Solid Earth Dynamics

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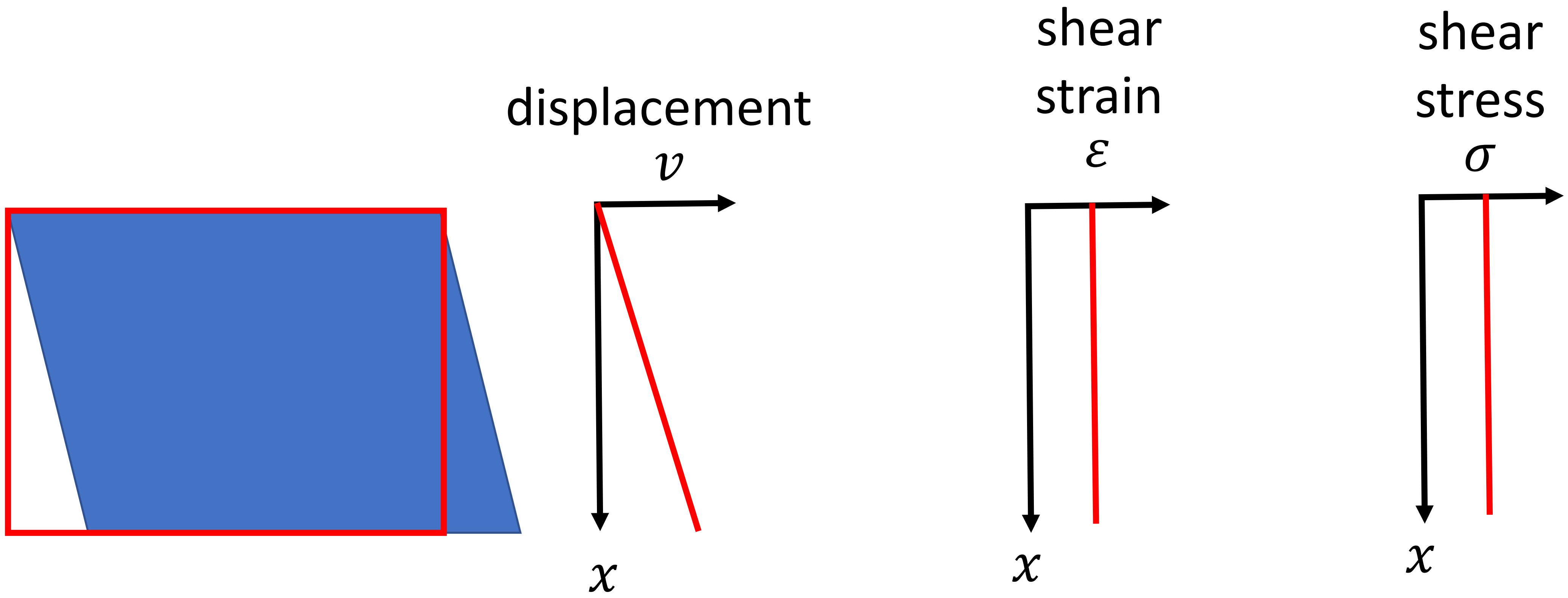
Lecture 13

Today

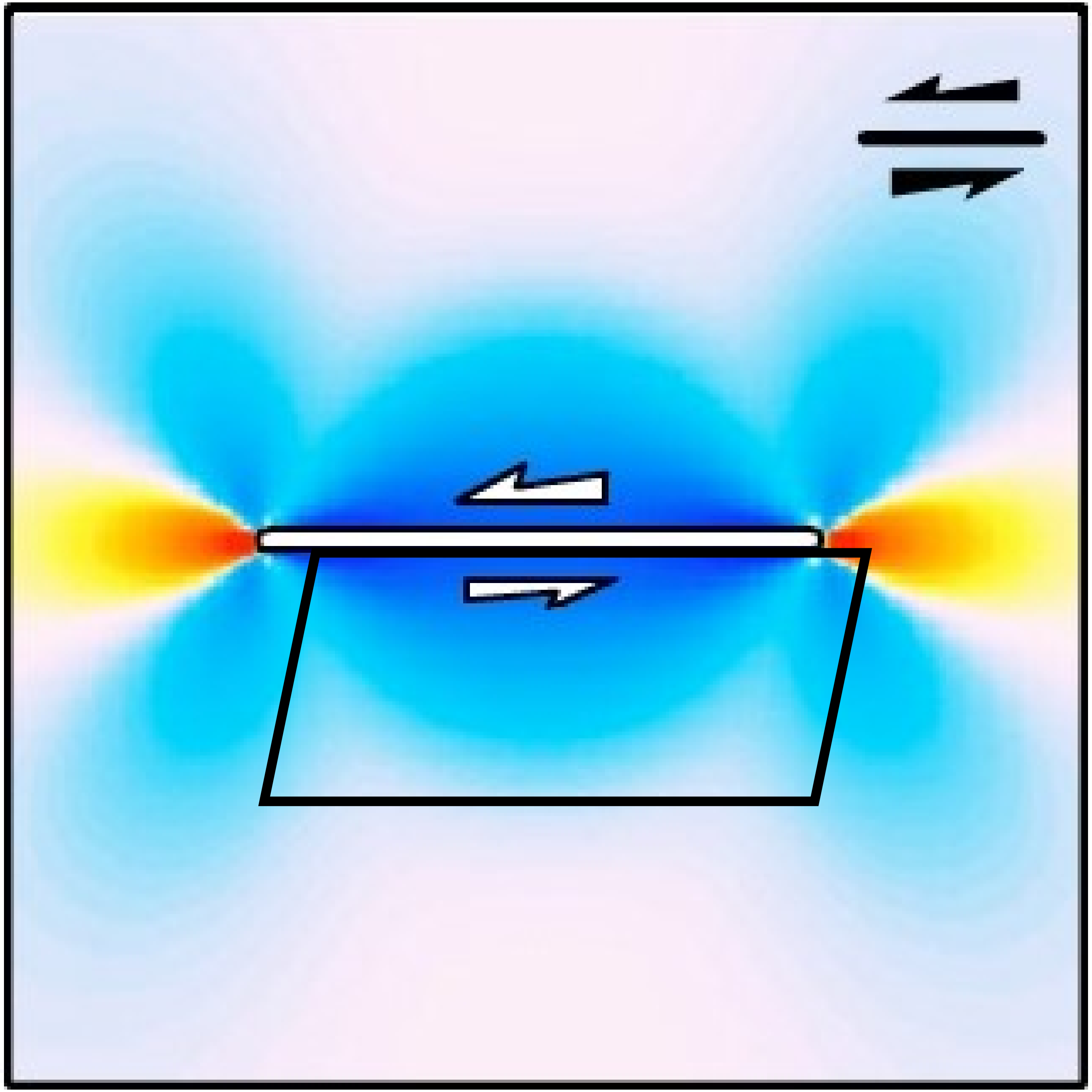
Shear Stress
Earthquake Stresses
Normal stress
angular momentum and torque
plate flexure

