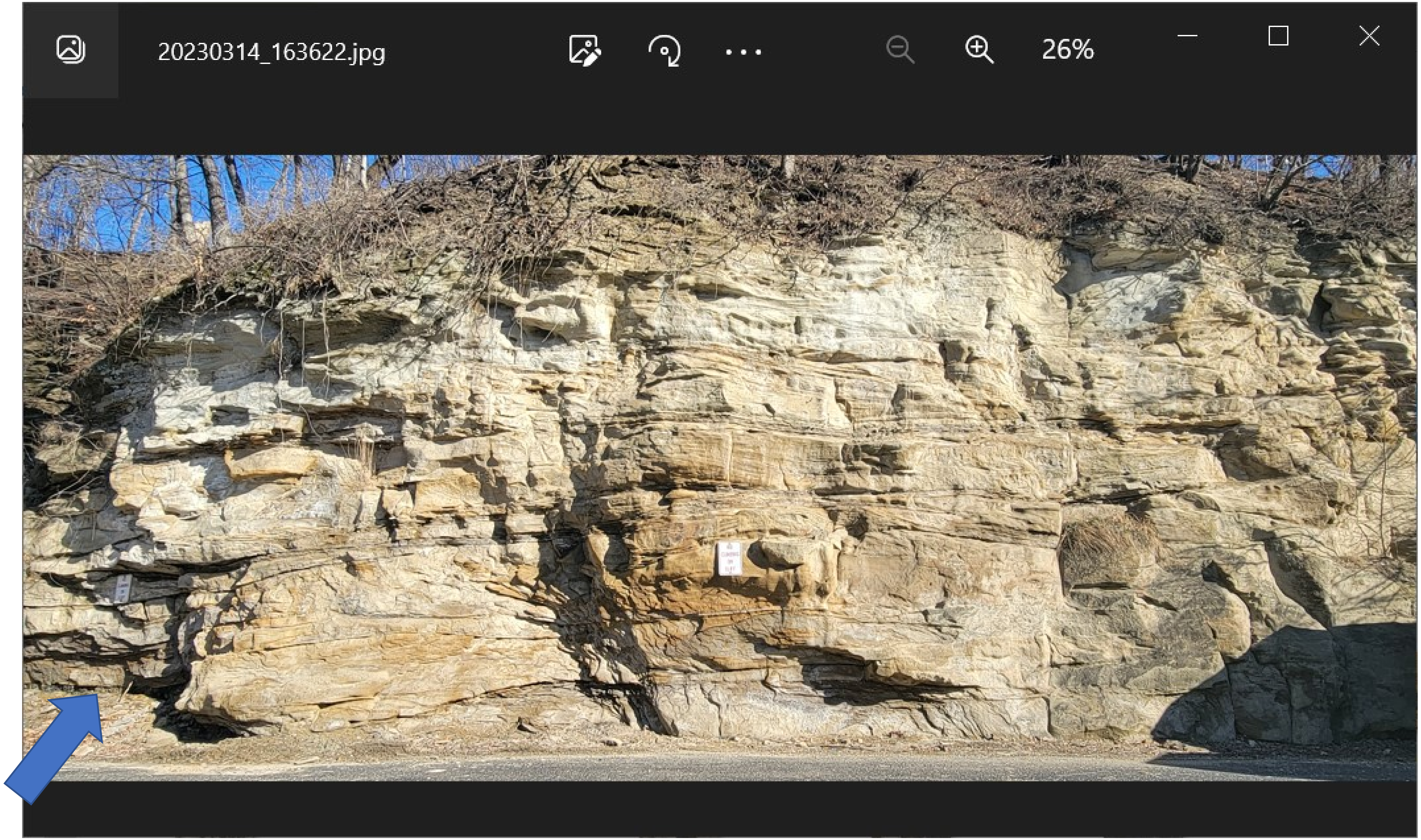
Solid Earth Dynamics

Bill Menke, Instructor

Lecture 16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Midterm

congratulations!

everyone did well

two letter grades, one each problem

Question 1

same densities, no thermal expansion, societal effects

same densities

no density stratification of the mantle/crust

no isostasy – no deep sedimentary basins, different topography profile

societal: less petroleum, less economic minerals

no thermal expansion

no convection, no plate tectonics

conductive temperature profile of earth, melt mantle (?)

societal: no ocean basins, water-world

Question 2

water world with thick cloud cover

centrifugal & Coriolis forces

rotation rate of planet, (match day-night cycle)

tides

solar tides (match day-night cycle)

other tides from planets, hints about their mass/distance, position

ocean depth soundings, gravity measurements

plate tectonics (depth age, trenches, etc), isostasy & lithospheric thickness

petrology, shape their islands

volcanic arcs? hot-spot chains?

Question 3

exoplanet rover

camera

topography, evidence of recent tectonism and volcanism

chemical analyses

infer densities, systematic differences between highlands and lowlands

gravimeter

size of anomalies (using inferred density)

isostasy, through troughs around highlands

heat flow

systematic differences between regions related to age of crust or thickness of lithosphere

