Solid Earth Dynamics

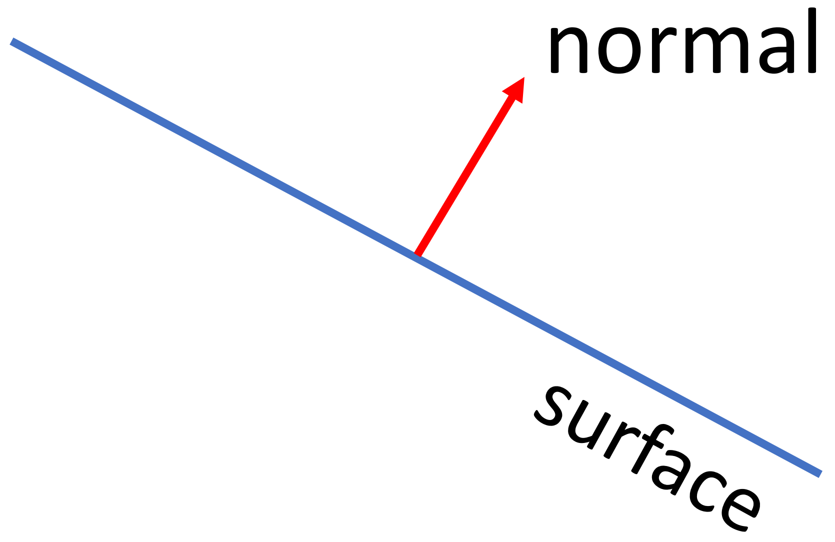
Bill Menke, Instructor

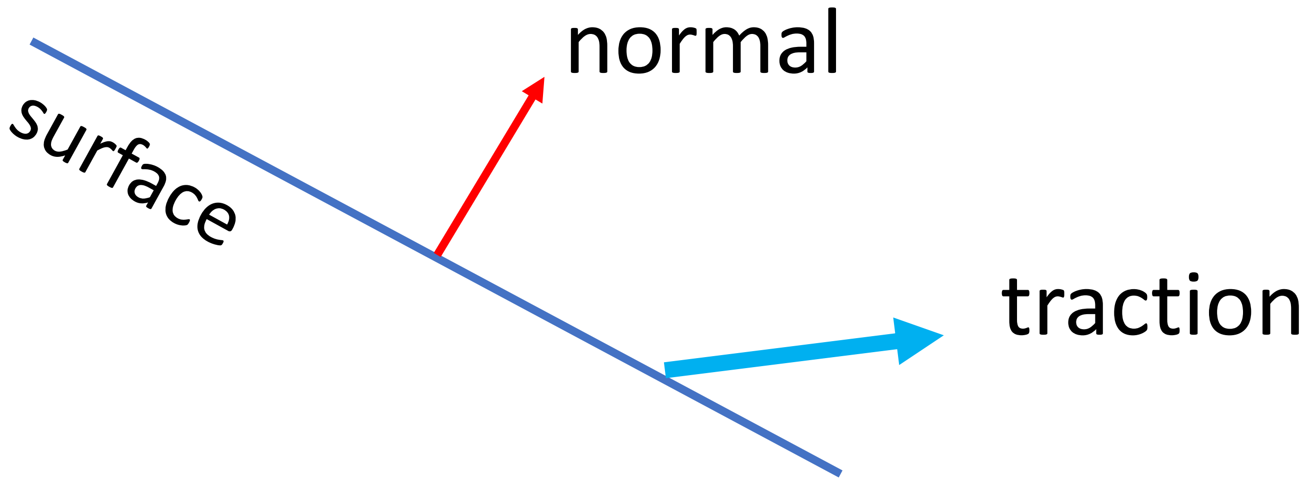
Lecture 15

Solid Earth Dynamics

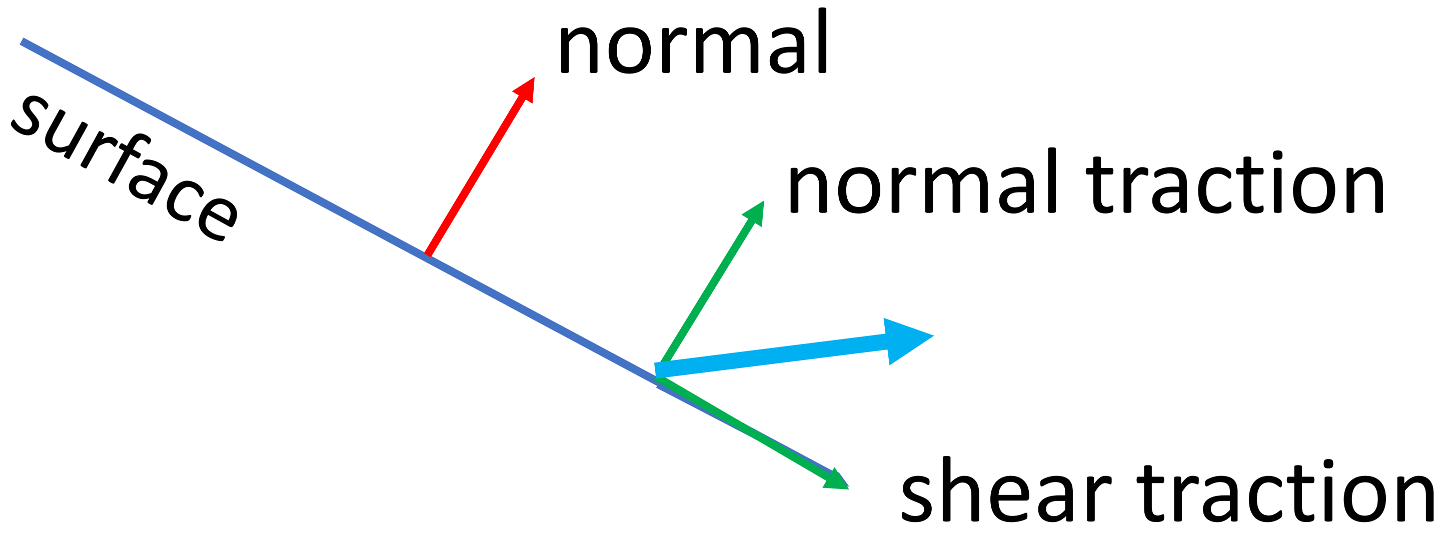