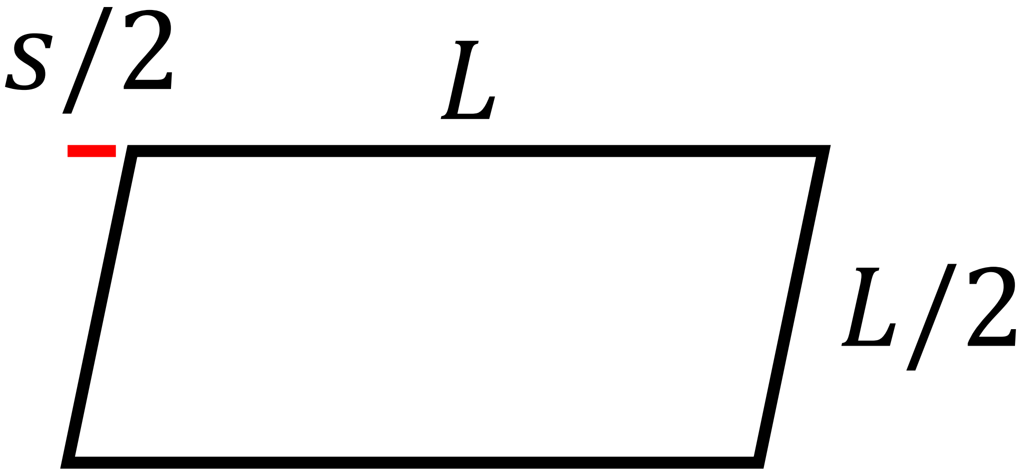
Shear Stress



displacement
v
x
shear
strain
ε
x
shear
stress
σ
x

Stress Change due to Earthquake



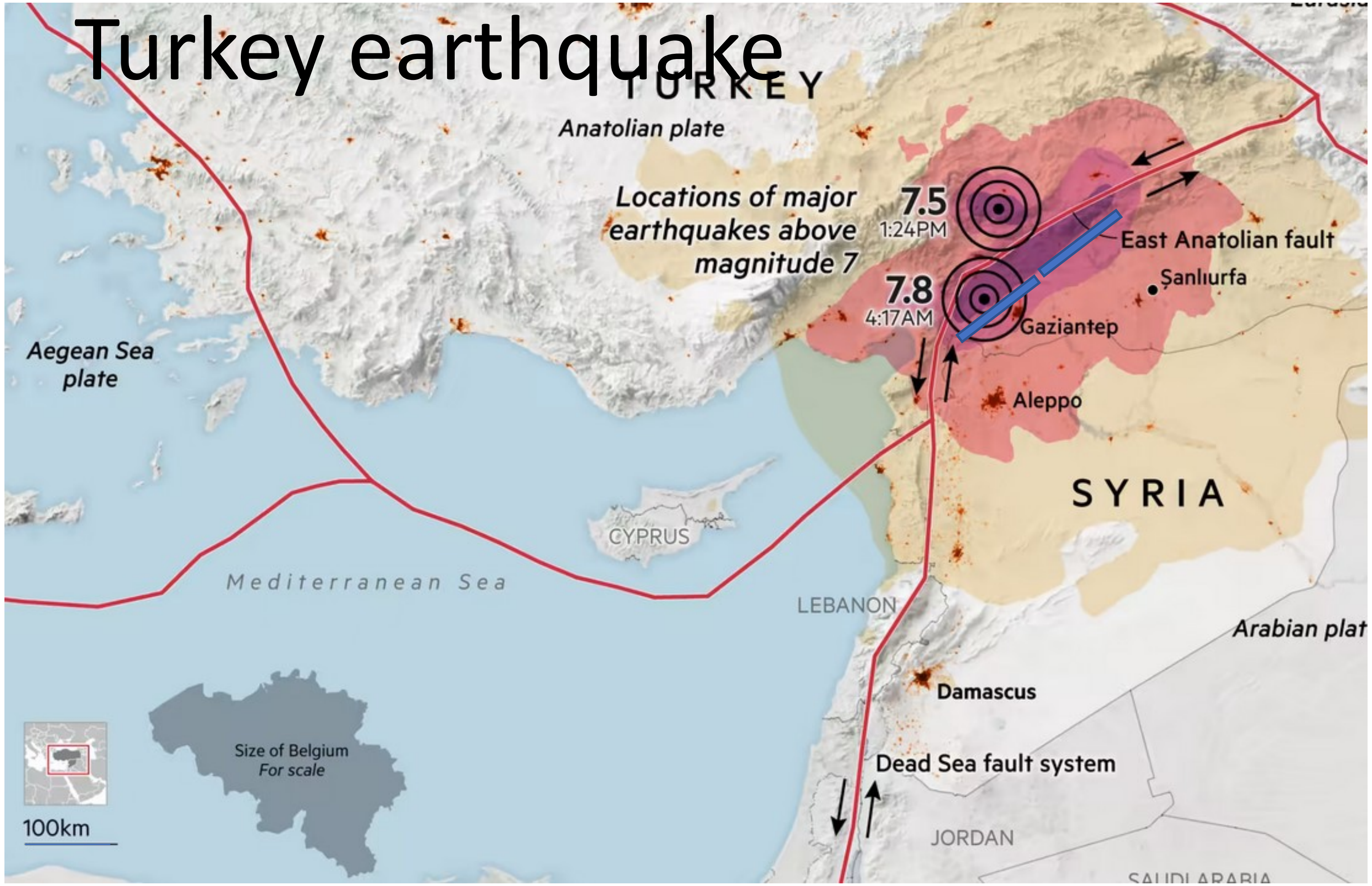


shear strain $\frac{s/2}{L/2} = \frac{s}{L}$

shear stress $\mu \frac{s}{L}$

μ shear modulus

Turkey earthquake



$$L = 200000\ m$$

$$s = 3\ m$$

$$maximum\ strain$$

$$s = 3/200000$$

maximum stress decrease

$$\sigma = 0.4\ Mpa$$

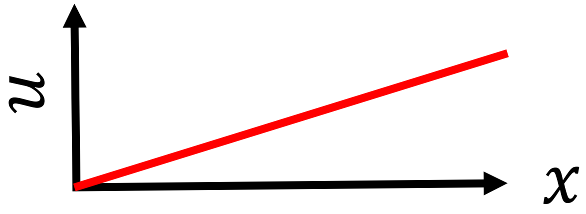
$$= 4\ bars = 4\ atmospheres$$

$$\mu = 27\ Gpa\ (granite)$$

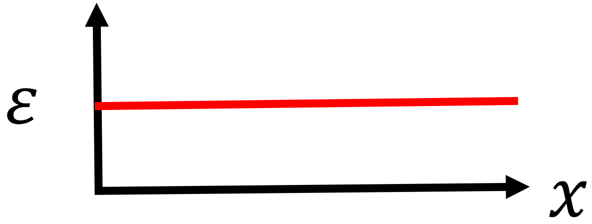
$$failure = a\ few\ Mpa$$

extensional stress

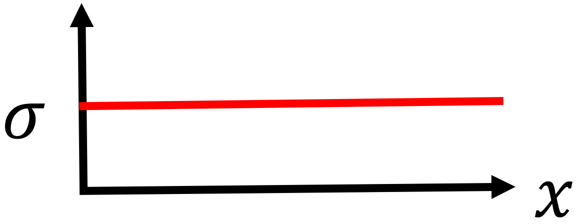
displacement



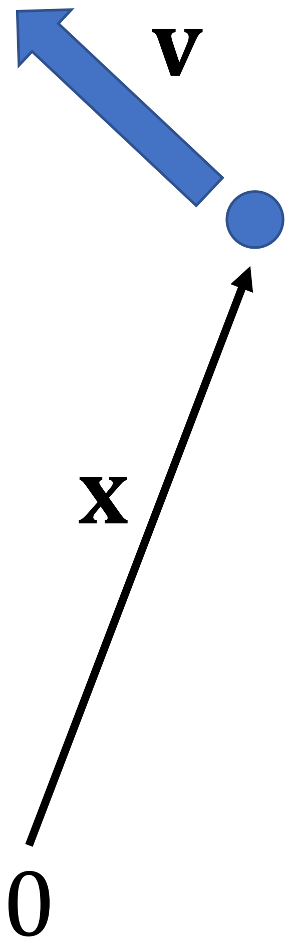
normal strain

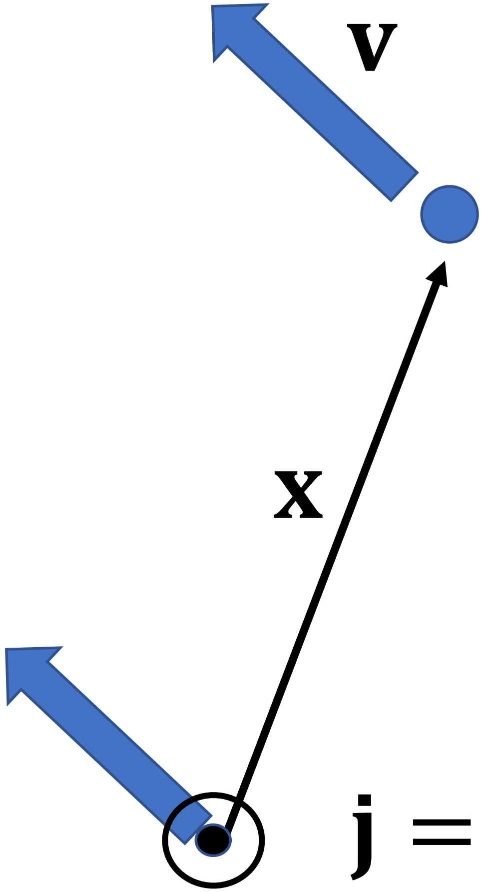


normal stress



Newton’s Law applied to rotation

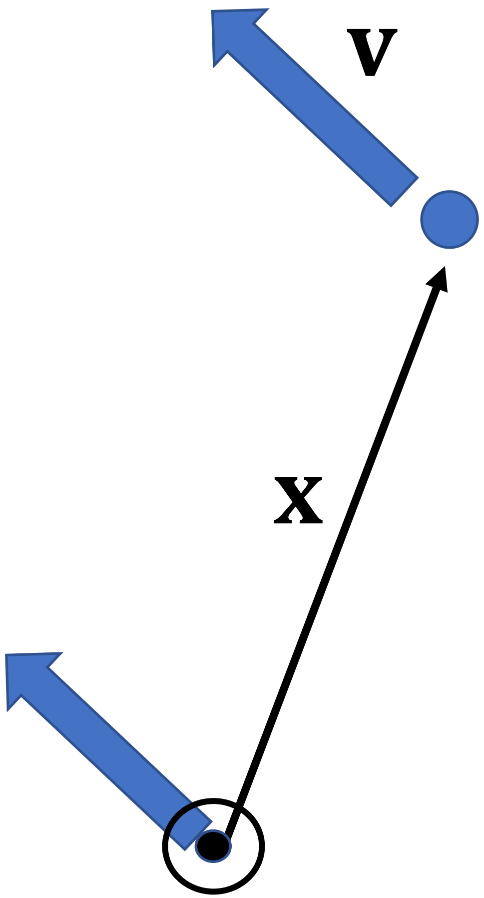


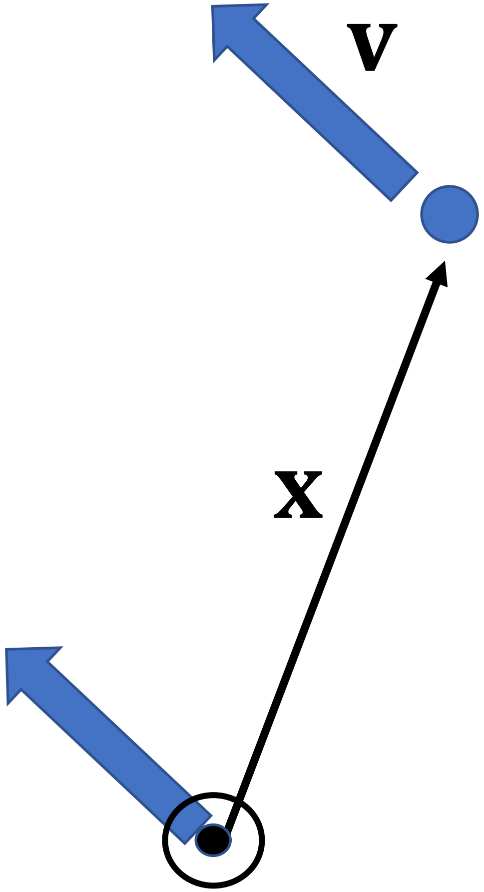


$$\mathbf{j} = \mathbf{x} \times m\mathbf{v}$$

vector parallel to rotation axis

$$\mathbf{j} = \mathbf{x} \times m\mathbf{v} = \mathbf{x} \times m\frac{d\mathbf{x}}{dt}$$





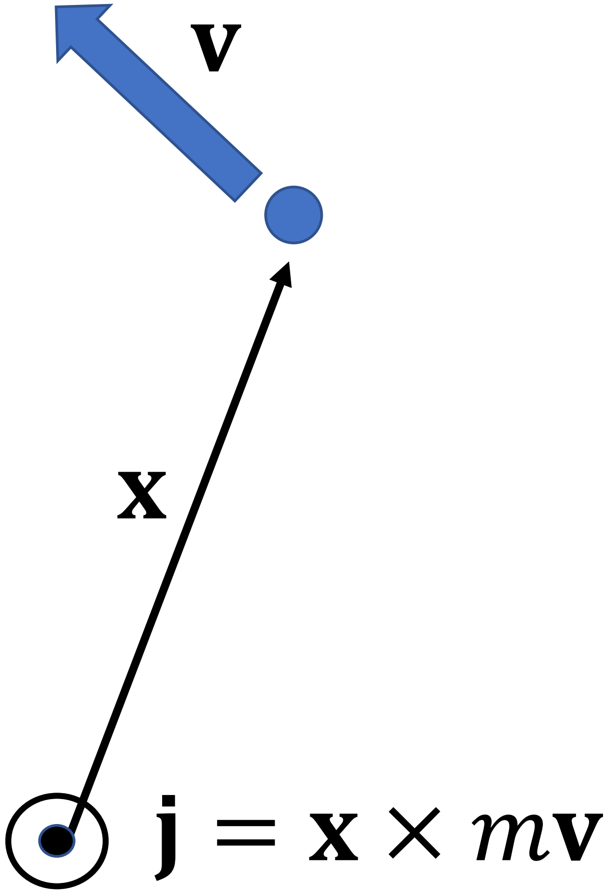
$$\frac{d\mathbf{j}}{dt} = \frac{d}{dt}\left(\mathbf{x} \times m\frac{d\mathbf{x}}{dt}\right)$$

$$\frac{d\mathbf{j}}{dt} = m\frac{d\mathbf{x}}{dt} \times \frac{d\mathbf{x}}{dt} + \mathbf{x} \times m\frac{d^2\mathbf{x}}{d^2t}$$