Solid Earth Dynamics

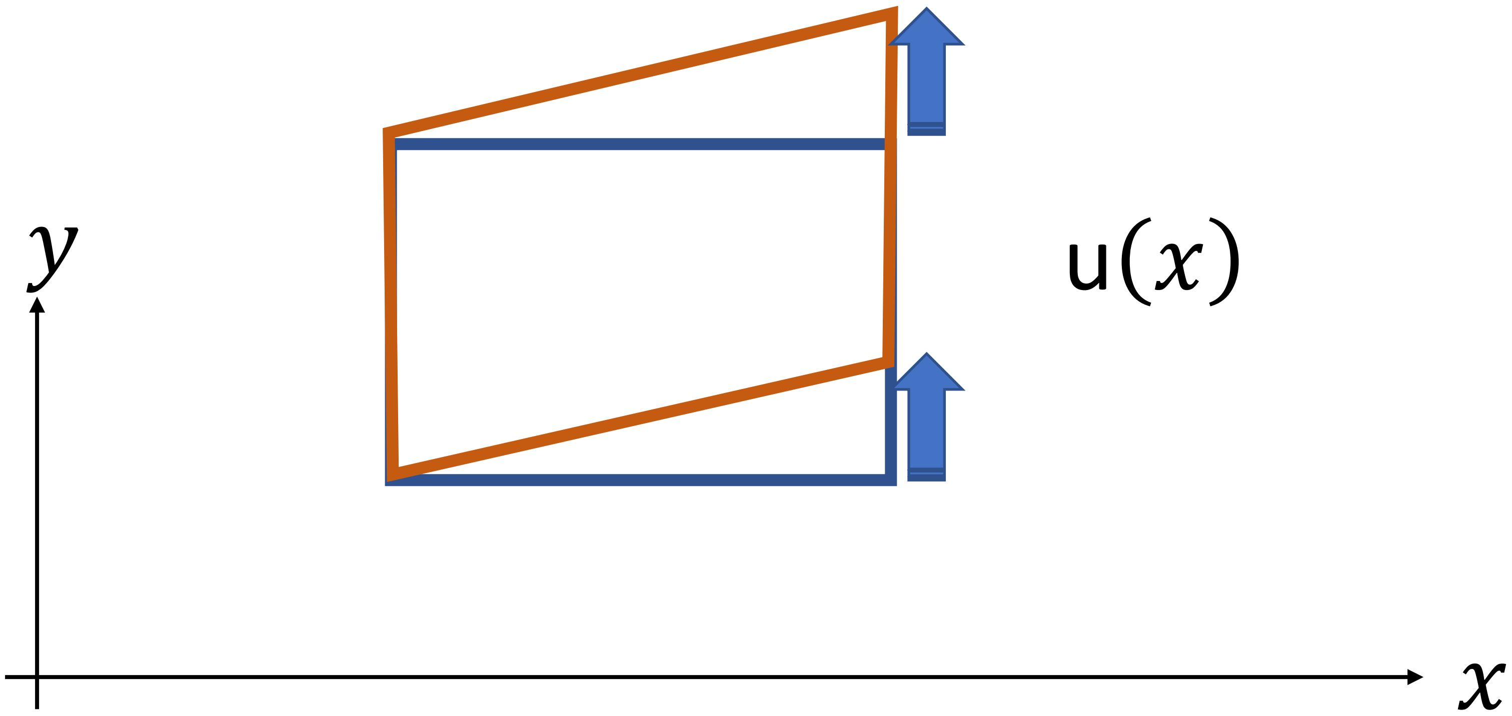
Vibrations in solids

shear wave
compressional wave
apparent velocity
refraction at interface

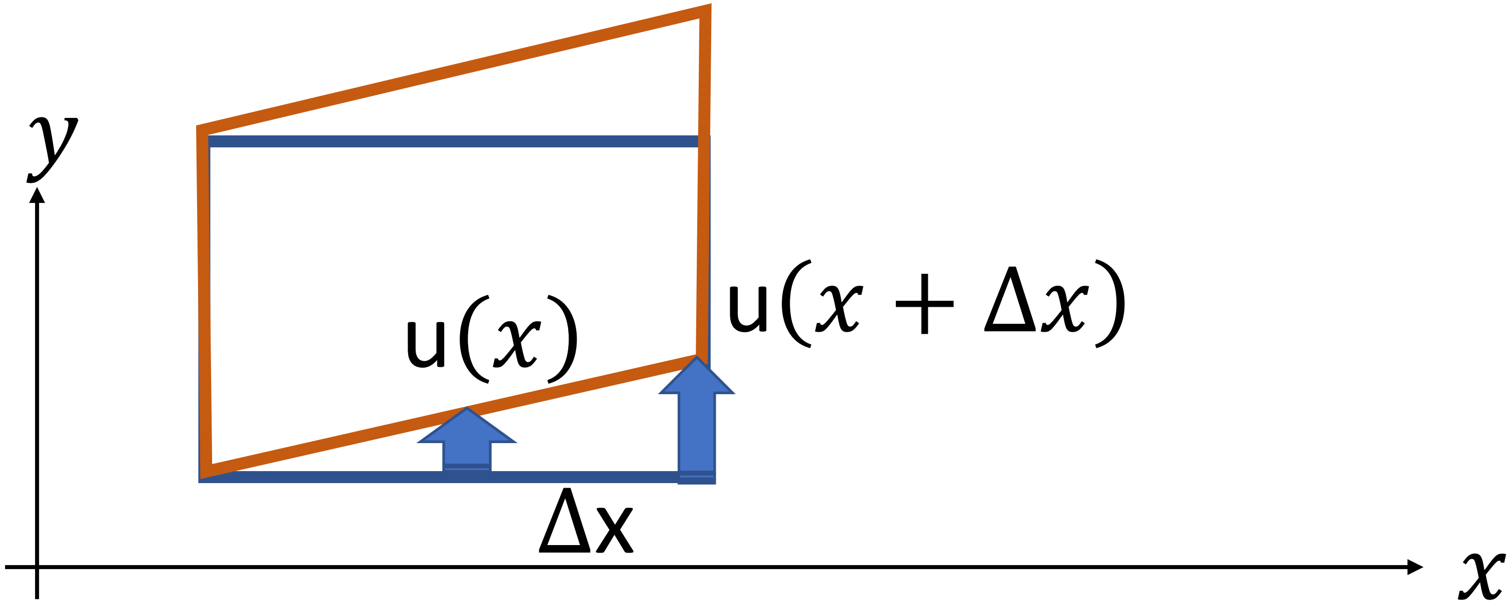
shear waves in a solid

shear deformation in a solid

displacement in the y-direction



shear strain $\varepsilon = \frac{\Delta u}{\Delta x} = \frac{du}{dx}$



linear elasticity

shear stress proportional to shear strain

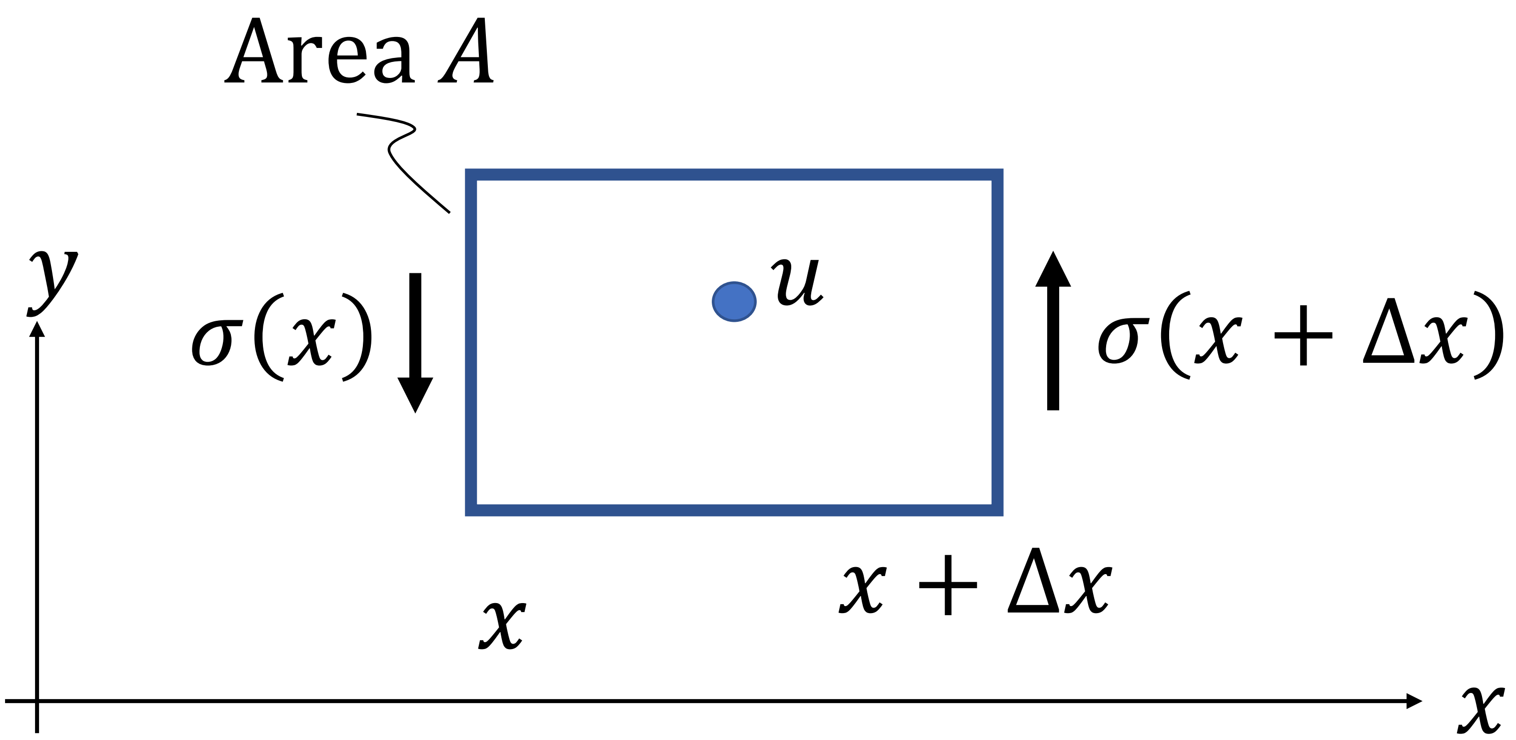
$$\text{shear stress } \sigma = \mu\varepsilon$$

rigidity

newton’s law for shear forces

(very similar to pressure case in Lec 15)

