Solid Earth Dynamics

Bill Menke, Instructor

Lecture 14

Midterm

In class, open book/notes

choose any 2 of 3 questions

“scenario” essay questions
focused at broad geodynamical questions

no “calculation”
but answers should involve quantitative thinking

Solid Earth Dynamics

Tides

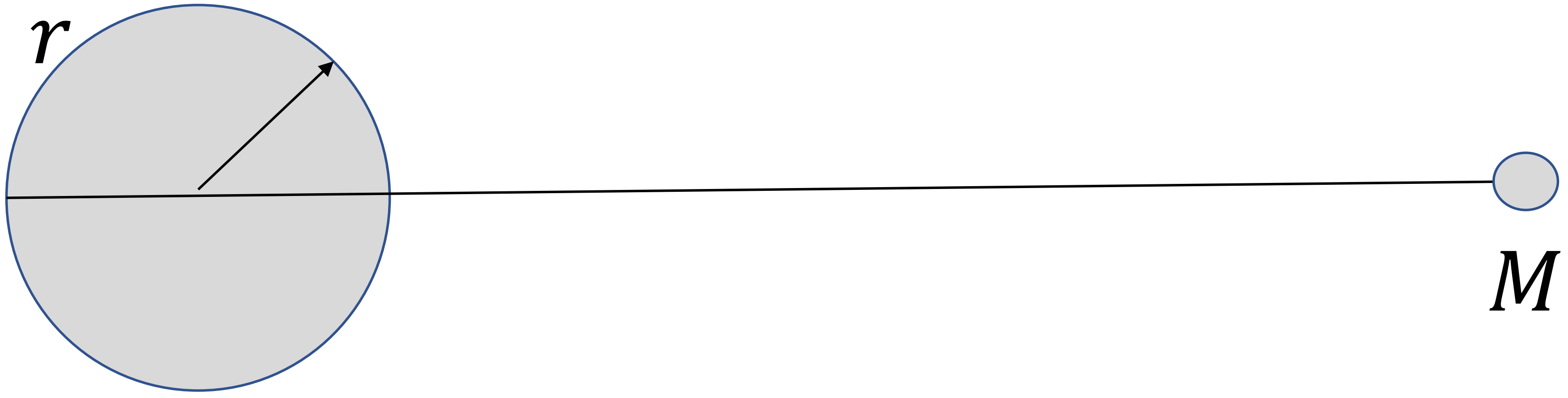
The Tide Rises, the Tide Falls
By Henry Wadsworth Longfellow

The tide rises, the tide falls,
The twilight darkens, the curlew calls;
Along the sea-sands damp and brown
The traveller hastens toward the town,
    And the tide rises, the tide falls.

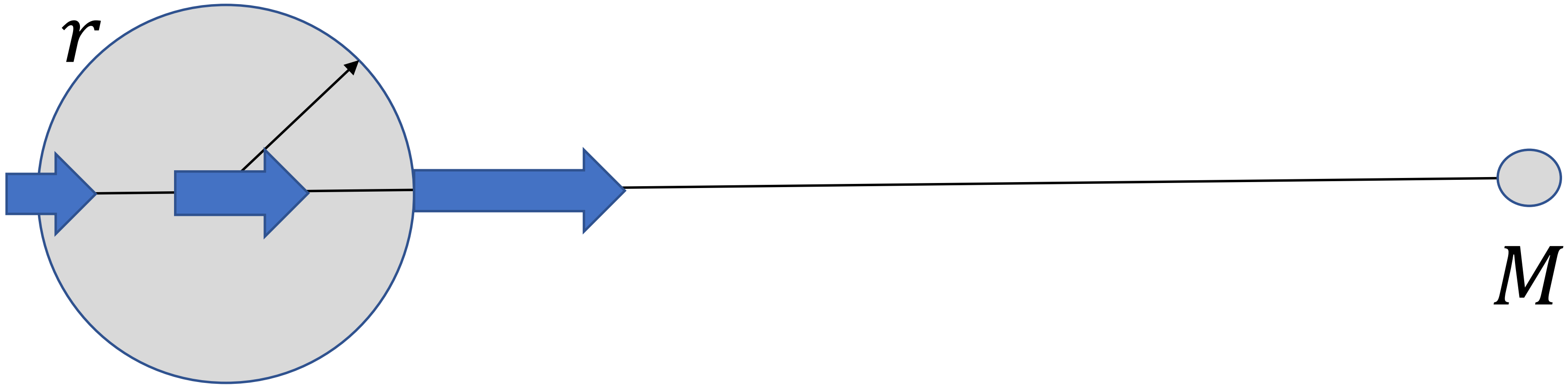
Darkness settles on roofs and walls,
But the sea, the sea in the darkness calls;
The little waves, with their soft, white hands,
Efface the footprints in the sands,
    And the tide rises, the tide falls.

The morning breaks; the steeds in their stalls
Stamp and neigh, as the hostler calls;
The day returns, but nevermore
eturns the traveller to the shore,

    And the tide rises, the tide falls.



$$\frac{\gamma M}{(R+r)^2} \qquad \frac{\gamma M}{R^2} \qquad \frac{\gamma M}{(R-r)^2}$$

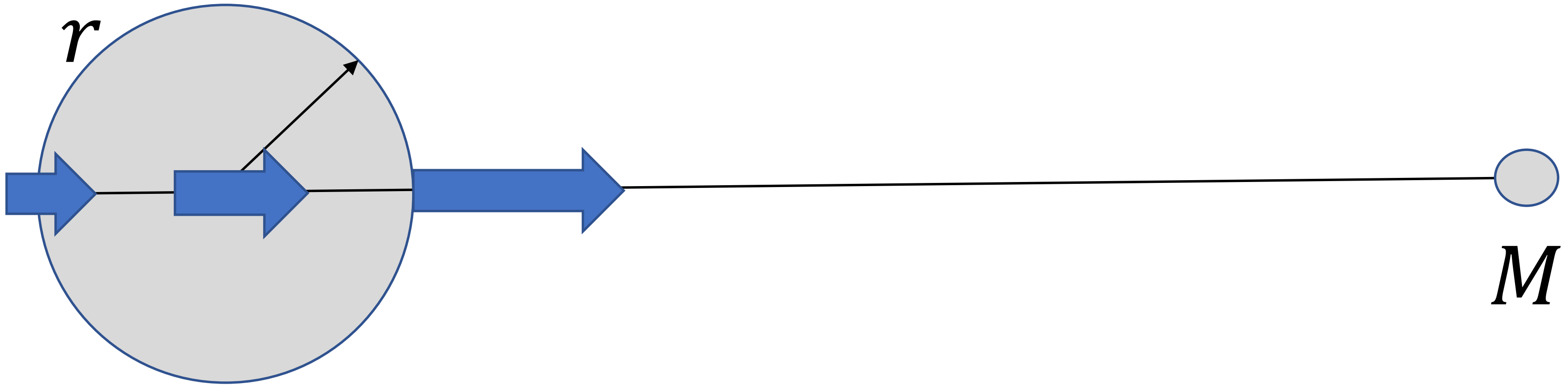


$$\frac{\gamma M}{(R+r)^2} \qquad \frac{\gamma M}{R^2} \qquad \frac{\gamma M}{(R-r)^2}$$

$$f_L = \gamma M(R+r)^{-2} = \frac{\gamma M}{R^2}\left(1+\frac{r}{R}\right)^{-2} = \frac{\gamma M}{R^2}\left(1-2\frac{r}{R}\right) = \frac{\gamma M}{R^2} - 2\frac{\gamma M r}{R^3}$$

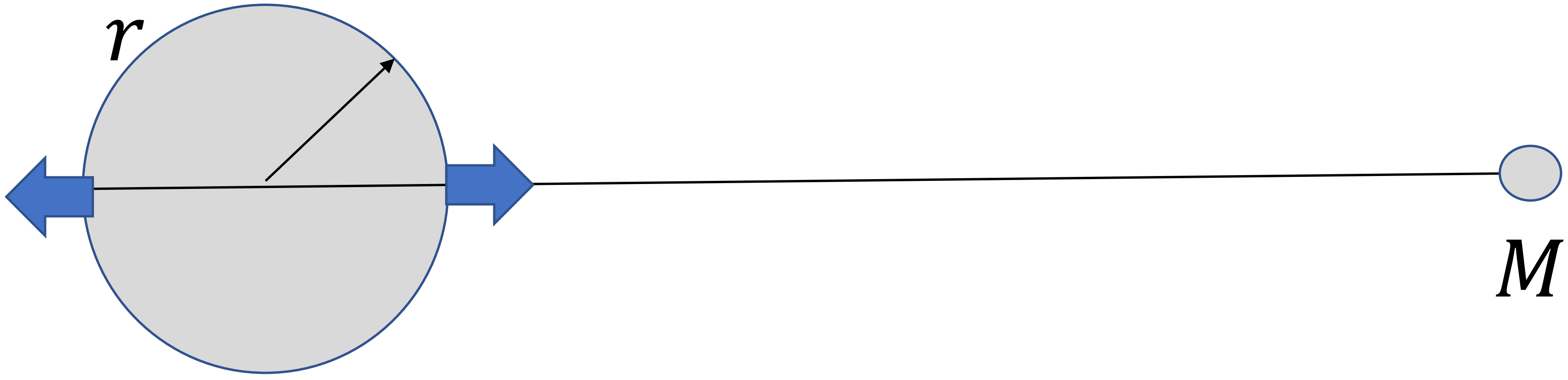
$$f_R = \gamma M(R-r)^{-2} = \frac{\gamma M}{R^2}\left(1-\frac{r}{R}\right)^{-2} = \frac{\gamma M}{R^2}\left(1+2\frac{r}{R}\right) = \frac{\gamma M}{R^2} + 2\frac{\gamma M r}{R^3}$$

$$\Delta f = f_R - f_L = \frac{4\gamma M r}{R^3}$$



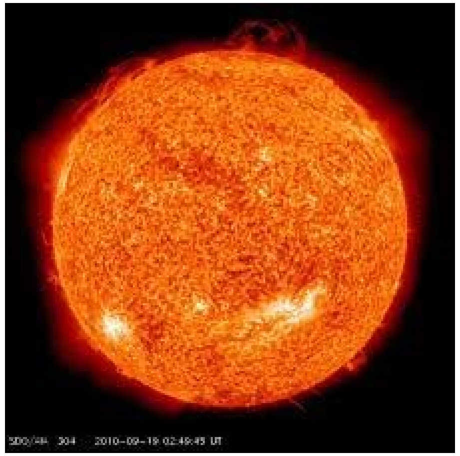
$$\frac{\gamma M}{(R+r)^2} \qquad \frac{\gamma M}{R^2} \qquad \frac{\gamma M}{(R-r)^2}$$

relative to force acting at the the center of the earth

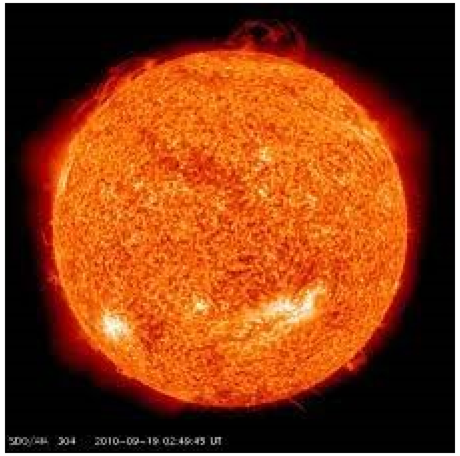


$$-\frac{2\gamma M r}{R^3} \qquad \frac{2\gamma M r}{R^3}$$

1	Mearth km	5.97E+24
2	rEarth m	6.37E+06
3	gamma	6.70E-11
4		
5	Mmoon kg	7.40E+22
6	Rmoon m	3.83E+08
7		
8	Msun kg	2.00E+30
9	Rsun m	1.48E+11
10		
11	Df moon	2.26E-06
12	DF sun	1.06E-06
13		



