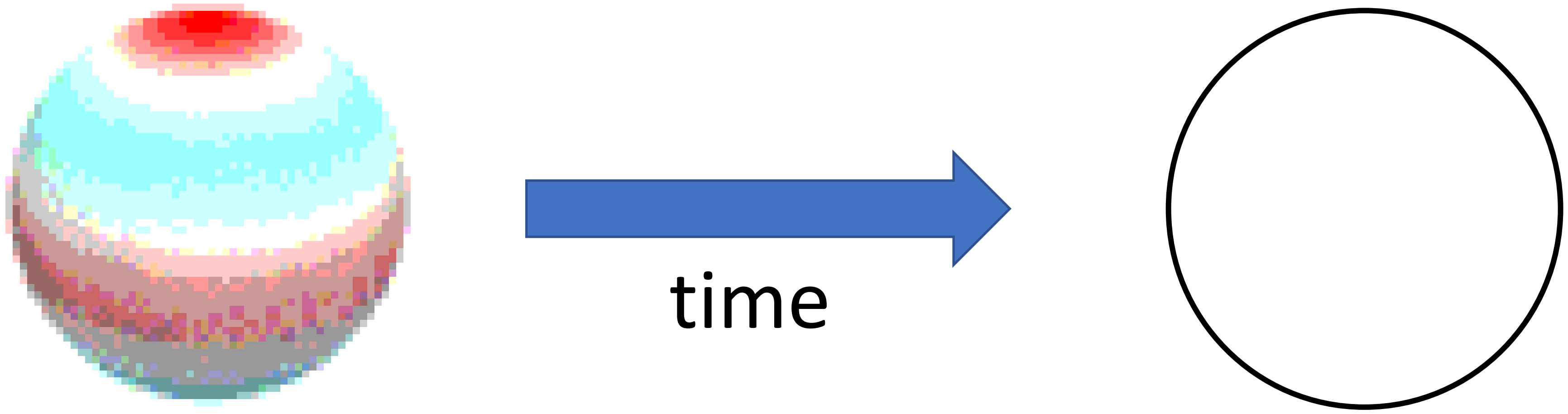
5


6


7


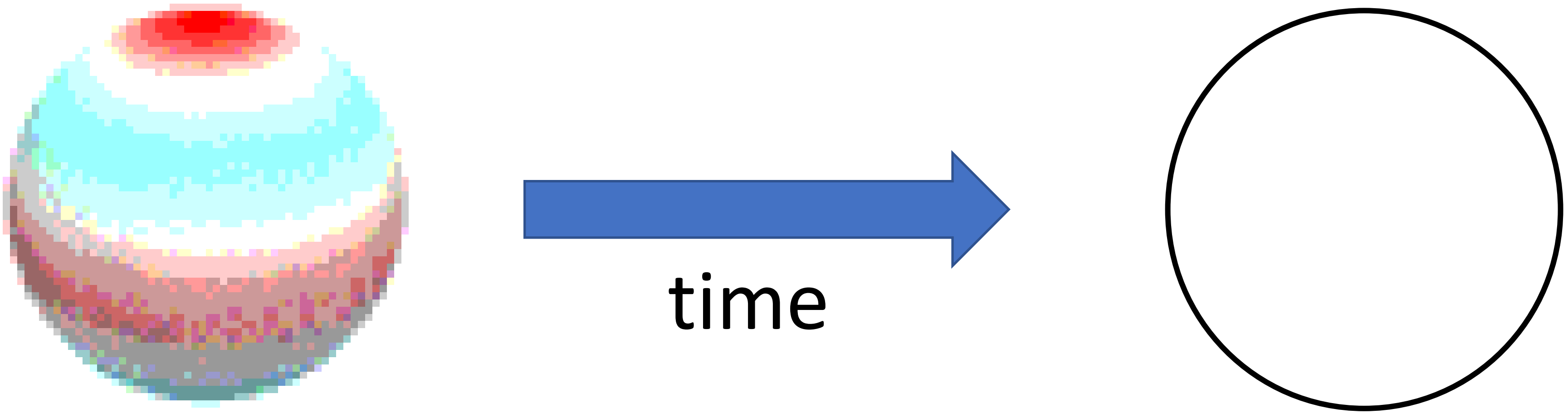
8


9

if a viscous sphere
was initially deformed to this shape

how long would it take to decay away?