surface force

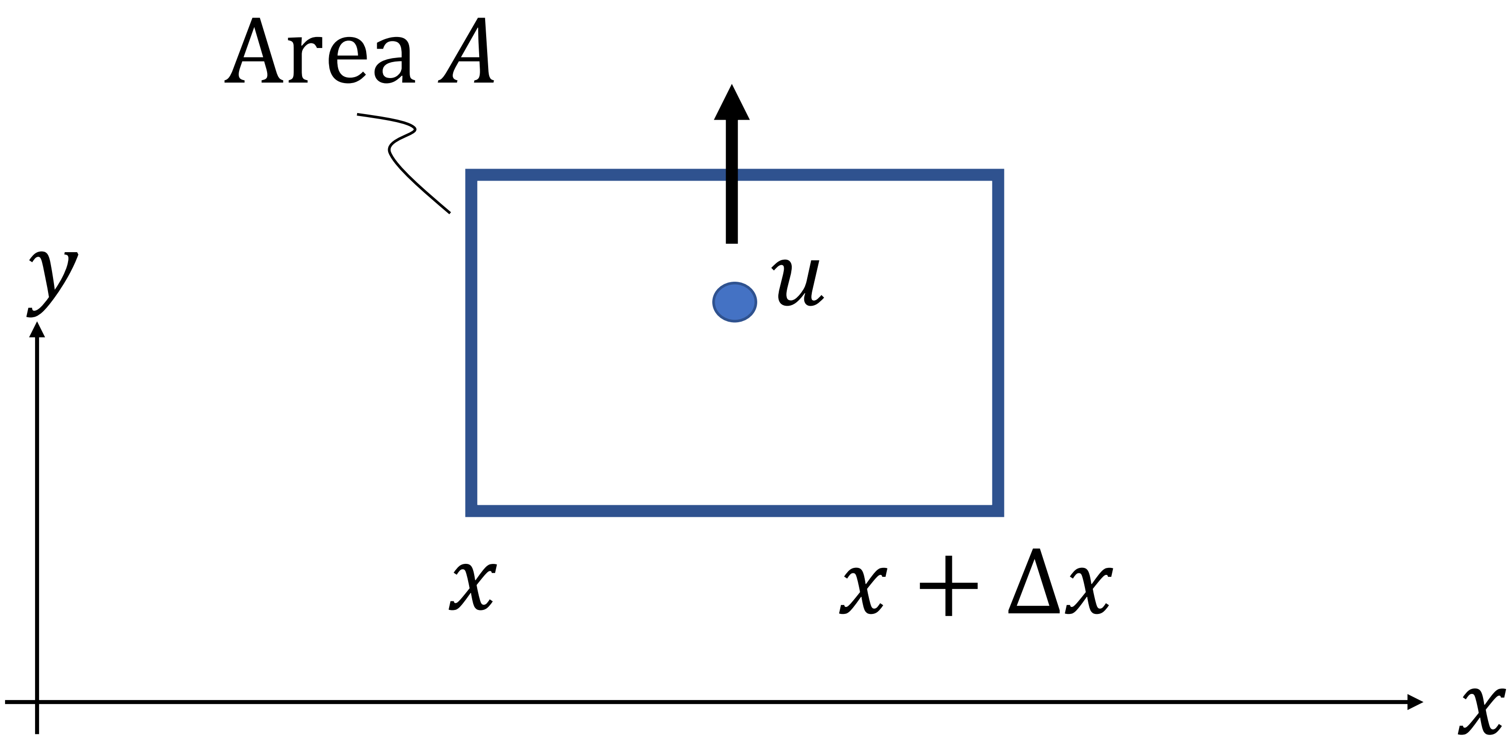


$$F(x) = -A\sigma(x) \qquad F(x + \Delta xp) = A\sigma(x + \Delta x)$$

force in y-direction

$$F(x) + F(x + \Delta xp) = A\sigma(x + \Delta x) - A\sigma(x)$$

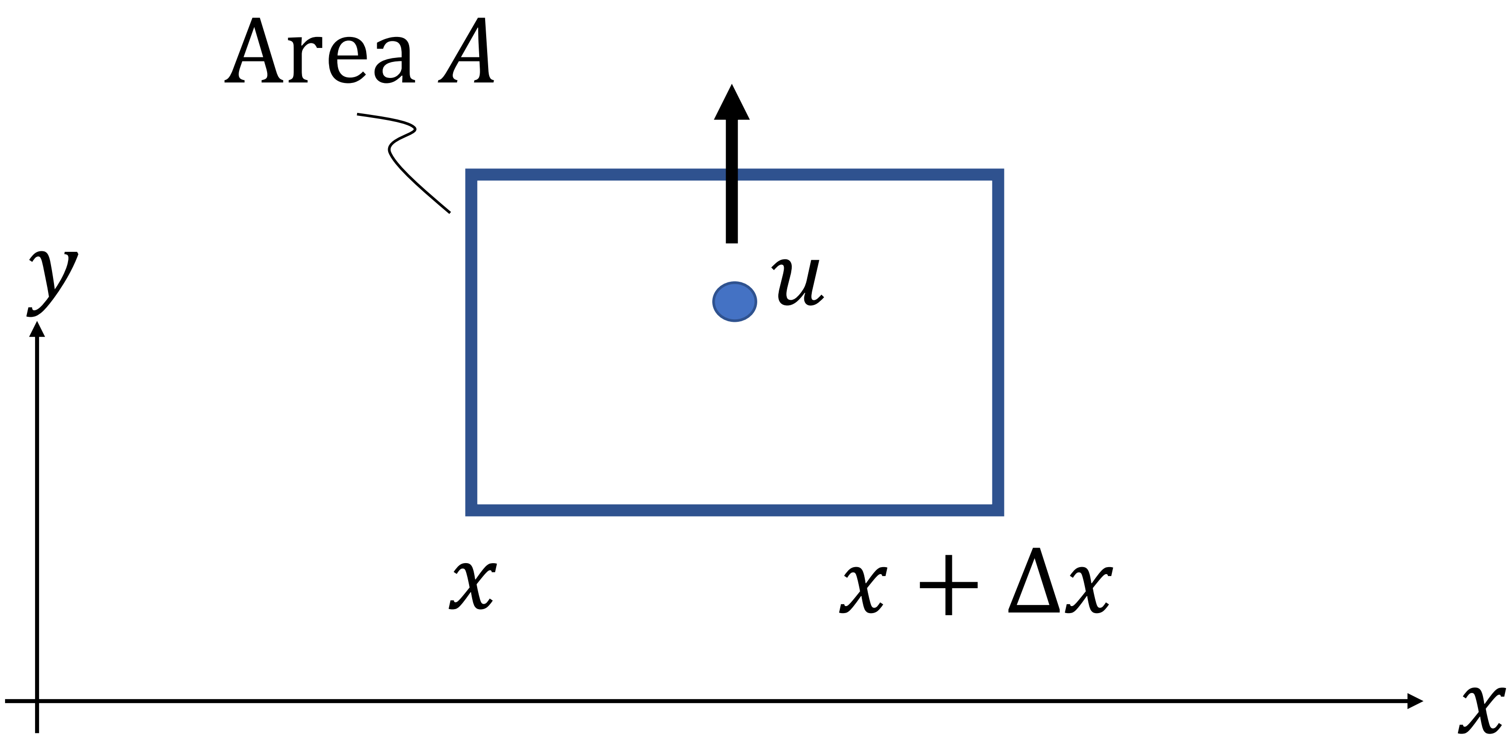
body force



force in y-direction

