Solid Earth Dynamics

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Lecture 12

Today:

more on glacial isostatic rebound

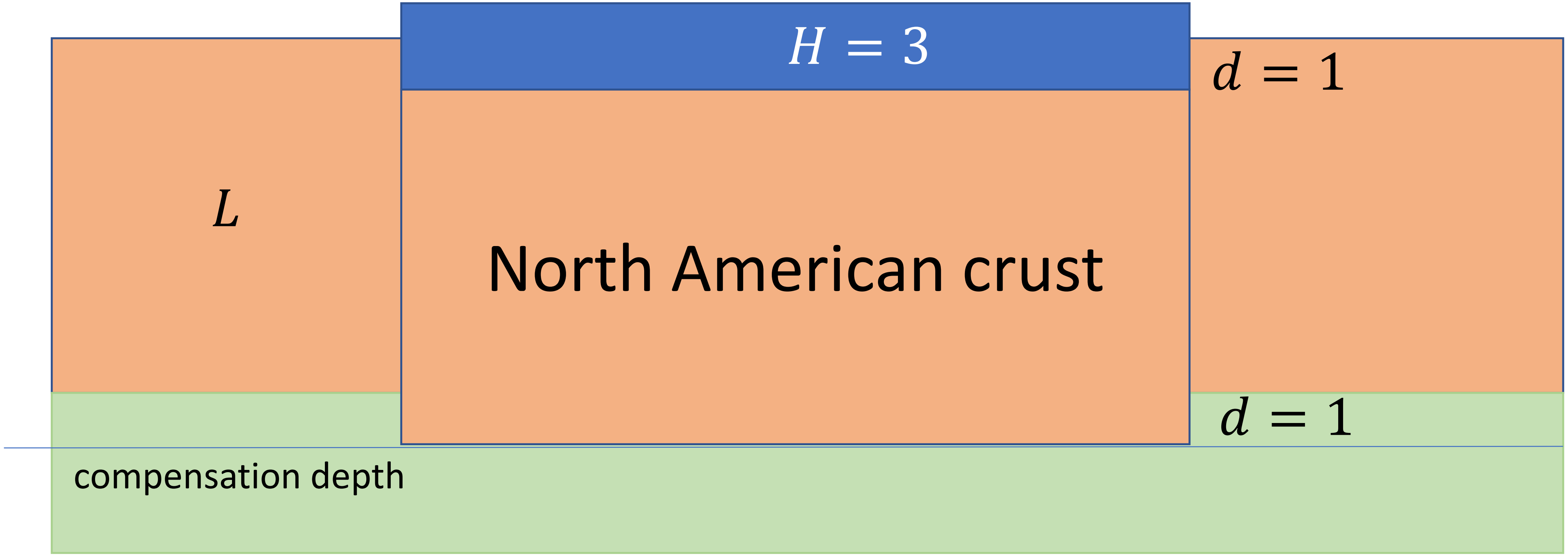
starting discussion of deformation

newton's law in a material

atmospheric pressure

volumetric and shear strain

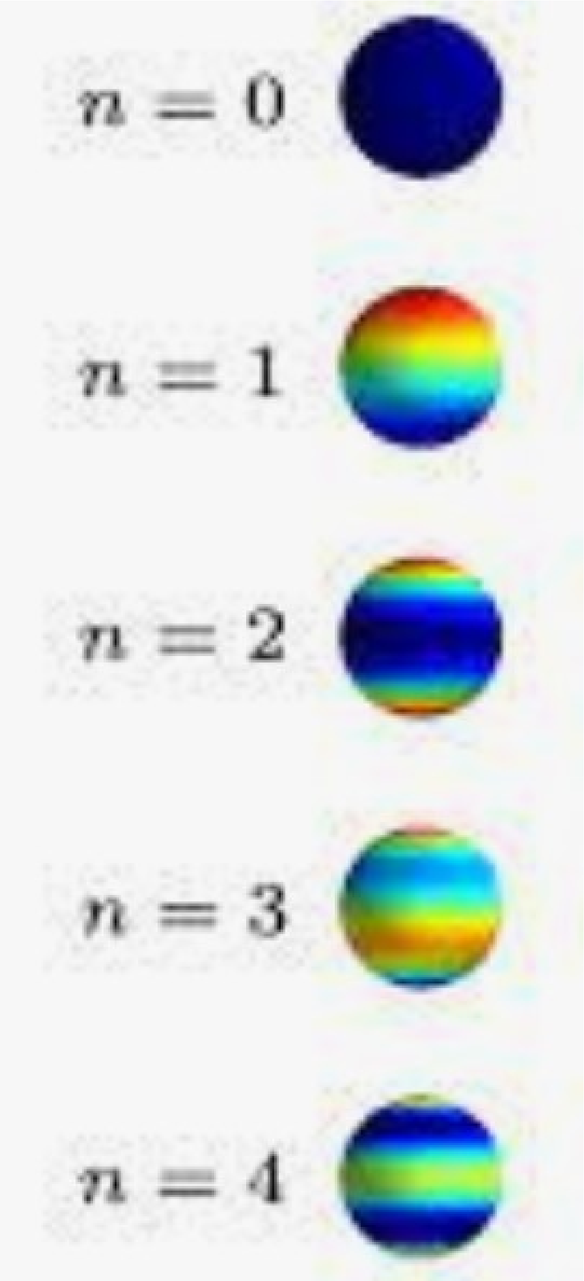
Glacial Isostatic Rebound



$$\rho_C L + \rho_M d = \rho_I H + \rho_C L$$

$$\rho_M d = \rho_I H$$

$$d = \frac{1000}{3000} H = \frac{1}{3} H = 1\,\text{km}$$

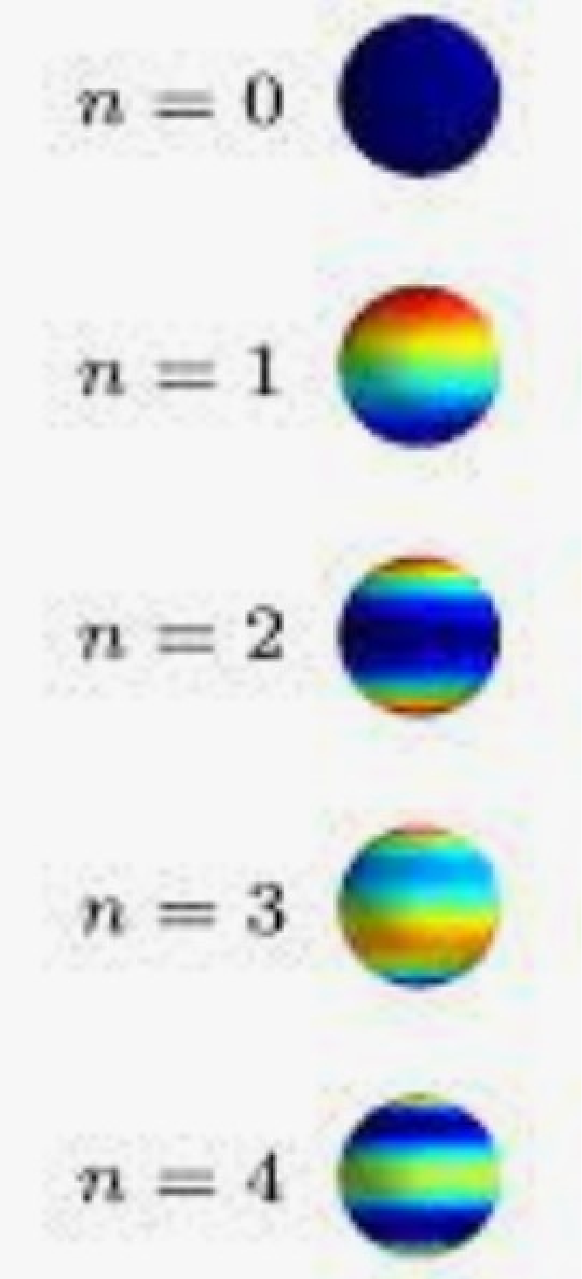


patterns with increasing number of wiggles

n = number of half-wavelengths

let's call them $P_n(\cos\theta)$

Legendre Polynomials



each decays with
characteristic decay time τ

t=0
$c_n P_n(\cos\theta)$

at a later time, t
$c_n \ \exp(-t/\tau) \ P_n(\cos\theta)$

formula for τ known, depends on n

$$F = \frac{\rho R \sqrt{gR}}{\mu}$$

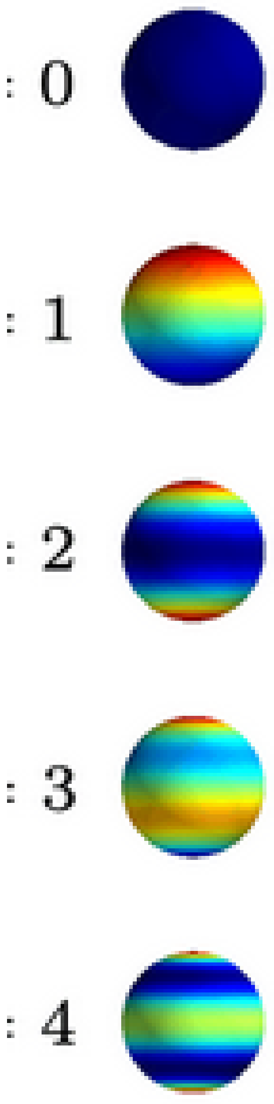
dimensionless constant

$$f = \sqrt{\frac{R}{g}}$$

time scale, units of s

$$\tau = \frac{f}{F} \frac{(2n^2 + 4n + 3)}{n}$$

decay time, units of s



$$n = 4 \quad \text{gives} \quad \tau = \frac{f}{F}\frac{(2n^2 + 4n + 3)}{n} \approx 2200 \text{ years}$$

because both viscous flow and gravity are linear processes

if load L_A leads to uplift U_A

and

if load L_B leads to uplift U_B

then

if load $L_A + L_B$ leads to uplift $U_A + U_B$

strategy for dealing with complicated load, $\mathrm{L}(\theta)$

(A) figure out the proportionality between Load and uplift at time t=0, for loads of shape $P_n(\cos\theta)$

at time t=0
Load $L = P_n(\cos\theta)$
leads to uplift $U = c_n P_n(\cos\theta)$

only work for some "the right" $P_n(\cos\theta)$s

c_n is known for this problem

$$c_n = \frac{2n+1}{2(n-1)}$$

when $P_n(\cos\theta)$ are the ones shown previously

(B) approximate Load L$(\theta, t = 0)$ as

as sum of the functions $P_n(\theta)$

$$f(\theta) \approx \sum_{n=1}^{N} b_n \, P_n(\cos\theta)$$

need to find b_n

(C) invoke superposition:

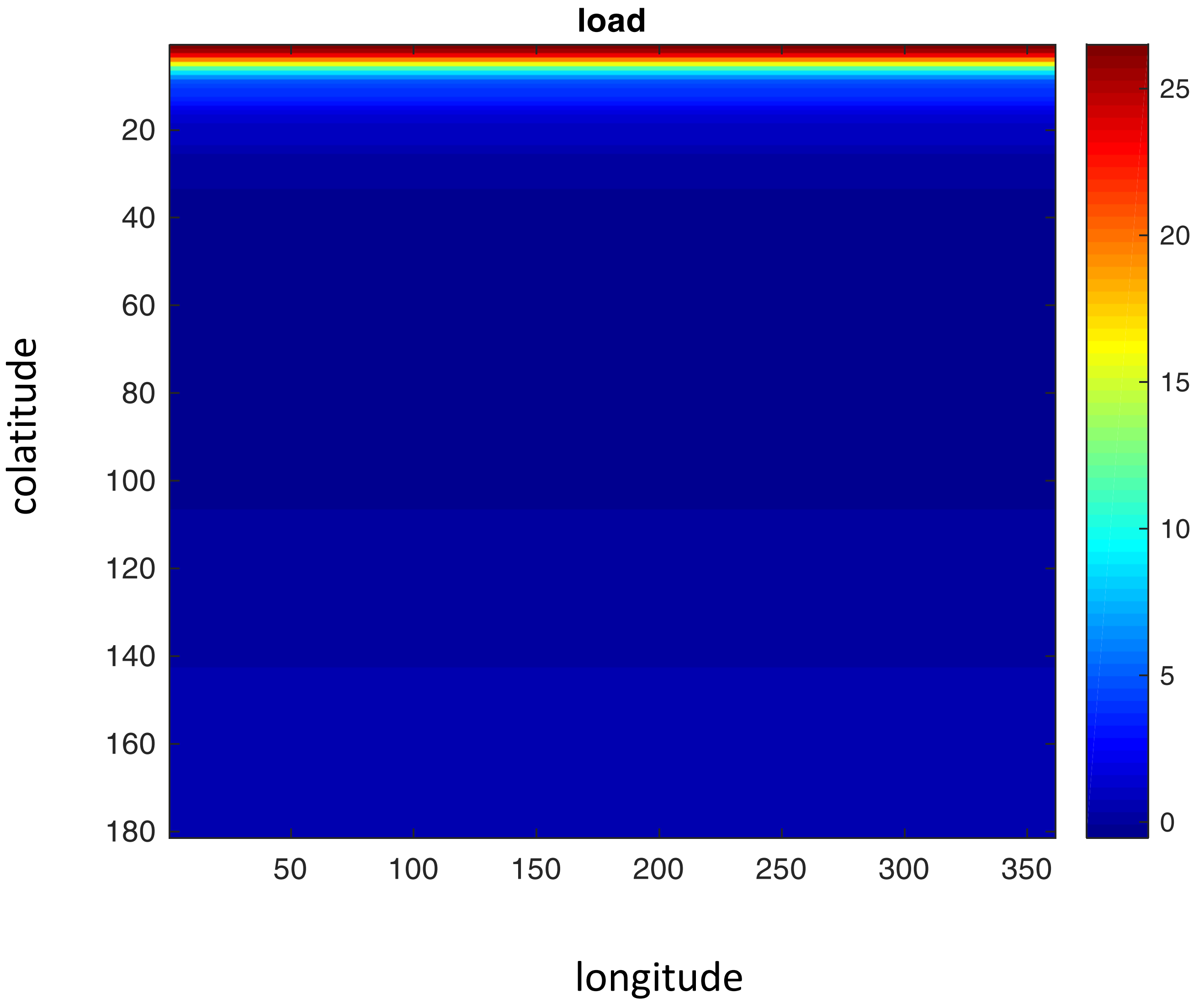
each c_n decays independently with time

$$c_n \quad \text{becomes} \quad c_n \exp(-t/\tau)$$

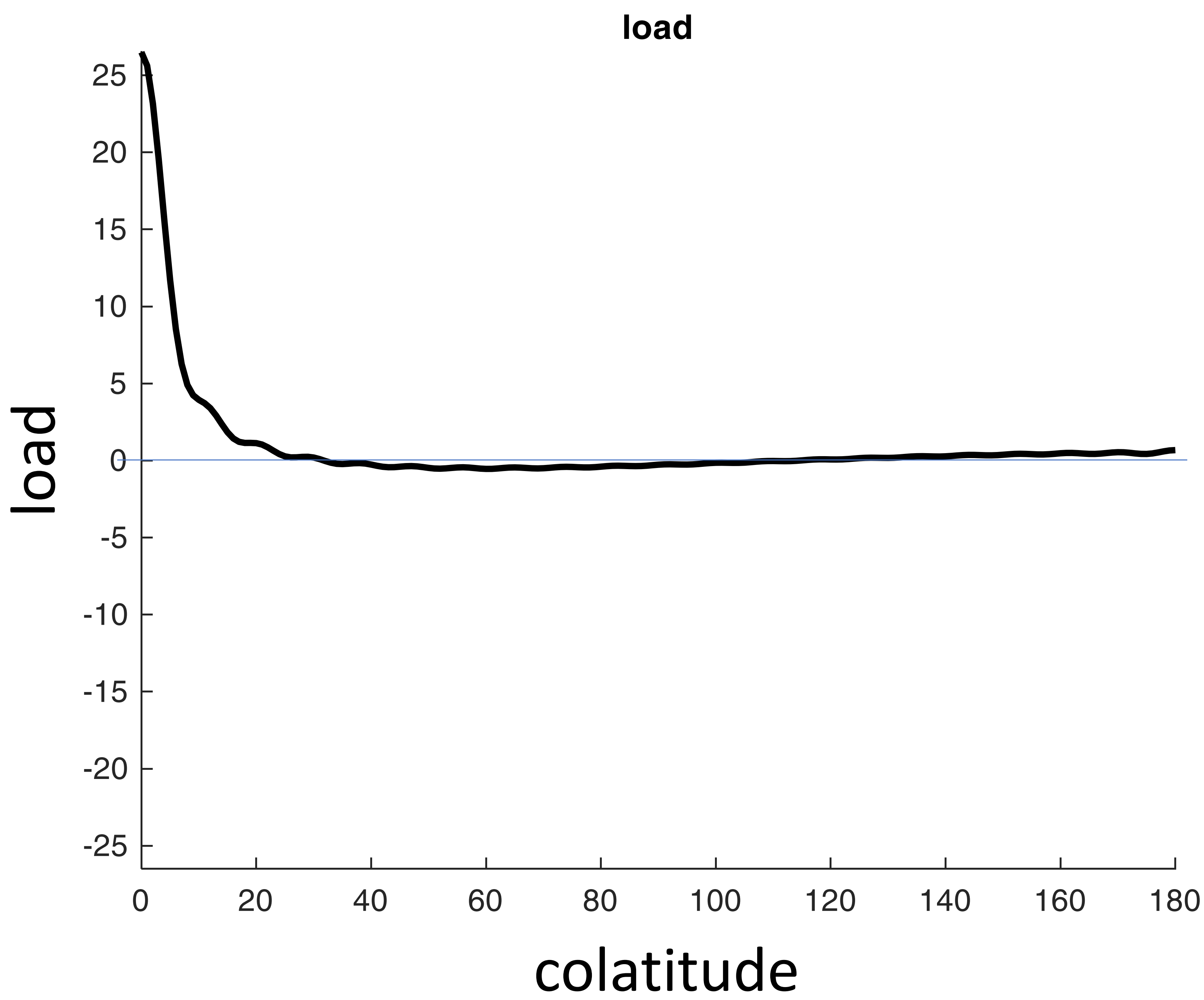
(D) perform the summation

$$R(\theta, t = 0) \approx \sum_{n=1}^{N} b_n c_n \exp(-t/\tau\,) P_n(\cos\theta)$$

I cooked up the b’s that give this load

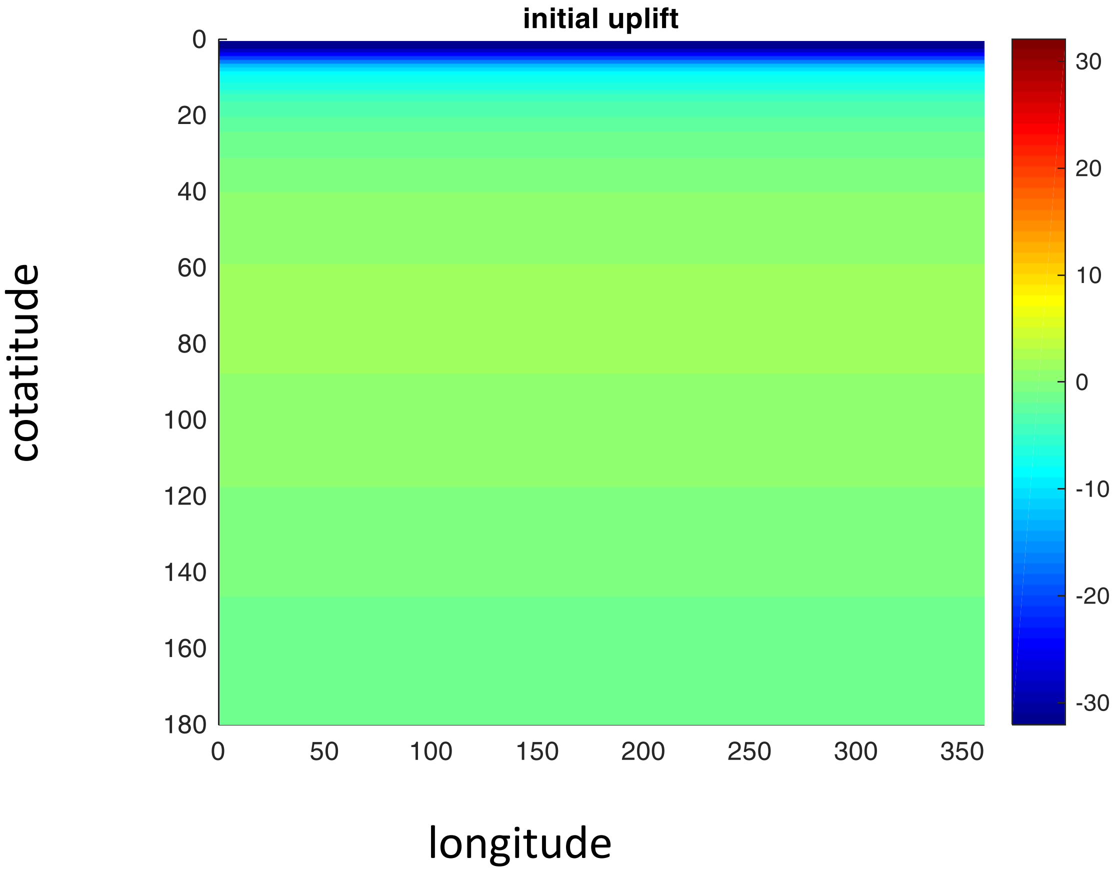


R = 6371000;
rho = 3000;
mu = 1e21;
g = 9.81;
ty=3000.0;

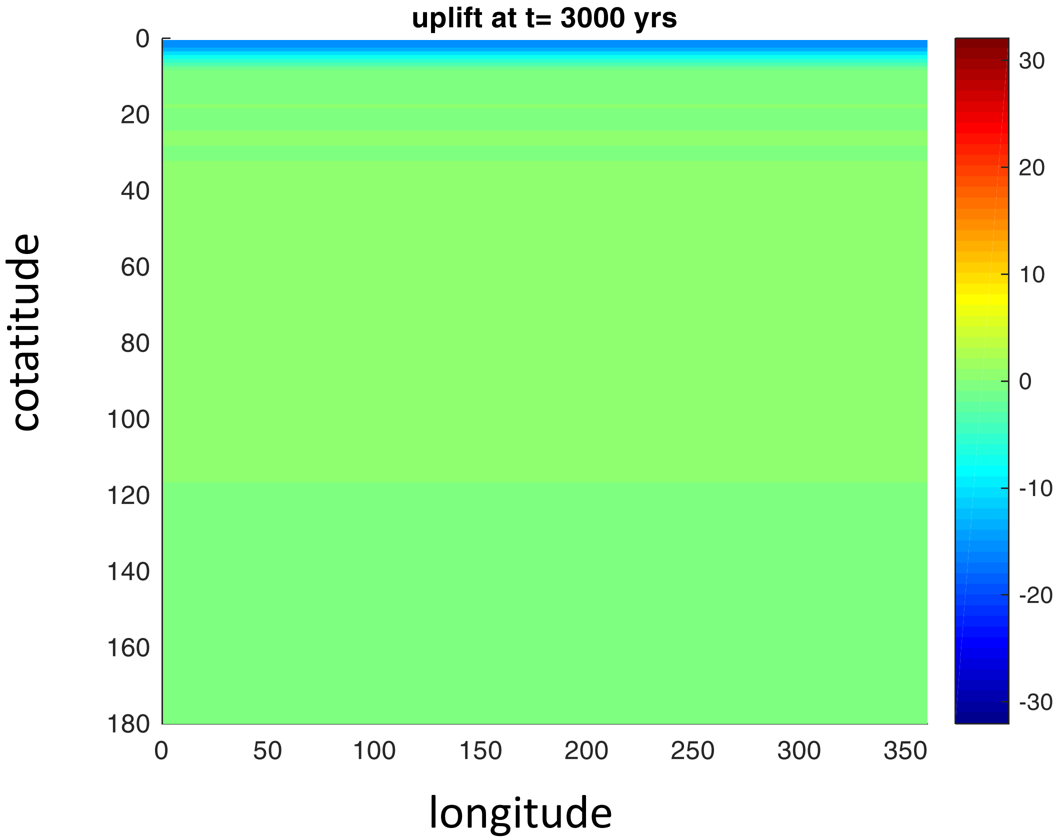

load
25
20
15
10
5
0
-5
-10
-15
-20
-25
load
0
20
40
60
80
100
120
140
160
180
colatitude

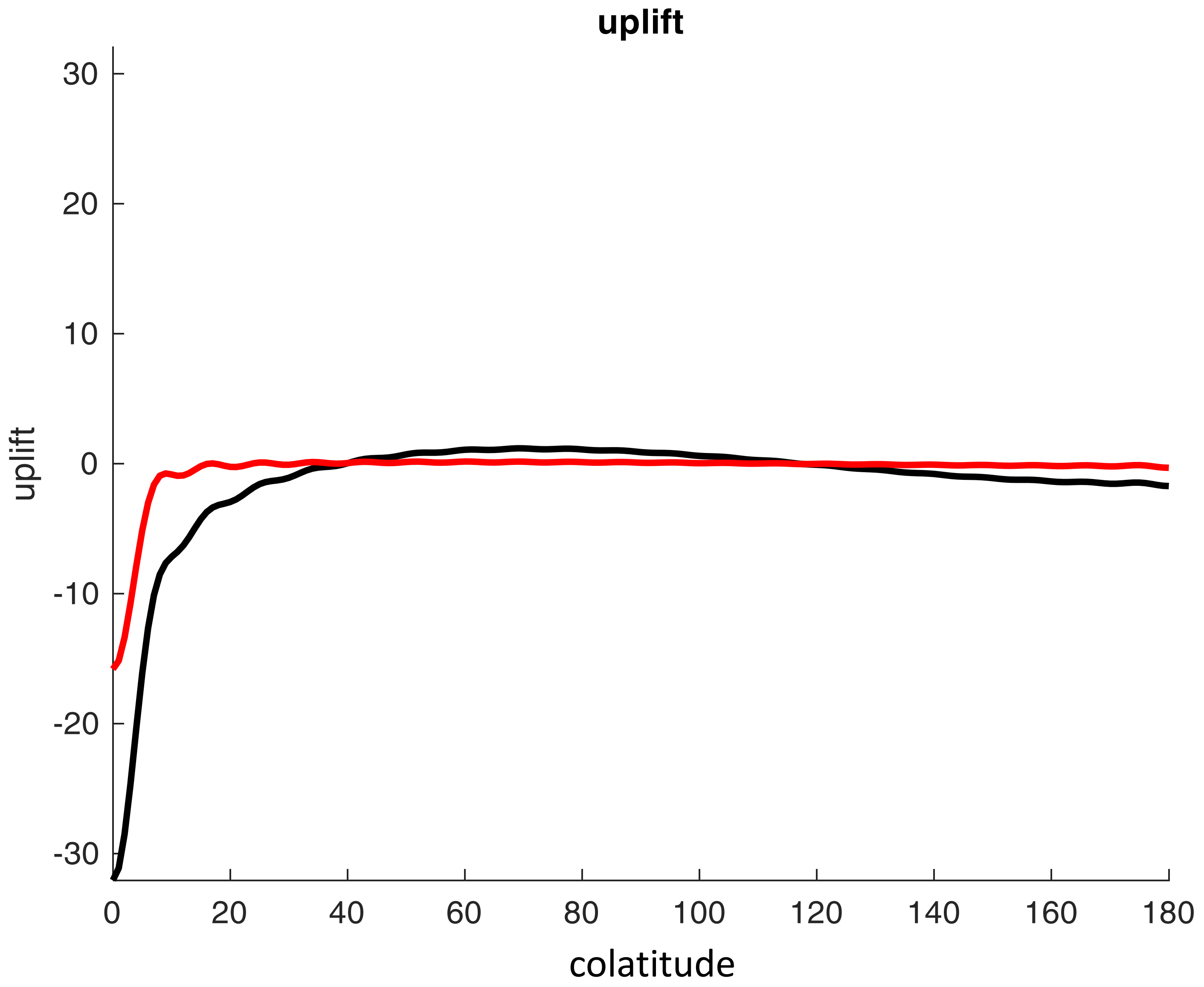
uplift

time=0



time=3000 yrs



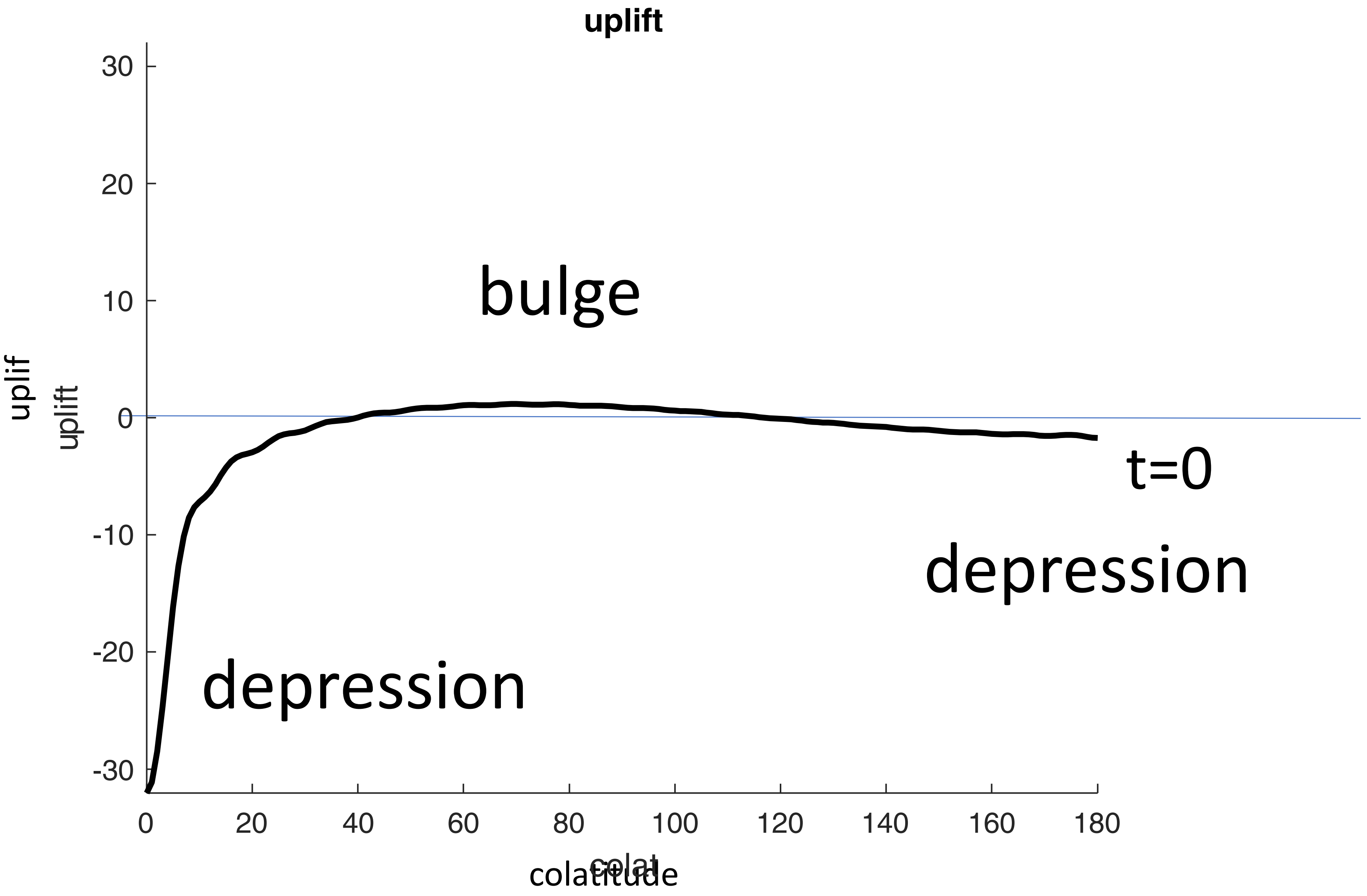


uplift
uplift
30
20
10
0
-10
-20
-30
0
20
40
60
80
100
120
140
160
180
colatitude