Vibrations in fluids

Part 1: Newton’s law applied to pressure fluctuations in a fluid

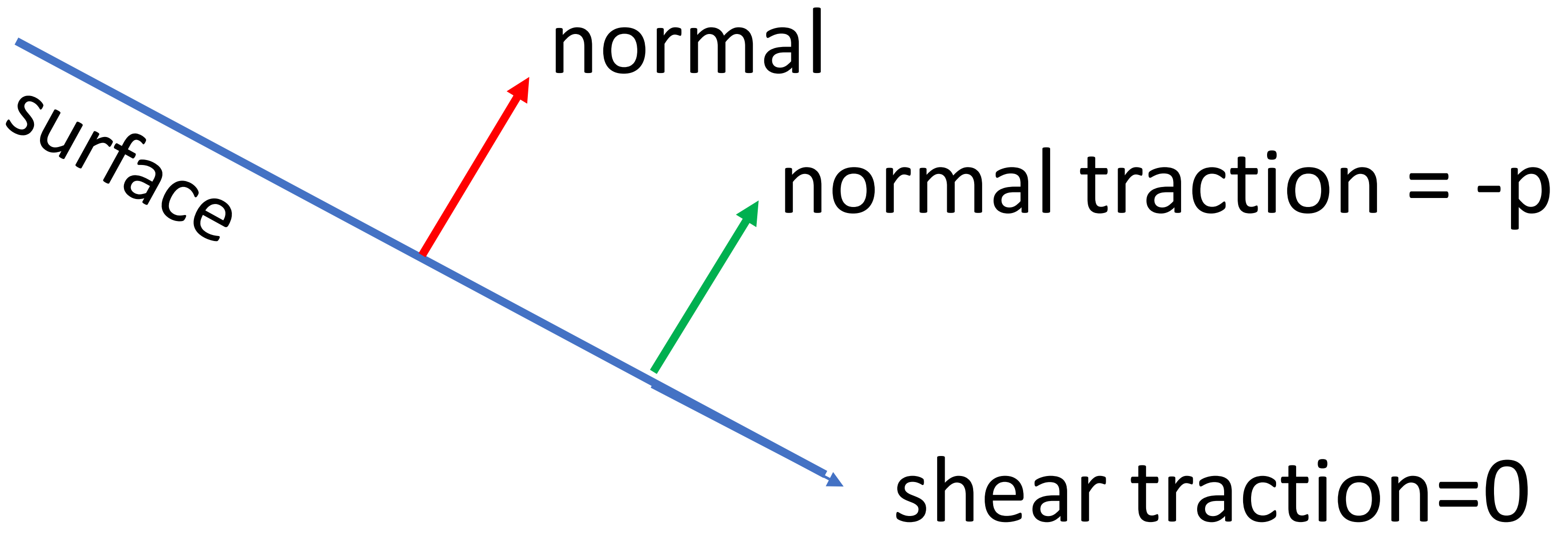

normal
surface



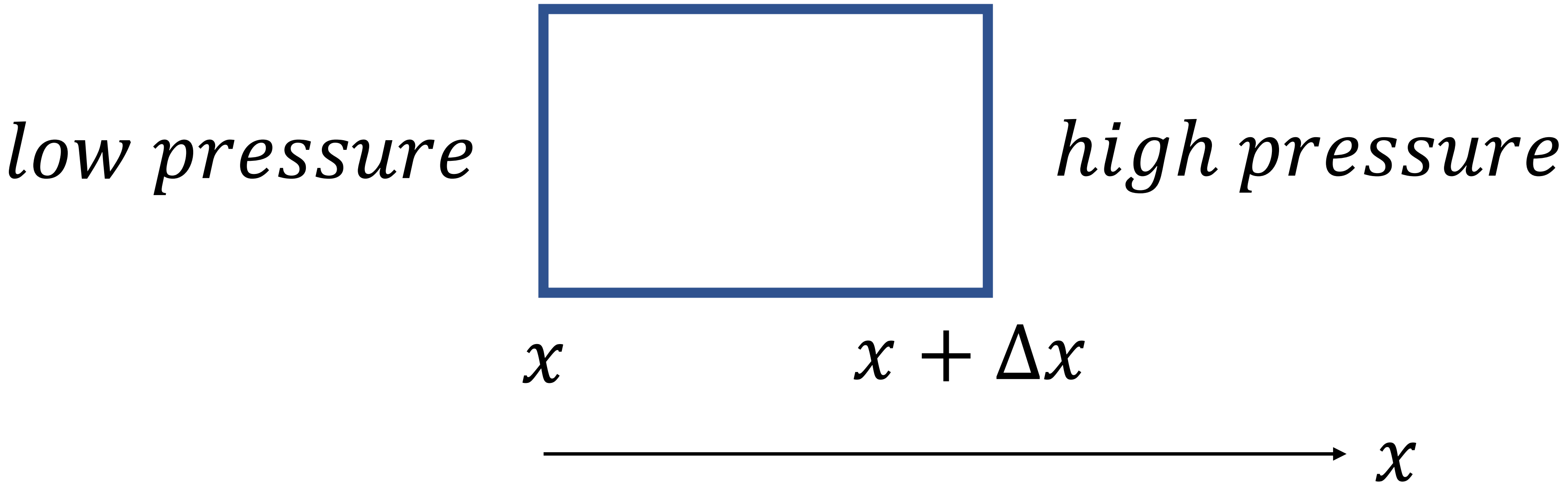
traction = force per unit area