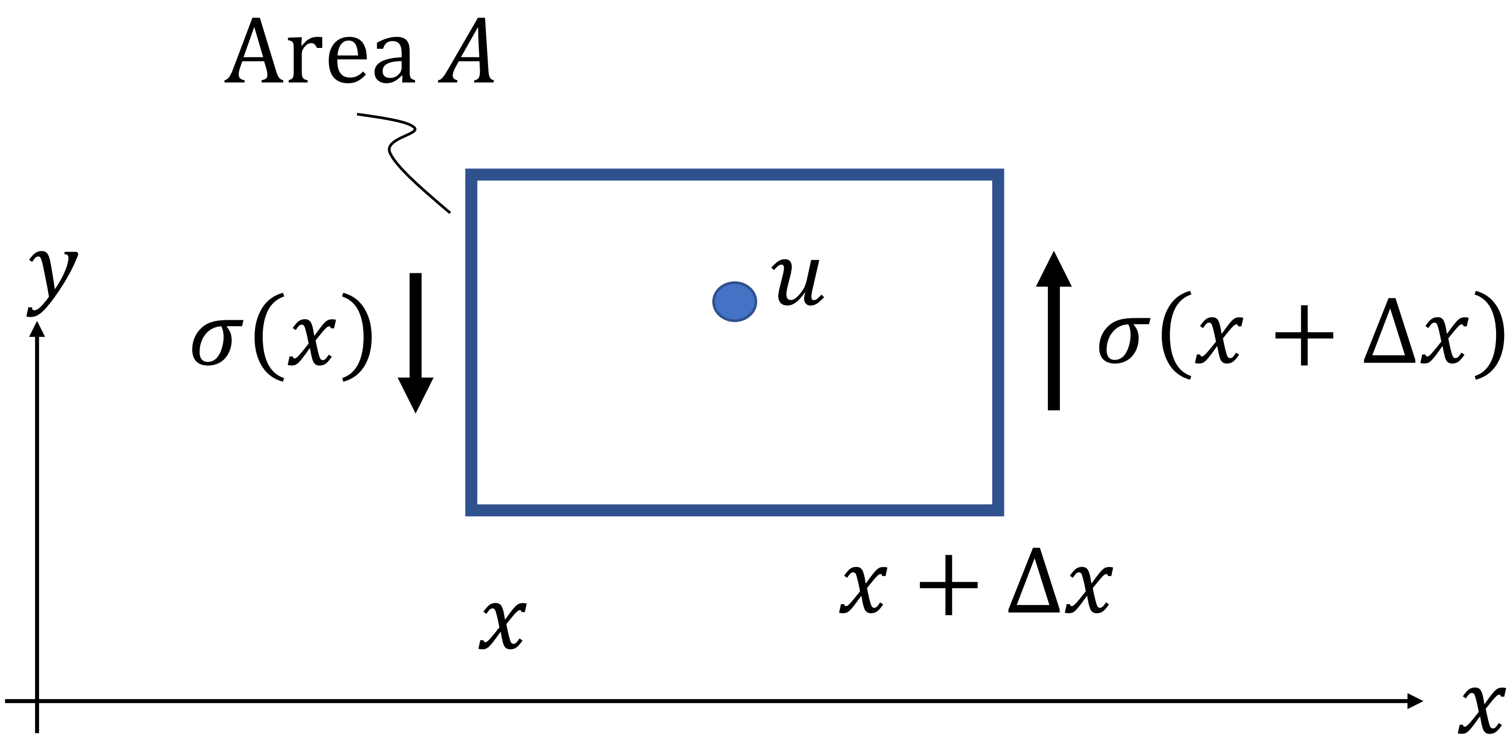
$$F(x) = f\Delta V = fA\Delta x$$

acceleration in y direction



mass × accerleration $\rho A \Delta x \dfrac{d^2 u}{dt^2}$

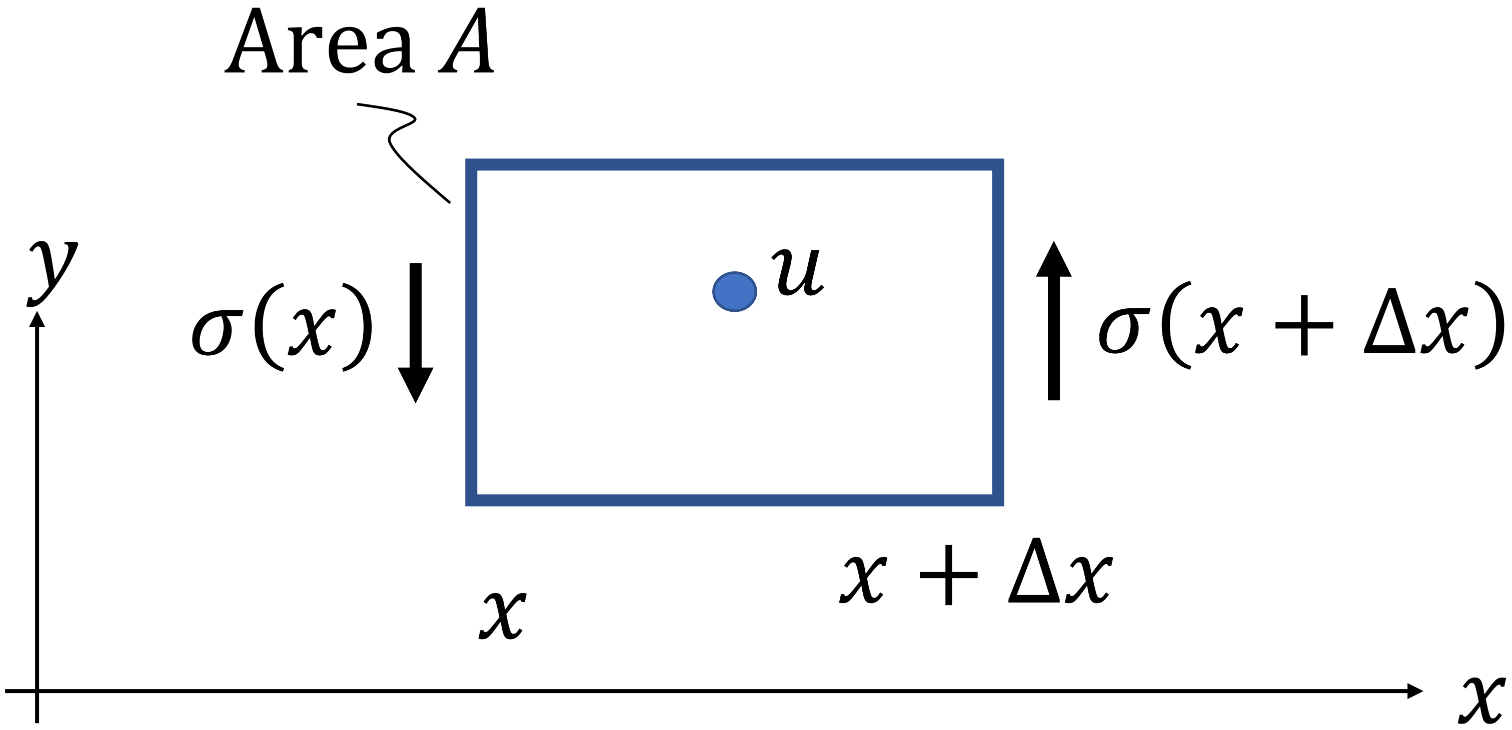
newton's law



motion in y-direction

$$A\sigma(x+\Delta x) - A\sigma(x) + fA\Delta x = \rho \Delta x A \frac{d^2u}{dt^2}$$