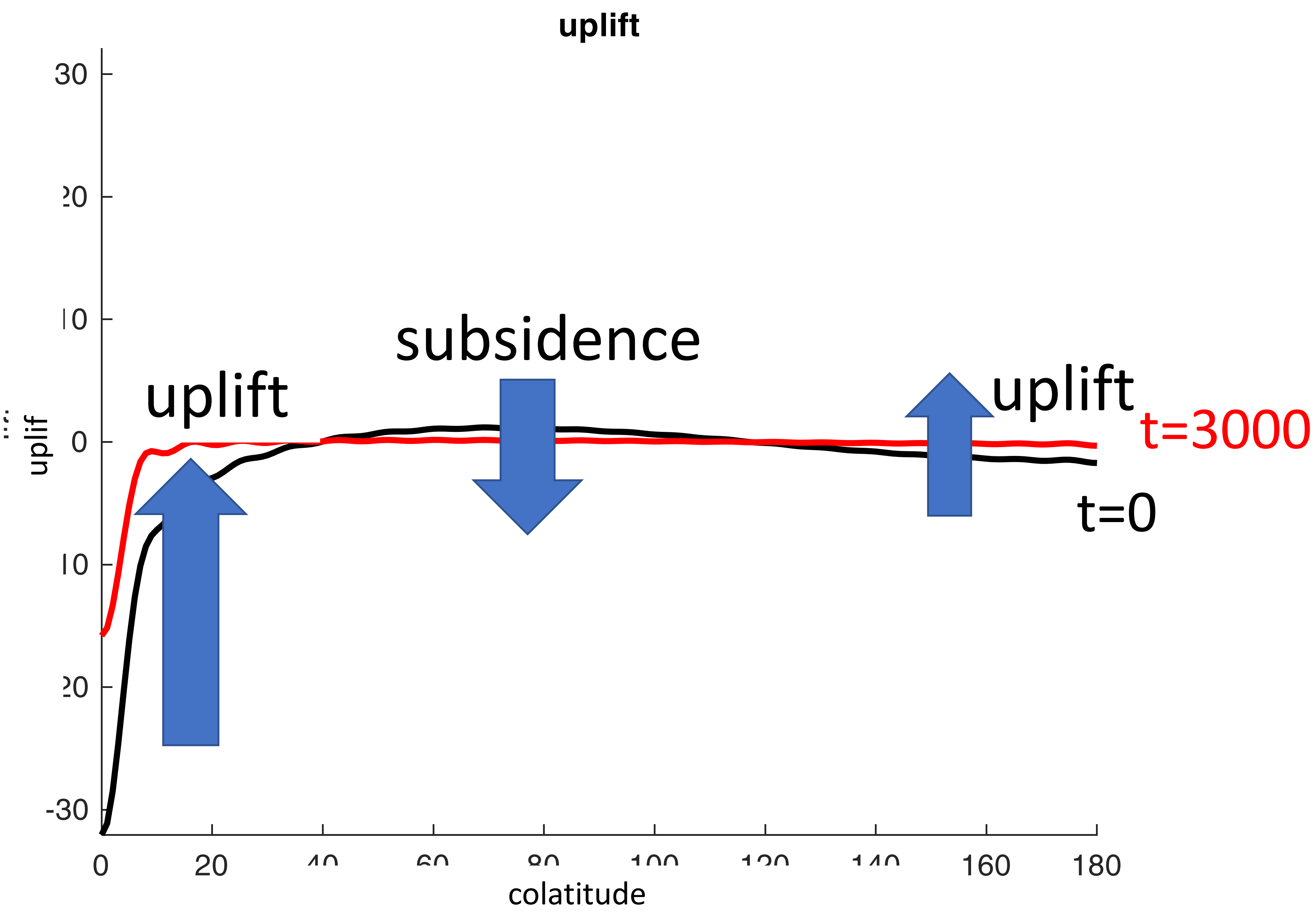
t=3000 yrs

t=0

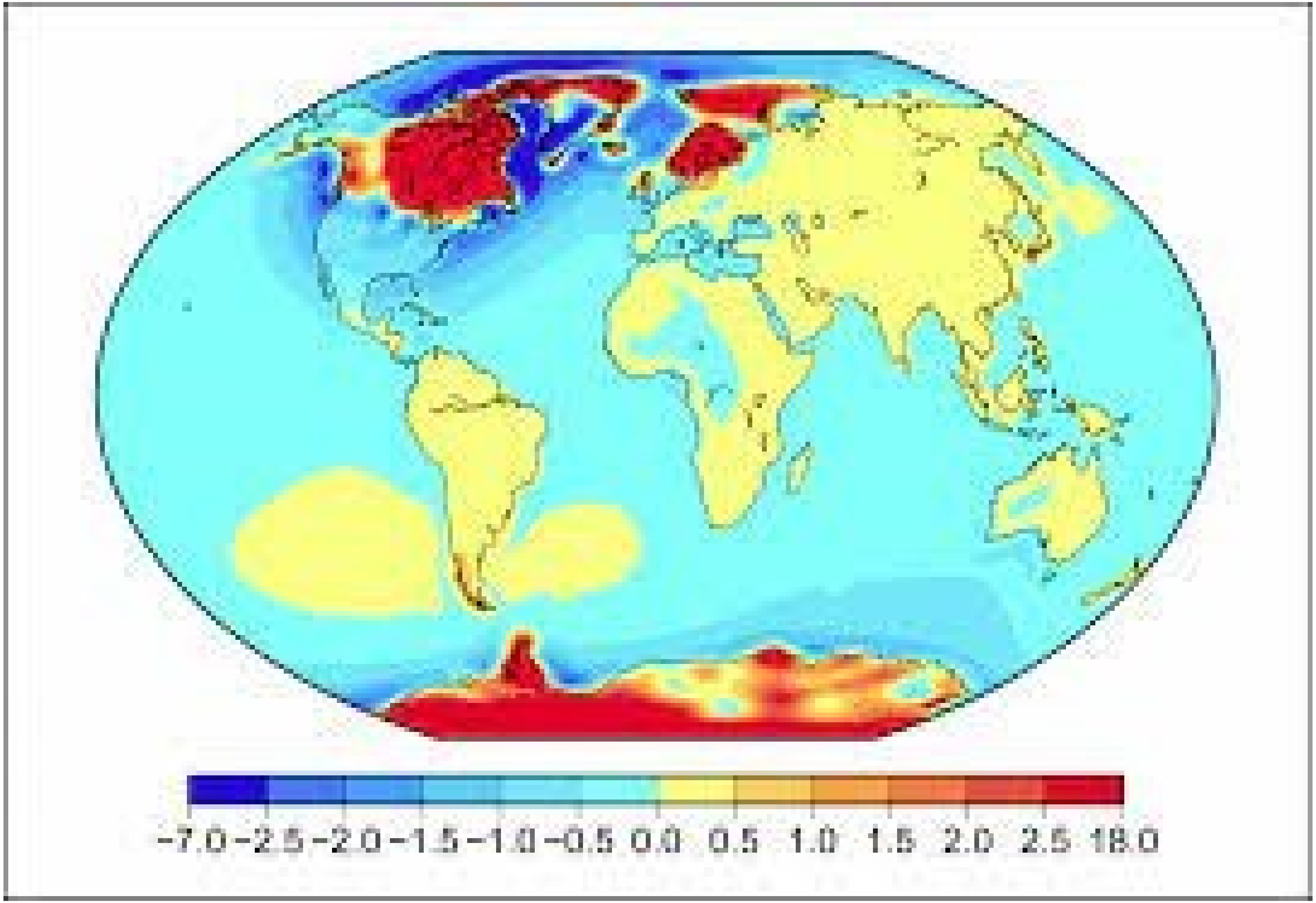
during the Ice Age



today



uplift
30
20
10
0
10
20
-30
uplif
0
20
160
180
colatitude
uplift
subsidence
uplift
t=3000
t=0