normal
normal traction
surface
shear traction

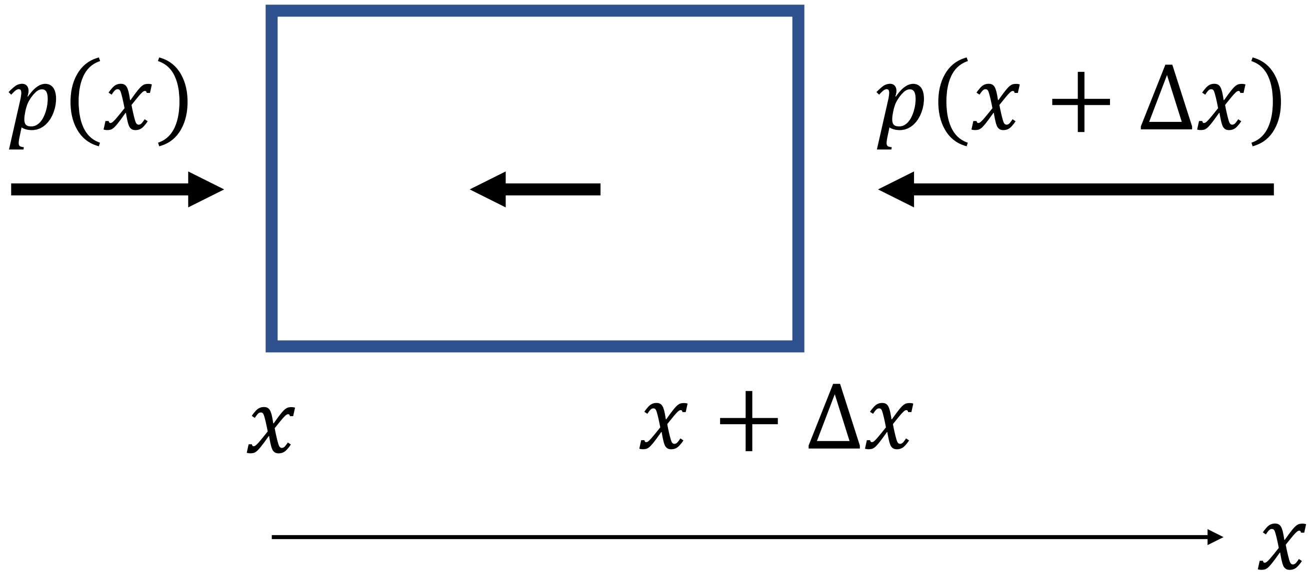
fluid



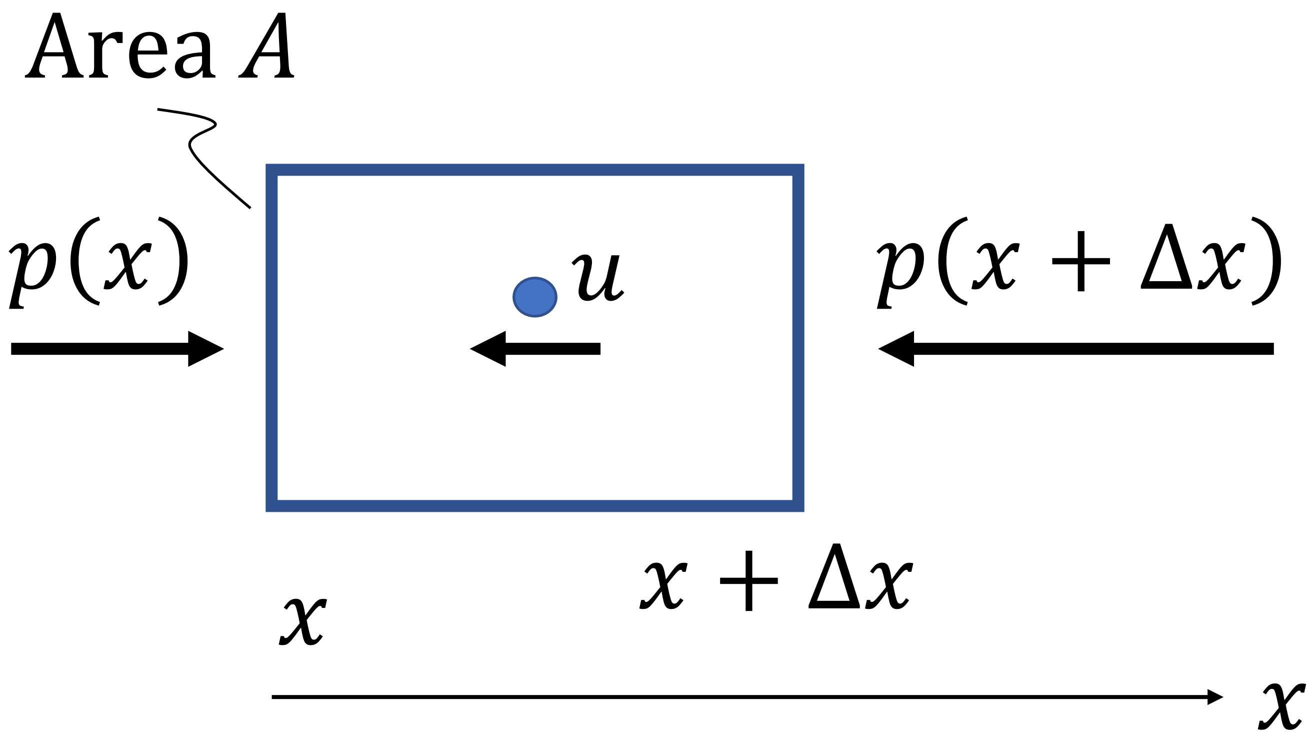
pressure = strength of inward pointing traction which way will it move?