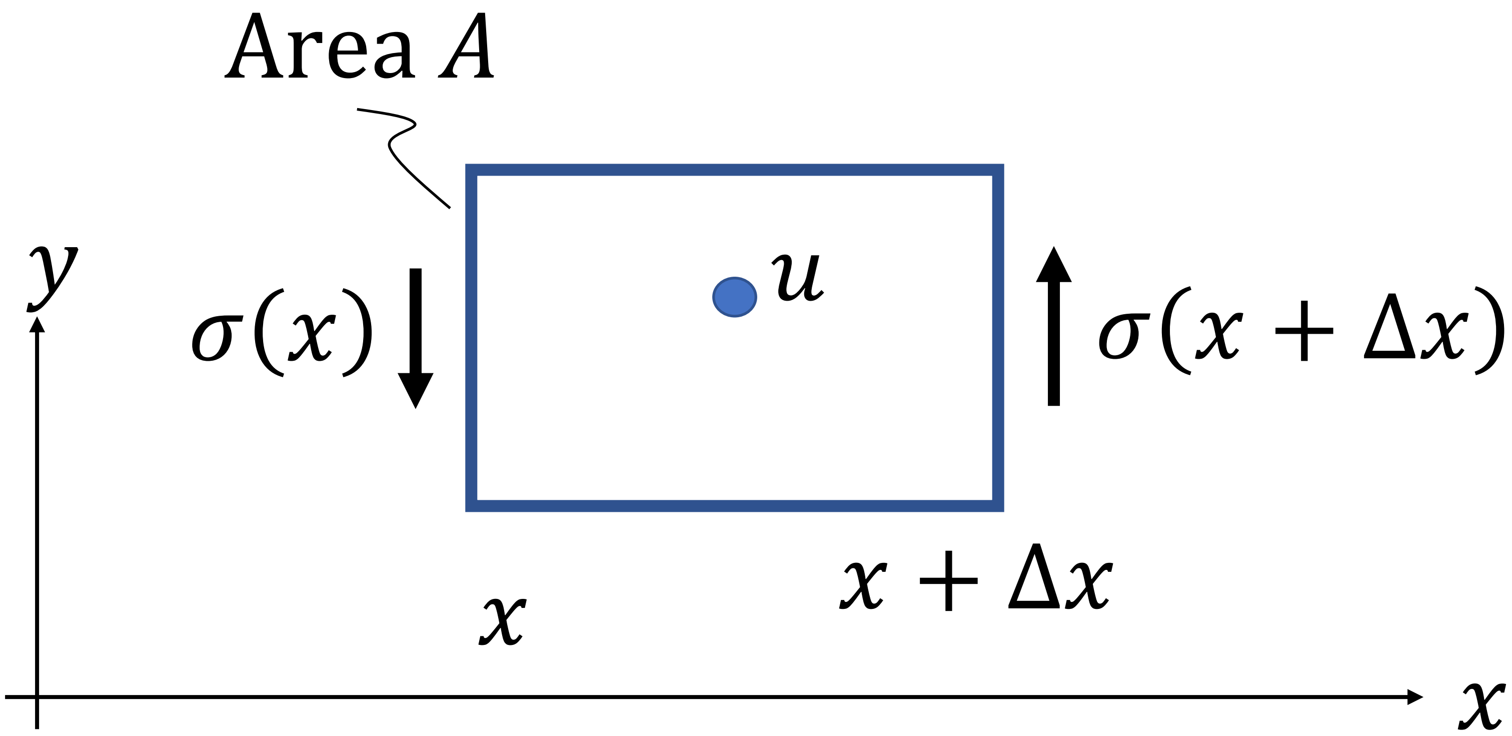
newton's law



motion in y-direction

$$\frac{\sigma(x + \Delta x) - \sigma(x)}{\Delta x} + f = \rho \frac{d^2 u}{dt^2}$$

newton's law



motion in y-direction

$$\frac{d\sigma}{dx} + f = \rho \frac{d^2u}{dt^2}$$

Part 3: Equation for pressure fluctuations in a fluid

$$\sigma = \mu\varepsilon = \mu \frac{du}{dx}$$

shear stress ∝ shear strain

$$\frac{d\sigma}{dx} = \rho \frac{d^2u}{dt^2}$$

Newton’s law

combined equation for displacement

$$\sigma = \mu\varepsilon = \mu\frac{du}{dx}$$

shear stress ∝ shear strain

$$\frac{d\sigma}{dx} = \rho\frac{d^2u}{dt^2}$$

Newton's law

$$\mu\frac{d^2u}{dx^2} = \rho\frac{d^2u}{dt^2}$$

combined equation for shear stress

$$\sigma = \mu\varepsilon = \mu\frac{du}{dx}$$

shear stress ∝ shear strain

$$\frac{d}{dx} \qquad \frac{d\sigma}{dx} = \rho\frac{d^2u}{dt^2} \quad \Rightarrow \quad \frac{d^2\sigma}{dx^2} = \rho\frac{d^2}{dt^2}\frac{du}{dx}$$