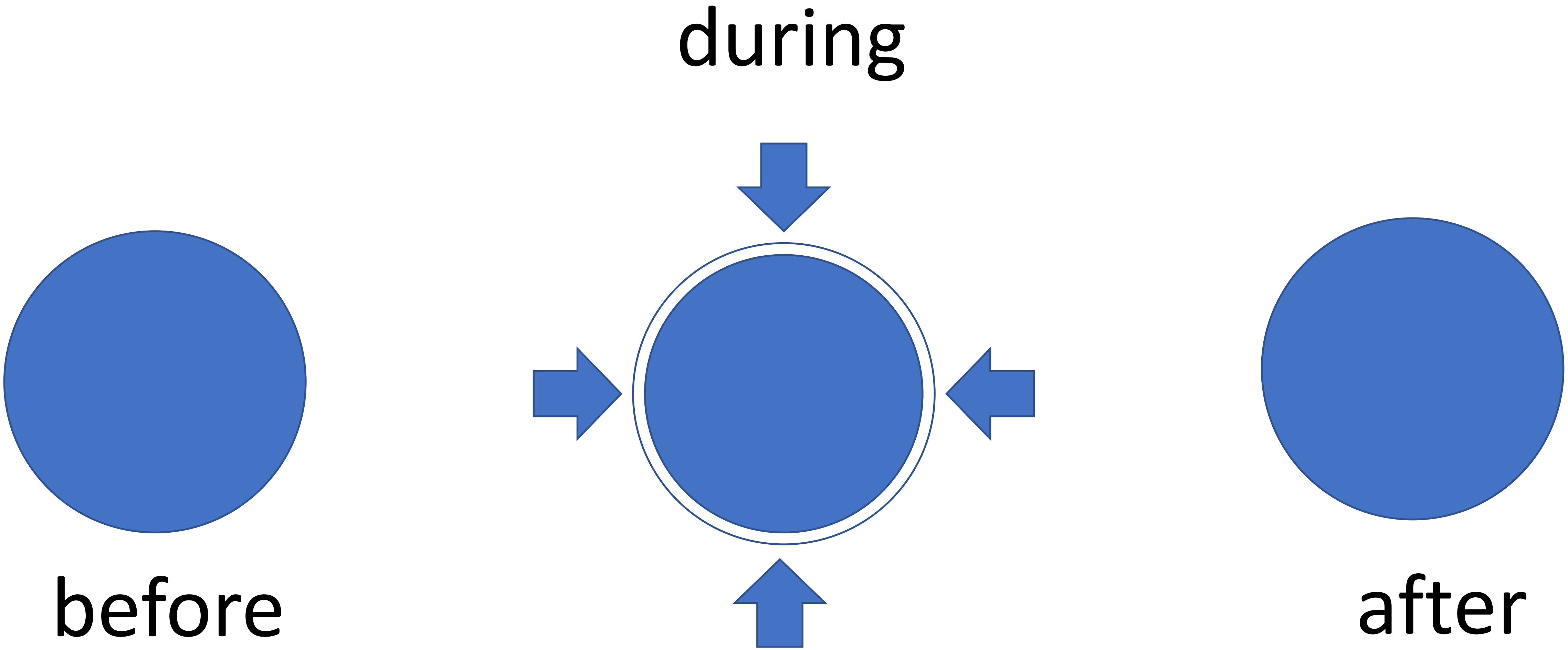

−7.0 −2.5 −2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0 2.5 18.0

linear elasticity

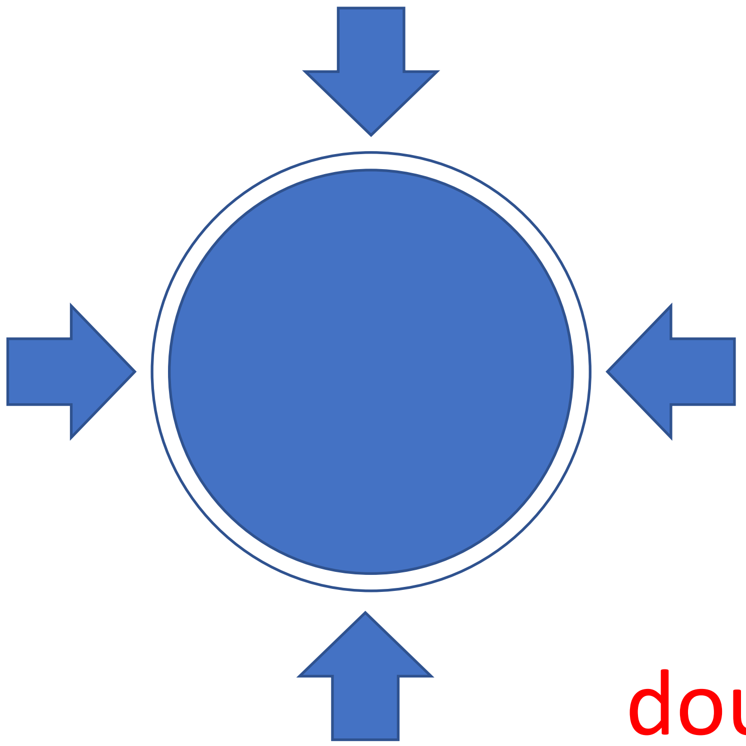
and

Newton’s Law in fluids

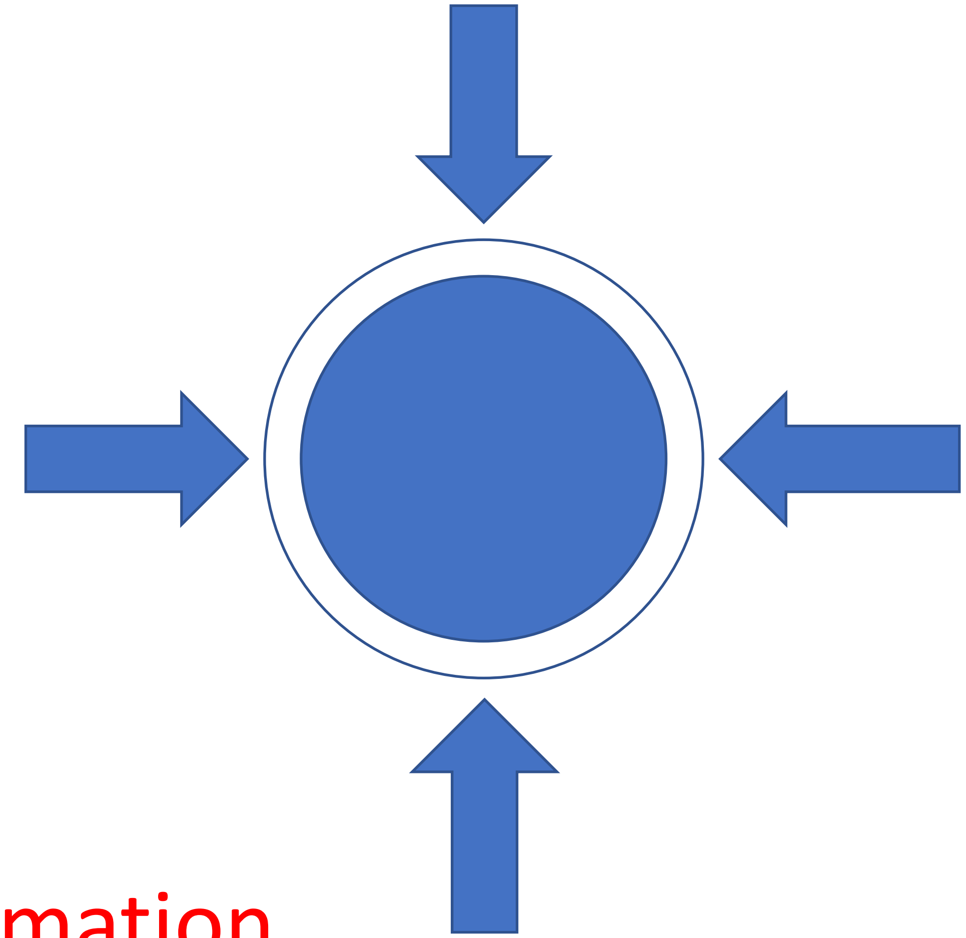
elasticity: reversible deformation

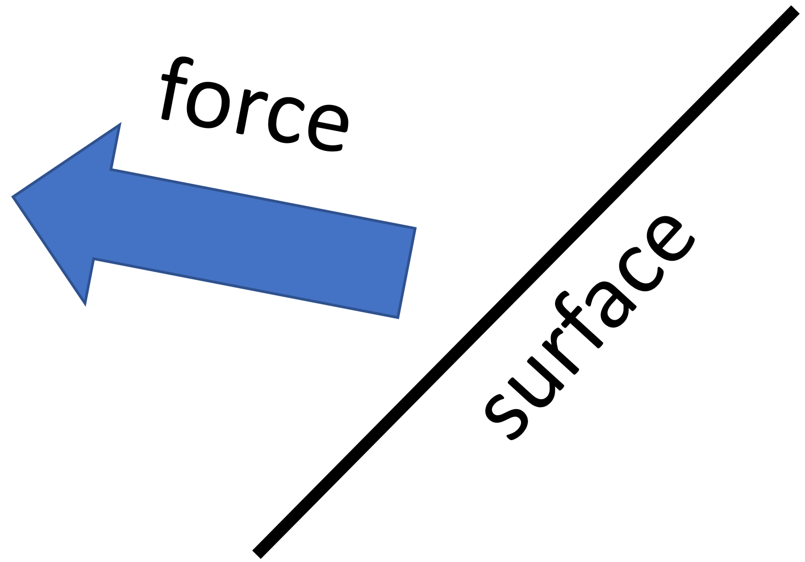


linear elasticity:
deformation proportional to force



double force
double deformation



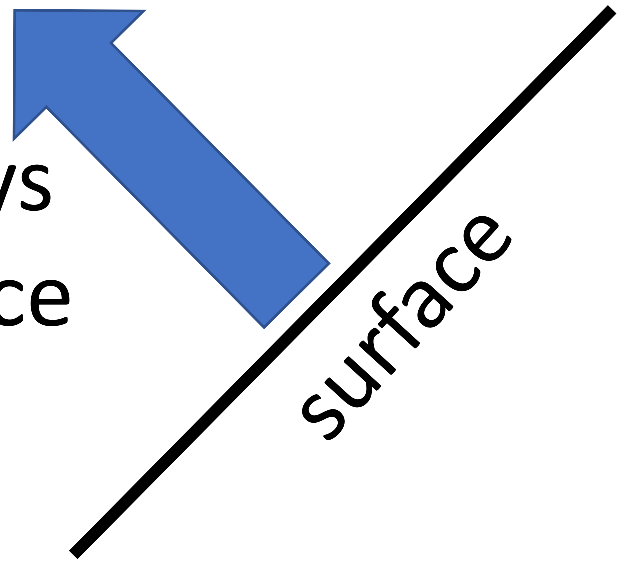


force per unit area
is called traction, T

positive when outward
pointing

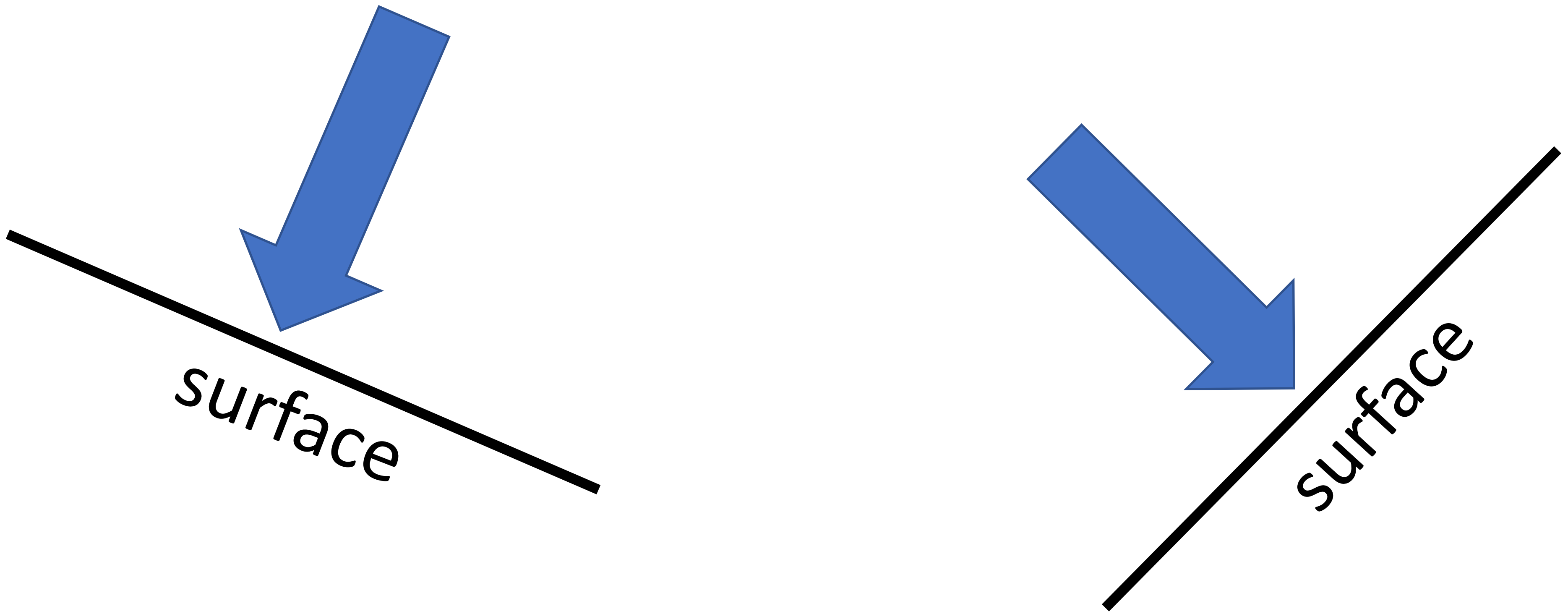
fluid

traction is always
normal to surface



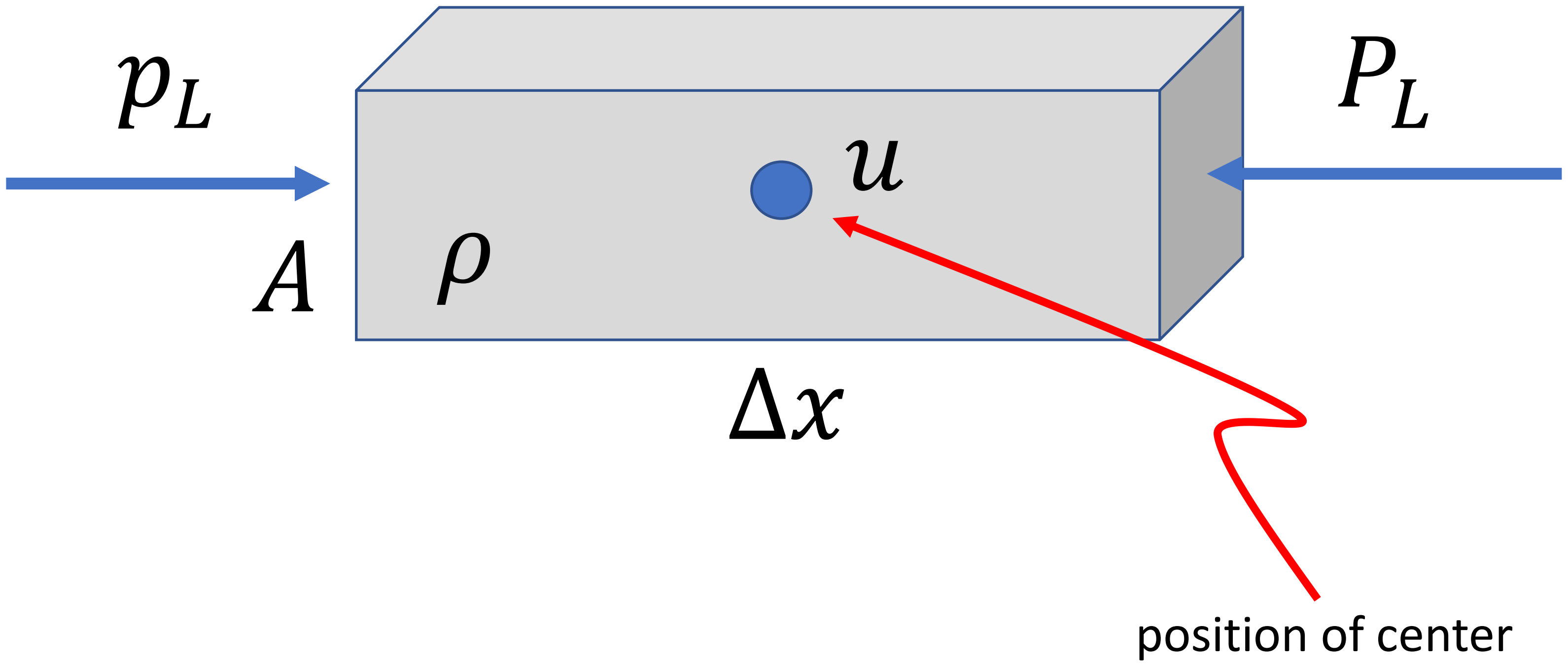
fluid

all (nearby) surfaces have the same traction

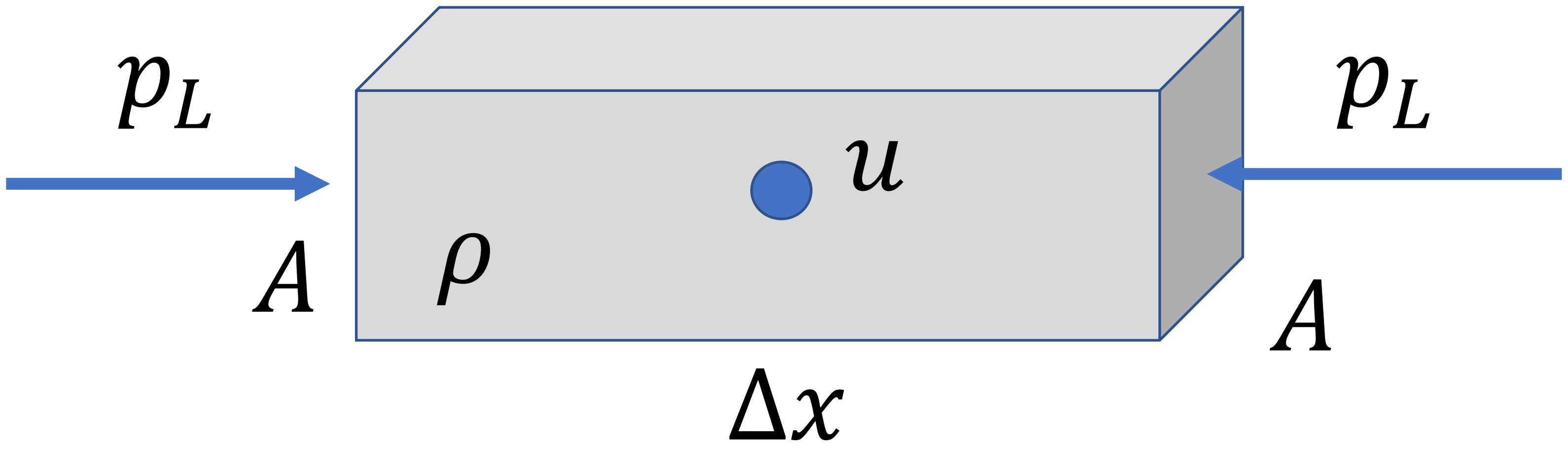


tractoin in fluid: minus the pressure, p

Newton's Law (horizontal motion only)