moves to left



moves to left

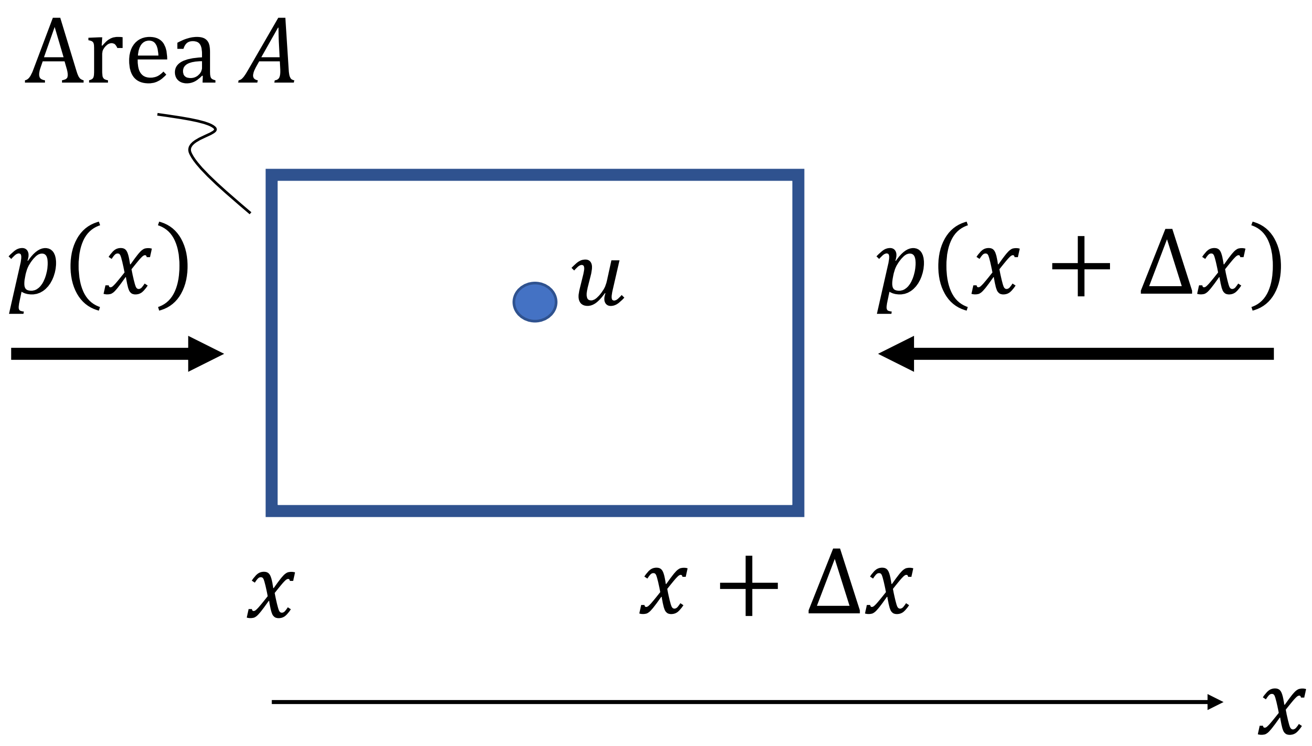


$$F(x) = Ap(x) \qquad F(x + \Delta xp) = -p(x + \Delta x)$$

total surface force

$$F(x) + F(x + \Delta xp) = Ap(x) - Ap(x + \Delta x)$$

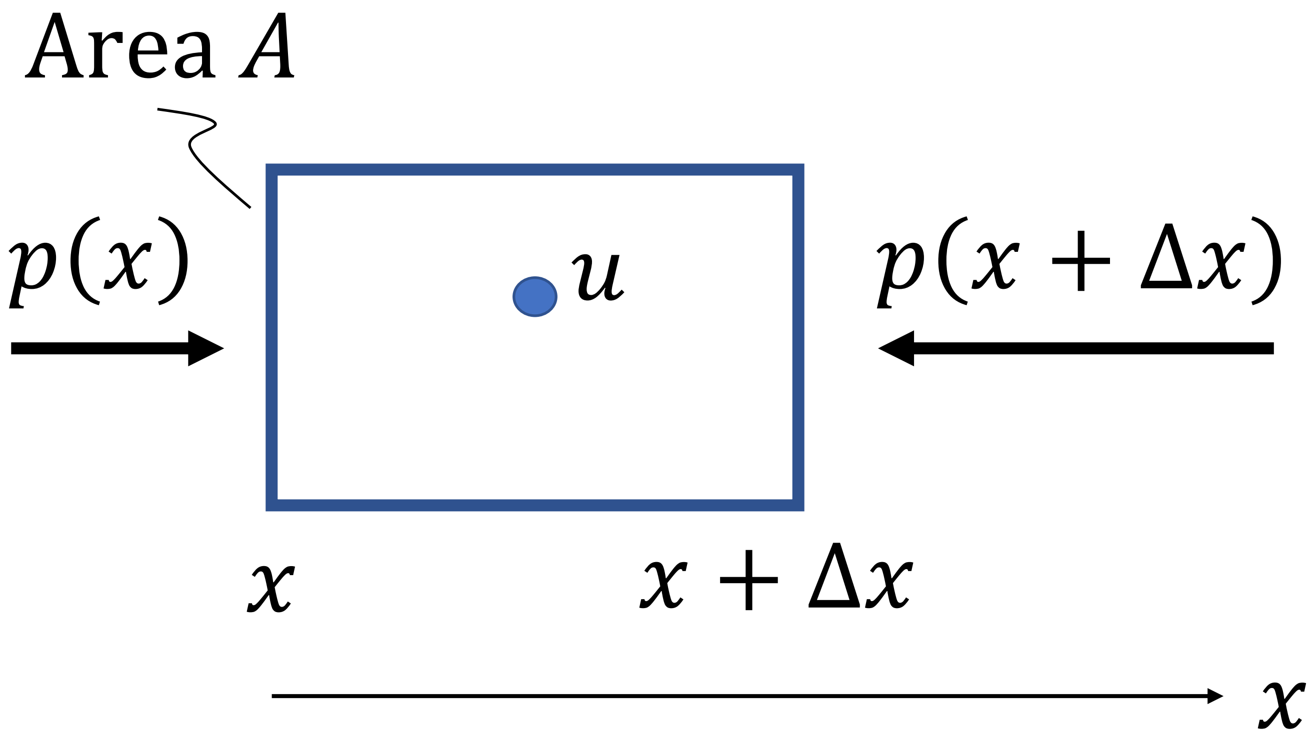
Newton' Law



Newton' Law $F_{surface} + F_{body}$ = mass times acceleration

$$Ap(x) - Ap(x + \Delta x) + fA\Delta x = A\Delta x \rho \frac{d^2u}{dt^2}$$

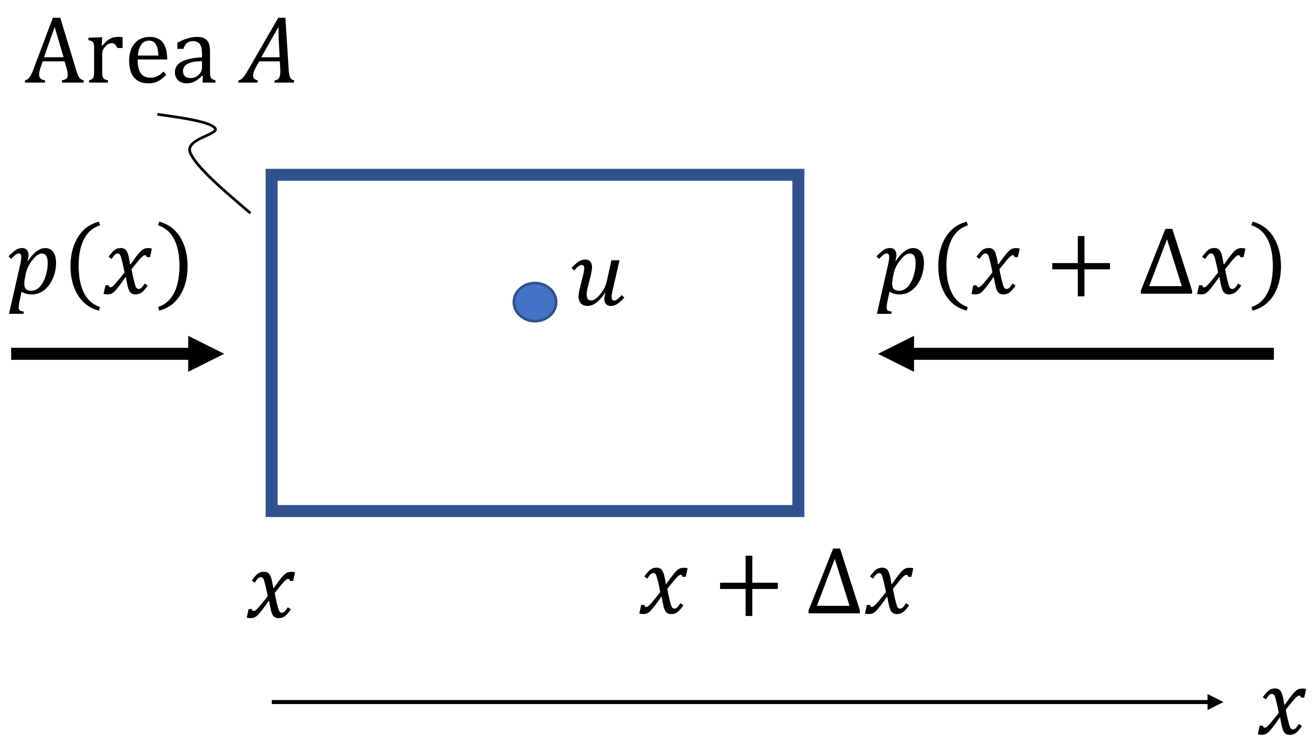