$$= m \quad 0 + \mathbf{x} \times m\frac{d^2\mathbf{x}}{d^2t}$$

$$= \mathbf{x} \times \mathbf{f}$$

$$= \mathbf{T}$$ torque

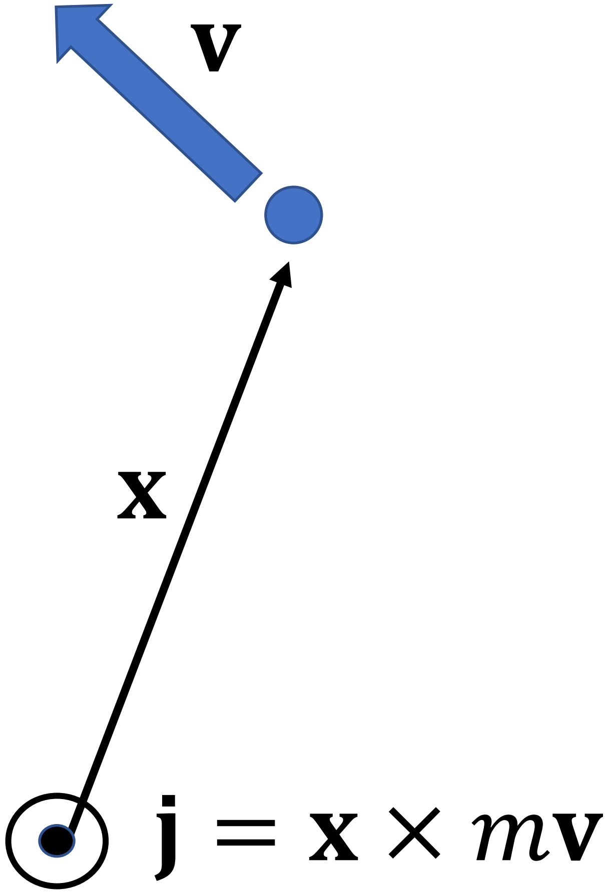


$\mathbf{j} = \mathbf{x} \times m\mathbf{v}$ angular momentum

$\mathbf{T} = \mathbf{x} \times \mathbf{f}$ torque

newton’s Law

$$\frac{d\mathbf{j}}{dt} = \mathbf{T}$$



$\mathbf{j} = \mathbf{x} \times m\mathbf{v}$ angular momentum

$\mathbf{T} = \mathbf{x} \times \mathbf{f}$ torque

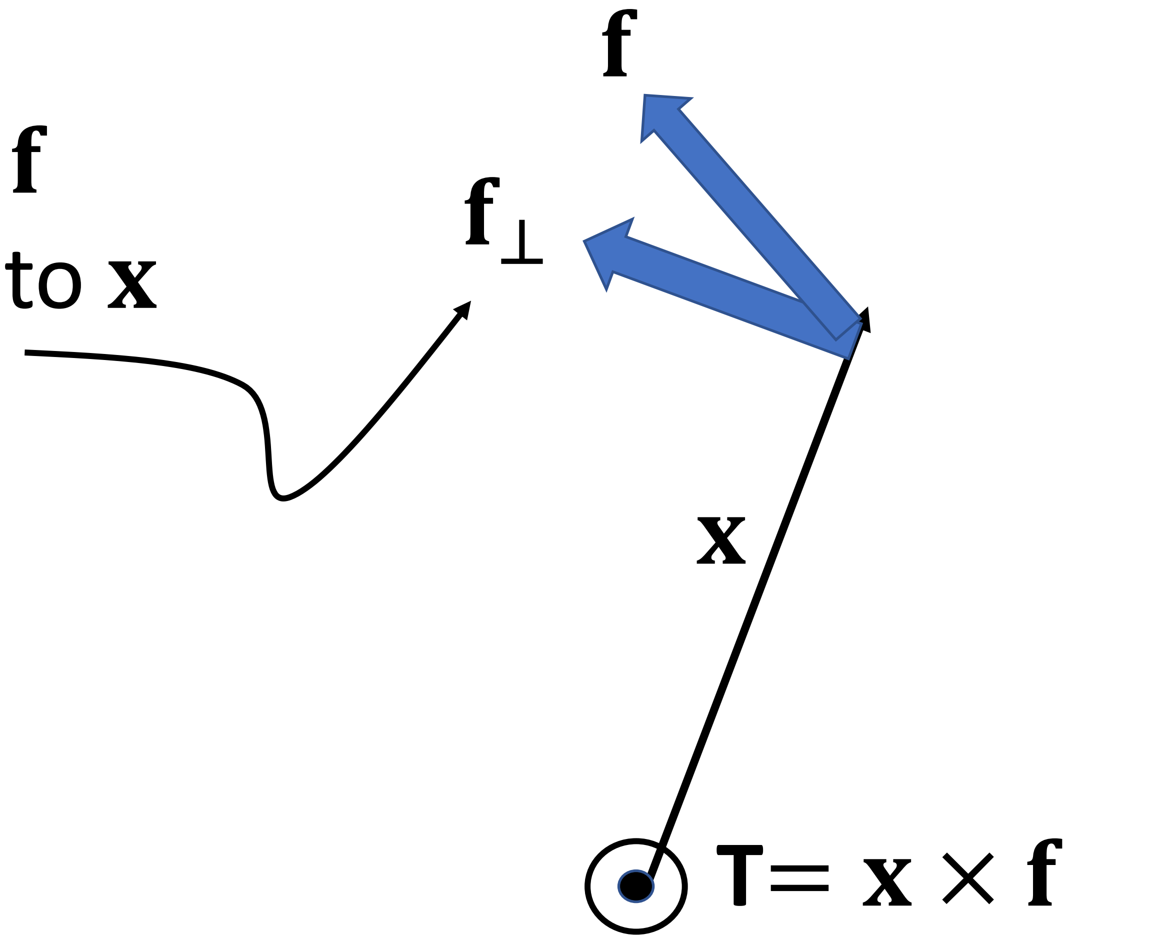
newton’s Law

$$\frac{d\mathbf{j}}{dt} = \mathbf{T}$$

no rotation
balance of torques

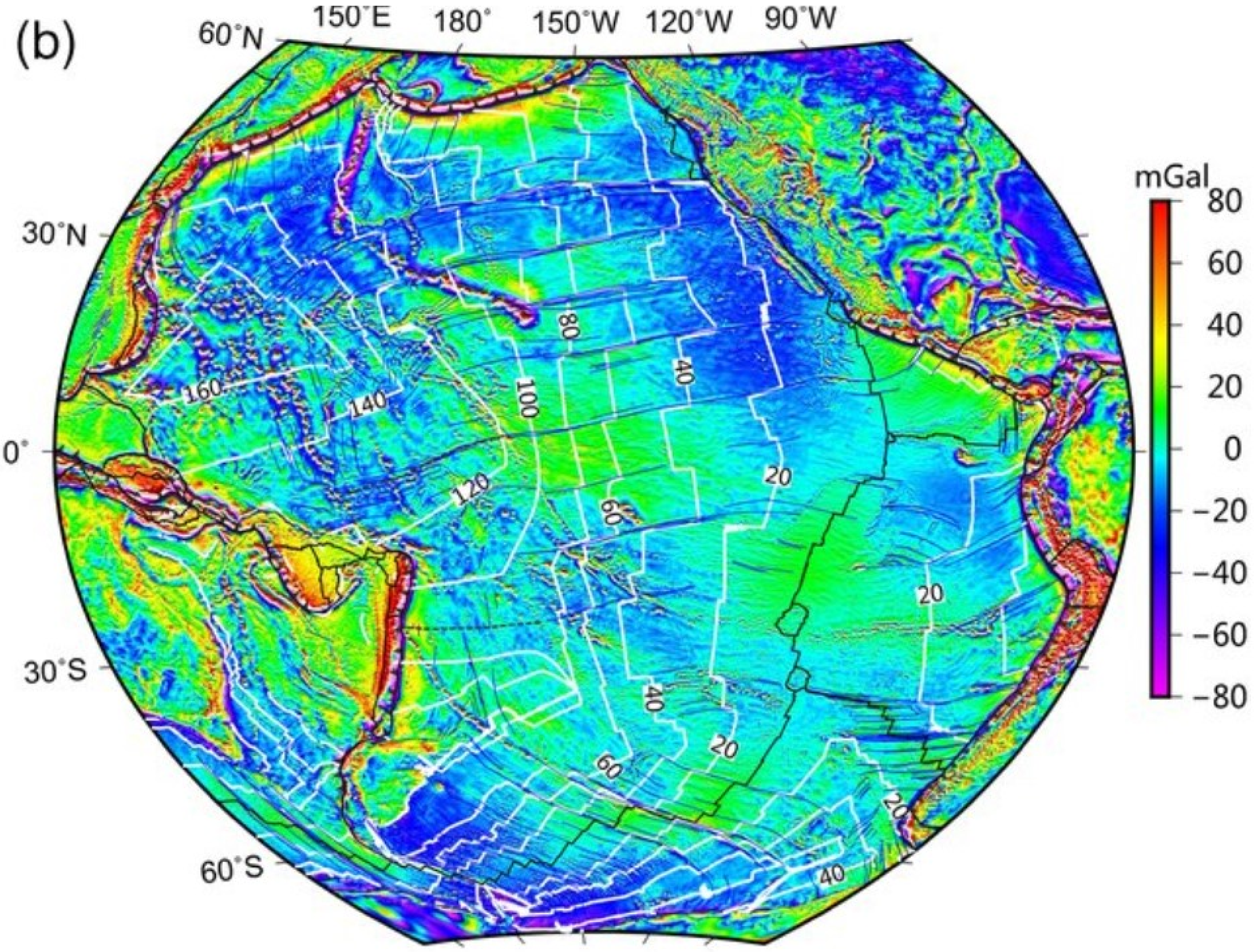
$$0 = \mathbf{T}$$

component of $\mathbf{f}$
perpendicular to $\mathbf{x}$

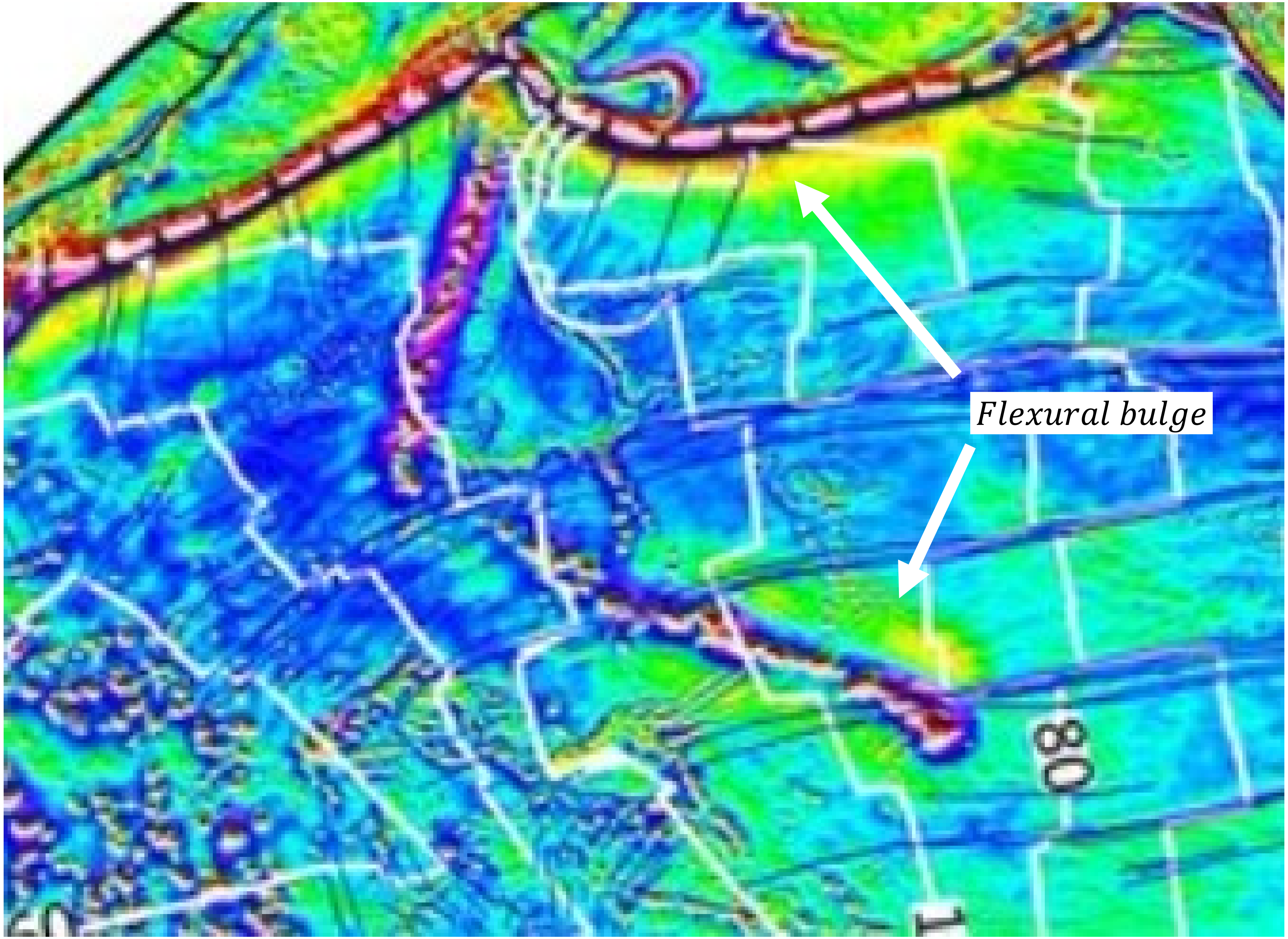


has magnitude $|\mathbf{T}| = |\mathbf{x}|\,|\mathbf{f}_{\perp}|$

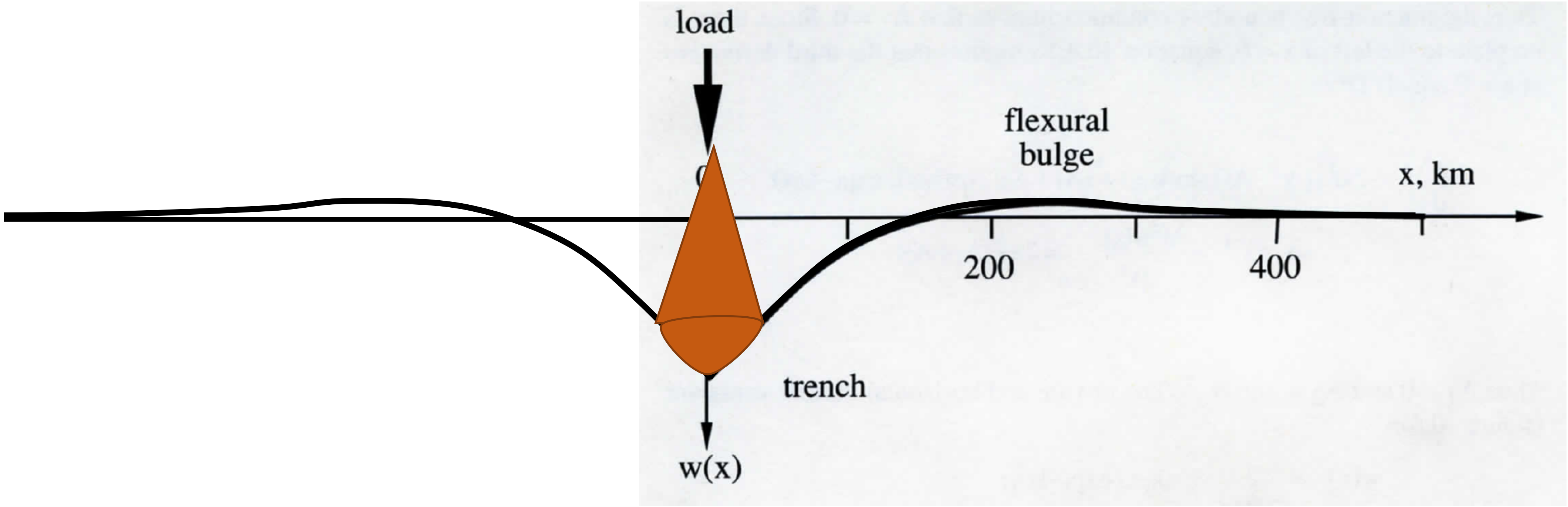
Lithospheric Flexure



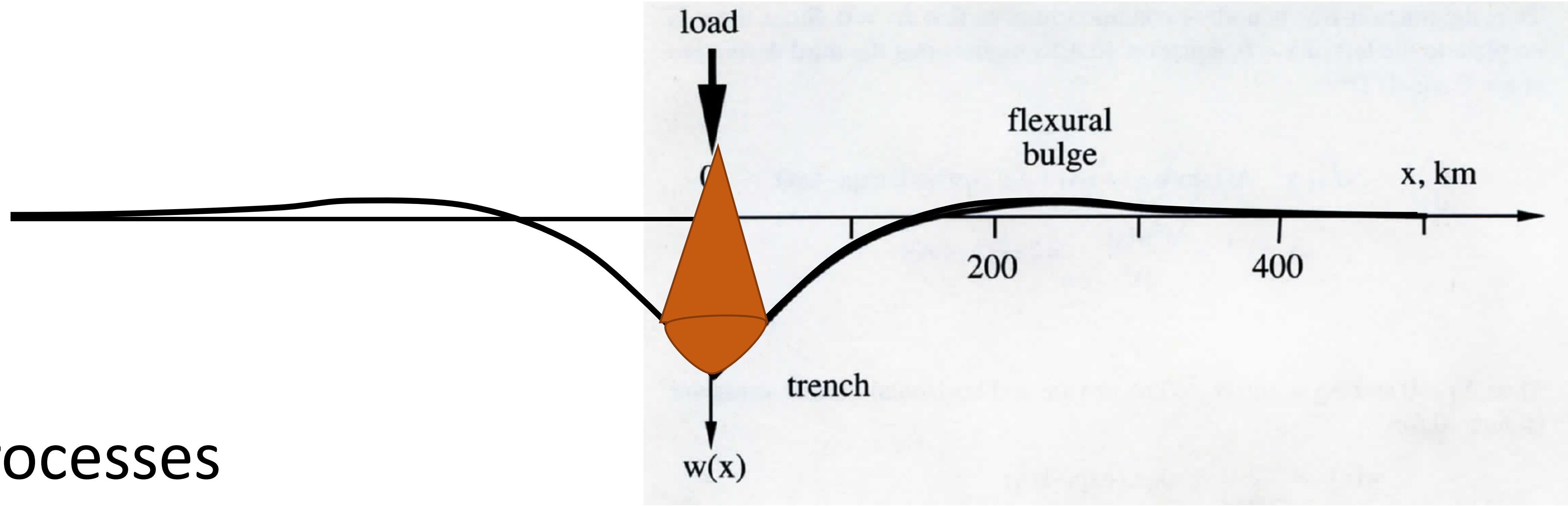
(b)
60°N
150°E
180°
150°W
120°W
90°W
30°N
0°
30°S
60°S
mGal
80
60
40
20
0
−20
−40
−60
−80
160
140
120
100
80
60
40
20