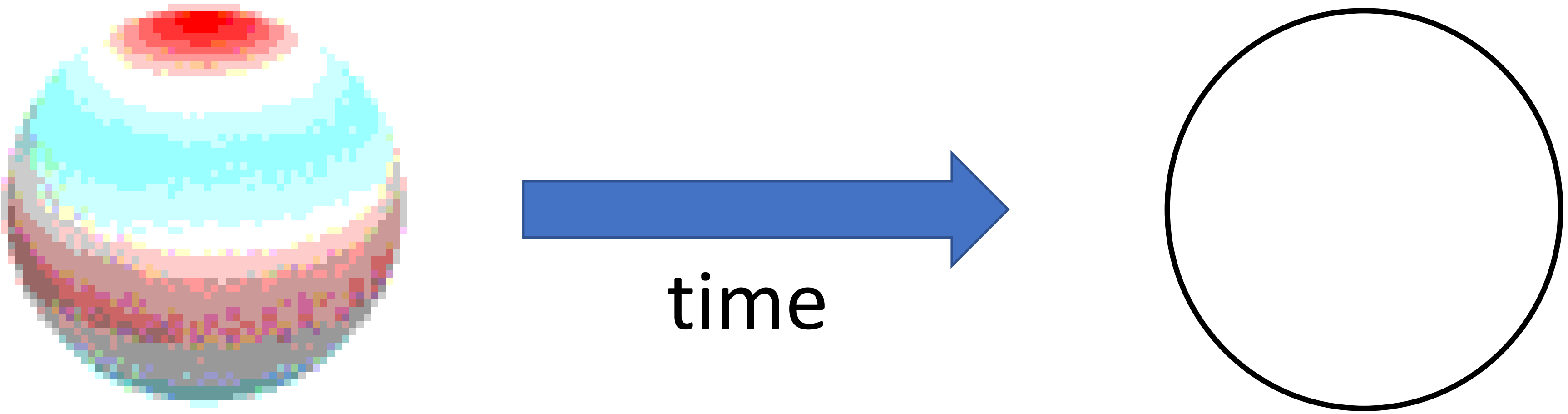


time

exponential decay

$$P_n(t) = P_n(t=0)\exp(-t/\tau)$$

with time constant, τ



exponential decay

$$P_n(t) = P_n(t = 0)\exp(-t/\tau)$$

with time constant, τ

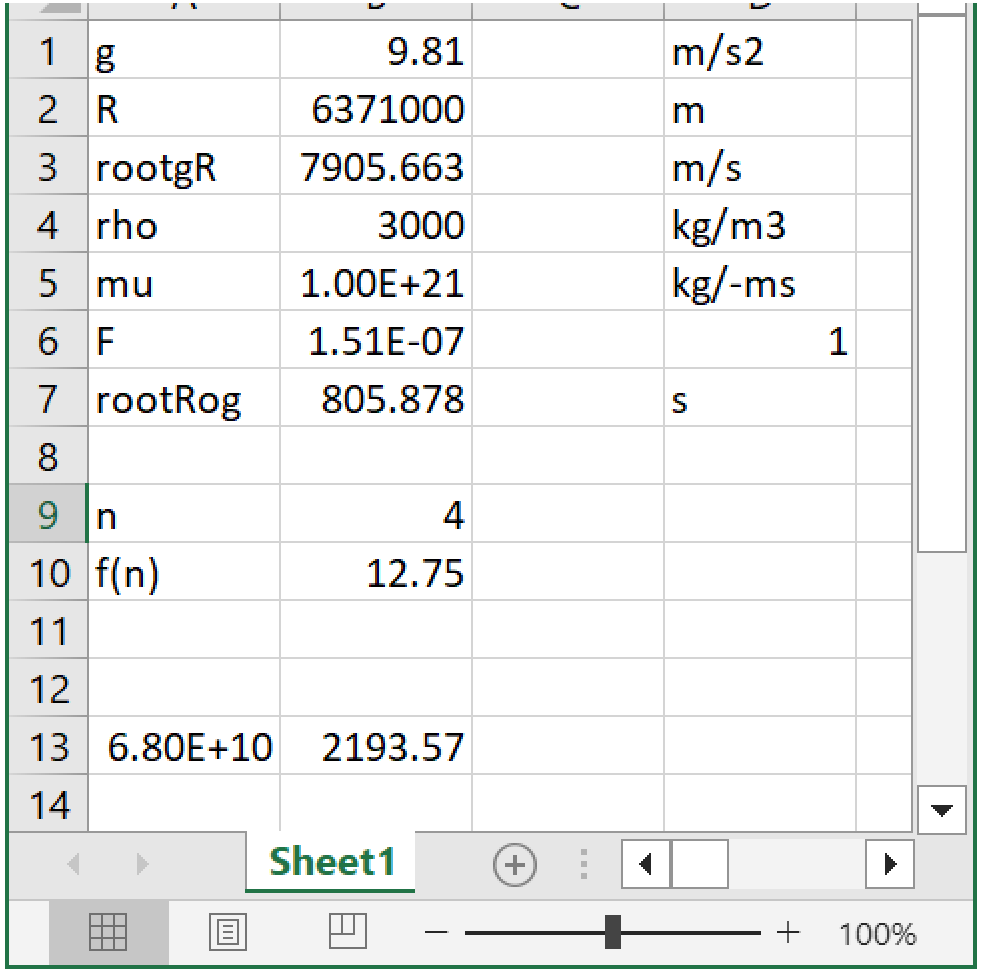
$$\tau = \frac{1}{F}\sqrt{\frac{R}{g}}\left(\frac{2n^2 + 4n + 3}{n}\right) \quad \text{with} \quad F = \frac{\rho R\sqrt{gR}}{\mu}$$