Newton' Law



Newton' Law $F_{surface} + F_{body}$ = mass times acceleration

$$-\frac{p(x+\Delta x)-p(x)}{\Delta x}+f=\rho\frac{d^2u}{dt^2}$$

Newton' Law

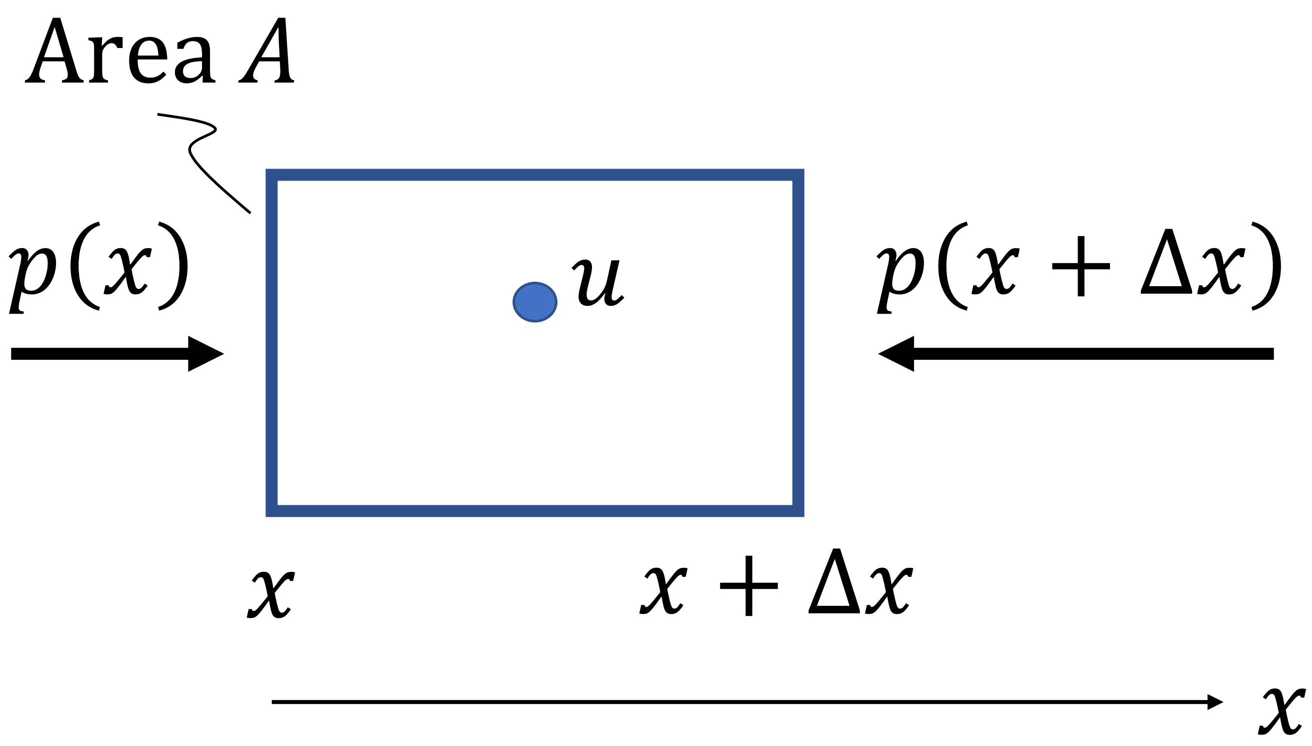


Newton' Law $F_{surface} + F_{body}$ = mass times acceleration

$$-\frac{dp}{dx} + f = \rho \frac{d^2u}{dt^2}$$

Newton' Law in a fluid

Newton' Law



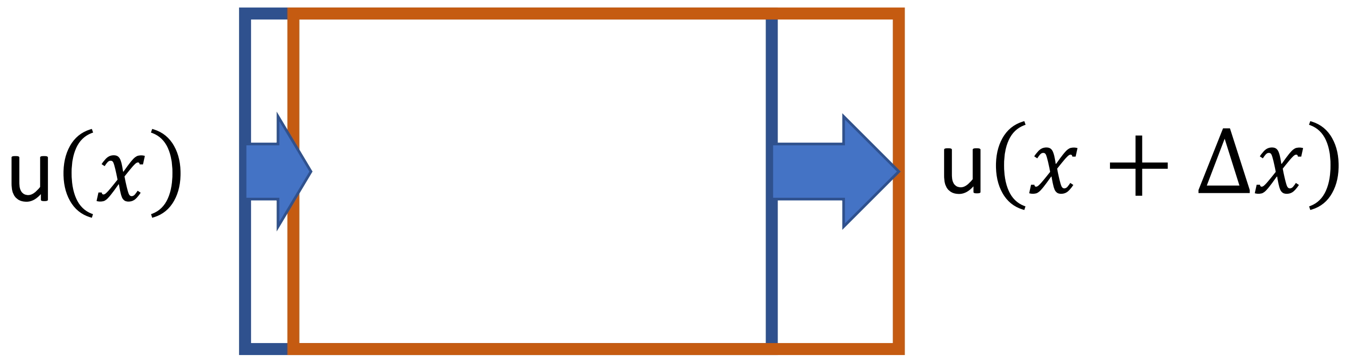
Newton' Law $F_{surface} + F_{body}$ = mass times acceleration