Flexural bulge

Ocean Island



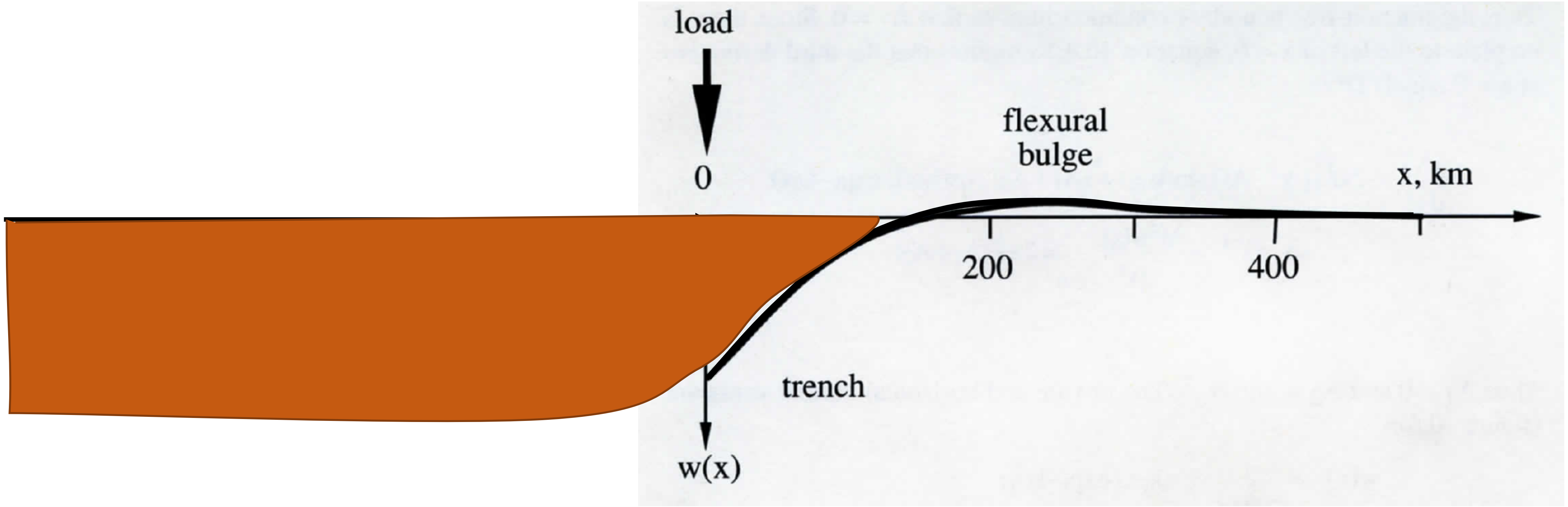
Ocean Island



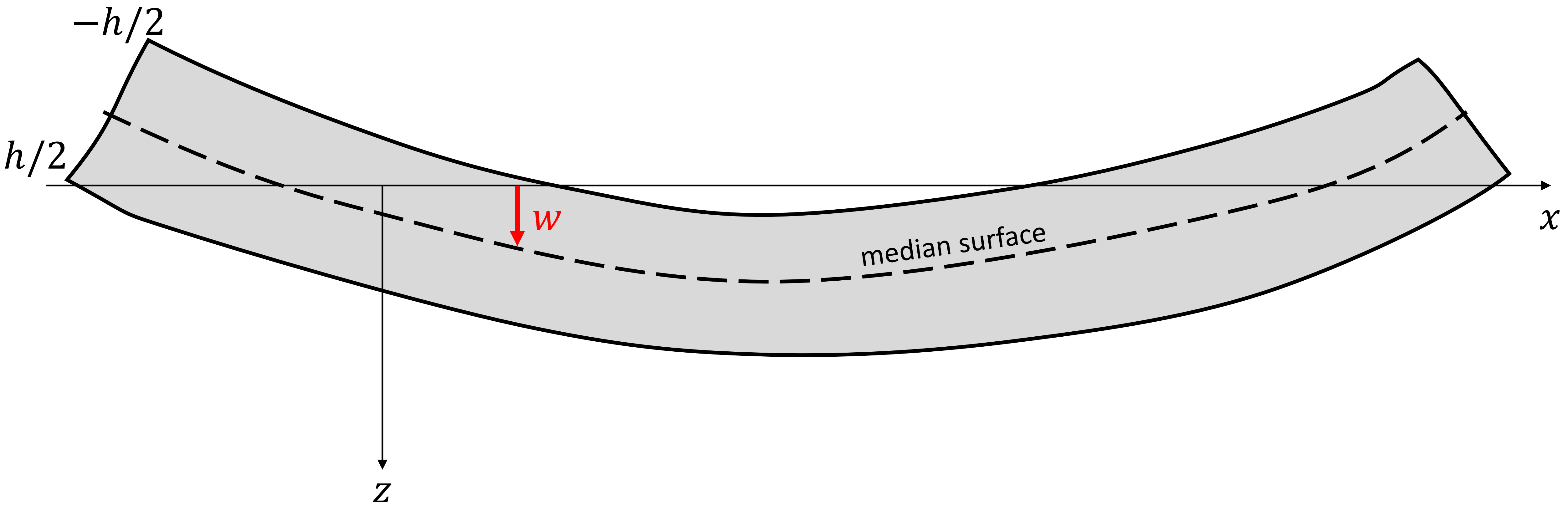
processes

- surface forces in plate
- isostacy
- gravitational loading (island)

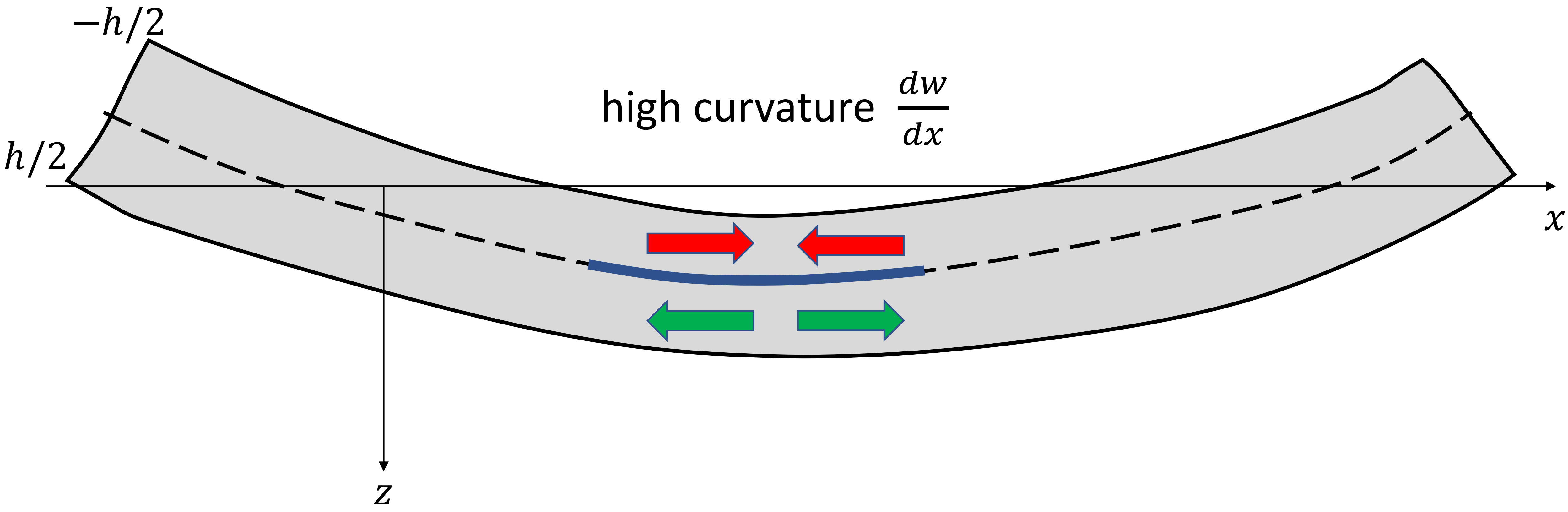
Subduction zone



Flexure $w(x)$ of the median surface



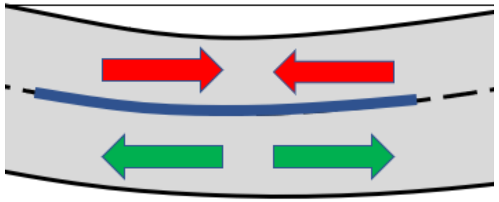
extension and compression related to curvature