pressure force

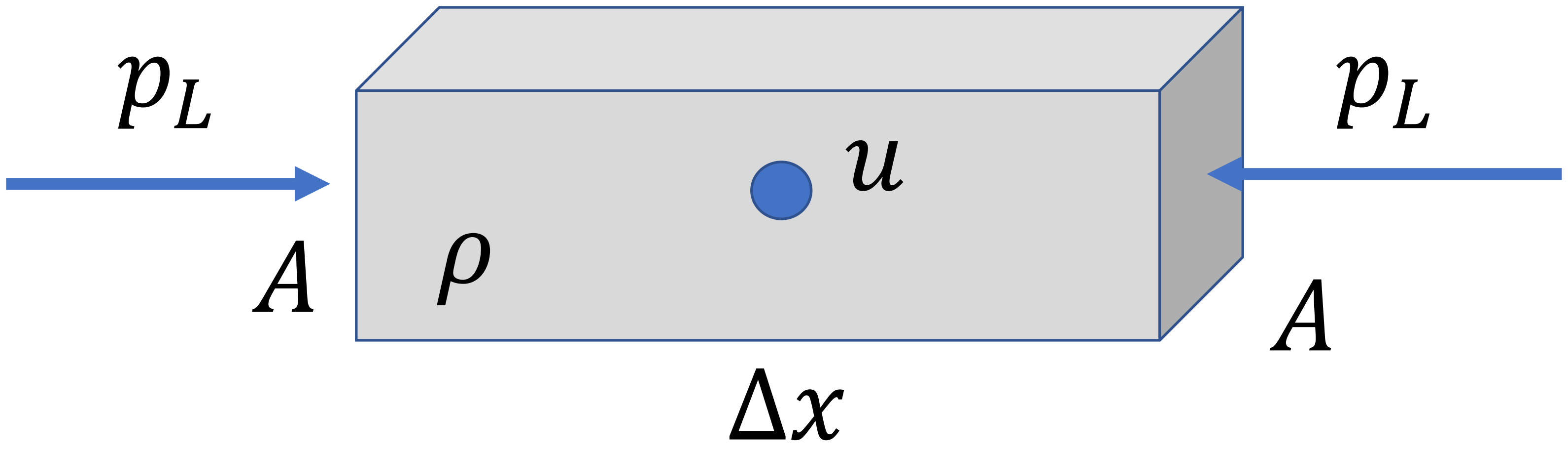


$$F_F = Ap_L$$

$$F_R = -Ap_L$$

$$F = F_L - F_R = A(p_L - p_R)$$

volumetric force, like gravity



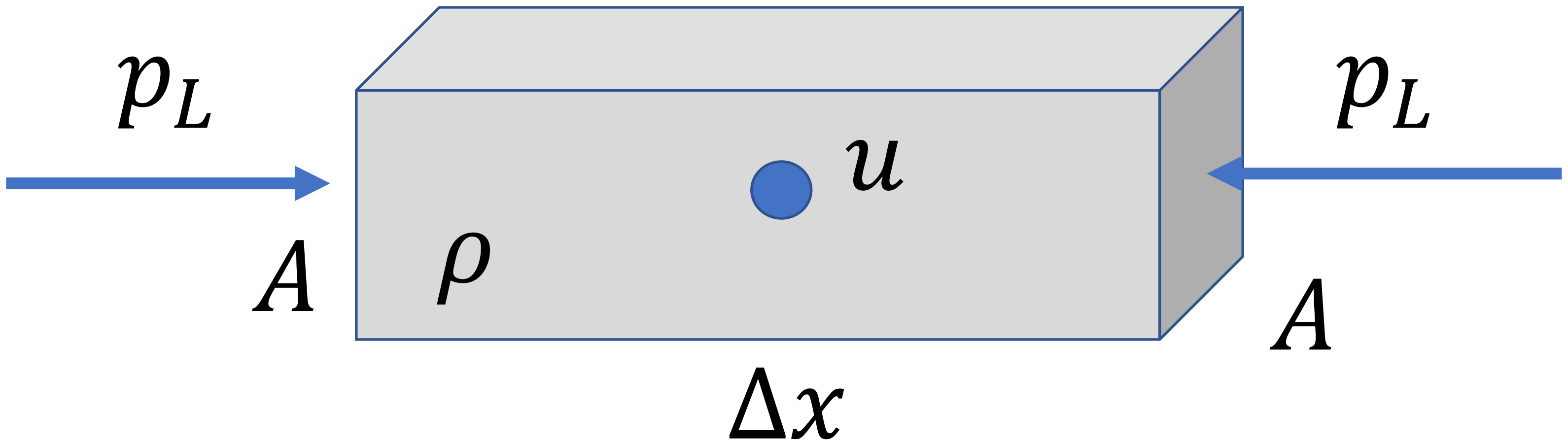
$$F_F = Ap_L$$

$$F_R = -Ap_L$$

$$F = fA\Delta x$$

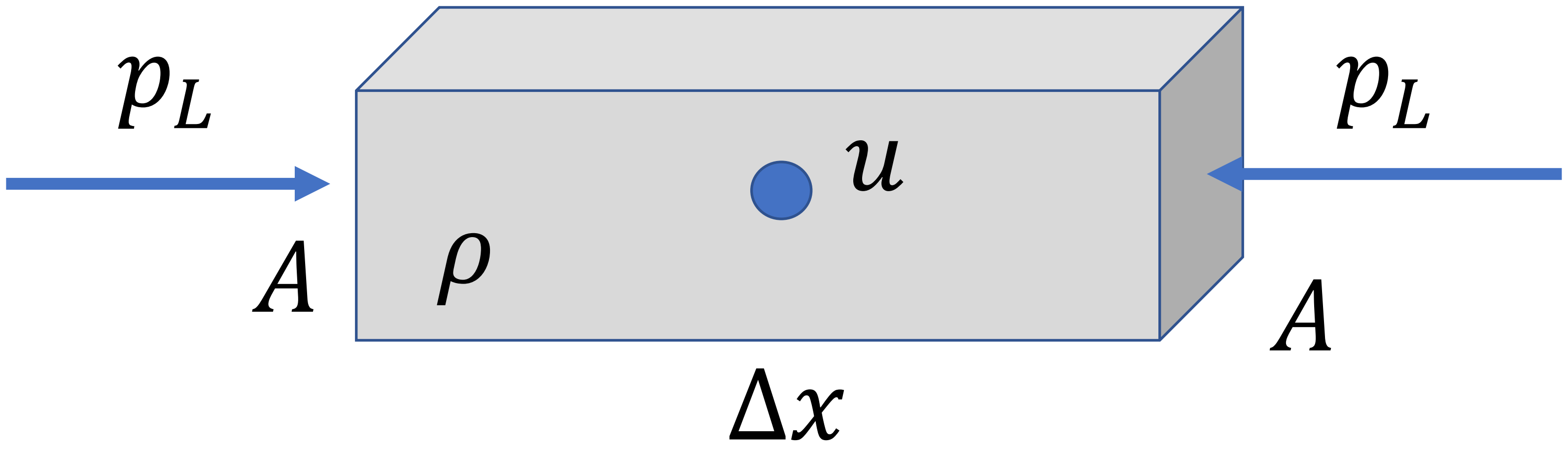
f force per unit volume

Mass



$$M = \rho A \Delta x$$

Acceleration



$$a = \frac{d^2u}{dt^2}$$

Newton's law $F = Ma$

$$F = A(p_L - p_R)$$

$$F = fA\,\Delta x$$

$$M = \rho A \Delta x$$

$$a = \frac{d^2u}{dt^2}$$

$$A(p_L - p_R) + fA\Delta x = \rho A\Delta x\frac{d^2u}{dt^2}$$

$$\frac{(p_L - p_R)}{\Delta x} + f = \rho\frac{d^2u}{dt^2}$$

$$f - \frac{dp}{dx} = \rho\frac{d^2u}{dt^2}$$

Linear Elasticity in a fluid

$$\frac{\Delta V}{V} = -c\Delta p$$

fractional change in volume is proportional to pressure

Linear Elasticity in a fluid

or if you prefer

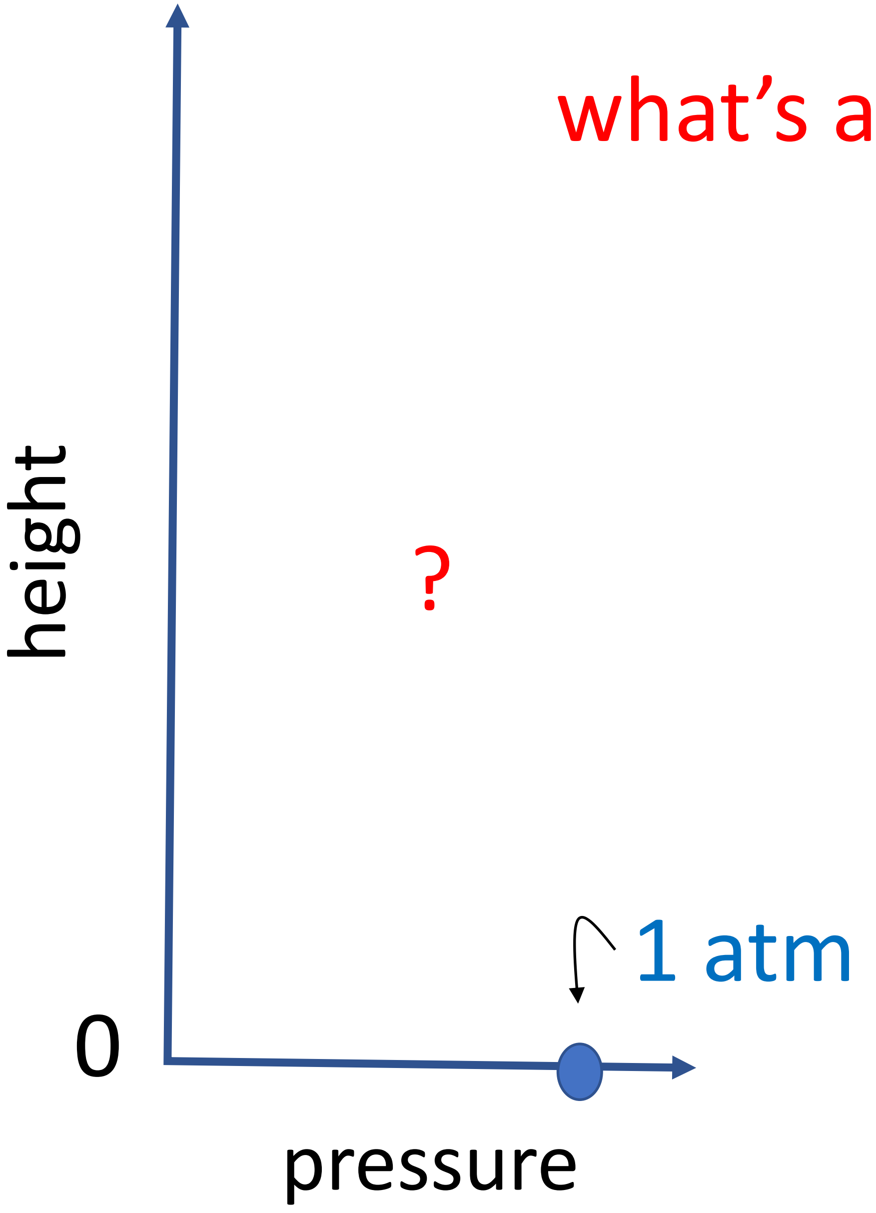
with $\mathrm{V} = \frac{M}{\rho}$

$$\frac{\Delta \frac{M}{\rho}}{\frac{M}{\rho}} = -c\Delta p$$

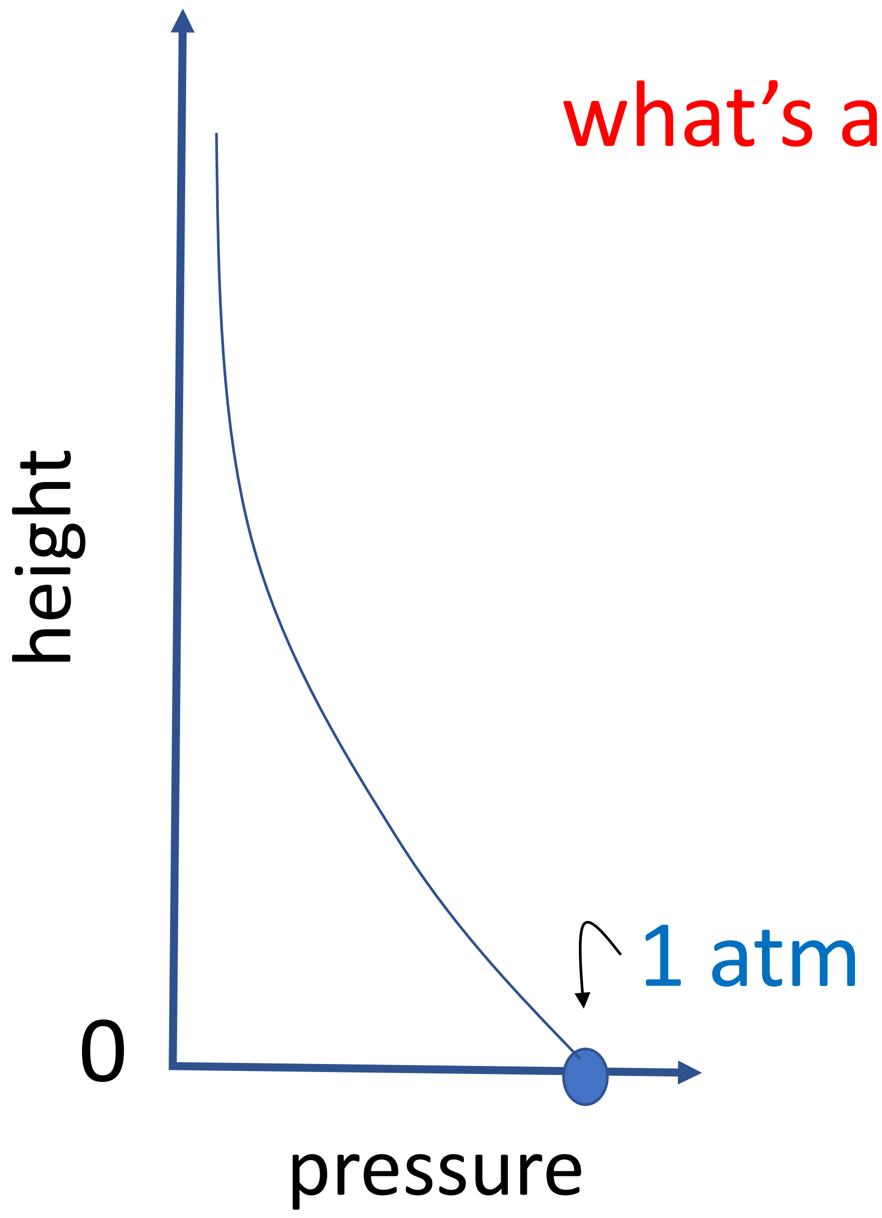
$$\rho \Delta \rho^{-1} = -c\Delta p$$

$$\rho \frac{d\rho^{-1}}{dp} = -c \qquad -\rho\rho^{-2}\frac{d\rho}{dp} = -c \qquad \rho^{-1}\frac{d\rho}{dp} = c$$

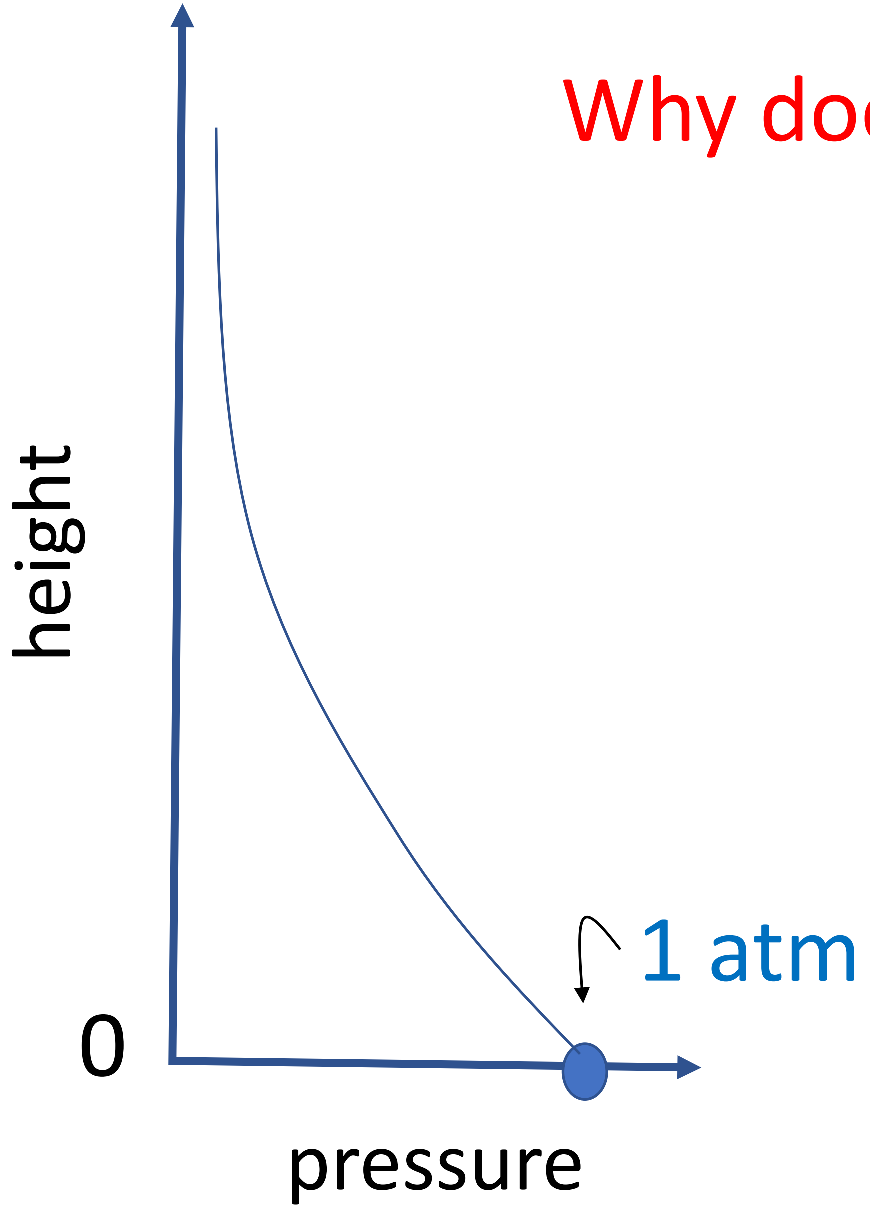
fractional change in density proportional to pressure what’s atmospheric pressure do with height?