1	Mearth km	5.97E+24
2	rEarth m	6.37E+06
3	gamma	6.70E-11
4		
5	Mmoon kg	7.40E+22
6	Rmoon m	3.83E+08
7		
8	Msun kg	2.00E+30
9	Rsun m	1.48E+11
10		
11	Df moon	2.26E-06
12	DF sun	1.06E-06
13		

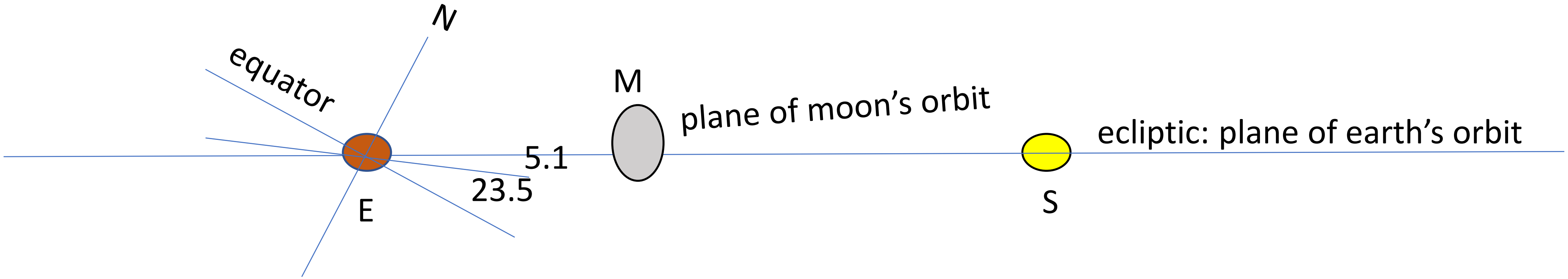


small compared
tooverall 9.8 m/s^2 gravity


Earth
Moon

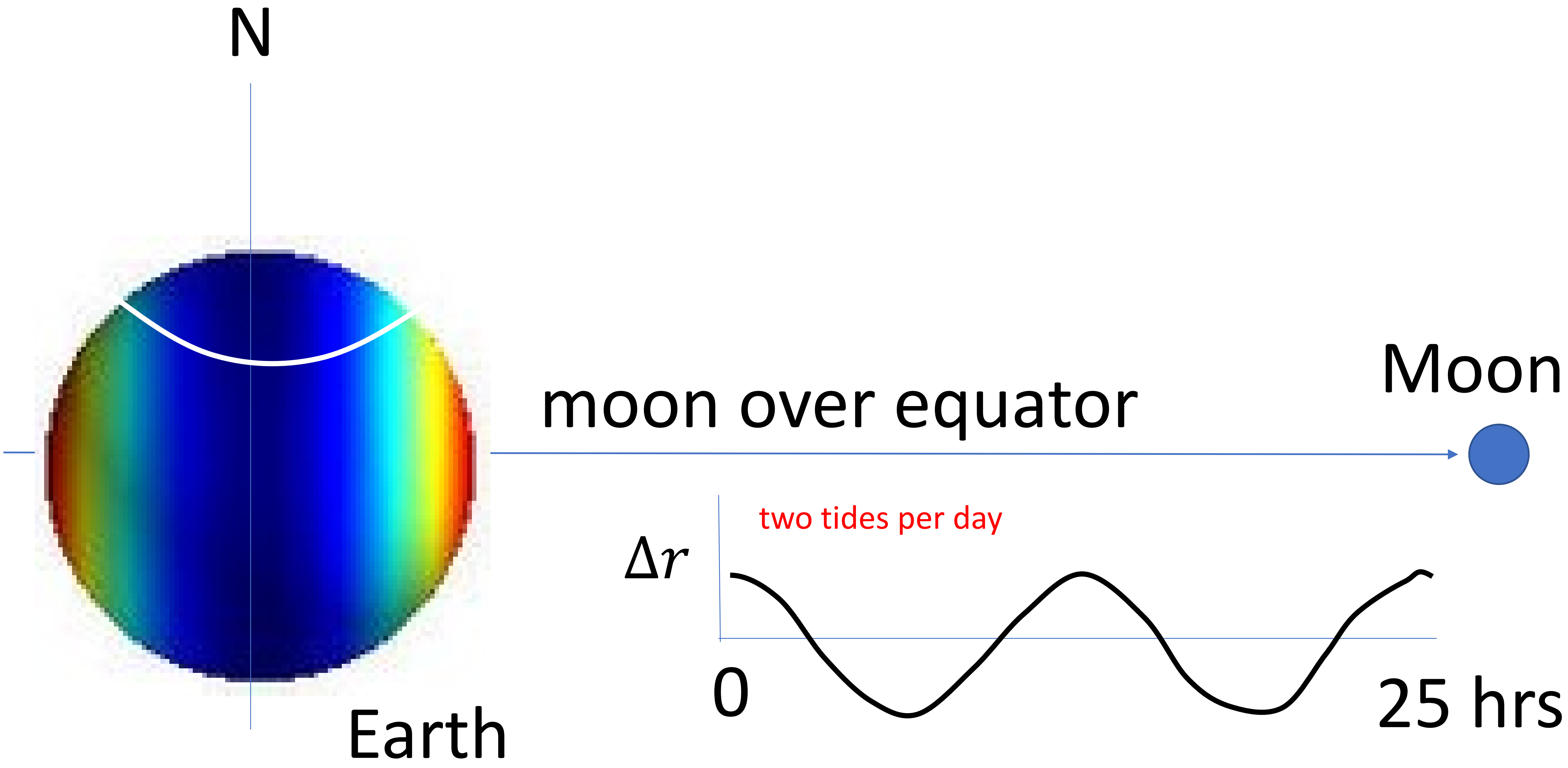
Where's the Earth's spin axis?

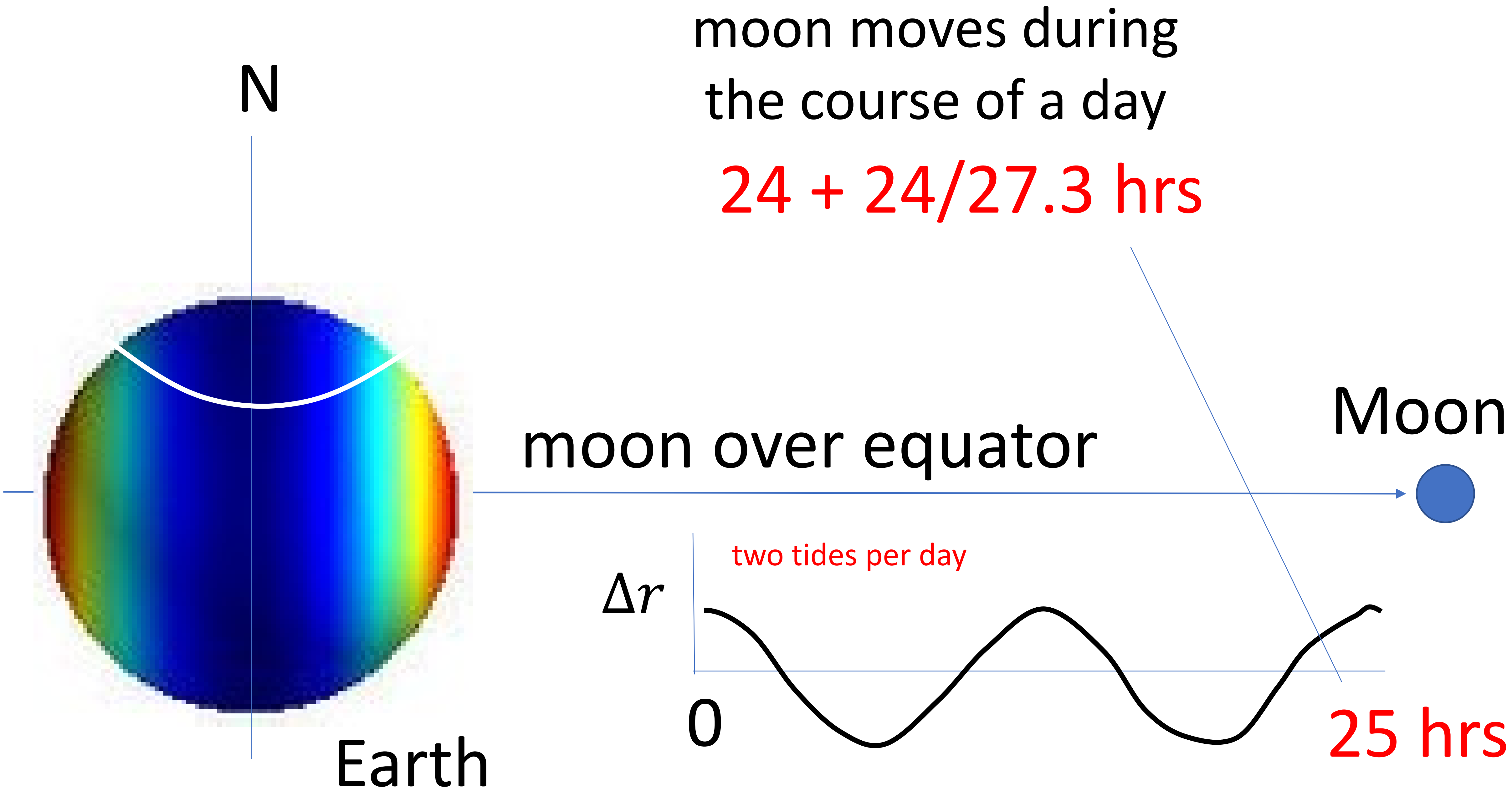
but moon's orbit precesses with period of 18.6 years