$$-\frac{dp}{dx} = \rho \frac{d^2u}{dt^2}$$

no body force

Part 2: Linear elasticity in a fluid

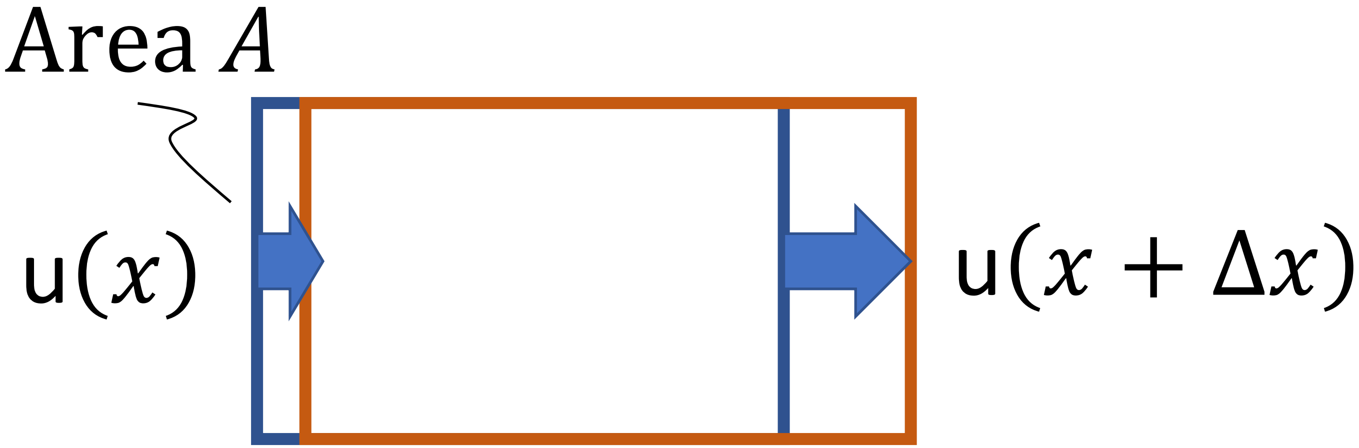
deformation causes pressure



did the volume get bigger or smaller?

did the pressure go up or down?

went down



volume $Au(x)$ $Au(x + \Delta x)$

volumetric strain $\Delta V / V$

$$\frac{Au(x + \Delta x) - Au(x)}{A\Delta x}$$

volume $Au(x)$ $Au(x+\Delta x)$

volumetric strain $\Delta V/V$

$$\frac{Au(x+\Delta x)-Au(x)}{A\Delta x}=\frac{du}{dx}$$

Part 3: Equation for pressure fluctuations in a fluid

$$p = -k\frac{du}{dx}$$ linear elasticity

pressure decreases
linearly
with volumetric strain

$$p = -k\frac{du}{dx}$$ linear elasticity

$$-\frac{dp}{dx} = \rho\frac{d^2u}{dt^2}$$ newton’s law

equation for pressure fluctuations in a fluid $$k\frac{d^2u}{dx^2} = \rho\frac{d^2u}{dt^2}$$

$$p = -k\frac{du}{dx}$$ linear elasticity

$$-\frac{dp}{dx} = \rho\frac{d^2u}{dt^2}$$ newton's law

what do we call pressure fluctuation in air?

$$k\frac{d^2u}{dx^2} = \rho\frac{d^2u}{dt^2}$$

$$p = -k\frac{du}{dx}$$ linear elasticity

$$-\frac{dp}{dx} = \rho\frac{d^2u}{dt^2}$$ newton's law

equation for sound $$k\frac{d^2u}{dx^2} = \rho\frac{d^2u}{dt^2}$$

(for displacement u)

To get pressure p as the variable …

$$p = -k\frac{du}{dx} \qquad \text{so} \qquad \frac{du}{dx} = -\frac{p}{k}$$

$$-\frac{dp}{dx} = \rho\frac{d^2u}{dt^2} \qquad \text{take } \frac{d}{dx} \text{ so} \qquad -\frac{d^2p}{dx^2} = \rho\frac{d^2}{dt^2}\frac{du}{dx}$$

equation for sound $$k\frac{d^2p}{dx^2} = \rho\frac{d^2p}{dt^2}$$

(for pressure p)

Part 4: propagation of pressure fluctuations

equation for sound $$k\frac{d^2p}{dx^2} = \rho\frac{d^2p}{dt^2}$$

a pressure fluctuation moving at speed $$c = \sqrt{\frac{k}{\rho}}$$

retains its shape

$$\mathrm{p}(x,t) = s\left(t - \frac{x}{c}\right)$$

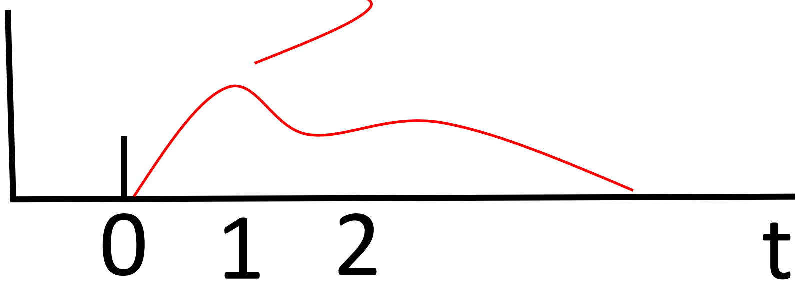
retains its shape as it moves

example with c=1

position 0

$$p(x=0,t) = s(t)$$

maximum when argument is 1 (and t=1)



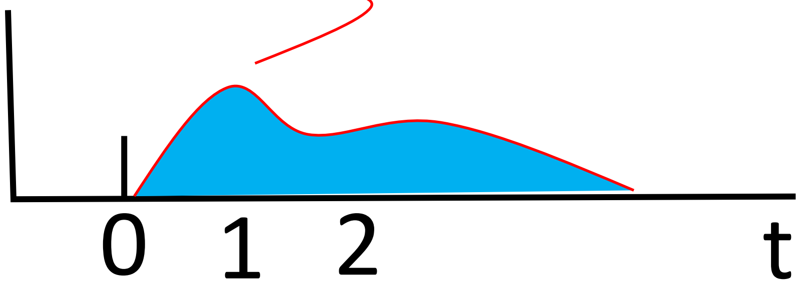
$$p(x,t) = s\left(t - \frac{x}{c}\right)$$ retains its shape as it moves

example with c=1

position=0

$$p(x=0,t) = s(t)$$

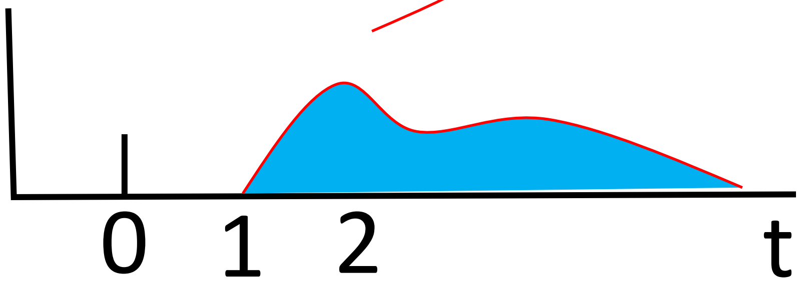
maximum when argument is 1 (and t=1)



position 1

$$\mathrm{p}(x=1,t) = s(t-1)$$

maximum when argument is 1 (and t=2)