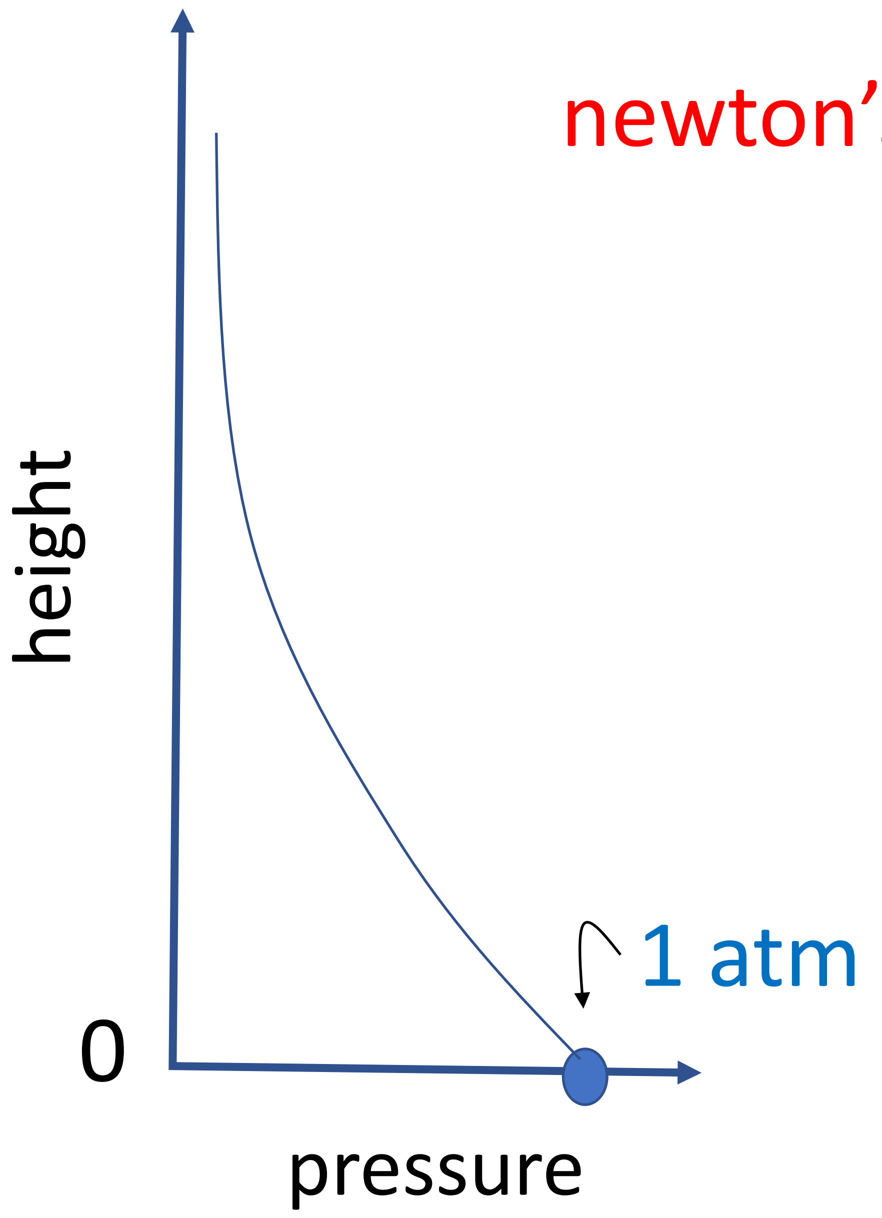
what’s atmospheric pressure do with height?



Why does it decrease?



newton's law



$$f - \frac{dp}{dx} = \rho \frac{d^2u}{dt^2}$$

$$-\rho g - \frac{dp}{dx} = 0$$

$$\frac{dp}{dx} = -\rho g$$

Linear Elasticity in an isothermal ideal gas

$$PV = nRT$$

$$n = number\ of\ moles$$

$$m = mass\ per\ mole$$

$$P = \frac{n}{V}\frac{RT}{1}$$

$$\text{mass} = nm$$

$$P = \frac{nm}{V}\frac{RT}{m}$$

$$\rho = \frac{mass}{volume} = \frac{nm}{V}$$

Linear Elasticity in an isothermal ideal gas

$$\mathrm{p}V = nRT$$

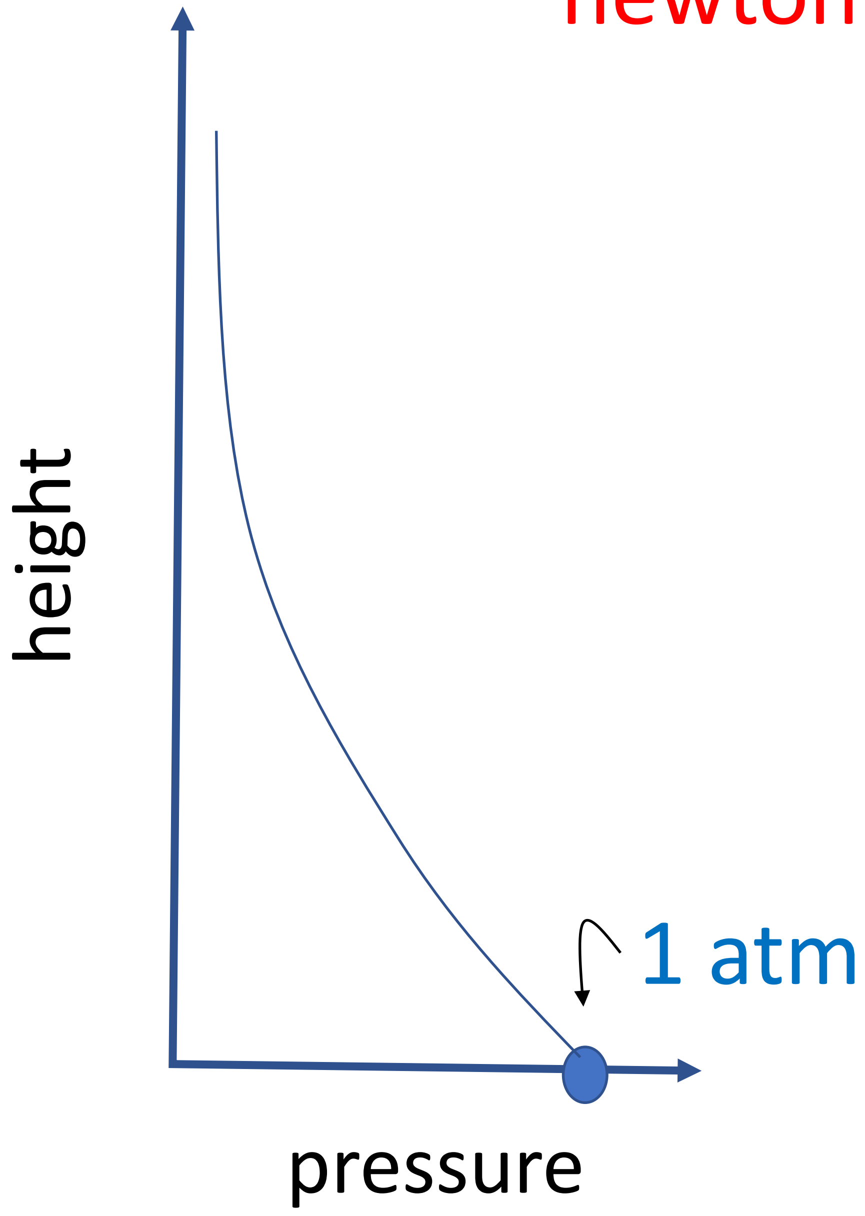
$$p = \frac{n}{V}\frac{RT}{1}$$

$$p = \frac{nm}{V}\frac{RT}{m}$$

$$\mathrm{p} = \frac{RT}{m}\rho$$

$$\rho = \frac{m}{RT}p$$

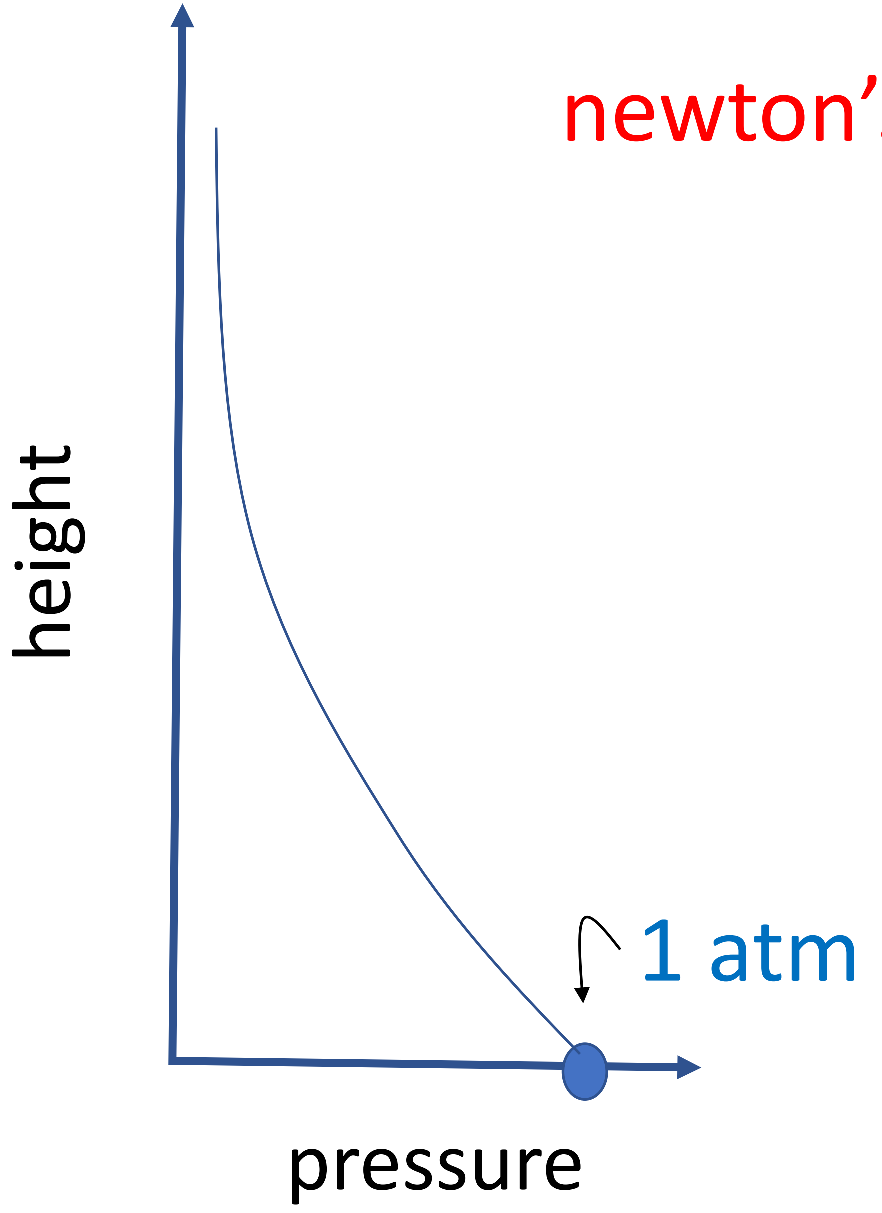
newton's law (no acceleration)



$$\frac{dp}{dx} = -g\rho$$

$$\frac{dp}{dx} = -\frac{gm}{RT}p$$

newton's law

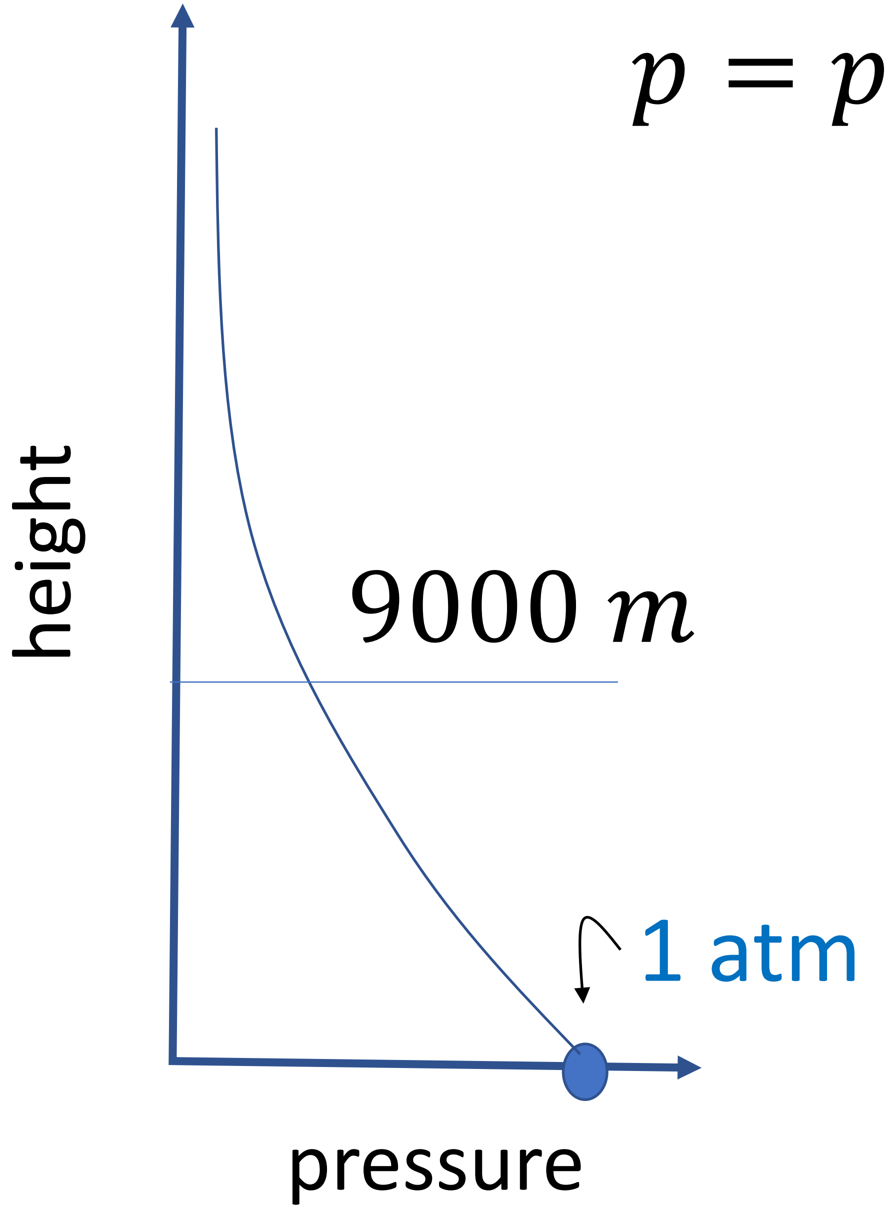


$$\frac{dp}{dx} = -\frac{gm}{RT}p$$

$$p = p_0 \exp(-x/x_o)$$

$$x_o = \frac{RT}{gm}$$

$$p = p_0 \exp(-x/x_0) \qquad x_o = \frac{RT}{gm}$$



	A	B	C	D	E
1	m	0.028	(nitrogen)	kg/mol	
2	g	9.81	m/s2		
3	R	8.3	J/mok-K		
4	T	300	K		
5	RT/gn	9065.094	m/s2		
6					
7					