$$F = \frac{\rho R \sqrt{gR}}{\mu} = 5 \times 10^{-9}$$

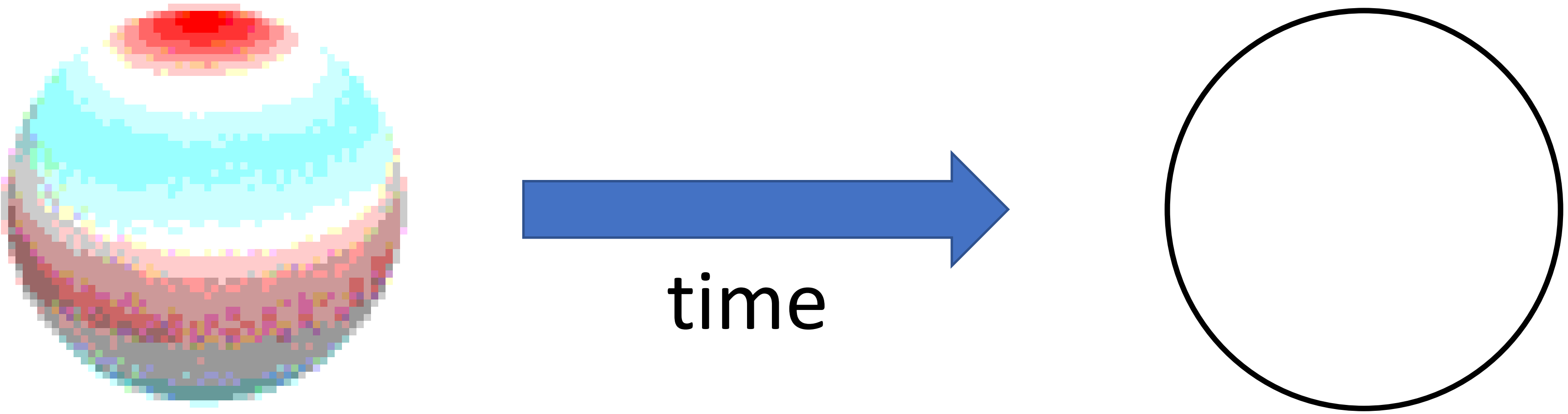
dimensionless number

$$\sqrt{\frac{R}{g}} = 806\ s$$

$$\left(\frac{2n^2 + 4n + 3}{n}\right) = 12.75$$



1	g	9.81		m/s2
2	R	6371000		m
3	rootgR	7905.663		m/s
4	rho	3000		kg/m3
5	mu	1.00E+21		kg/-ms
6	F	1.51E-07		1
7	rootRog	805.878		s
8				
9	n	4		
10	f(n)	12.75		
11				
12				
13	6.80E+10	2193.57		
14				