$$\mu\frac{d^2\sigma}{dx^2} = \rho\frac{d^2\sigma}{dt^2}$$

$$\mu \frac{d^2u}{dx^2} = \rho \frac{d^2u}{dt^2}$$

$$\mu \frac{d^2\sigma}{dx^2} = \rho \frac{d^2\sigma}{dt^2}$$

shear stress
and
displacement
satisfy similar equations

$$\mu \frac{d^2u}{dx^2} = \rho \frac{d^2u}{dt^2}$$

$$\mu \frac{d^2\sigma}{dx^2} = \rho \frac{d^2\sigma}{dt^2}$$

solution

$$u(x,t) = s\left(t - \frac{x}{\beta}\right)$$

$$\sigma\,(x,t) = -\frac{1}{\beta}\dot{s}\left(t - \frac{x}{\beta}\right)$$

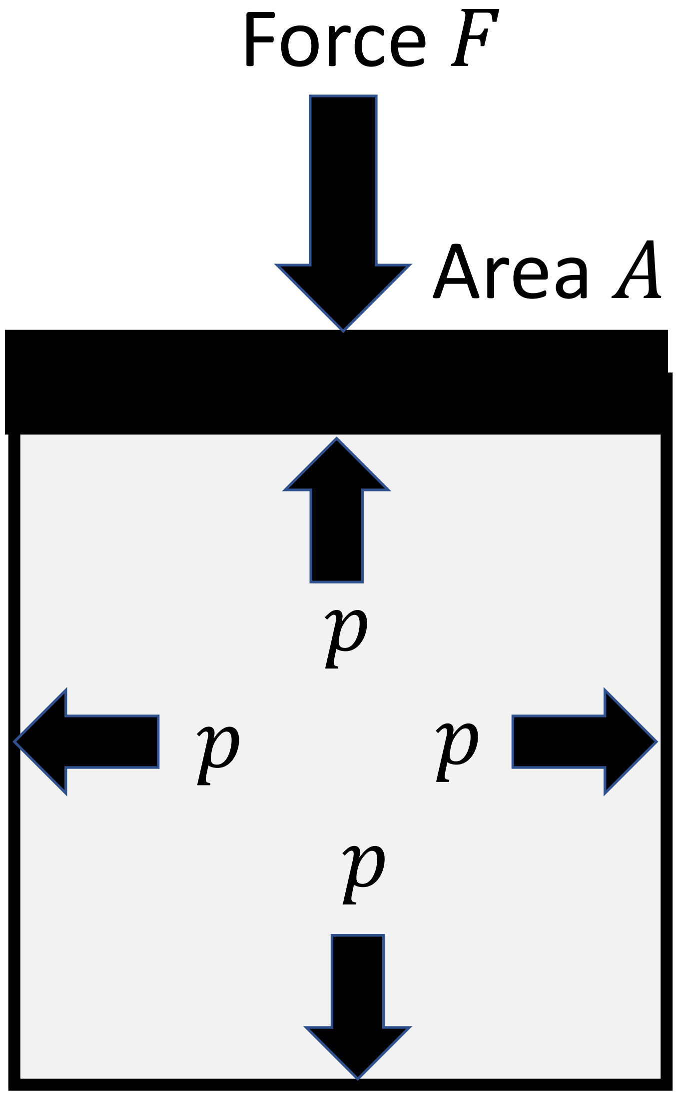
s: any function

shear velocity

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

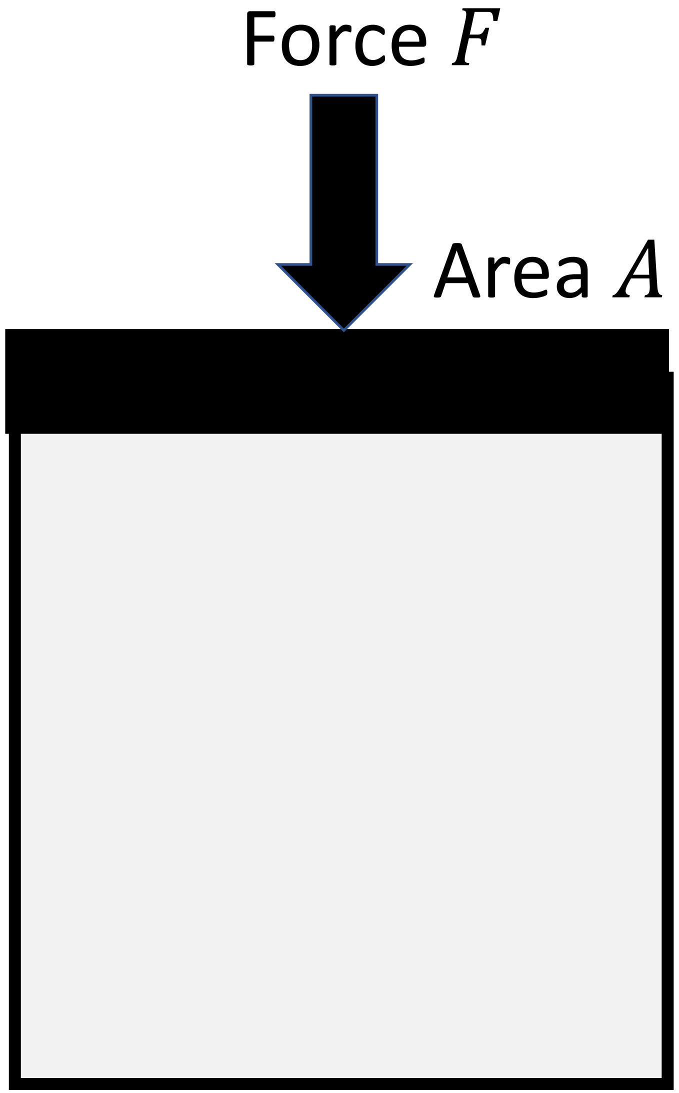
a pulse
maintains its shape
as it propagates
at the shear velocity β

Difference in the way a fluid and a solid responds to a pressure-generating force