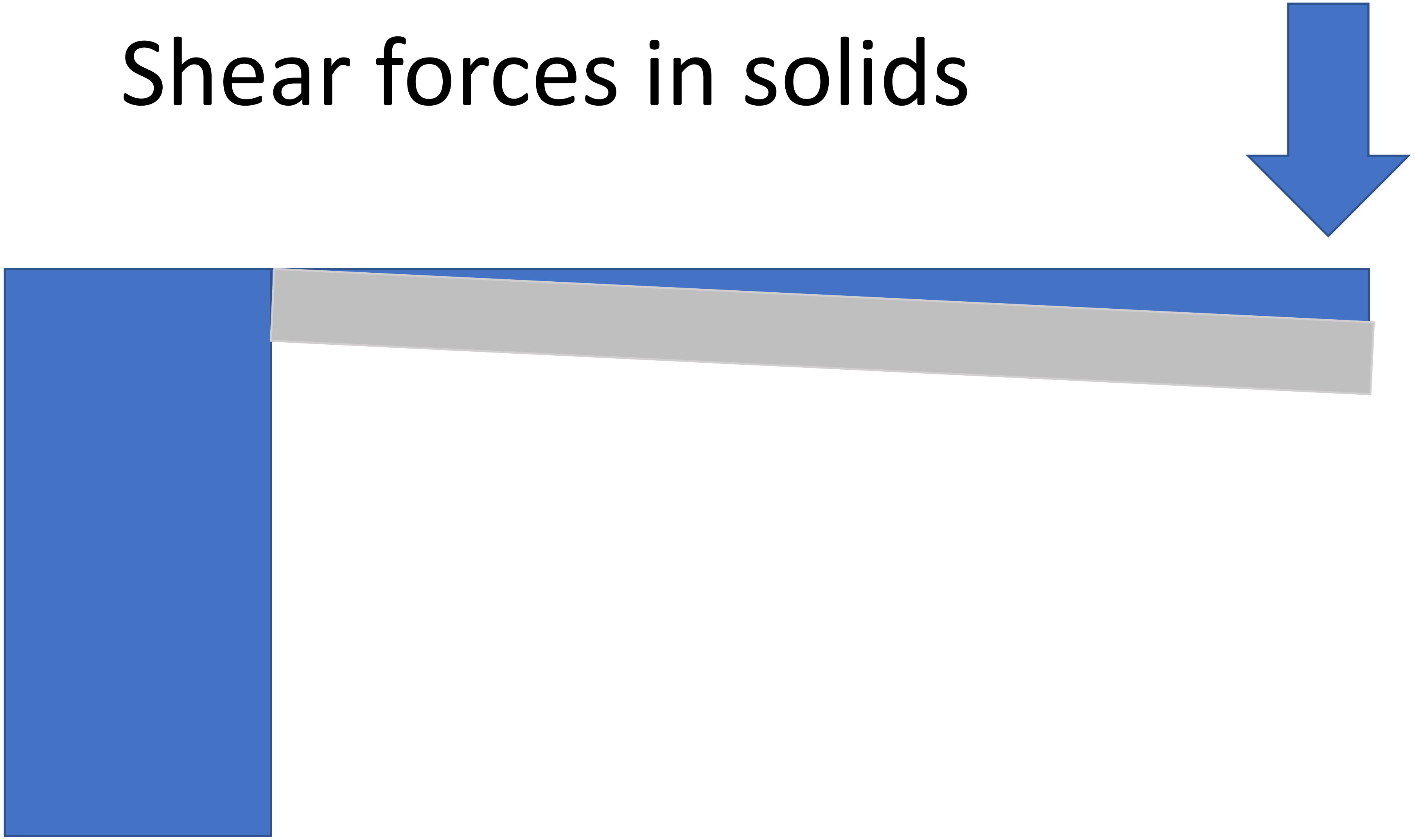
Sheet1

100%

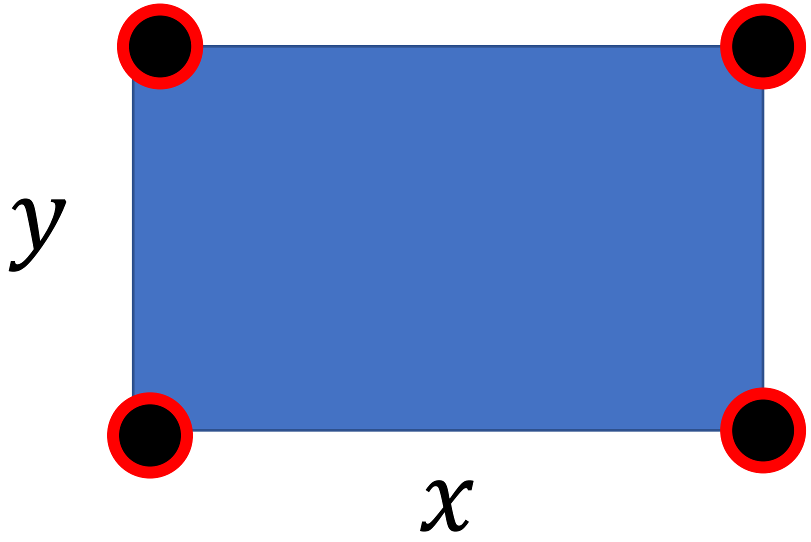
Shear forces in solids

Shear forces in solids

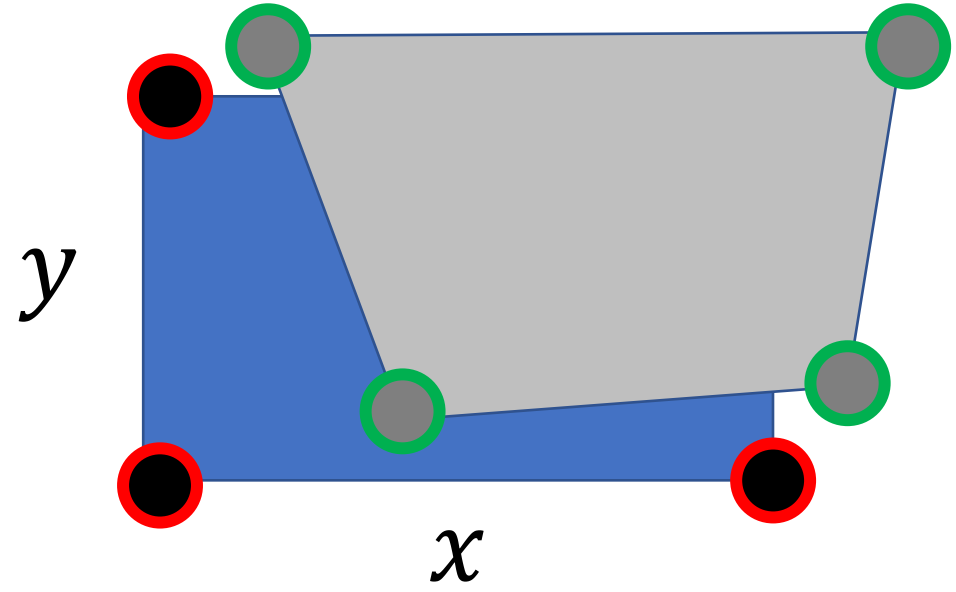
Shear forces in solids



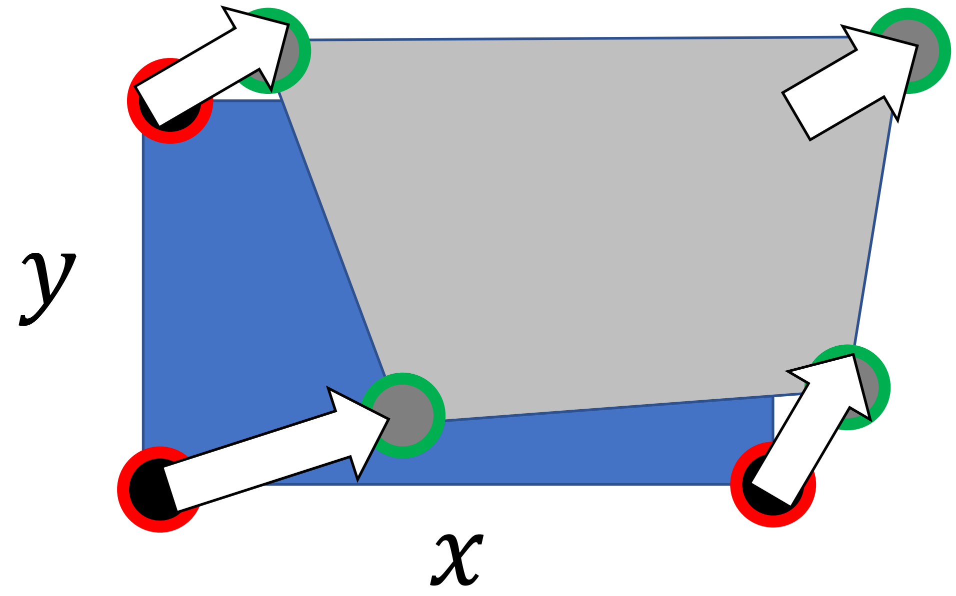
undeformed object



deformed object

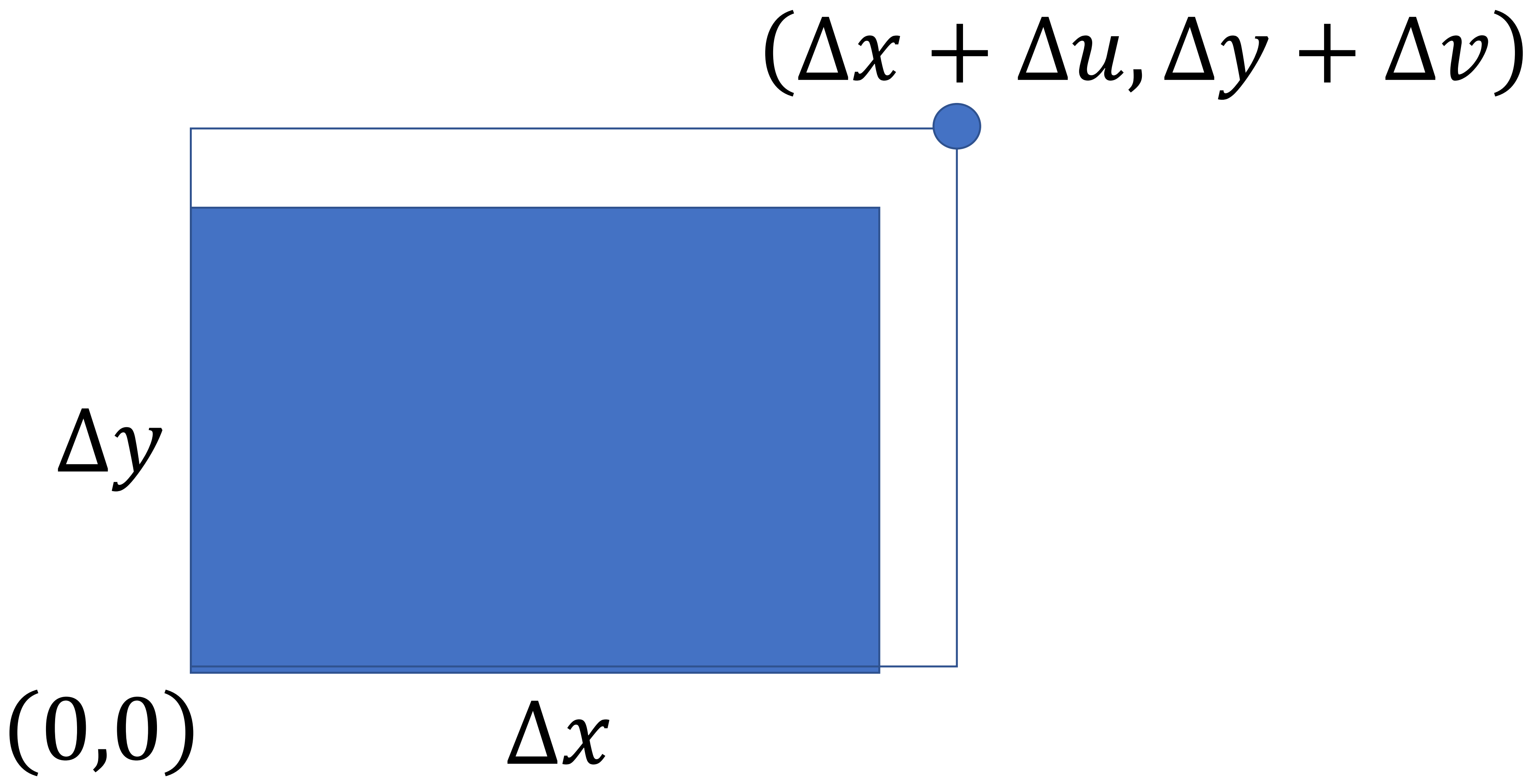


displacement



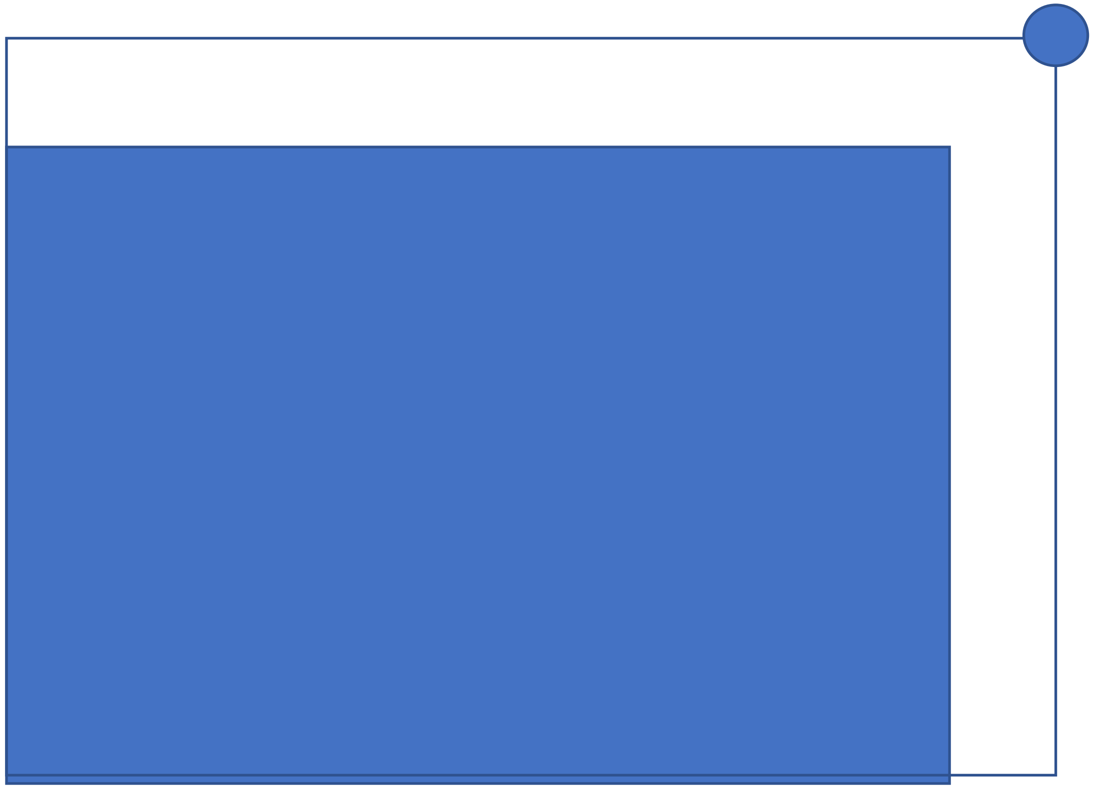
a point initially at (x, y)
moves to $(x + u, y + v)$