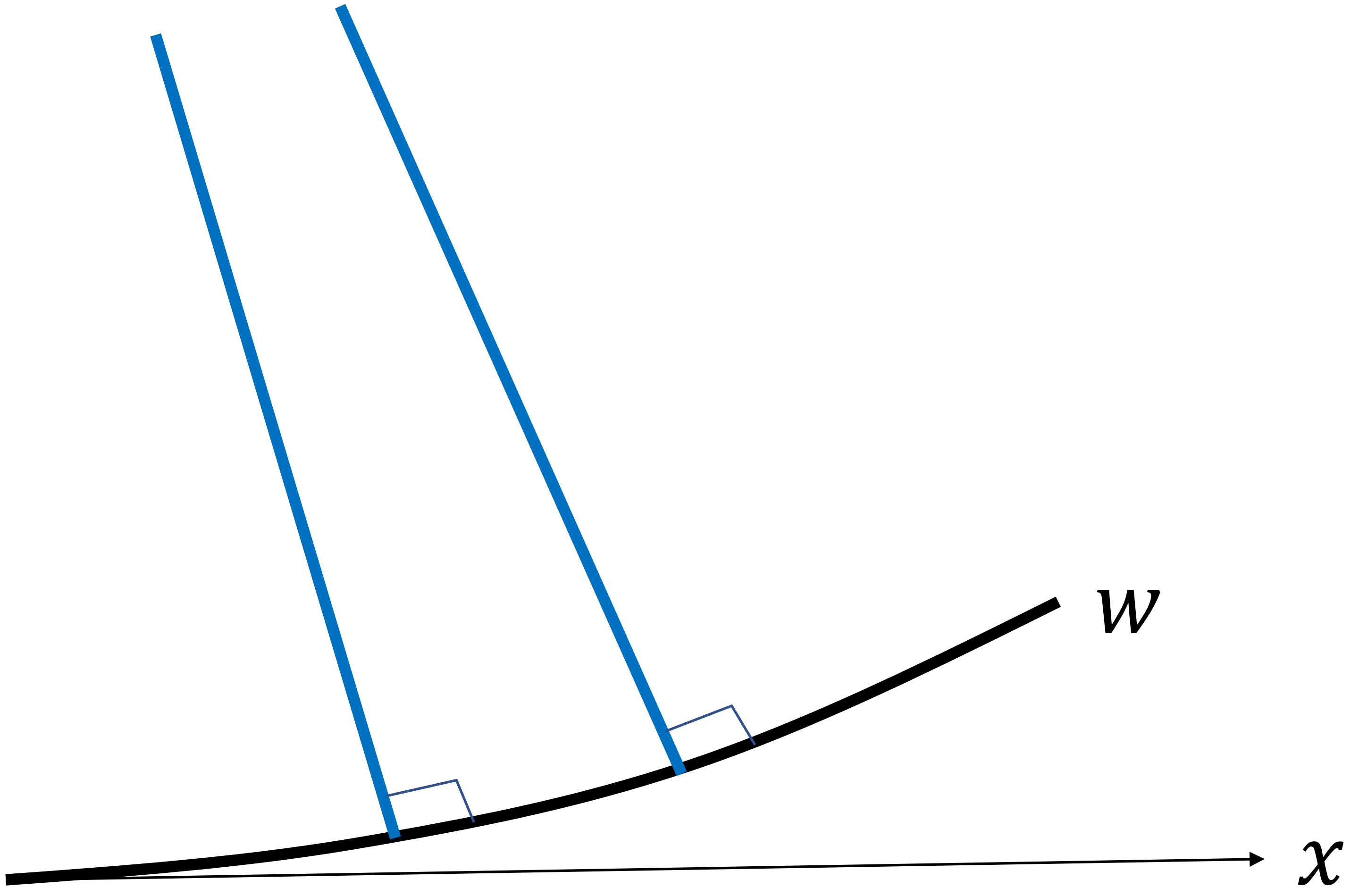


Step 1: relating normal stress to flexure

extension and compression related to curvature

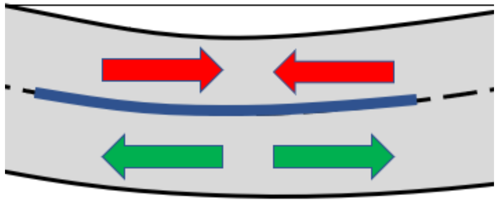
high curvature $\frac{dw}{dx}$

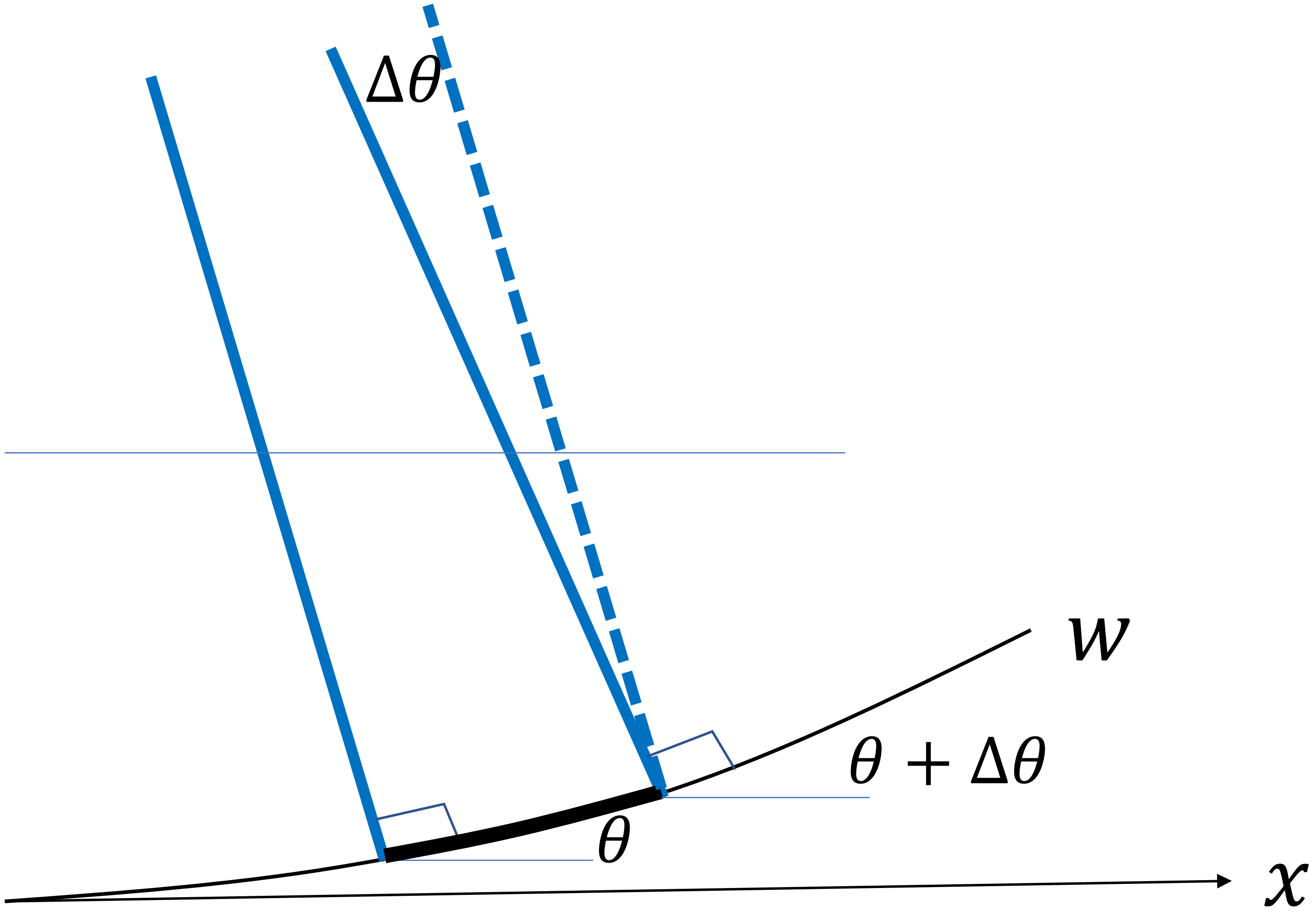




extension and compression related to curvature

high curvature $\frac{dw}{dx}$

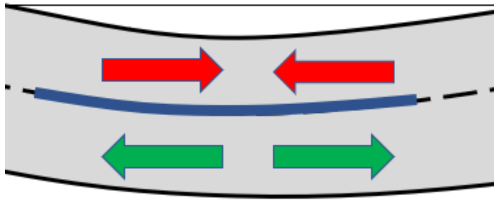


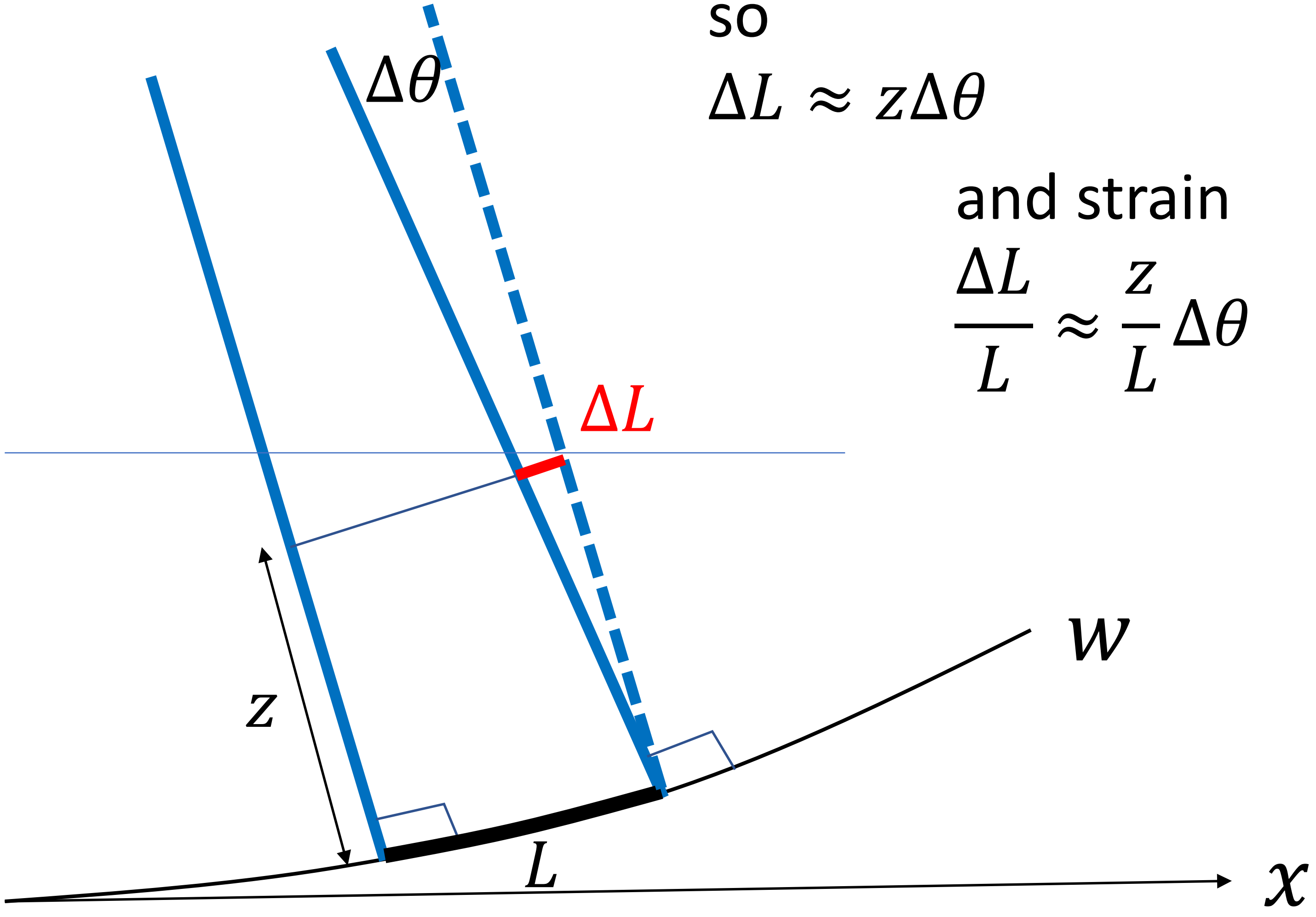


for small angles

$$\sin \Delta\theta \approx \tan \Delta\theta \approx \Delta\theta$$

high curvature $\frac{dw}{dx}$





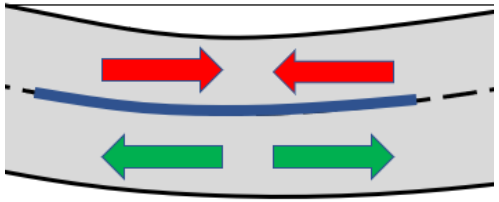
so

$$\Delta L \approx z\Delta\theta$$

and strain

$$\frac{\Delta L}{L} \approx \frac{z}{L}\Delta\theta$$

high curvature $\frac{dw}{dx}$



slope of curve

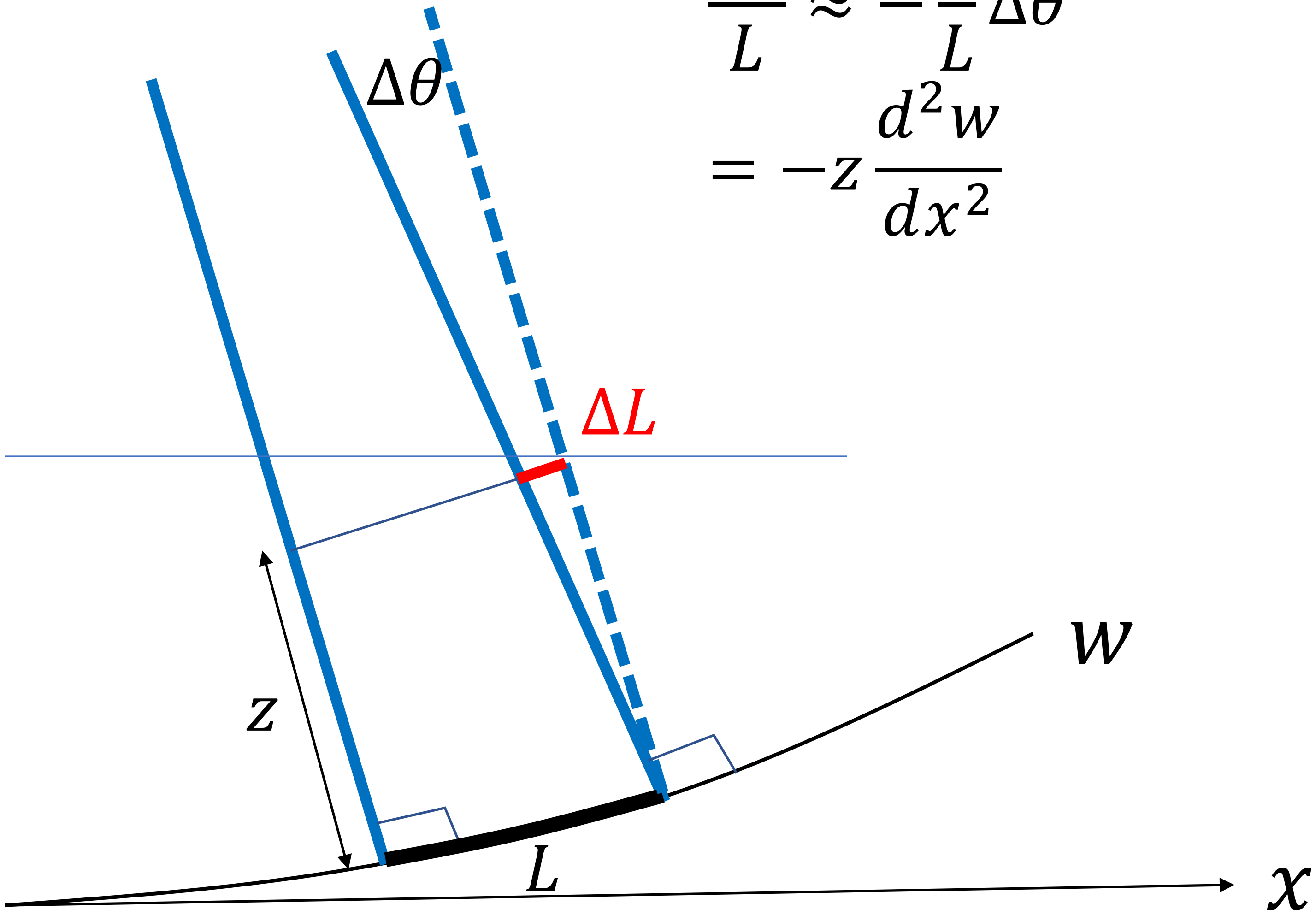
$$\frac{dw}{dx} \approx \tan\theta \approx \theta$$

$$\Delta\theta = \frac{d\theta}{dx}L = \frac{d^2w}{dx^2}L$$

so strain

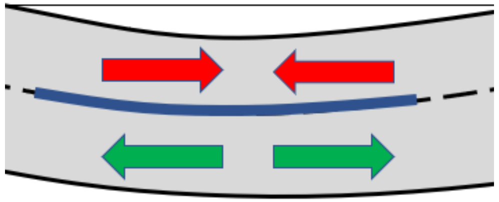
$$\frac{\Delta L}{L} \approx -\frac{z}{L}\Delta\theta$$