demonstration that

$$\mathrm{p}(x,t) = s\left(t - \frac{x}{c}\right)$$

solves equation

$$c^2 \frac{d^2 p}{dx^2} = \frac{d^2 p}{dt^2}$$

let $y = t - c^{-1}x$ so $\frac{dy}{dx} = -c^{-1}$ $\frac{dy}{dt} = 1$

demonstration that $\mathrm{p}(x,t) = s(y)$ solves equation

$$c^2 \frac{d^2 p}{dx^2} = \frac{d^2 p}{dt^2}$$

with $y = t - c^{-1}x$ so $\frac{dy}{dx} = -c^{-1}$ $\frac{dy}{dt} = 1$

Employ chain rule

$$\frac{d^2 p}{dx^2} = \frac{d^2 s}{dx^2} = \frac{d}{dx}\frac{d}{dx}s = \frac{dy}{dx}\frac{d}{dy}\frac{dy}{dx}\frac{d}{dy}s = c^{-2}\frac{d^2 s}{dy^2}$$

$$\frac{d^2 p}{dt^2} = \frac{d^2 s}{dt^2} = \frac{d}{dt}\frac{d}{dt}s = \frac{dy}{dt}\frac{d}{dy}\frac{dy}{dt}\frac{d}{dy}s = \frac{d^2 s}{dy^2}$$

demonstration that

$$p(x,t) = s(y)$$

solves equation

$$c^2 \frac{d^2p}{dx^2} = \frac{d^2p}{dt^2}$$

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$$\frac{d^2p}{dx^2} = \frac{d^2s}{dx^2} = \frac{d}{dx}\frac{d}{dx}s = \frac{dy}{dx}\frac{d}{dy}\frac{dy}{dx}\frac{d}{dy}s = c^{-2}\frac{d^2s}{dy^2}$$

$$\frac{d^2p}{dt^2} = \frac{d^2s}{dt^2} = \frac{d}{dt}\frac{d}{dt}s = \frac{dy}{dt}\frac{d}{dy}\frac{dy}{dt}\frac{d}{dy}s = \frac{d^2s}{dy^2}$$

equation becomes

$$\frac{d^2s}{dy^2} = \frac{d^2s}{dy^2}$$ ✓

Part 5: sound speed in air

$$p = -k\frac{du}{dx}$$

k is pressure divided by volumetric strain

$$k = -\frac{p}{\left(\frac{du}{dx}\right)}$$

pressure fluctuation in an ideal gas

$$pV = nRT$$

$$(p_0 + \Delta p)(V_0 + \Delta V) = nRT$$

fluctuation around reference pressure

$$p_0V_0 + p_0\Delta V + V_0\Delta p = nRT$$

multiply out, discard small term

$$p_0V_0 \qquad\qquad = nRT$$

Subtract ideal gas law
Which is true at the reference pressure

$$p_0\Delta V + V_0\Delta p = 0$$

pressure fluctuation in an ideal gas

$$p_0 \Delta V + V_0 \Delta p = 0$$

$$\frac{\Delta V}{V_0} = -\frac{\Delta p}{p_0}$$ rearrange

$$-\frac{\Delta p}{\frac{\Delta V}{V_0}} = p_0$$ rearrange

$$k = p_0$$ Since $k = -p/\left(\left(\frac{du}{dx}\right)\right)$

$$c = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{p_0}{\rho}}$$

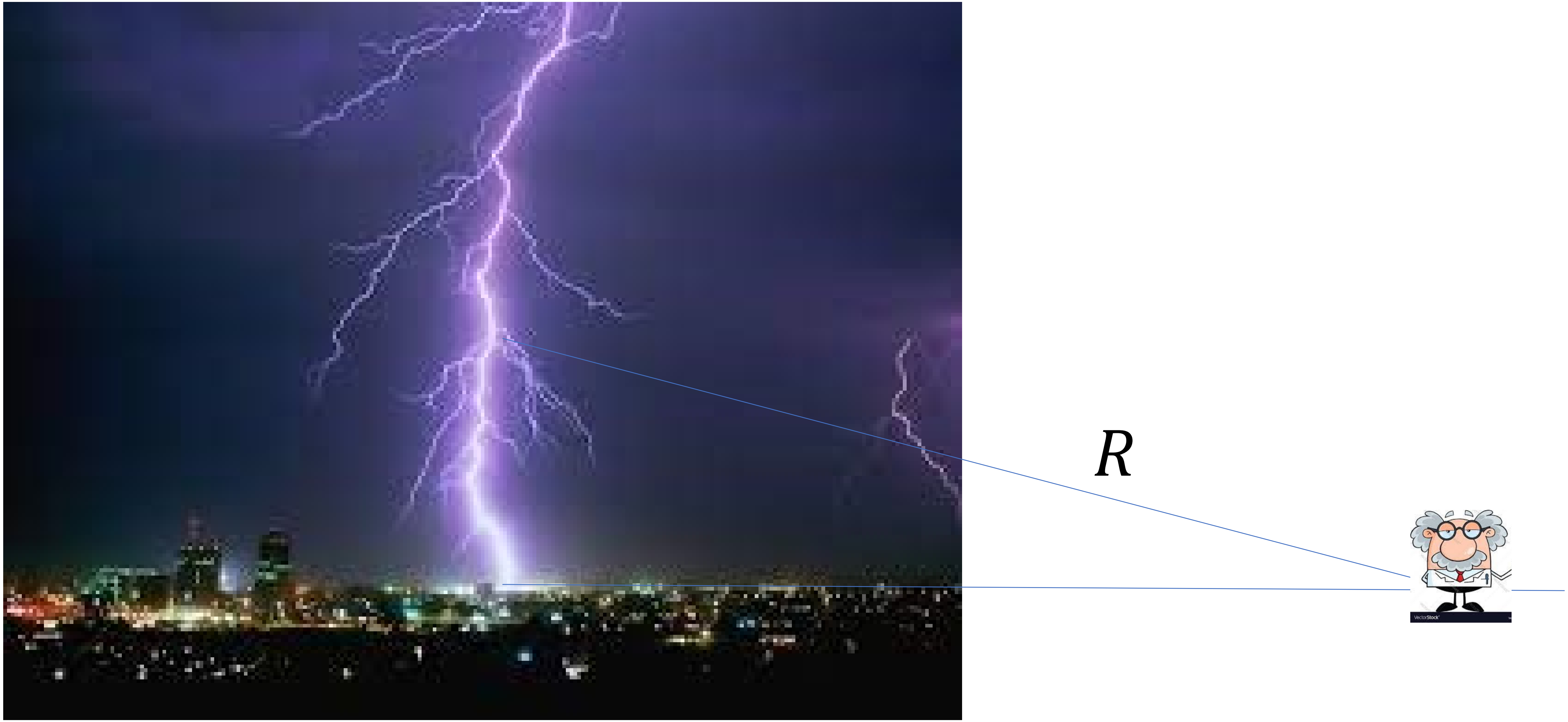
	A	B	C	D
1	p0	101325		
2	rho	1.293		
3				
4	sqrt(p0/rho)	279.9362	m/s	
5				