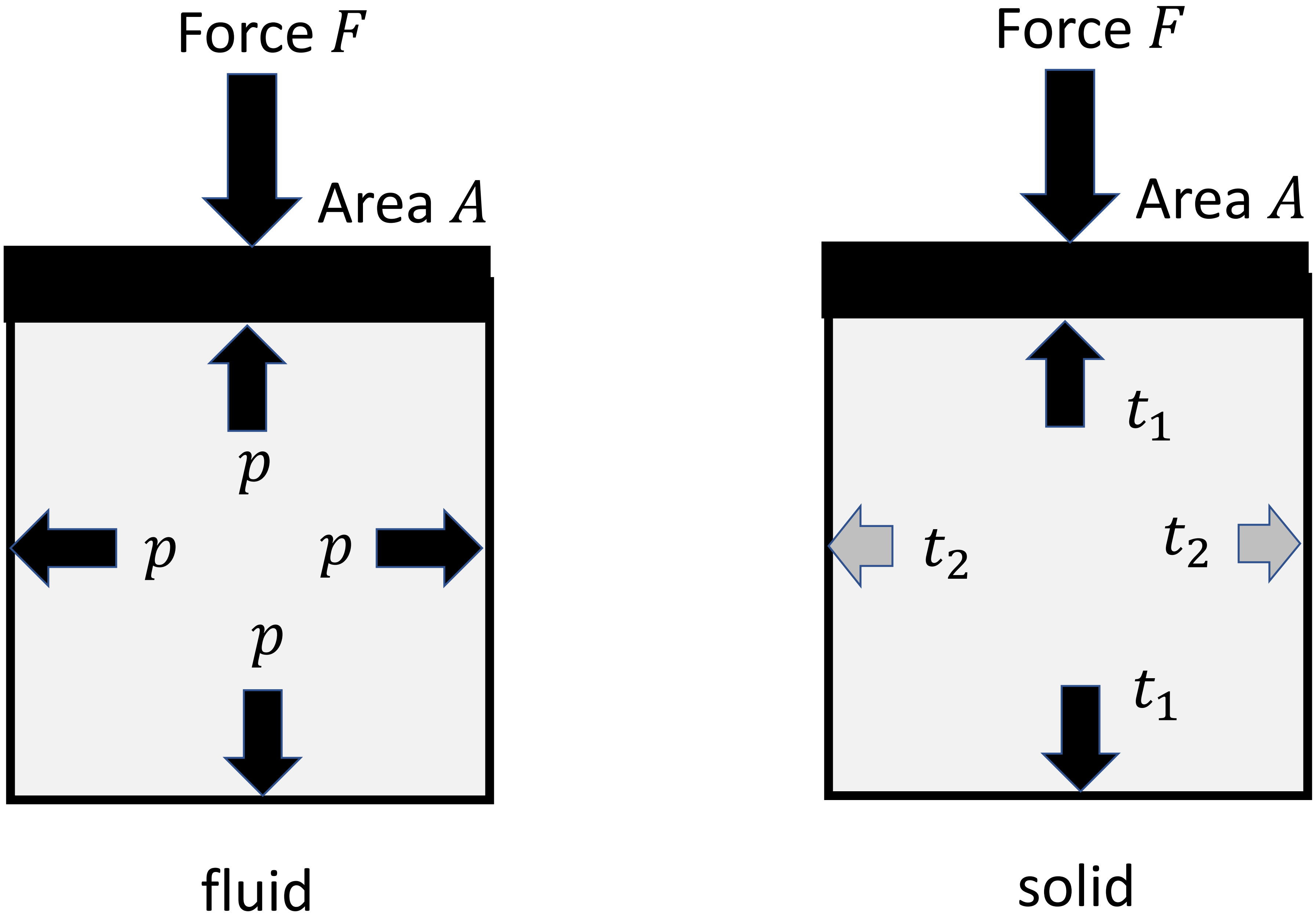


$$p = \frac{F}{A}$$

fluid



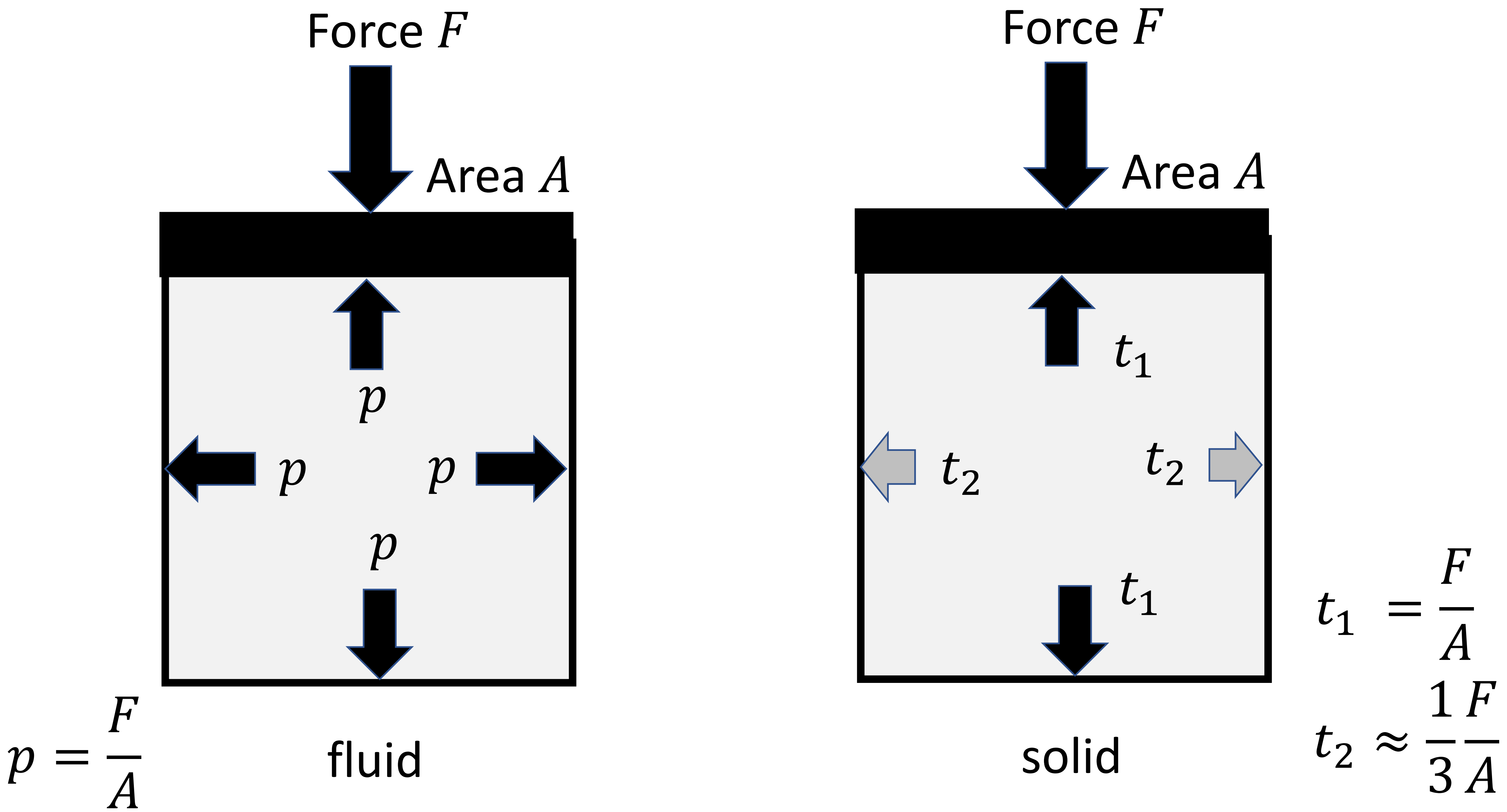
solid



$$p = \frac{F}{A}$$

$$t_1 = \frac{F}{A}$$

$$t_2 < \frac{F}{A}$$



Force F
Area A
p
p
p
p
$p = \frac{F}{A}$
fluid
Force F
Area A
t_1
t_2
t_2
t_1
$t_1 = \frac{F}{A}$
$t_2 \approx \frac{1}{3}\frac{F}{A}$
solid

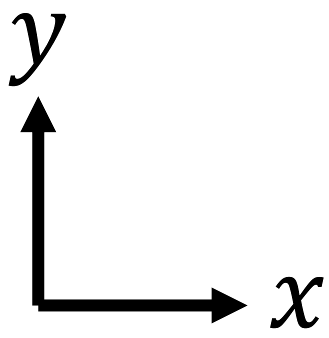
upshot

while there is a “sound-like” wave in a solid

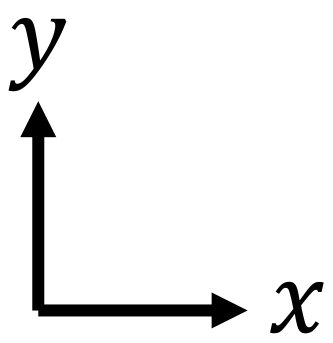
called a “compressional wave”

it doesn’t behave exactly like a sour wave in a fluid “compressional wave”

direction of propagation x



“compressional wave”

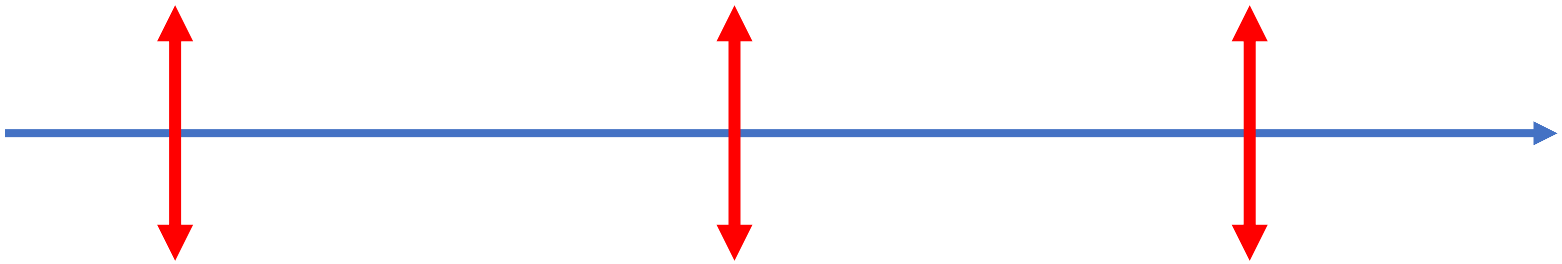


direction of propagation x

direction of displacement x