$$= -z\frac{d^2w}{dx^2}$$



Assume linear elasticity

high curvature $\frac{dw}{dx}$

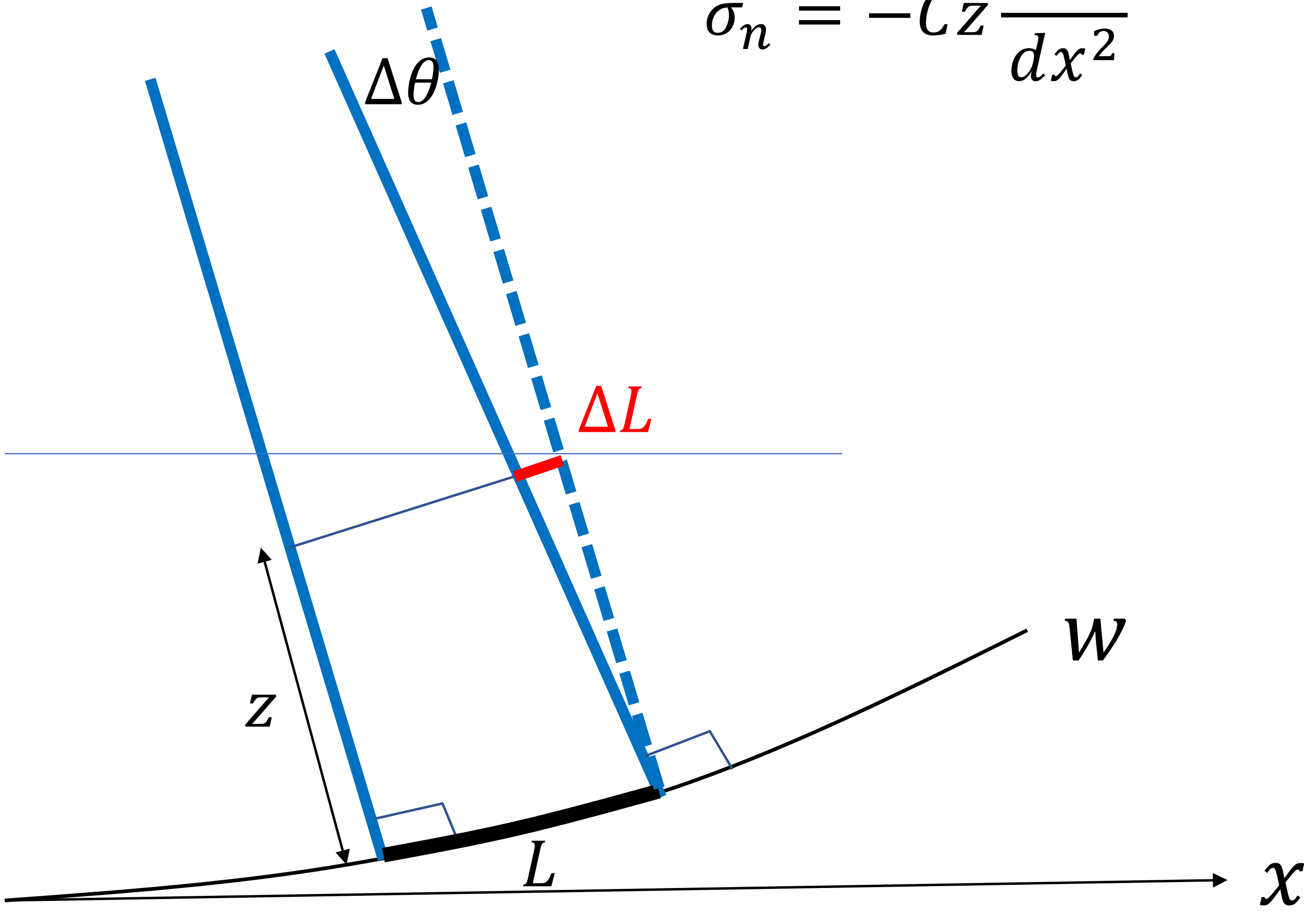


stress proportional to strain

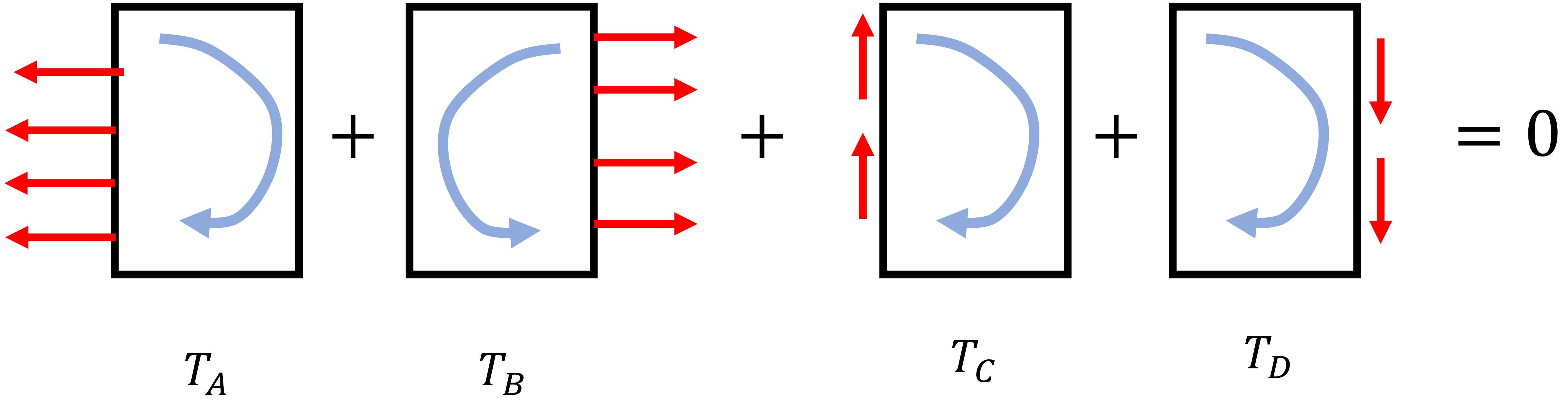
stress = C times strain

so horizontal stress

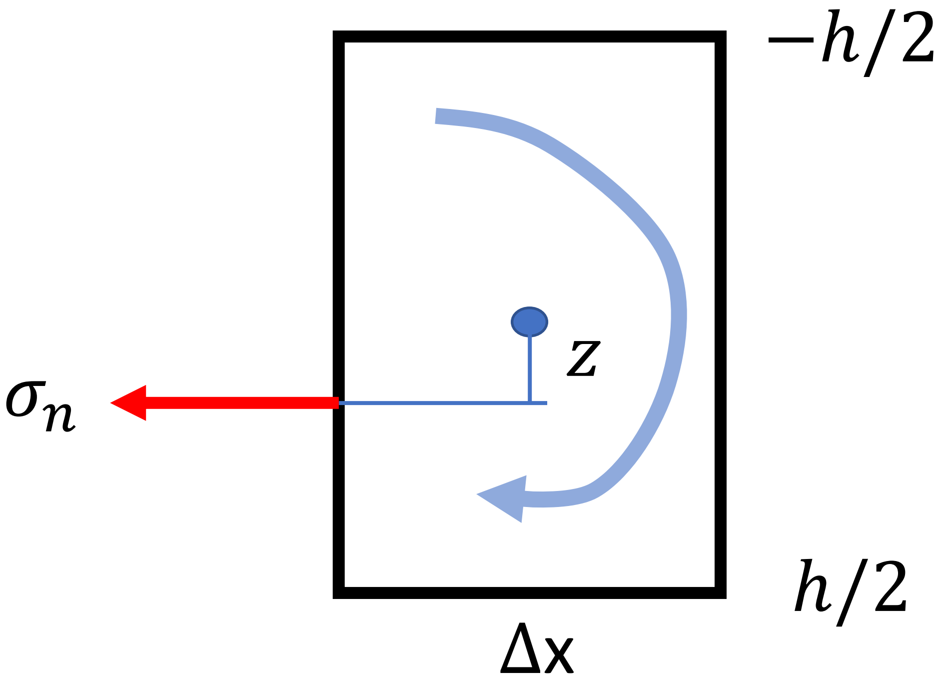
$$\sigma_n = -Cz\frac{d^2w}{dx^2}$$



Step 2: Balance of vertical forces

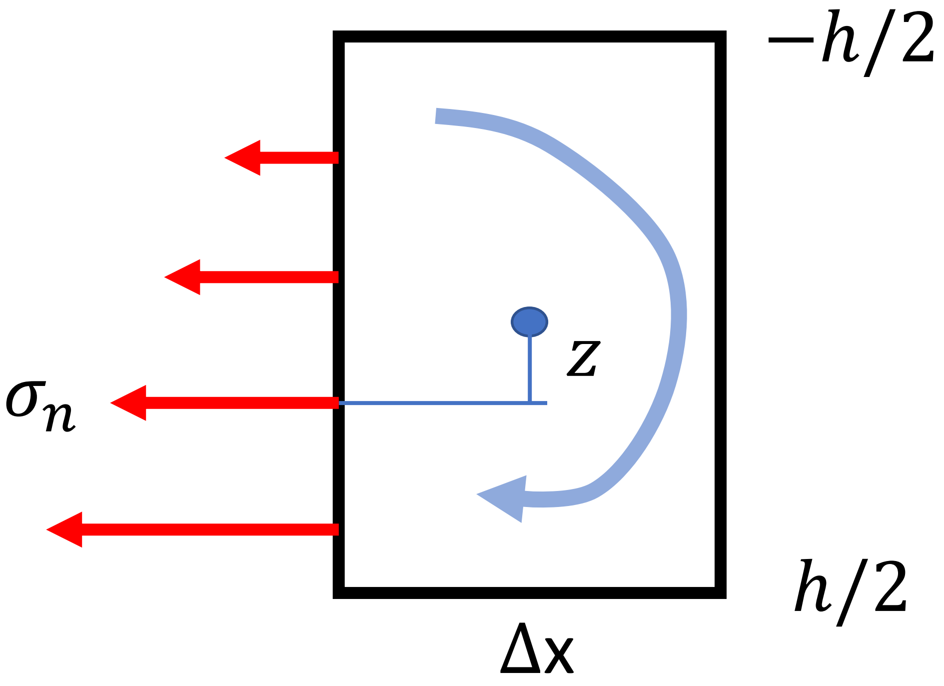


Torque T_A on left surface from normal stress $\sigma_n(x)$



Torque: $T_A = -z\sigma_n Y$

Y: distance into plane of drawing

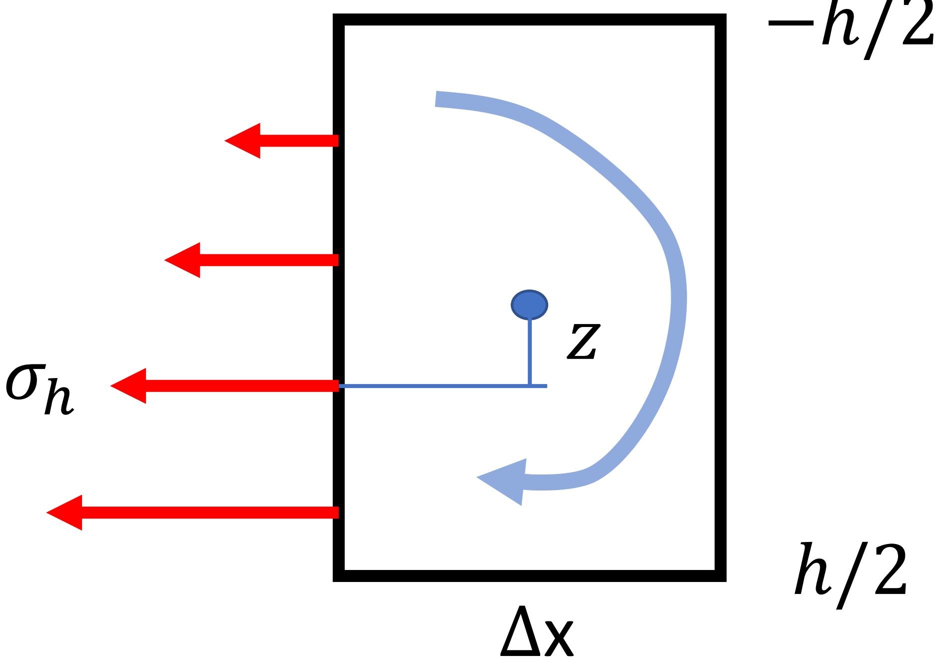


Torque: hY time average value of $-(z\sigma_n)$

$T_A(x) = -hY\,\mathrm{avg}[z\sigma_n(x)]$

but

$$\sigma_n = -Cz\frac{d^2w}{dx^2}$$



Flexural rigidity

$$D = \frac{Ch^3}{12}$$

$$T_A(x) = -hY\,\text{avg}[z\sigma_n(x)] = -ChY\frac{d^2w}{dx^2}\text{avg}\,z^2 =$$

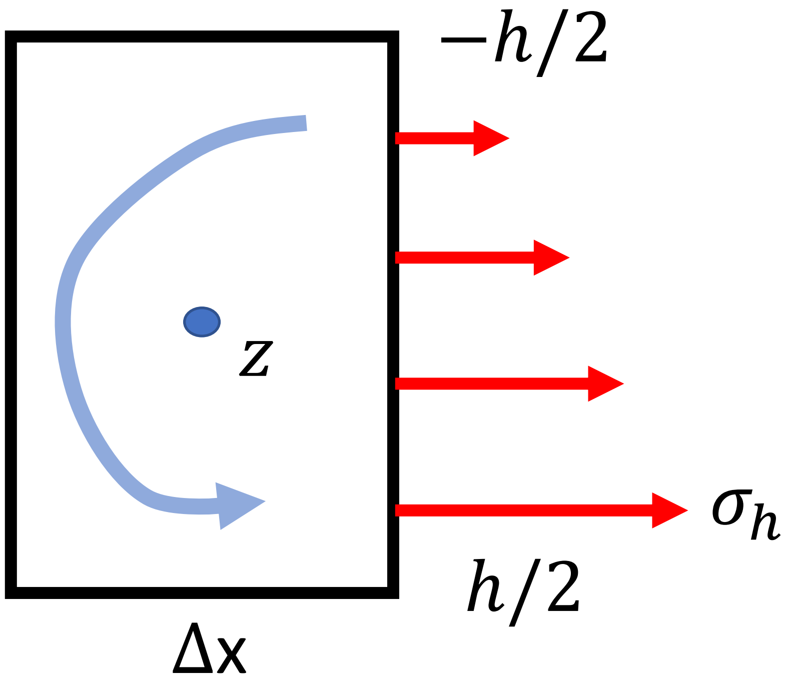
$$-\frac{Ch^3}{12}Y\frac{d^2w}{dx^2} = -DY\left[\frac{d^2w}{dx^2}\right]_x$$