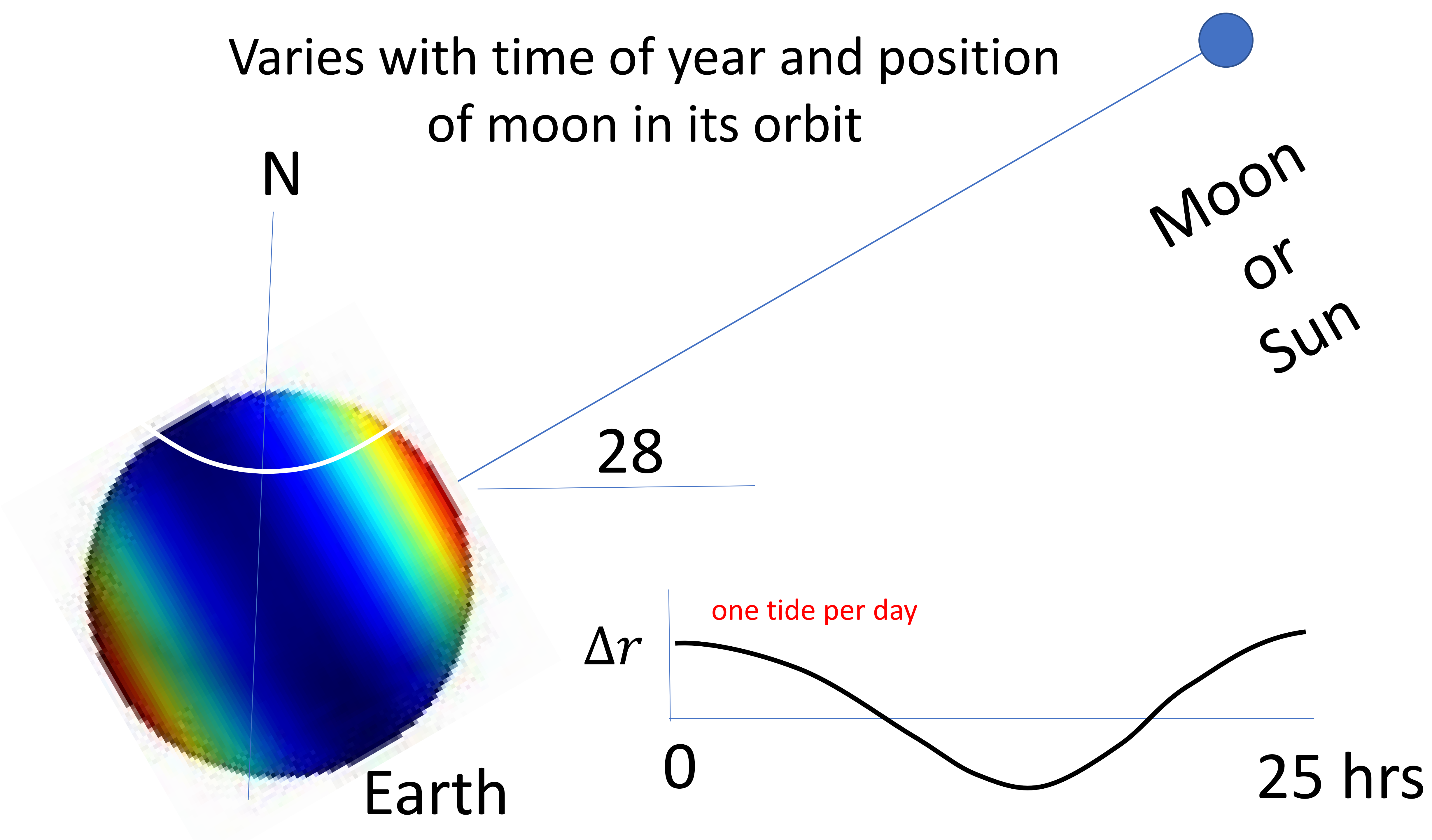
moon 18.4 degrees above the equator

Varies with time of year and position of moon in its orbit



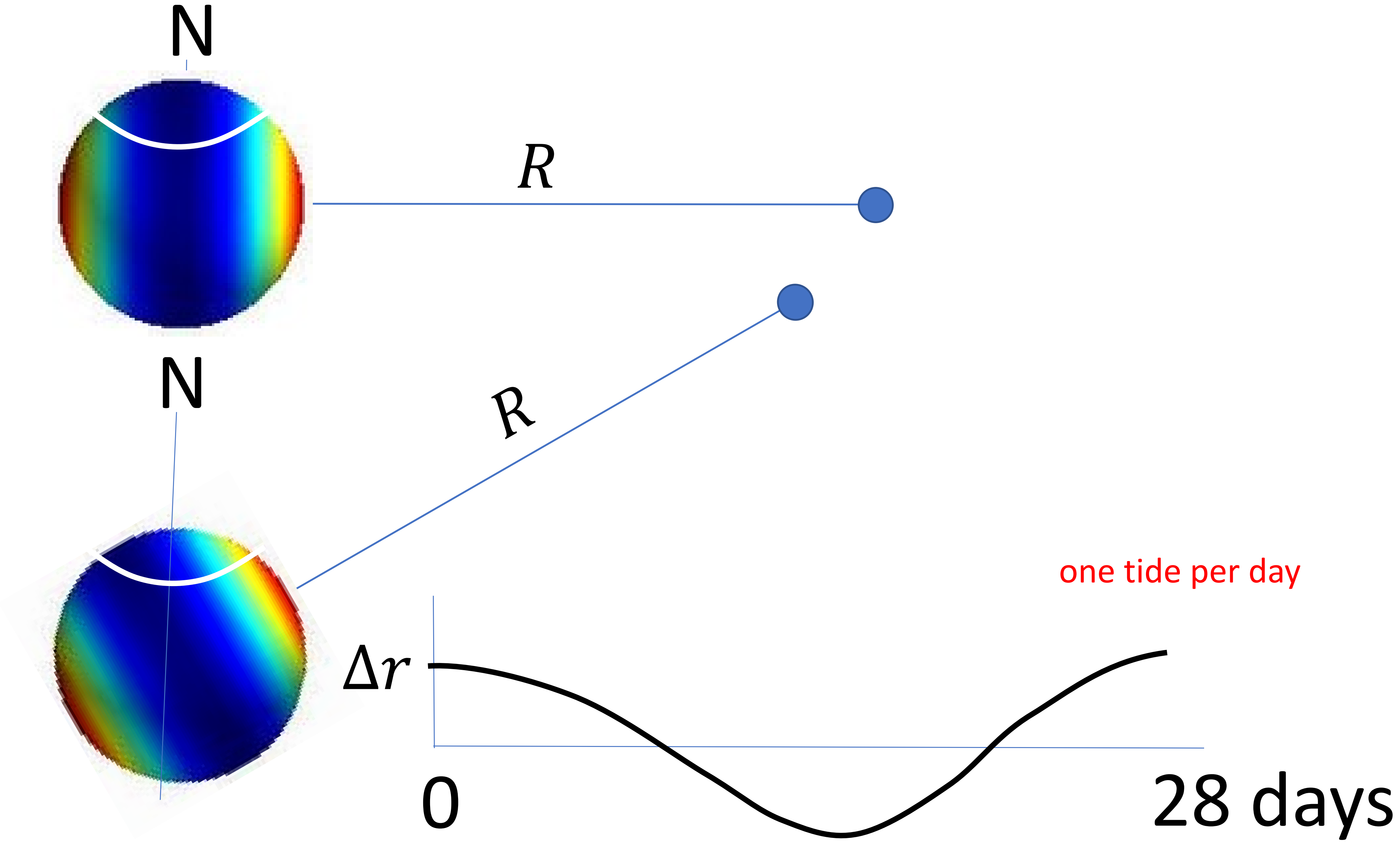


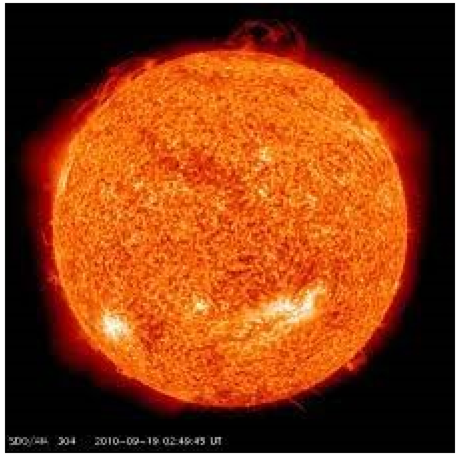
moon moves during
the course of a day
24 + 24/27.3 hrs
N
Moon
moon over equator
two tides per day
Δr
0
25 hrs
Earth



Varies with time of year and position of moon in its orbit
N
Moon or Sun
28
one tide per day
Δr
0
25 hrs
Earth

Monthly modulation





But what about the sun

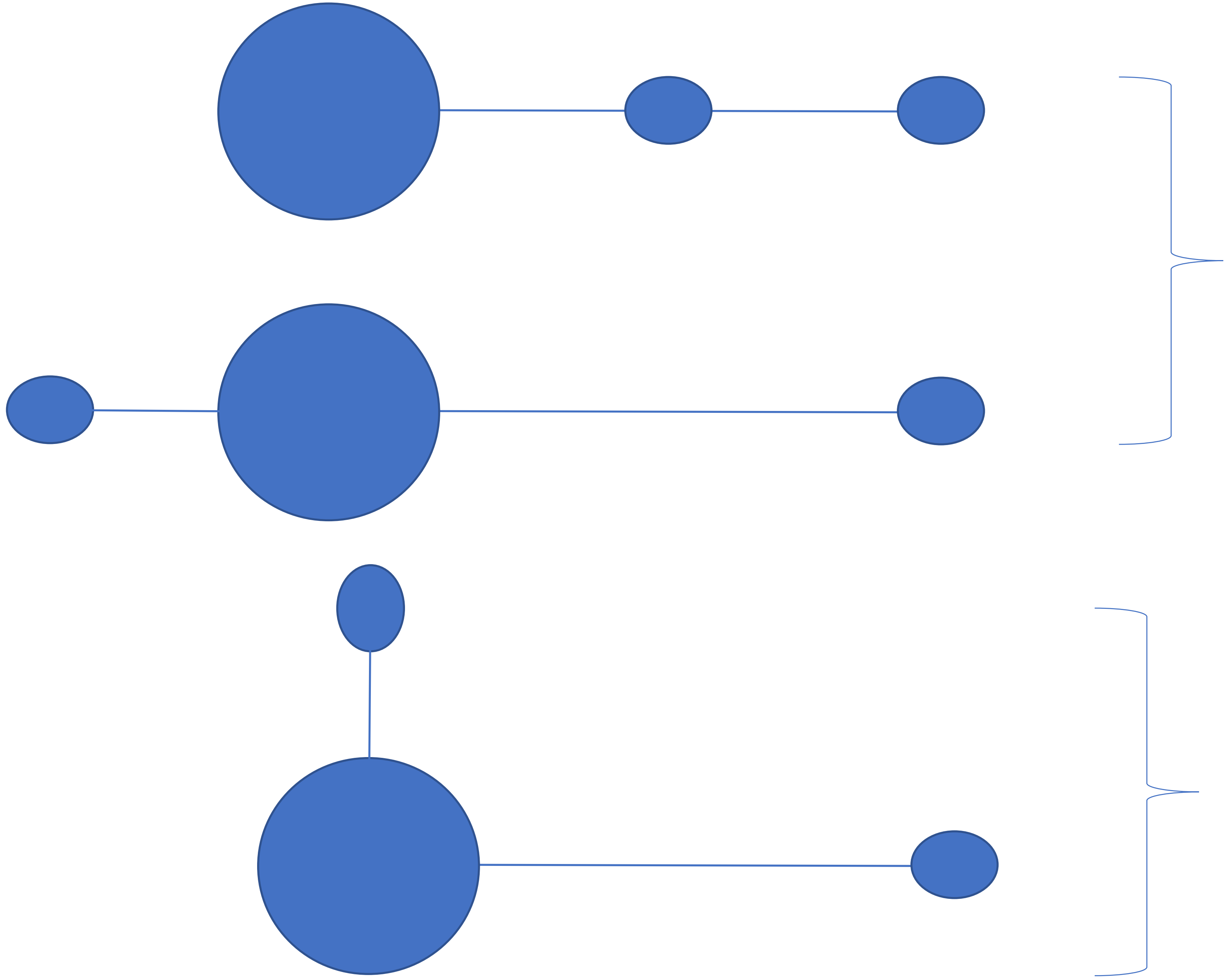
same tidal patterns,
but periods a little different