time

exponential decay

$$P_n(t) = P_n(t = 0)\exp(-t/\tau)$$

with time constant, τ

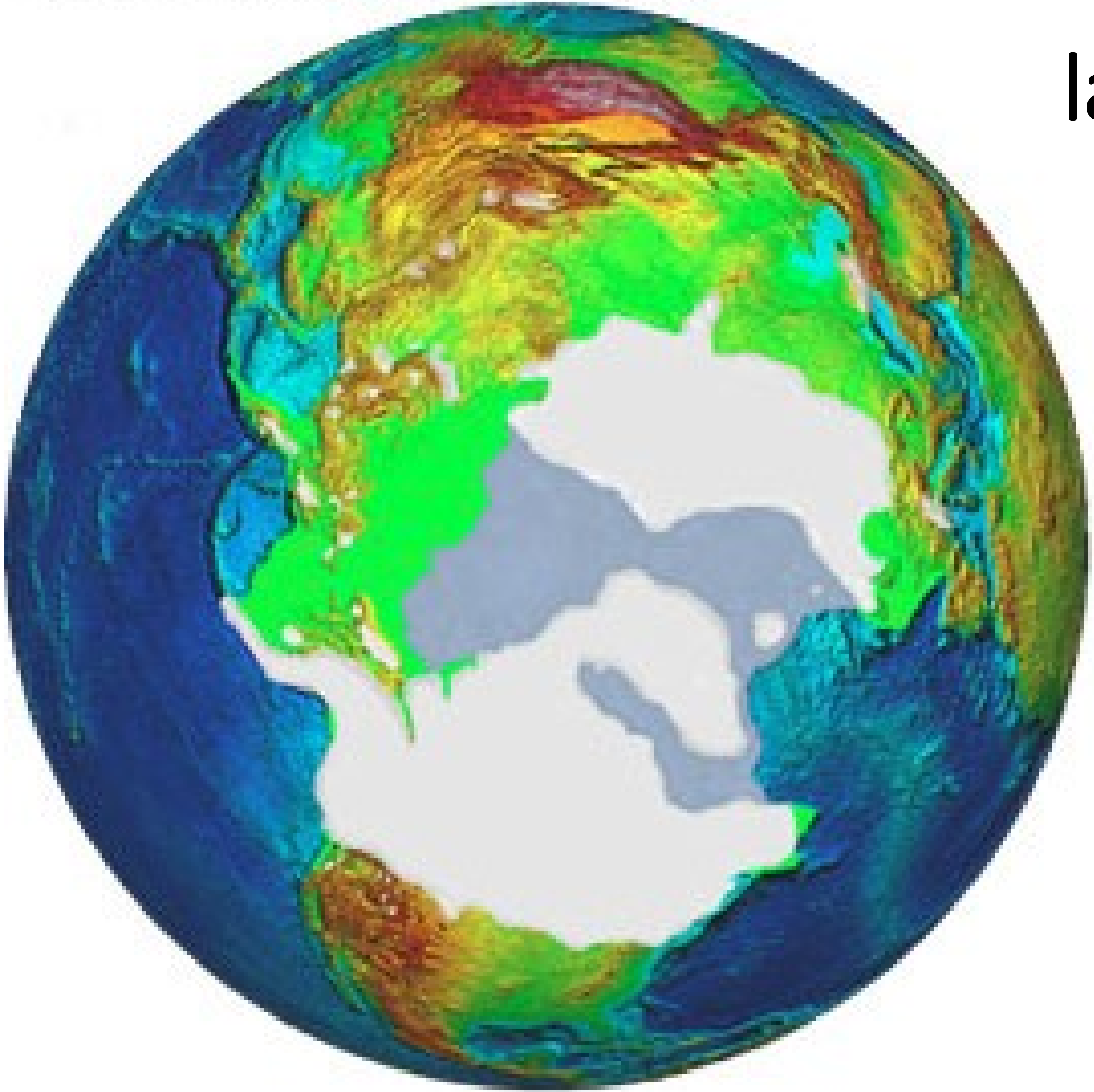
$\tau = 2100\ \mathrm{yr}$

conservation of volume

subsidence under ice sheet

must cause

uplift in other placed



last glacial maximum