You're not responsible for this, but here's how to compute the average

$$\mathrm{a}vg\; z^2 = \frac{1}{h}\int_{-h/2}^{h/2} z^2 dz = \frac{2}{h}\int_{0}^{h/2} z^2 dz = \frac{2}{3h} z^3 \Big|_0^{h/2} =$$

$$\frac{2}{3h}\left(\frac{h}{2}\right)^3 = \frac{h^2}{12}$$

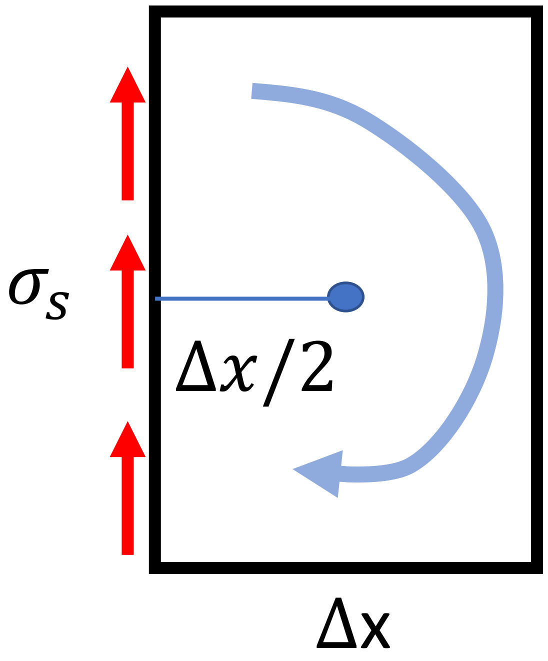
Torque T_B on right surface due to σ_h



Torque:

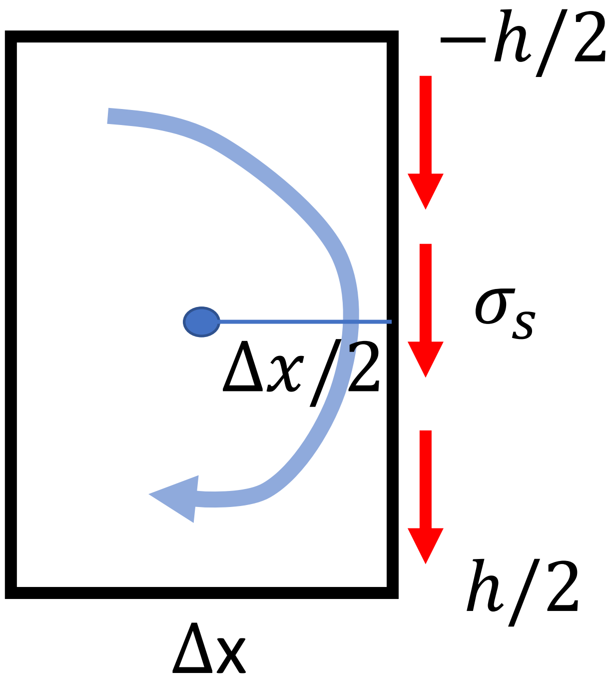
$$T_h(x) = DY\left[\frac{d^2w}{dx^2}\right]_{x+\Delta x}$$

Torque T_C on left surface from shear stress $\sigma_s(x)$



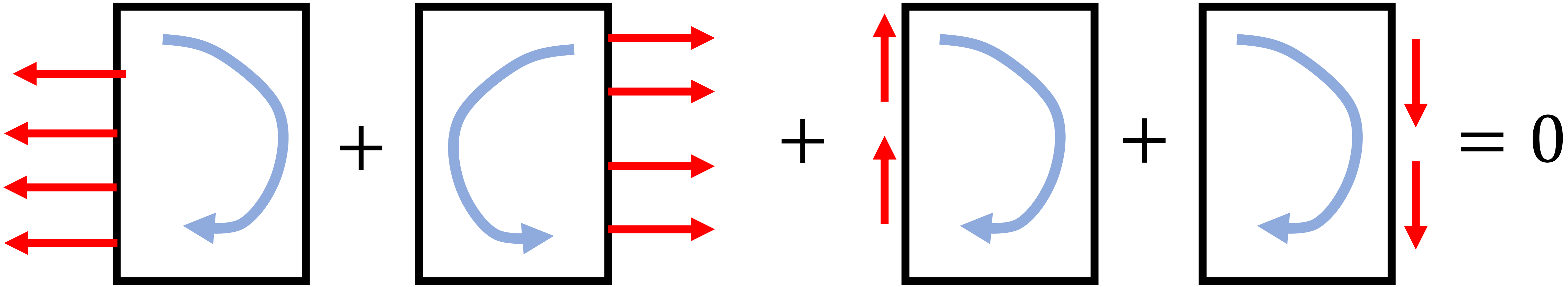
Torque: $T_C = -\frac{\Delta x}{2} hY\sigma_s(x)$

Torque T_D on right surface from shear stress $\sigma_s(x + \Delta x)$



Torque: $T_D = -\frac{\Delta x}{2} hY\sigma_s(x + \Delta x)$

Balance of Torques

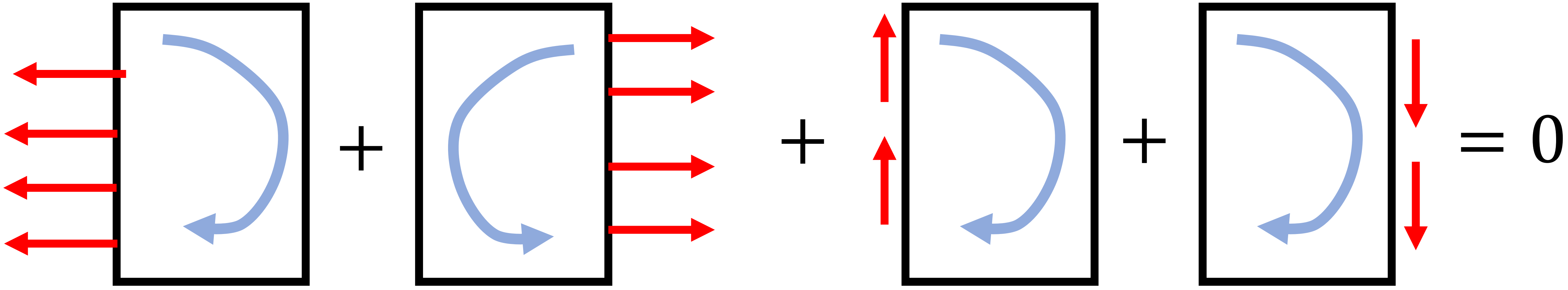


T_A T_B T_C T_D

$$-DA\left[\frac{d^2w}{dx^2}\right]_x + DA\left[\frac{d^2w}{dx^2}\right]_{x+\Delta x} - \frac{\Delta x}{2}hY\sigma_s(x) + -\frac{\Delta x}{2}hY\sigma_s(x+\Delta x) = 0$$

$$D\frac{d^3w}{dx^3} - h\sigma_s(x) = 0$$

Balance of Torques

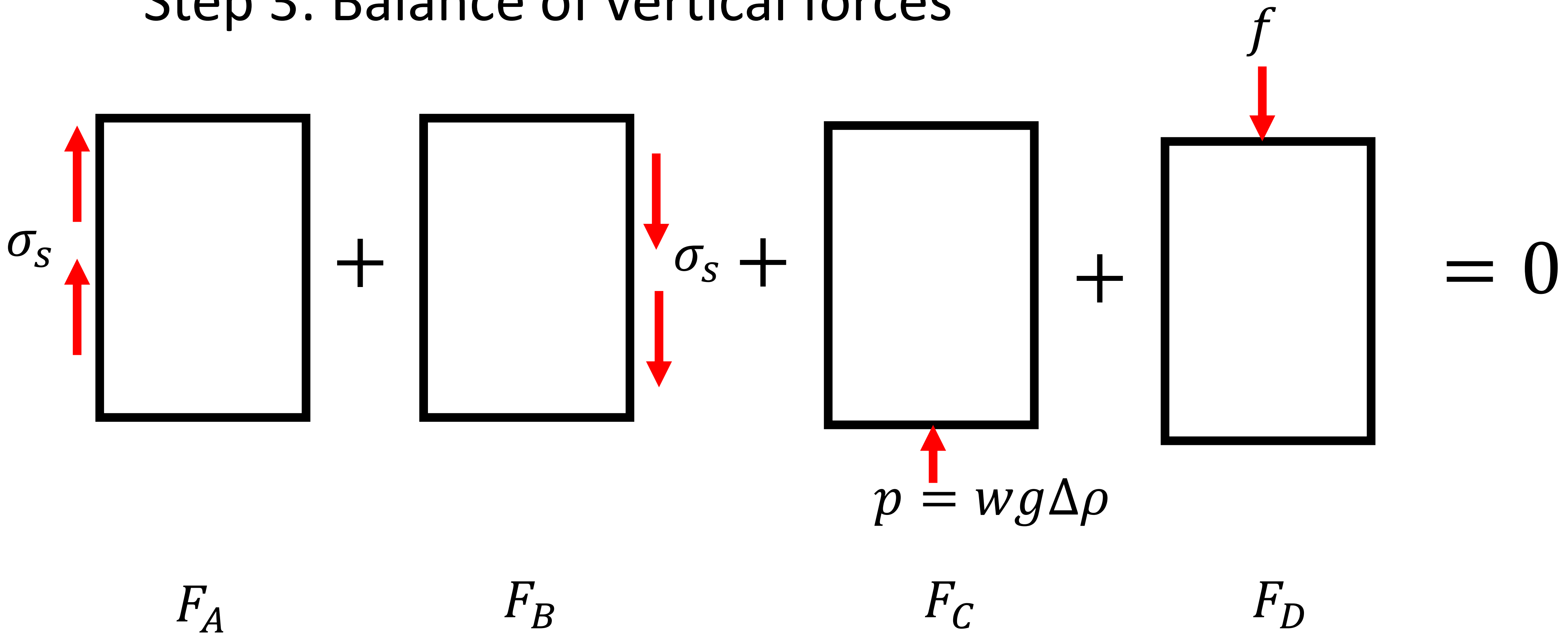


T_A T_B T_C T_D

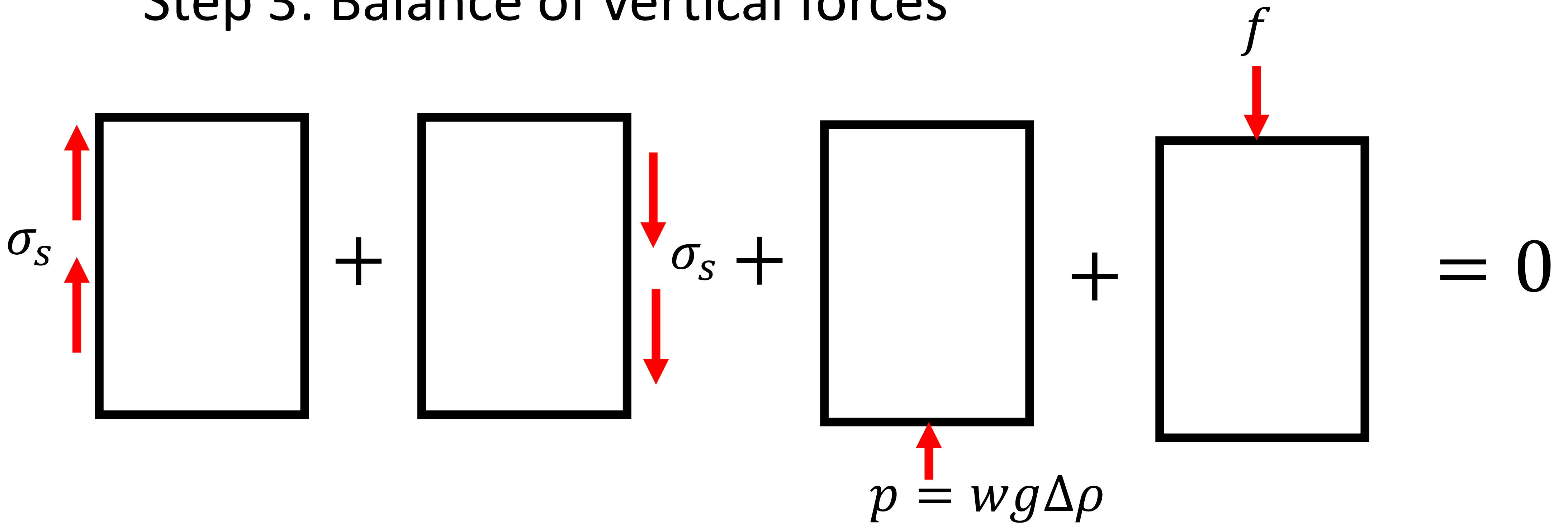
$$-DA\left[\frac{d^2w}{dx^2}\right]_x + DA\left[\frac{d^2w}{dx^2}\right]_{x+\Delta x} - \frac{\Delta x}{2} hY\sigma_s(x) + -\frac{\Delta x}{2} hY\sigma_s(x+\Delta x) = 0$$

$$D\frac{d^3w}{dx^3} - h\sigma_s(x) = 0 \Rightarrow D\frac{d^4w}{dx^4} - h\frac{d\sigma_s}{dx} = 0$$