daily, semi-daily tides

24+24/365 hours

annual tides

I year, not one month

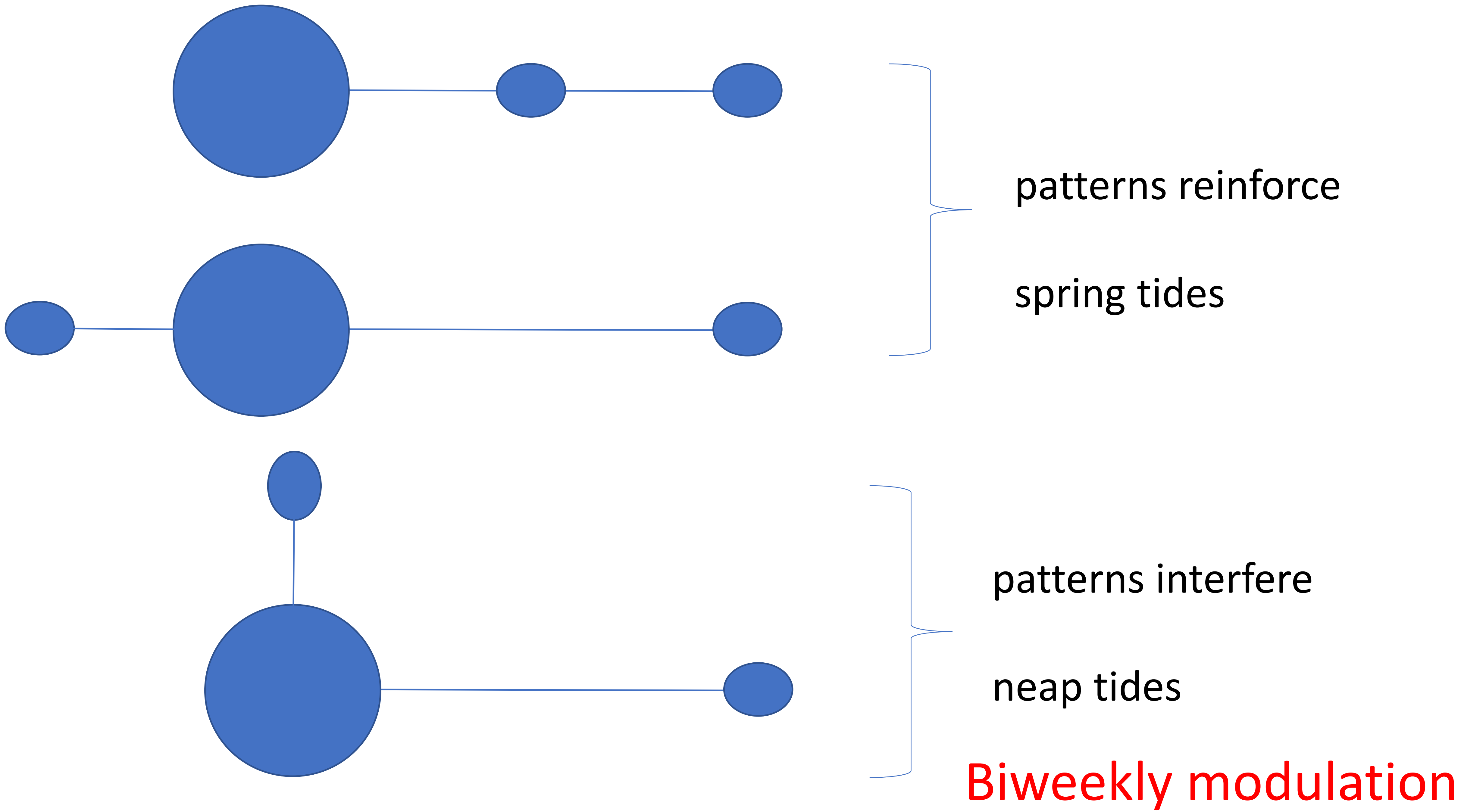


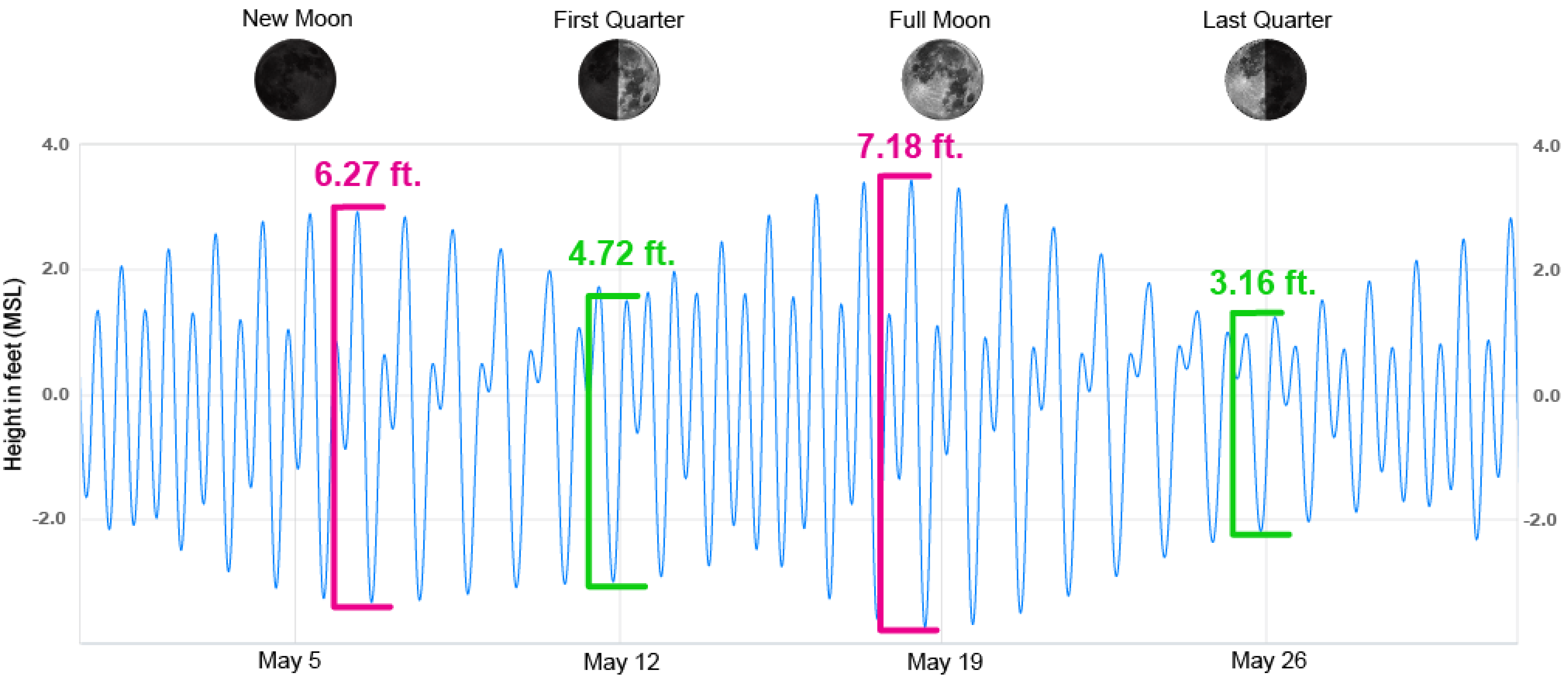
patterns reinforce

spring tides

patterns interfere

neap tides


patterns reinforce
spring tides
patterns interfere
neap tides
Biweekly modulation


New Moon
First Quarter
Full Moon
Last Quarter
6.27 ft.
4.72 ft.
7.18 ft.
3.16 ft.
Height in feet (MSL)
4.0
2.0
0.0
-2.0
May 5
May 12
May 19
May 26