volumetric strain: change in volume



volumetric strain: change in volume

$(\Delta x + \Delta u, \Delta y + \Delta v)$



Δy

$(0,0)$

original area

$\Delta x \Delta y$

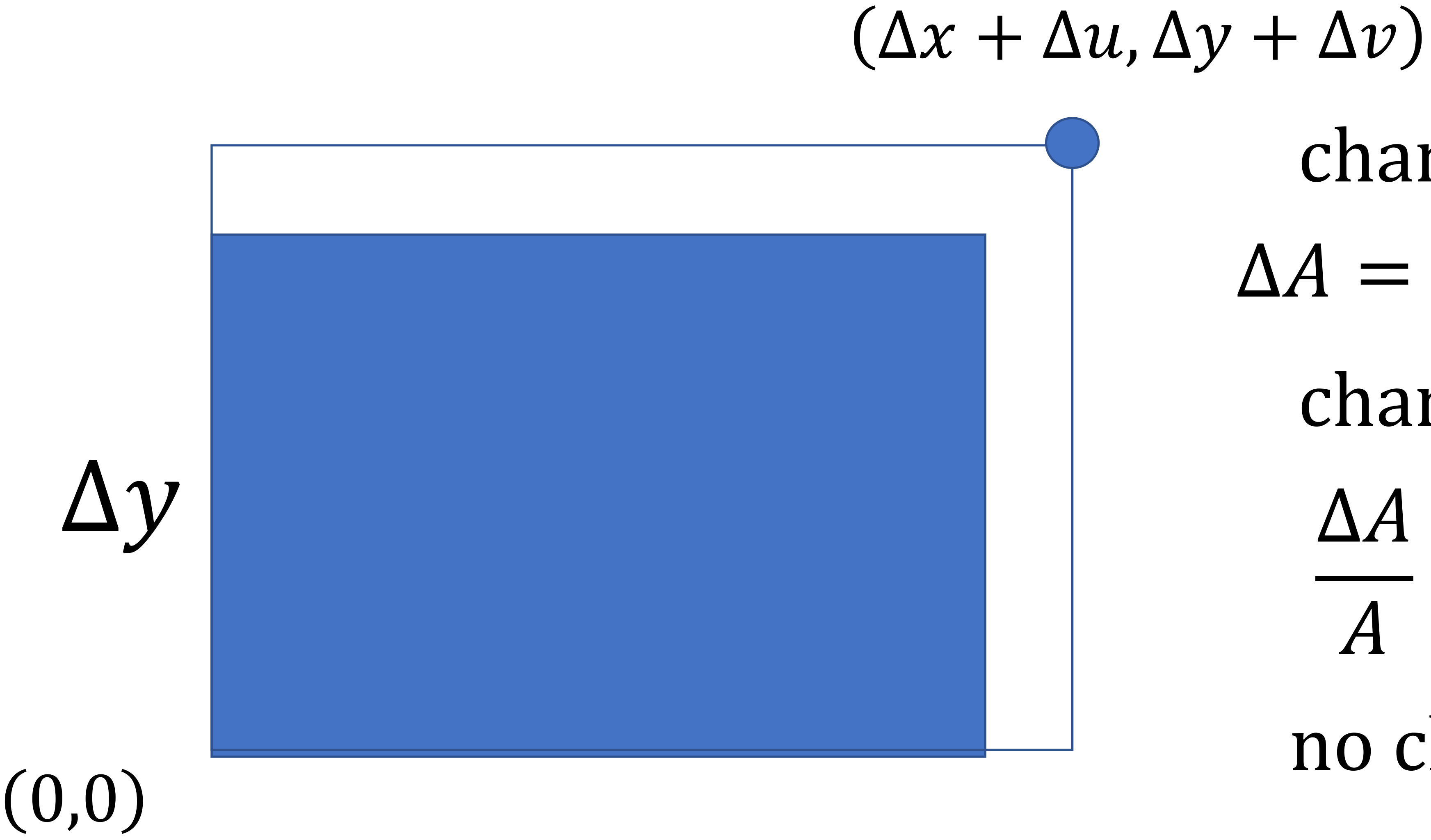
new area

$(\Delta x + \Delta u)(\Delta y + \Delta v)$

$\approx \Delta x \Delta y + \Delta x \Delta v + \Delta y \Delta u$

change in area $\Delta A = \Delta x \Delta v + \Delta y \Delta u$

volumetric strain: change in volume



change in area

$$\Delta A = \Delta x \Delta v + \Delta y \Delta u$$

change in area

$$\frac{\Delta A}{A} = \frac{\Delta v}{\Delta y} + \frac{\Delta u}{\Delta x}$$

no change in area

$$0 = \frac{dv}{dy} + \frac{du}{dx}$$

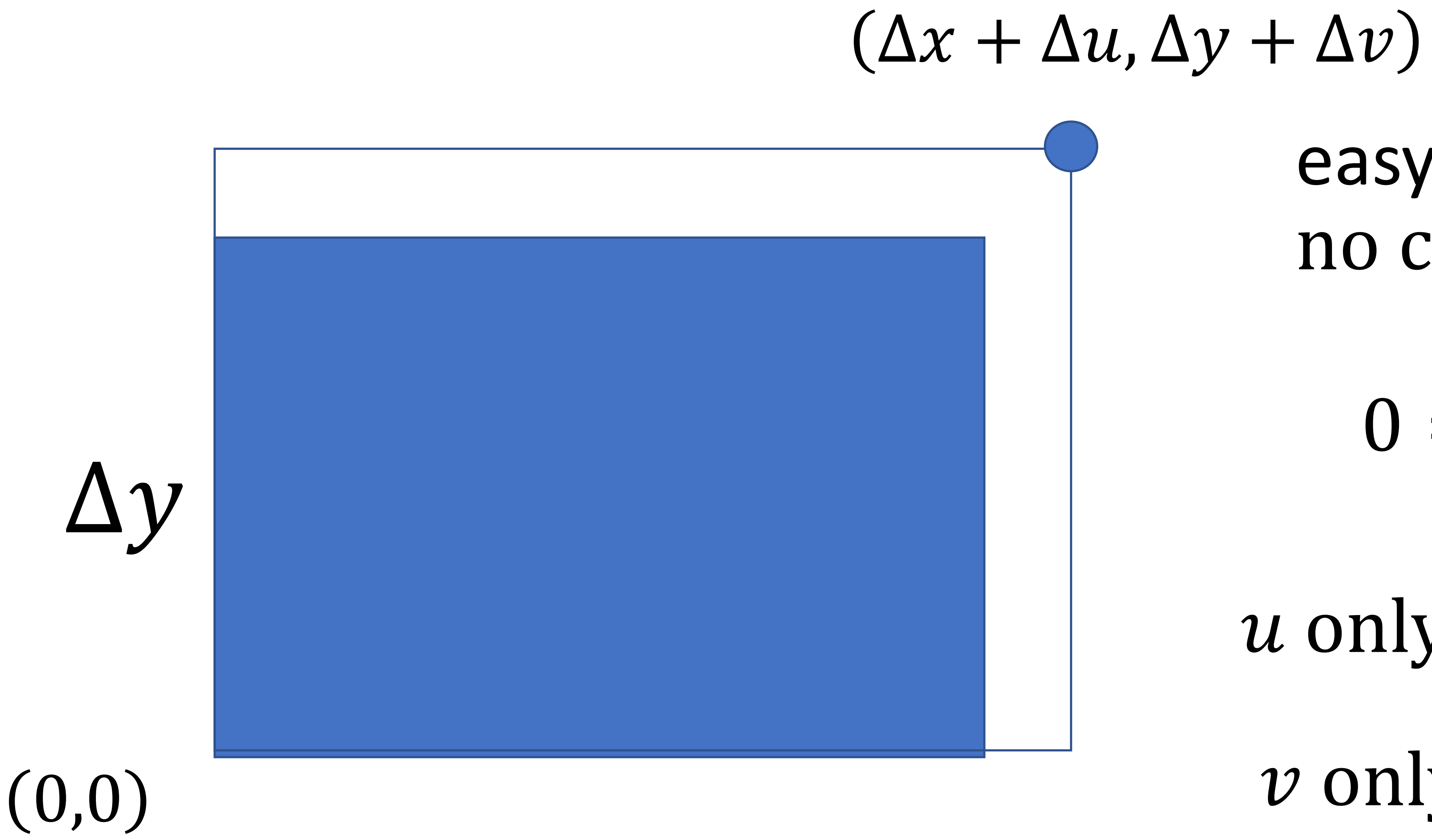
no change in volume

no change in area

$$0 = \frac{dv}{dy} + \frac{du}{dx}$$

called
shear strain

volumetric strain: change in volume



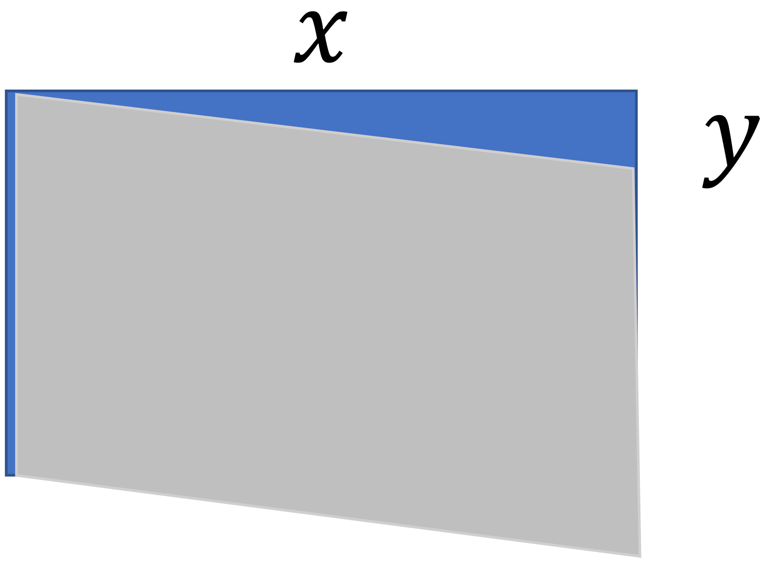
easy way to satisfy
no change in area

$$0 = \frac{dv}{dy} + \frac{du}{dx}$$

u only varies with y

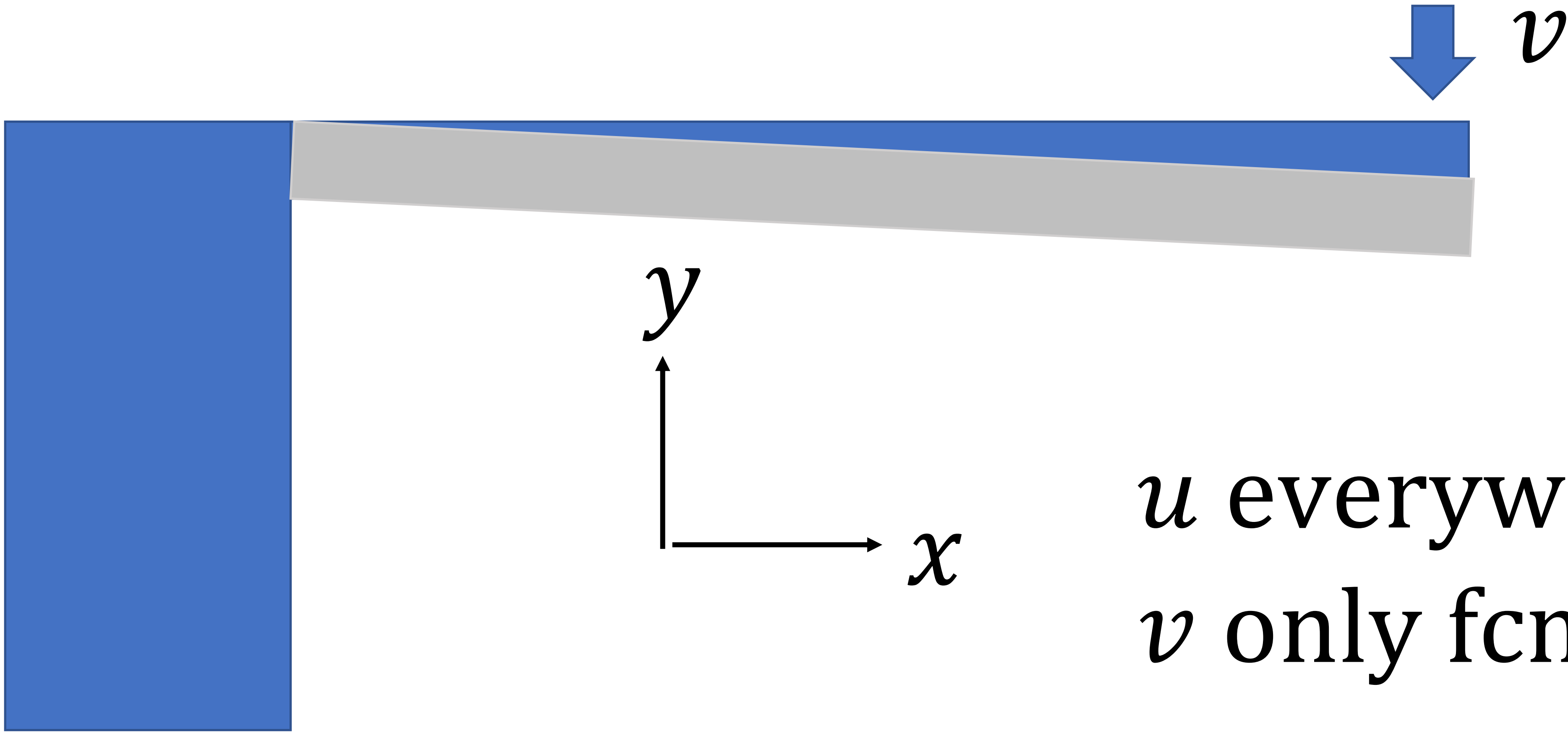
v only varies with x

undeformed object



undeformed object

shear strain only



u everywhere 0

v only fcn of x