$p_0 = 1\text{ atm} = 101325$ Pa = 101325 kg/m-s^2

$\rho = 1\text{ atm} = 1.293$ kg/m^3

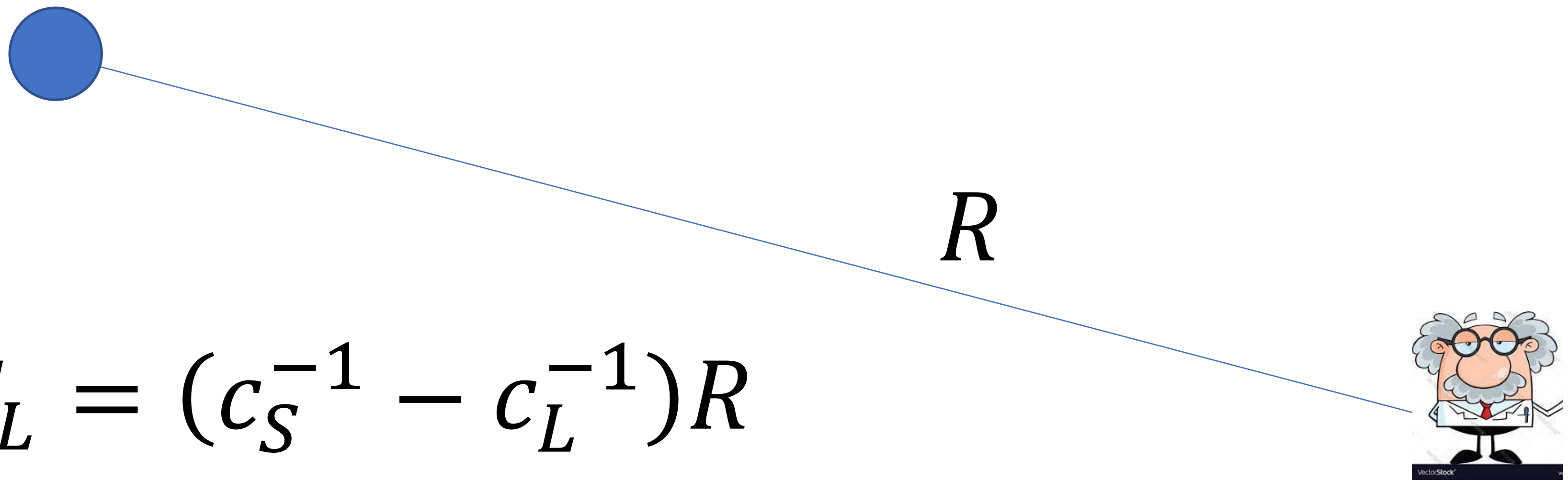
$$units\text{:}\quad \frac{p_0}{\rho} : \frac{kg}{m{-}s^2}\frac{m^3}{kg} = \frac{m^2}{s^2} \quad \text{so units: } \sqrt{\frac{p_0}{\rho}} : \frac{m}{s}$$

Part 5: Lightning and Thunder


R

$$T_L = t_0 + c_L^{-1} R$$

$$T_T = t_0 + c_S^{-1} R$$



$$\Delta T = T_T - T_L = (c_S^{-1} - c_L^{-1}) R$$

$$R = \frac{\Delta T}{(c_S^{-1} - c_L^{-1})}$$

$$c_S = 278 \text{ m/s}$$

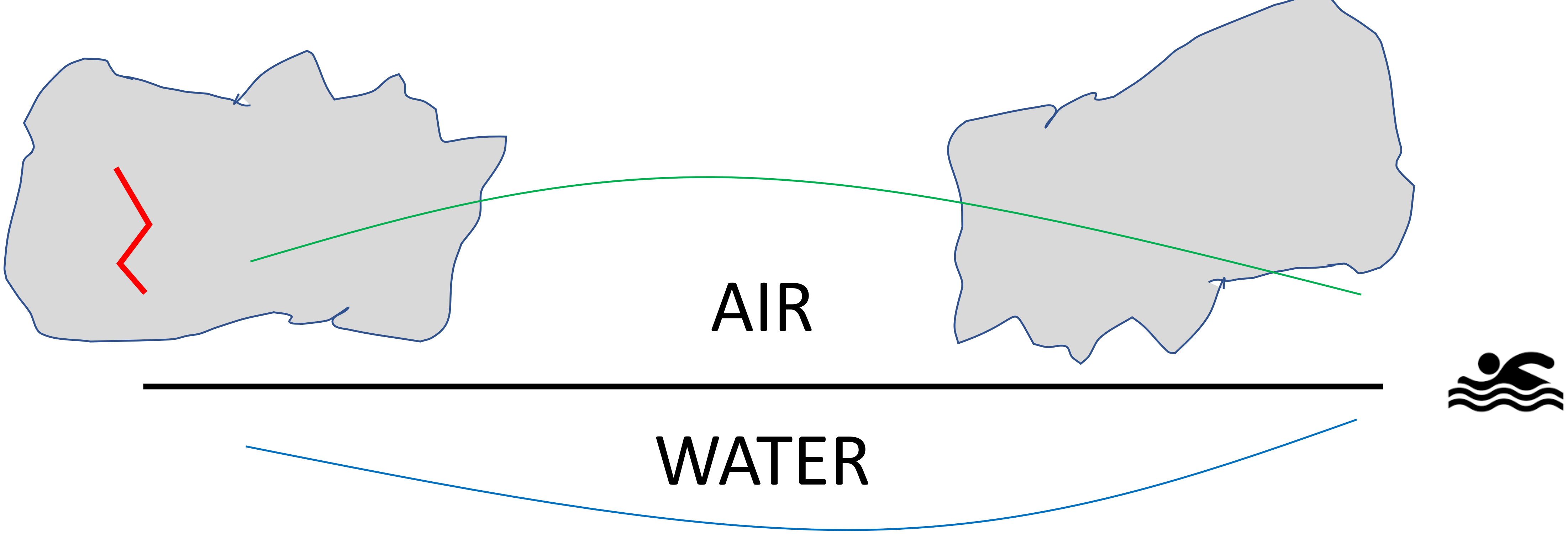
$$c_L = 299{,}792{,}458 \text{ m/s}$$

$$\frac{1}{(c_S^{-1} - c_L^{-1})} \approx \frac{1}{(c_S^{-1})} = c_S = 278 \text{ m/s}$$

$$R = 278\,\Delta T \ \text{ m}$$

or approximately

$$R \approx 1000\,\Delta T \text{ feet}$$


AIR
WATER

$$c_S = 278 \text{ m/s}$$

$$c_W = 1500 \text{ m/s}$$

$$\frac{1}{(c_S^{-1} - c_W^{-1})} \approx 341\, m/s$$

$$R = 341\, \Delta T \ \text{ m}$$