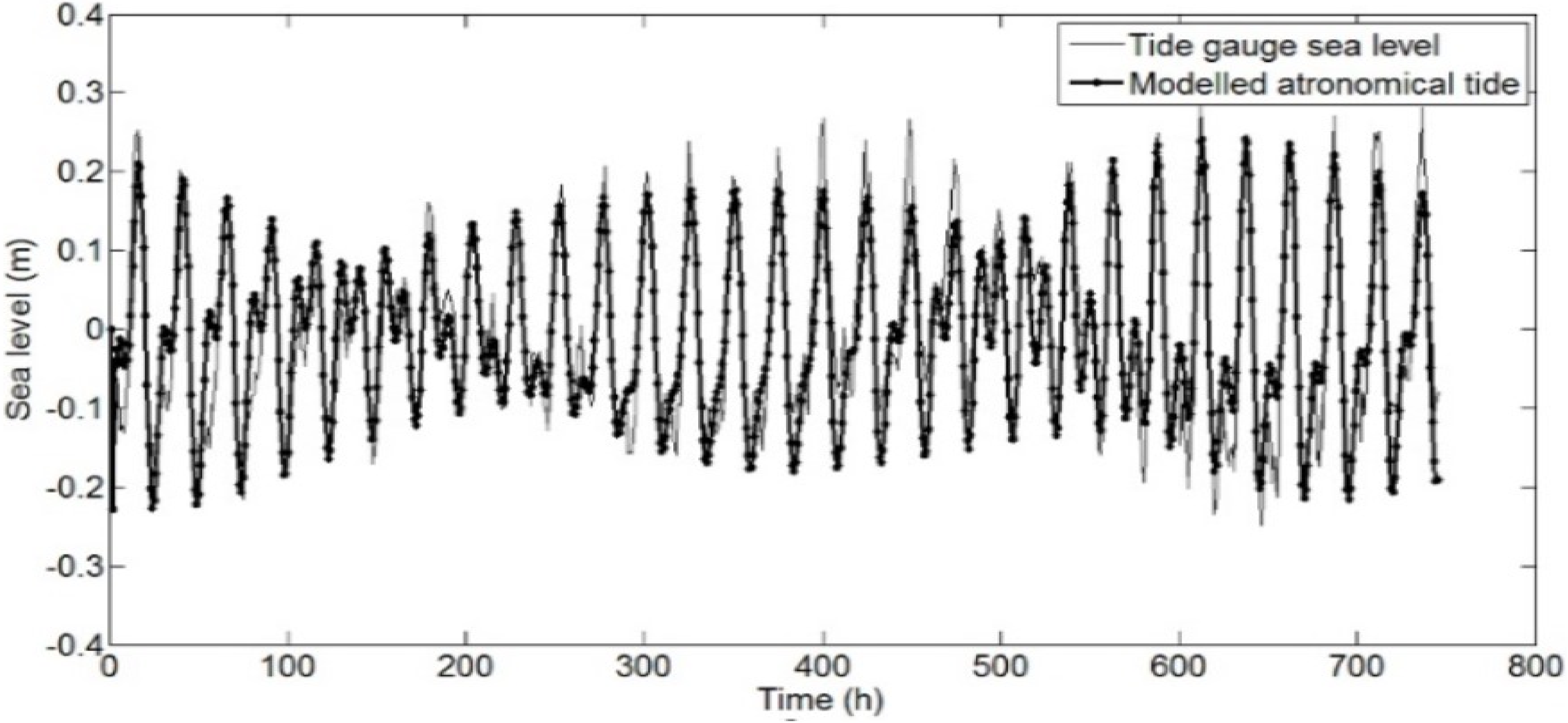


Figure 2. Comparison of modeled astronomical tide vs. sea level recorded by the tidal gauge in the Bay of Cartagena.
Source: The authors.

Tides

Sea surface

surface of equal potential energy

Agenda
just like equatorial bulge of earth

equatorial bulge

earth’s gravity

plus

centrifugal force

tides

earth’s gravity

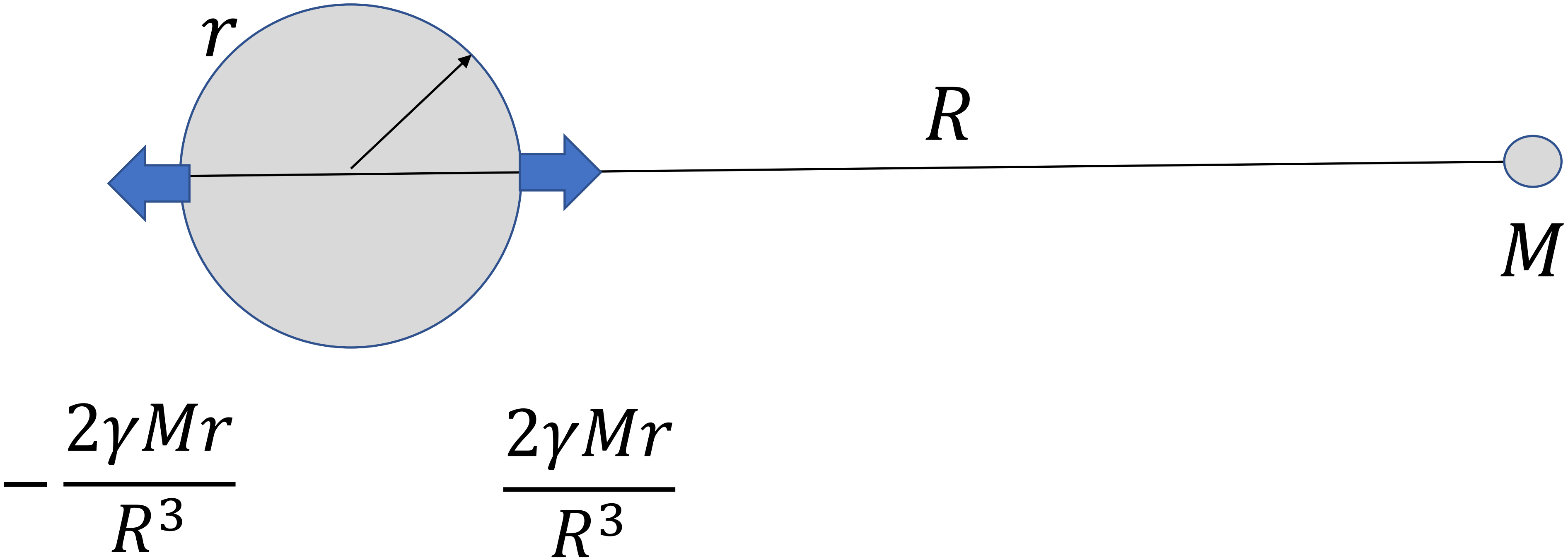
plus

moon’s (or sun’s) gravity

equatorial bulge
earth’s surface
is a surface
of equal potential energy

tides
sea surface
is a surface
of equal potential energy

relative to force acting at the the center of the earth



tricky part

surface of equal potential energy of the moon/sun

minus

potential of moon/sun acting on Earth as a whole
(as if all the Earth was at the position of its center)

(since only the difference makes tides)