Step 3: Balance of vertical forces

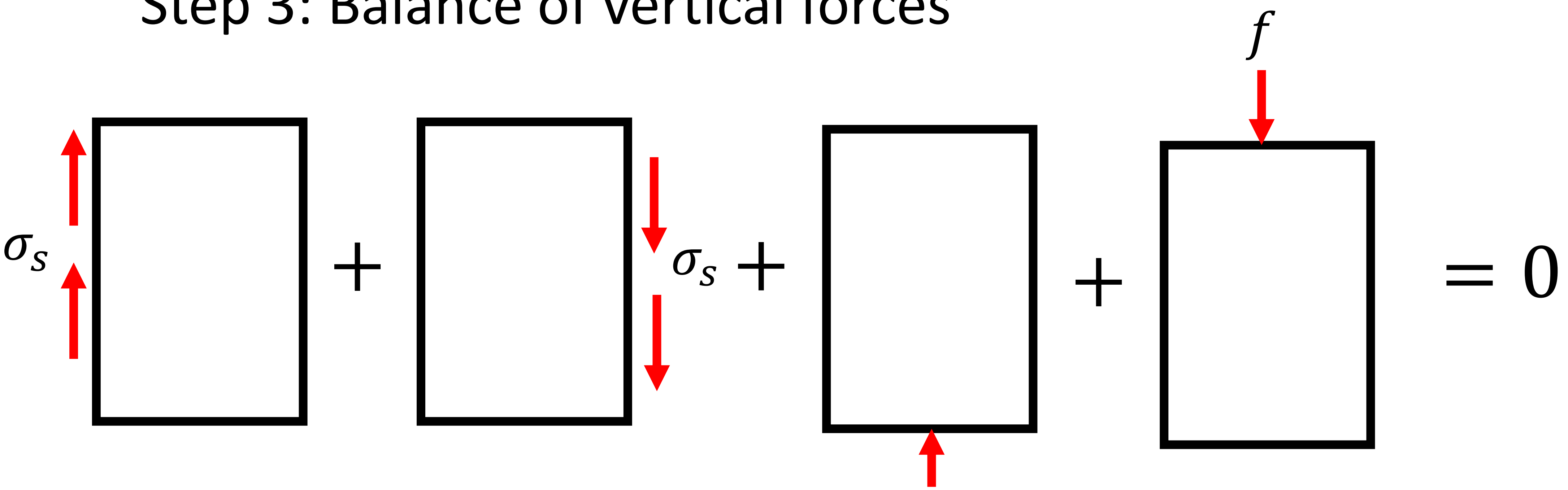


Step 3: Balance of vertical forces



F_A	F_B	F_C	F_D
$-hY\sigma_s(x)$	$hY\sigma_s(x + \Delta x)$	$-\Delta x Y w g \Delta\rho$	$\Delta x Y f$

Step 3: Balance of vertical forces



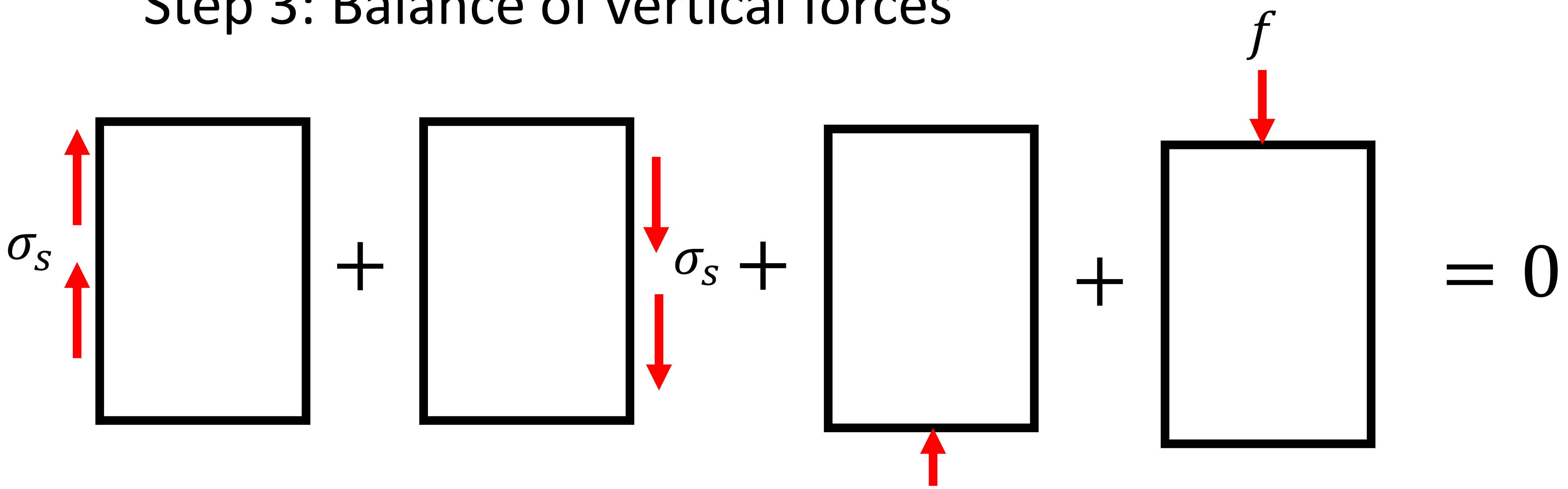
$$p = -wg\Delta\rho$$

F_A	F_B	F_C	F_D	
$-hY\sigma_s(x)$	$+hY\sigma_s(x+\Delta x)$	$-\Delta x Y wg\Delta\rho$	$+\Delta x Y f$	$= 0$

$$h\frac{d\sigma_s}{dx} + wg\Delta\rho + f = 0$$

$$h\frac{d\sigma_s}{dx} = +wg\Delta\rho - f$$

Step 3: Balance of vertical forces



$$p = -wg\Delta\rho$$

$$h\frac{d\sigma_s}{dx} = +wg\Delta\rho - f$$

$$D\frac{d^4w}{dx^4} - h\frac{d\sigma_s}{dx} = 0$$

$$D\frac{d^4w}{dx^4} + g\Delta\rho\, w = f$$

Equation of flexure

Equation of flexure

$$D\frac{d^4w}{dx^4} + g\Delta\rho\, w = f$$

Solution for f=0

$$w(x) = A\cos\left(\frac{2\pi}{\lambda}x\right)\exp\left(-\frac{2\pi}{\lambda}x\right)$$

Flexural wavelength $$\lambda = \frac{1}{2\pi}\left(\frac{D}{g\Delta\rho}\right)^{1/4}$$

Flexural wavelength

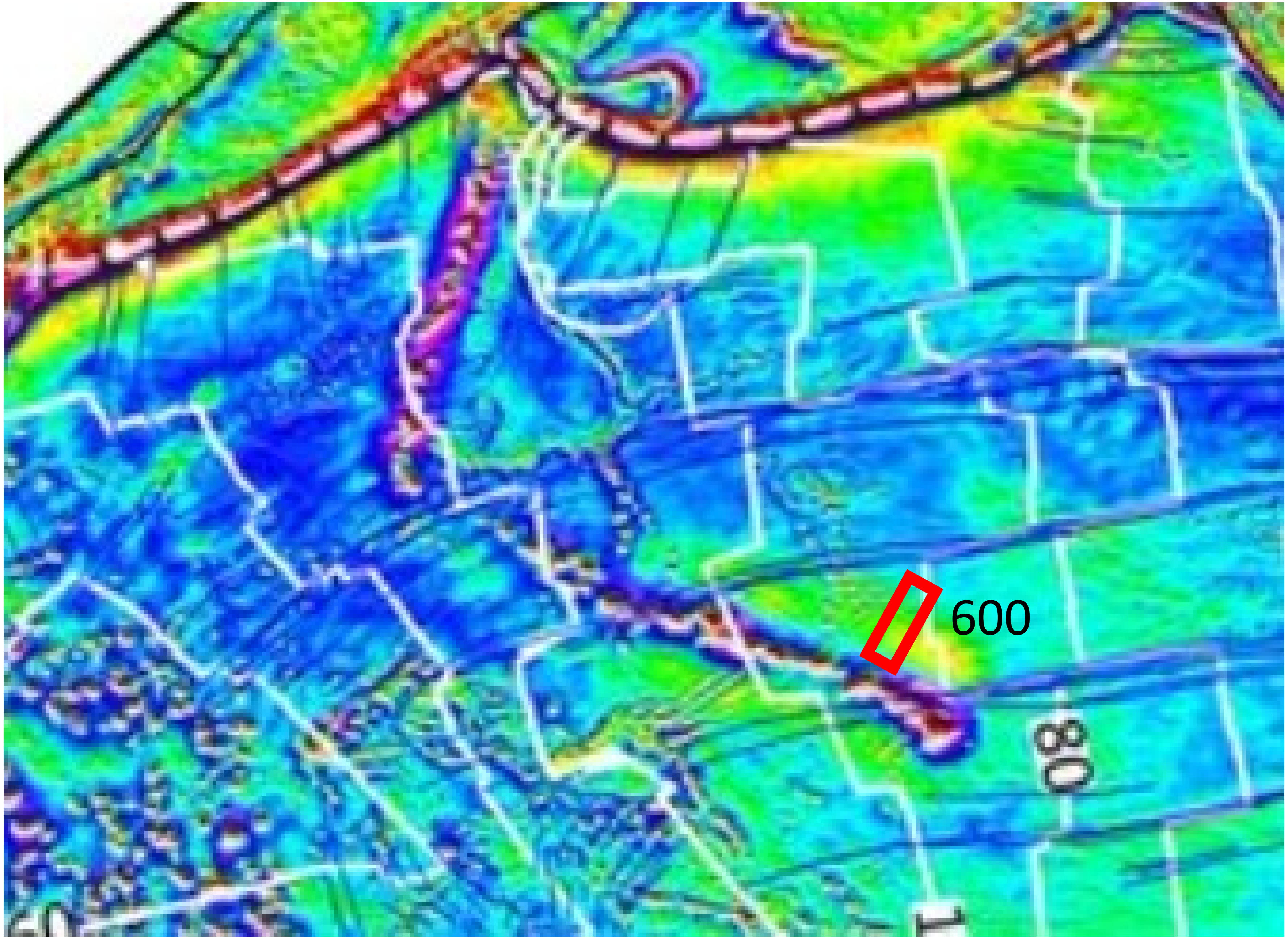
$$\lambda = \frac{1}{2\pi}\left(\frac{D}{g\Delta\rho}\right)^{\frac{1}{4}} = \frac{1}{2\pi}\left(\frac{Ch^3}{12g\Delta\rho}\right)^{\frac{1}{4}}$$

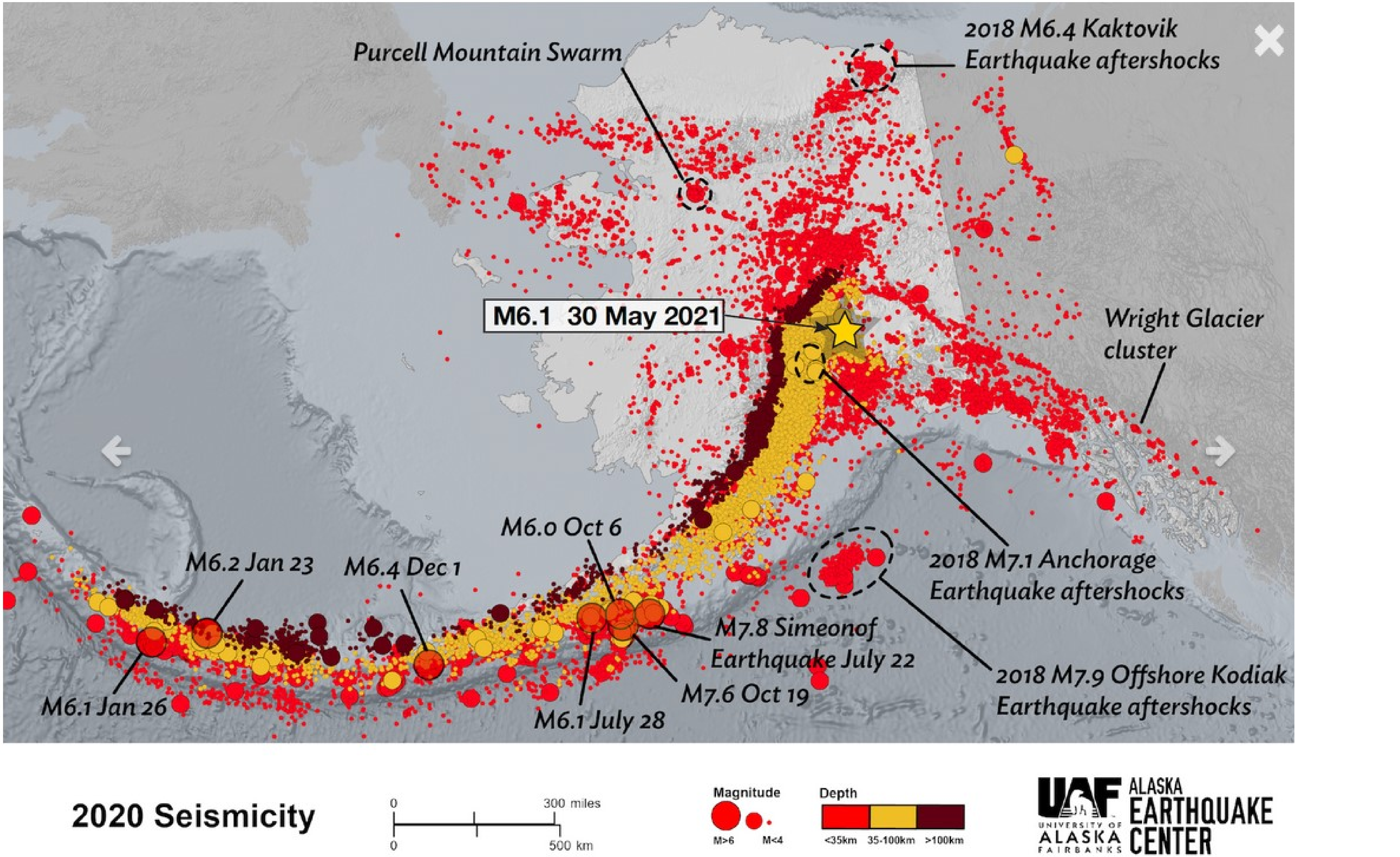
typical $\lambda = 600\ km$

$$D = \frac{Ch^3}{12}$$

$$h = \left(\frac{24\pi g\Delta\rho}{C}\right)^{1/3} \lambda^{4/3}$$

use measurements of λ
to determine thickness h
of lithosphere


600



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