30% of earth’s surface
covered by ice

but really thick ice much
less than that
say 10%

conservation of volume of displaced crust

$$V_{sub} = -V_{up}$$

$$fAd \ = \ (1 - f)hA$$

$$h = \ \frac{f}{(1 - f)}d$$

$$h = \ \frac{0.1}{1 - 0.1}1000 = 110 \text{ m}$$

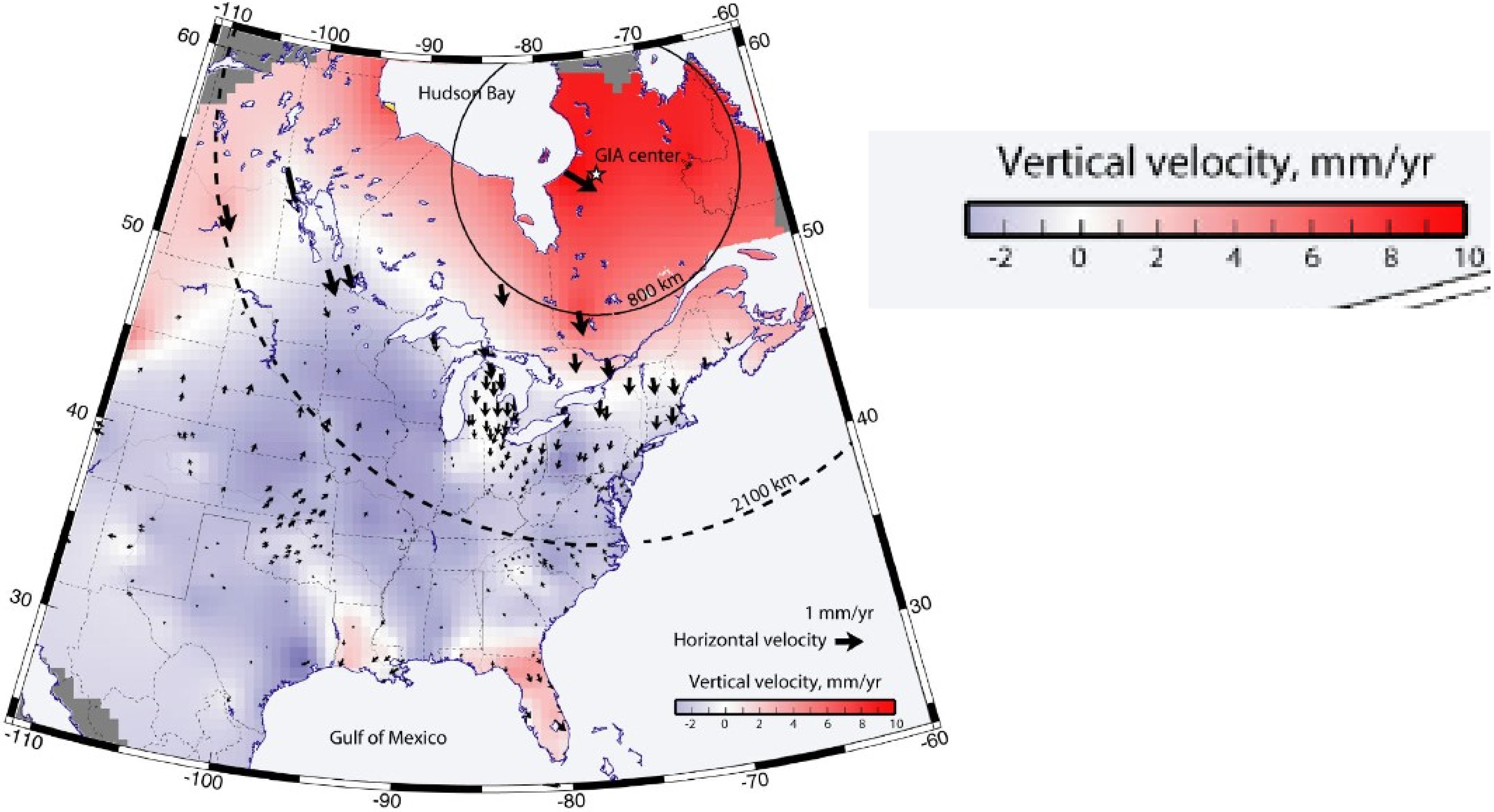
last glacial maximum

- land beneath ice sheets down
- rest of world up

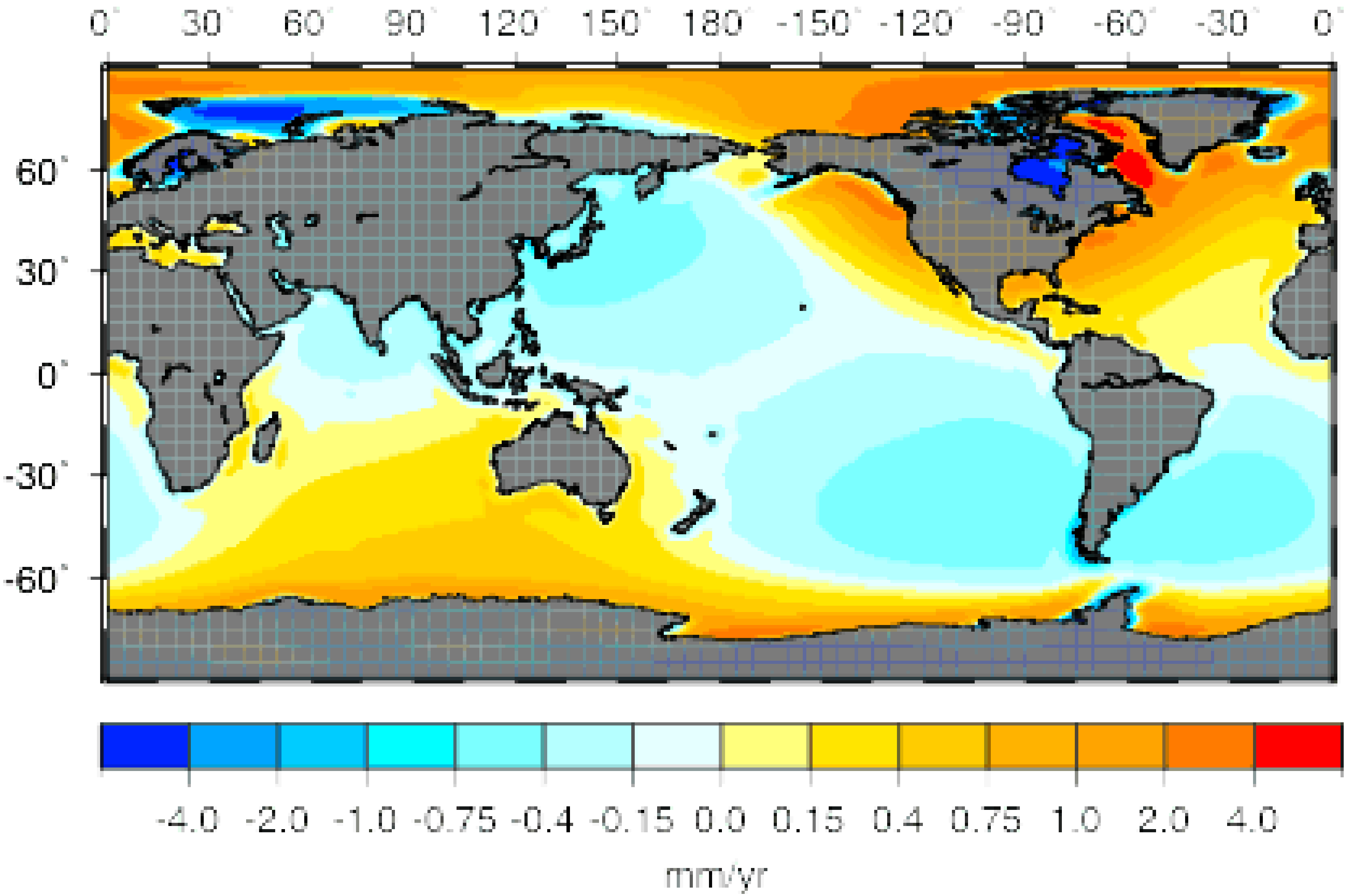
present day

- land once covered by ice sheets up
- rest of world down

Uplift Rate from GPS



Relative sea level



Relative sea level