Four part agenda

1. Energy of Moon as if all of earth was concentrated at its center

2. Energy of Moon

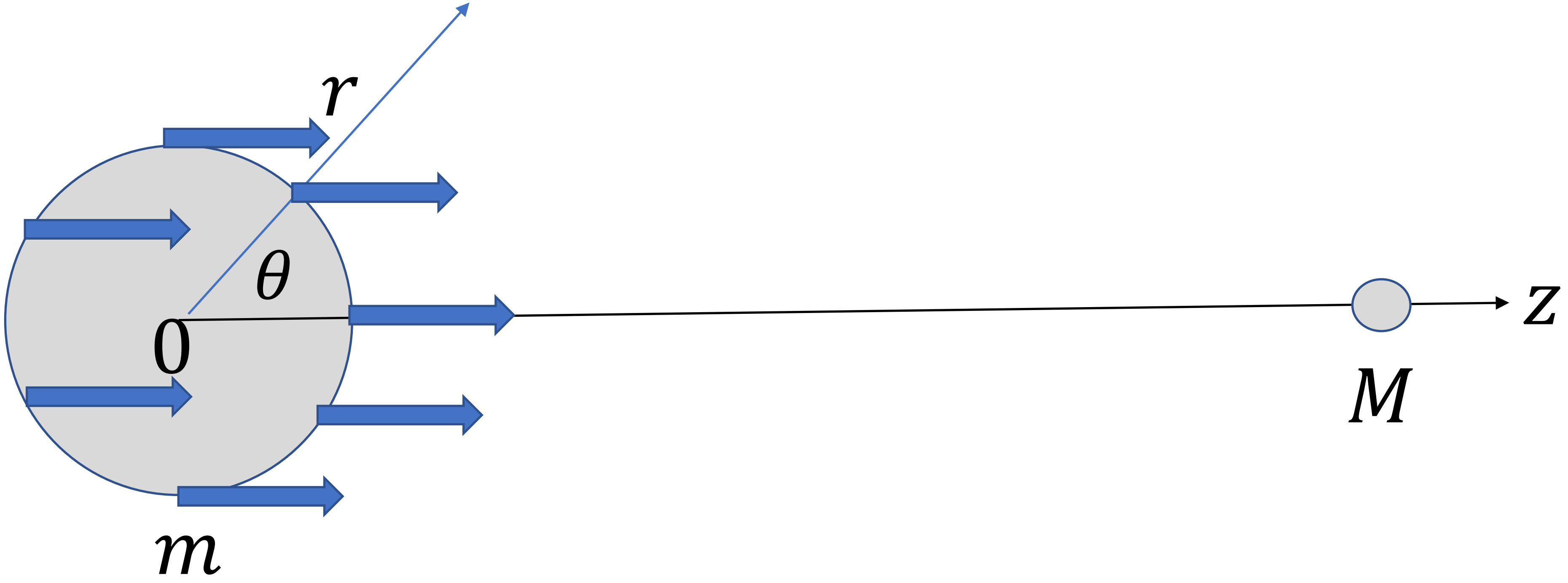
3. Energy of Earth

4. Infer formula for tides from sum

Part 1 energy of moon/sun acting on Earth as a whole

must vary with z, not r

derivative must give constant force $\frac{-\gamma M}{R^2}$

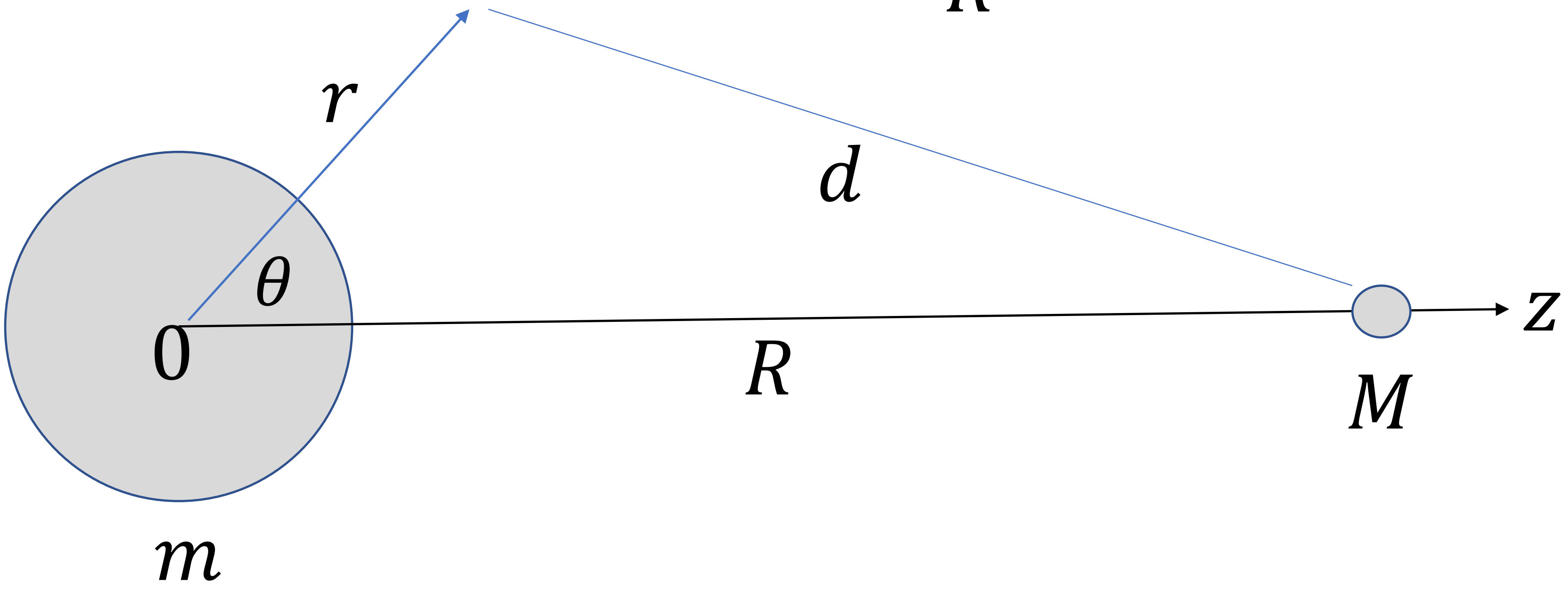


Part 1 energy of moon/sun acting on Earth as a whole

must vary with z, not r

so that direction of force is always parallel to z

derivative must give constant force $\frac{-\gamma M}{R^2}$

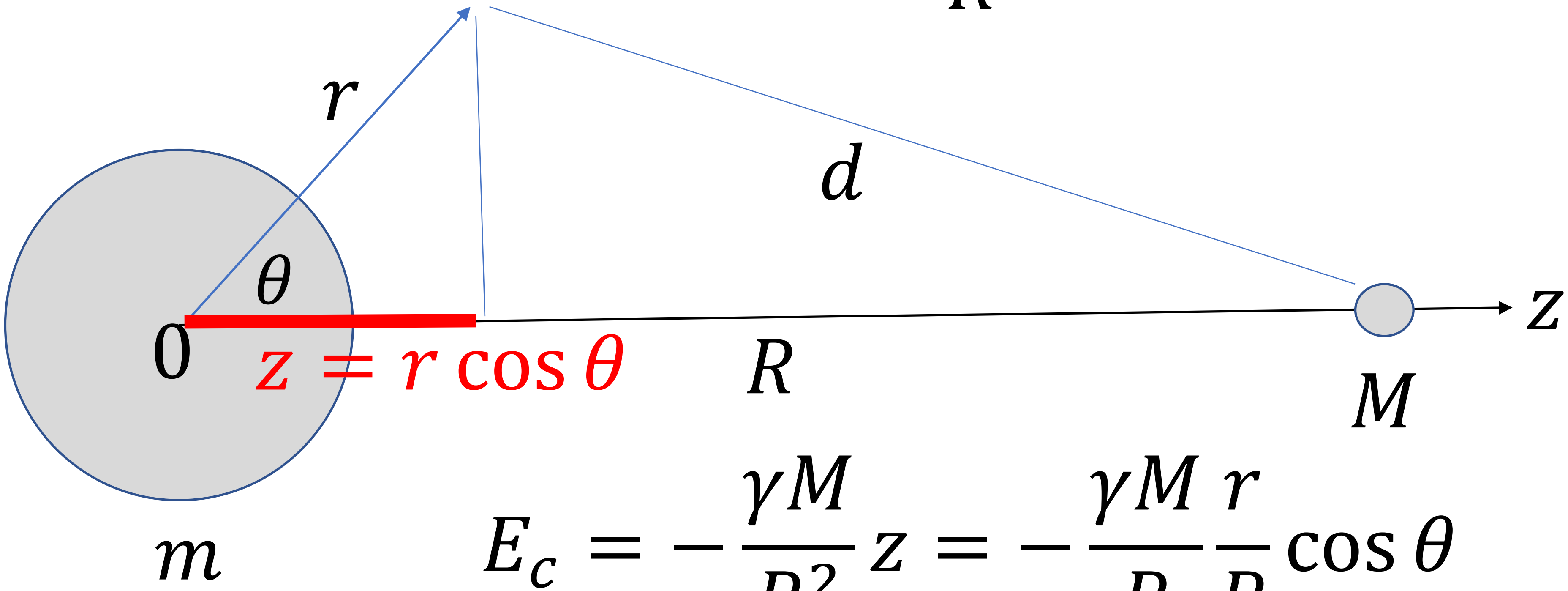


Part 1 energy of moon/sun acting on Earth as a whole

must vary with z, not r

so that direction of force is always parallel to z

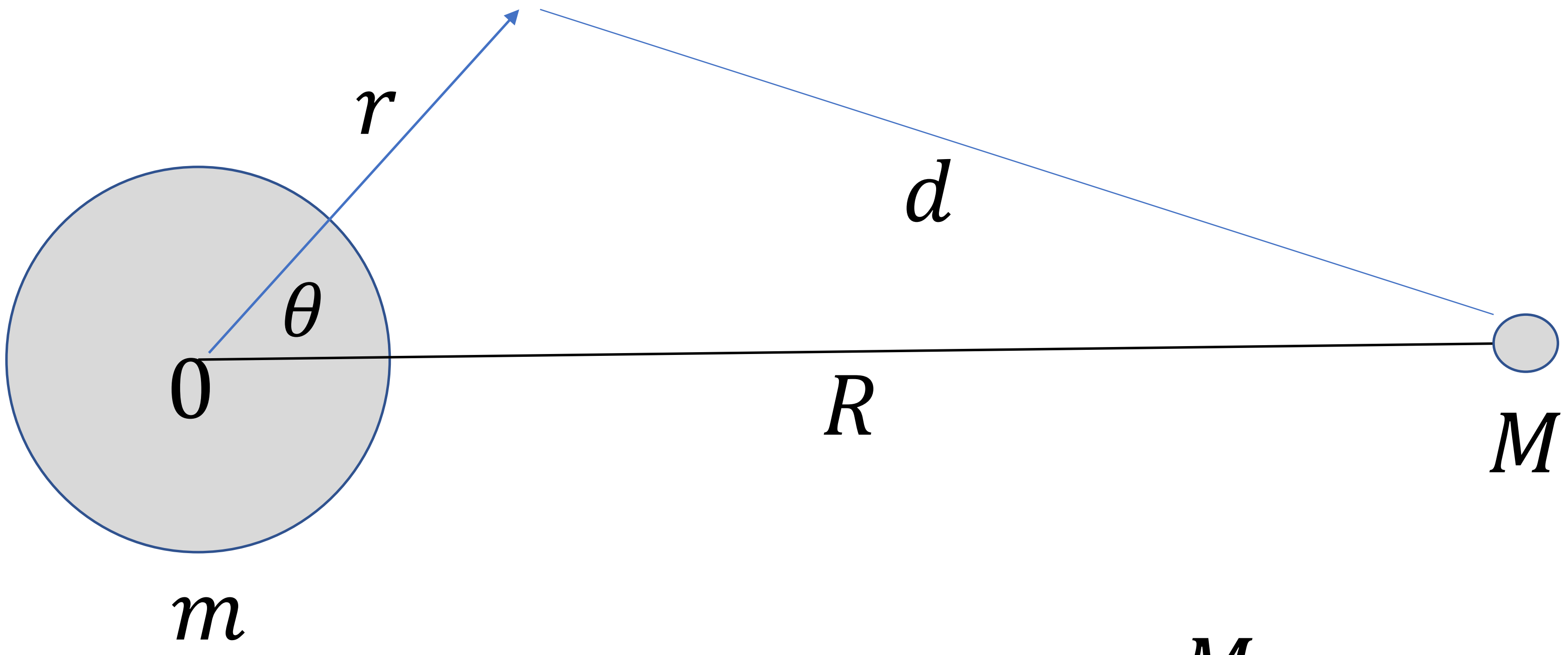
derivative must give constant force $\frac{-\gamma M}{R^2}$



$$E_c = -\frac{\gamma M}{R^2} z = -\frac{\gamma M}{R}\frac{r}{R}\cos\theta$$

Part 2: energy E_M of the moon

released taking unit mass from indefinitely far away to a position r, θ



$$\frac{-\gamma M}{d}$$

Part 2:

So “all” we need to is to take a formula

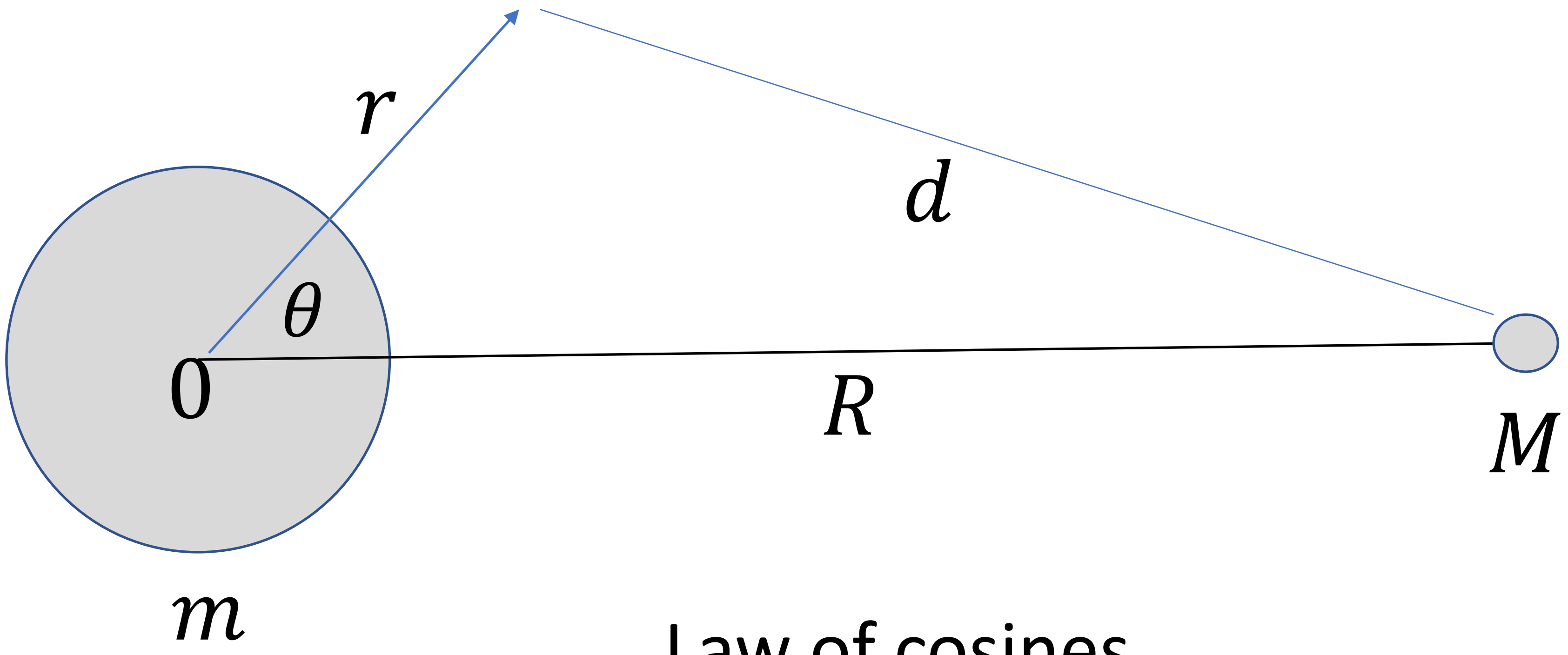
$$\frac{-\gamma M}{d}$$

that is written in terms of a coordinate system centered on the moon

and re-write it in terms of coordinate system centered on the earth

Part 2

need to write d as a function of r, θ



Law of cosines

$$d^2 = R^2 + r^2 - 2rR\cos\theta$$

Part 2

Law of cosines

$$d^{-1} = [R^2 + r^2 - 2rR\cos\theta]^{-1/2}$$

$$= R^{-1}\left[1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\theta\right]^{-1/2}$$

Part 2

Law of cosines

$$d^{-1} = [R^2 + r^2 - 2rR\cos\theta]^{-1/2}$$

$$= R^{-1}\left[1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\theta\right]^{-1/2}$$

$$[1 + x]^{-1/2} \approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 \cdots$$

binomial theorem

Part 2

Law of cosines

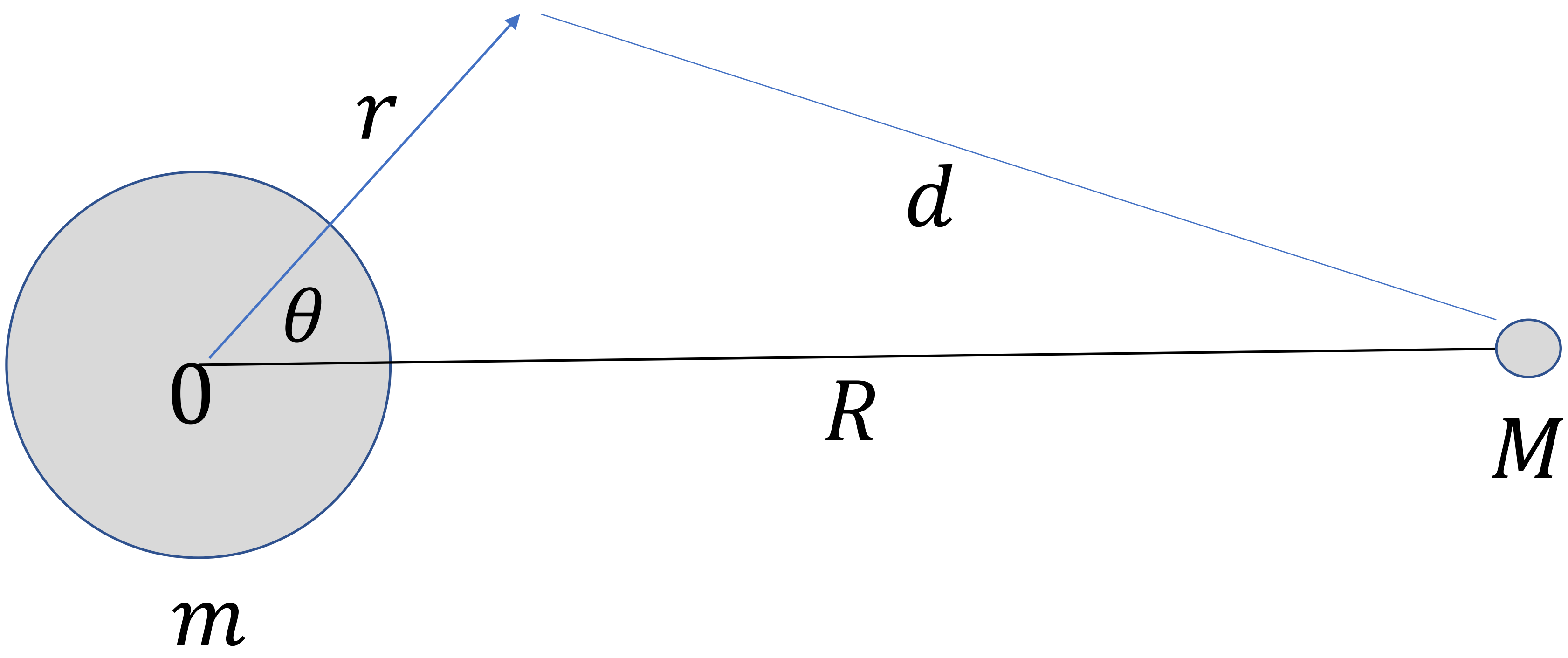
$$d^{-1} = [R^2 + r^2 - 2rR\cos\theta]^{-1/2}$$

$$= R^{-1}\left[1 + \underbrace{\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\theta}_{x}\right]^{-1/2}$$

$$[1 + x]^{-1/2} \approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 \cdots$$

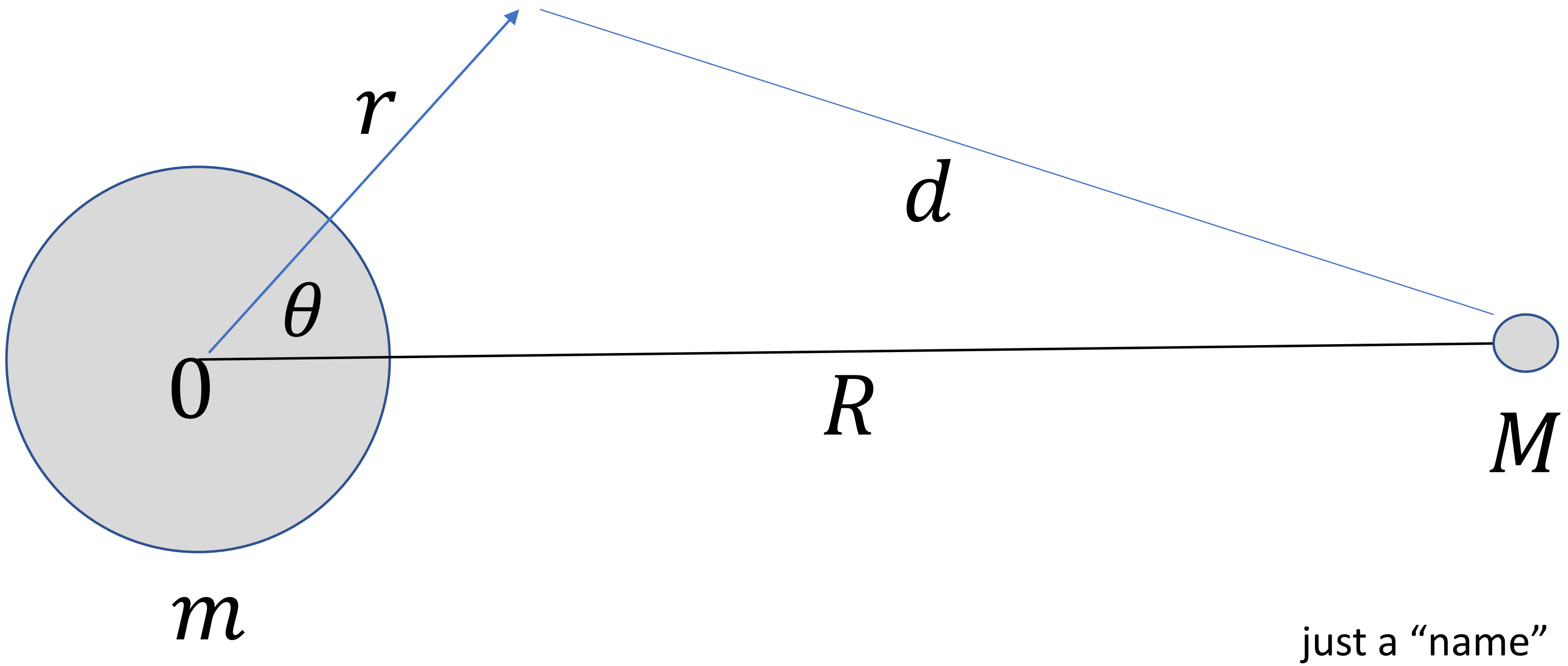
$$\approx R^{-1}\left[1 + \frac{r}{R}\cos\theta - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \frac{3}{2}\left(\frac{r}{R}\right)^2\cos^2\theta\right]$$

Part 2



$$E_M = \frac{-\gamma M}{d} = \frac{-\gamma M}{R}\left[1 + \frac{r}{R}\cos\theta - \left(\frac{r}{R}\right)^2 \frac{1}{2}\left(3\cos^2\theta - 1\right)\right]$$

Part 2



just a "name"

$$E_M = \frac{-\gamma M}{d} = \frac{-\gamma M}{R}\left[1 + \frac{r}{R}\cos\theta - \left(\frac{r}{R}\right)^2 P_2(\cos\theta)\right]$$

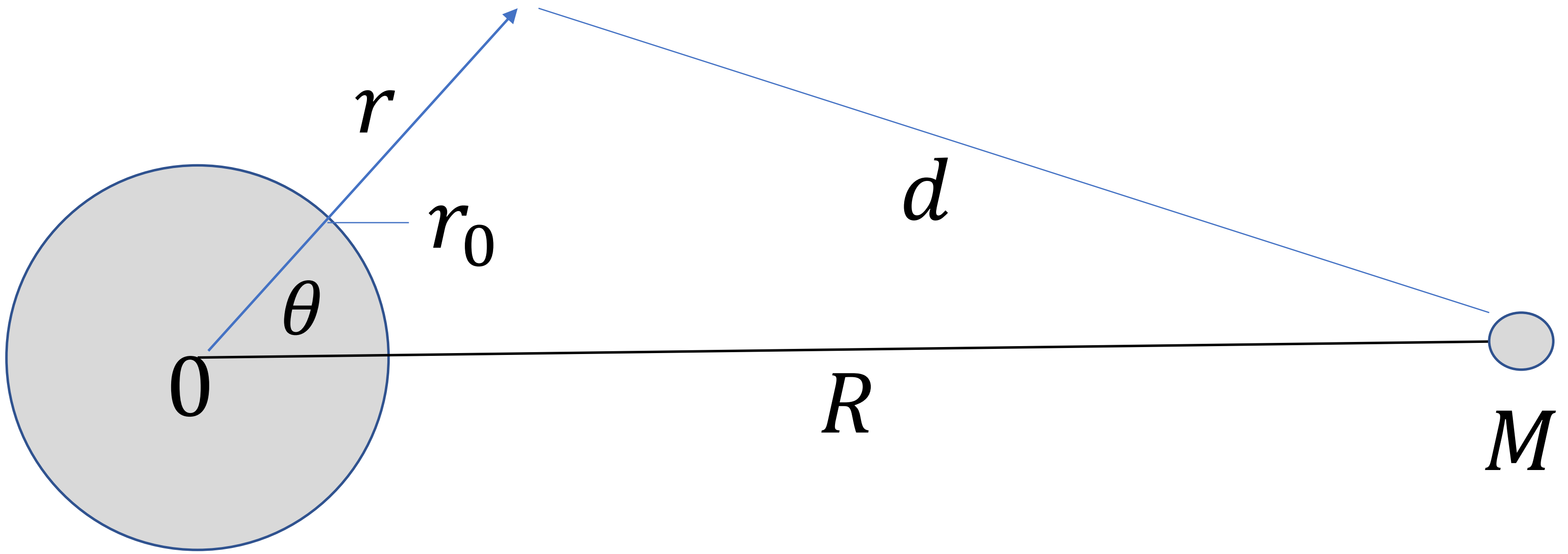
Part 2

$$E_M = \frac{-\gamma M}{d} = \frac{-\gamma M}{R}\left[1 + \frac{r}{R}\cos\theta - \left(\frac{r}{R}\right)^2 P_2(\cos\theta)\right]$$

$$E_c = \frac{-\gamma M}{R}\frac{r}{R}\cos\theta$$

$$E_M - E_c = \frac{-\gamma M}{R}\left[1 - \left(\frac{r}{R}\right)^2 P_2(\cos\theta)\right]$$

Part 2



tides: deviation from average radius

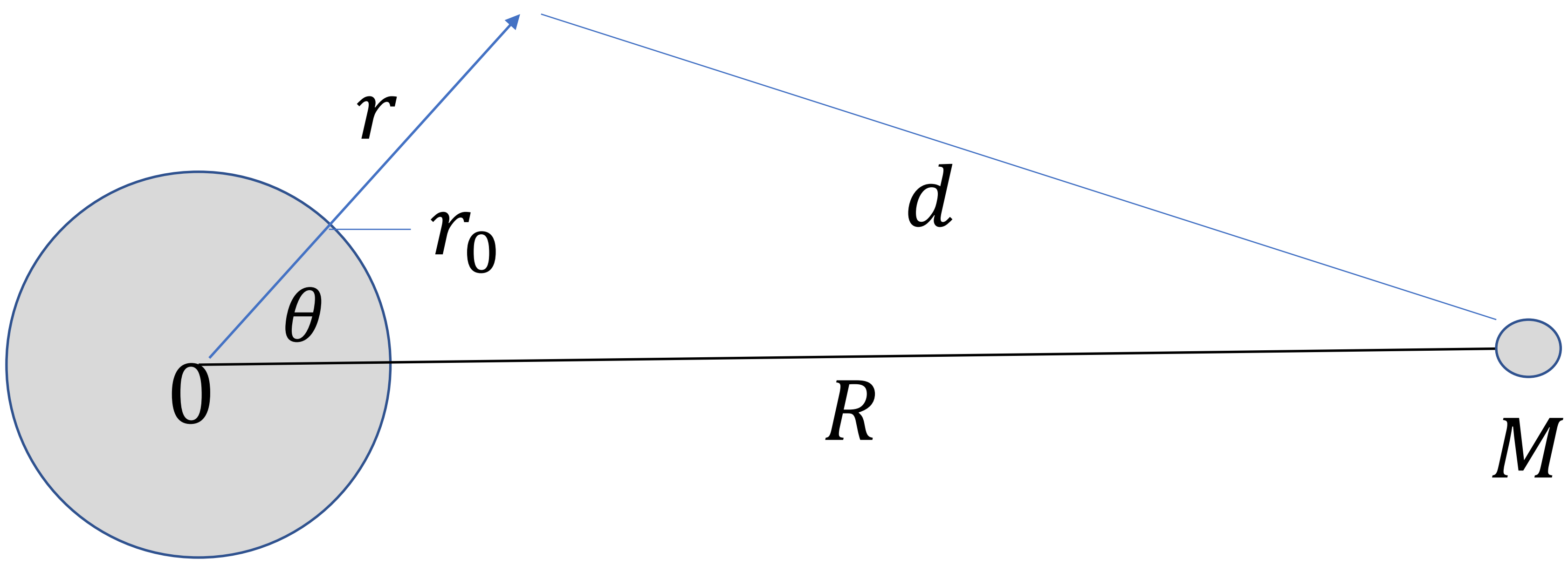
measure distance with respect to Earth's radius, r_0

$$r = r_0 + \Delta r$$

$$r^2 \approx r_0^2 \left(1 + \frac{2\Delta r}{r_0}\right)$$

$$\frac{1}{r} \approx \frac{1}{r_0}\left(1 - \frac{\Delta r}{r_0}\right)$$

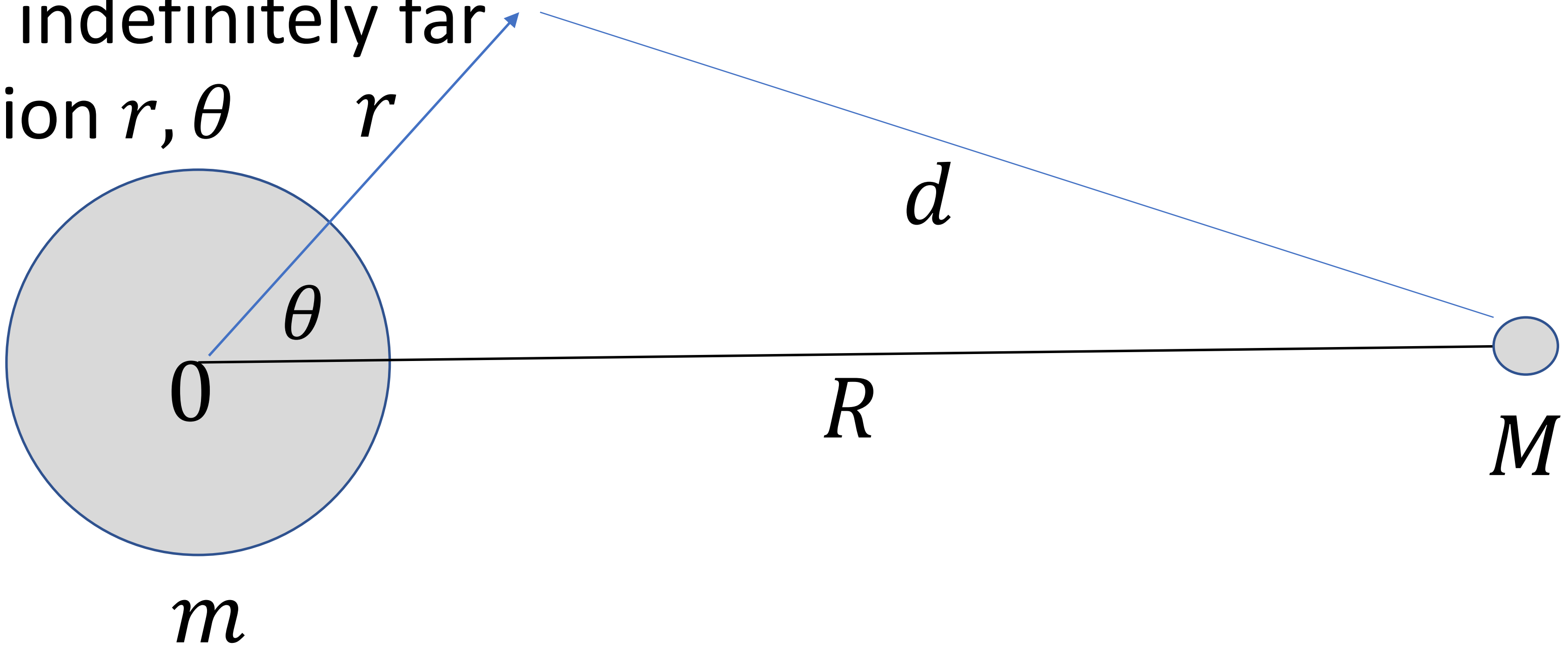
Part 2



$$E_M - E_c = \frac{-\gamma M}{R}\left[1 - \frac{1}{2}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) - 2\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0}\right]$$

Part 3

Earth: energy E_e released taking unit mass from indefinitely far away to a position r, θ



$$E_e = \frac{-\gamma m}{r} = -\frac{-\gamma m}{r_0}\left(1 - \frac{\Delta r}{r_0}\right)$$

Part 4 combined potential energy surfaces to get tides

$$E_e + E_M - E_c =$$

$$\frac{-\gamma m}{r_0}\left(1 - \frac{\Delta r}{r_0}\right) + \frac{-\gamma M}{R}\left[1 - \left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) - 2\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0}\right] =$$

$$-\frac{\gamma m}{r_0} - \frac{\gamma M}{R} + \frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) + 2\frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0} + \frac{\gamma m}{r_0}\frac{\Delta r}{r_0} =$$

constant part

part that varies with Δr and θ

Part 4 combined potential energy surfaces to get tides

$$E_e + E_M - E_c =$$

$$\frac{-\gamma m}{r_0}\left(1-\frac{\Delta r}{r_0}\right)+\frac{-\gamma M}{R}\left[1-\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)-2\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0}\right]=$$

$$-\frac{\gamma m}{r_0}-\frac{\gamma M}{R}+\boxed{\frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)+2\frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0}+\frac{\gamma m}{r_0}\frac{\Delta r}{r_0}}$$

set to zero to and solve for $\Delta r(\theta)$ to get ocean surface

$$-\frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) + 2\frac{\gamma M}{R}\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0} + \frac{\gamma m}{r_0}\frac{\Delta r}{r_0} = 0$$

$$-\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) + 2\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0} + \frac{m}{M}\frac{R}{r_0}\frac{\Delta r}{r_0} = 0$$

$$-\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta) + 2\left(\frac{r_0}{R}\right)^2 P_2(\cos\theta)\frac{\Delta r}{r_0} + \frac{m}{M}\frac{R}{r_0}\frac{\Delta r}{r_0} = 0$$

small since r_0<<R

$$\frac{\Delta r}{r_0} = \left(\frac{r_0}{R}\right)^3 \frac{M}{m} P_2(\cos\theta)$$

Formula for height of tides

with $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1 = 1)$$

$$P_2(\cos 0) = \frac{1}{2}(3 - 1) = 1$$

$$P_2(\cos 90) = \frac{1}{2}(-1) = -\frac{1}{2}$$

peak-to-peak tides

$$\frac{\Delta r}{r_0} = \frac{3}{2}\left(\frac{r_0}{R}\right)^3 \frac{M}{m}$$

Moon

	A	B	C
1	Mearth kg	5.97E+24	
2	Mmoon	7.40E+22	
3	rEarth m	6.37E+06	
4	Rmoon m	3.83E+08	
5			
6	Tides m	0.55	
7	1.5*B3*((B3/B4)^3)*(B2/B1)		

peak-to-peak tides

$$\frac{\Delta r}{r_0} = \frac{3}{2}\left(\frac{r_0}{R}\right)^3 \frac{M}{m}$$

more massive moon, bigger tides

peak-to-peak tides

$$\frac{\Delta r}{r_0} = \frac{3}{2}\left(\frac{r_0}{R}\right)^3 \frac{M}{m}$$

more distant moon, smaller tides

How big would tides be if moon was twice as close to earth?

$$\left(\frac{r_0}{R}\right)^3 \; so \; \left(\frac{1}{0.5}\right)^3 = 8$$

peak-to-peak tides

$$\frac{\Delta r}{r_0} = \frac{3}{2}\left(\frac{r_0}{R}\right)^3 \frac{M}{m}$$