speed of propagation α

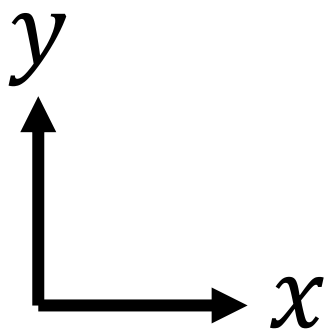
“shear wave”



direction of propagation x

direction of displacement y

speed of propagation β



solid

fluid

$$p = k\frac{\Delta V}{V}$$

$$\frac{t_1 + t_2 + t_3}{3} = k\frac{\Delta V}{V}$$

$$\sigma = \mu\frac{d\varepsilon}{dv}$$

speed of sound

$$v = \sqrt{\frac{k}{\rho}}$$

compressional wave speed

$$\alpha = \sqrt{\frac{k + 4\mu/3}{\rho}}$$

shear wave speed

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

k: bulk modulus

μ: shear modulus

solid

fluid

$$p = k\frac{\Delta V}{V}$$

$$\frac{t_1 + t_2 + t_3}{3} = k\frac{\Delta V}{V}$$

$$\sigma = \mu\frac{d\varepsilon}{dv}$$

speed of sound

$$v = \sqrt{\frac{k}{\rho}}$$

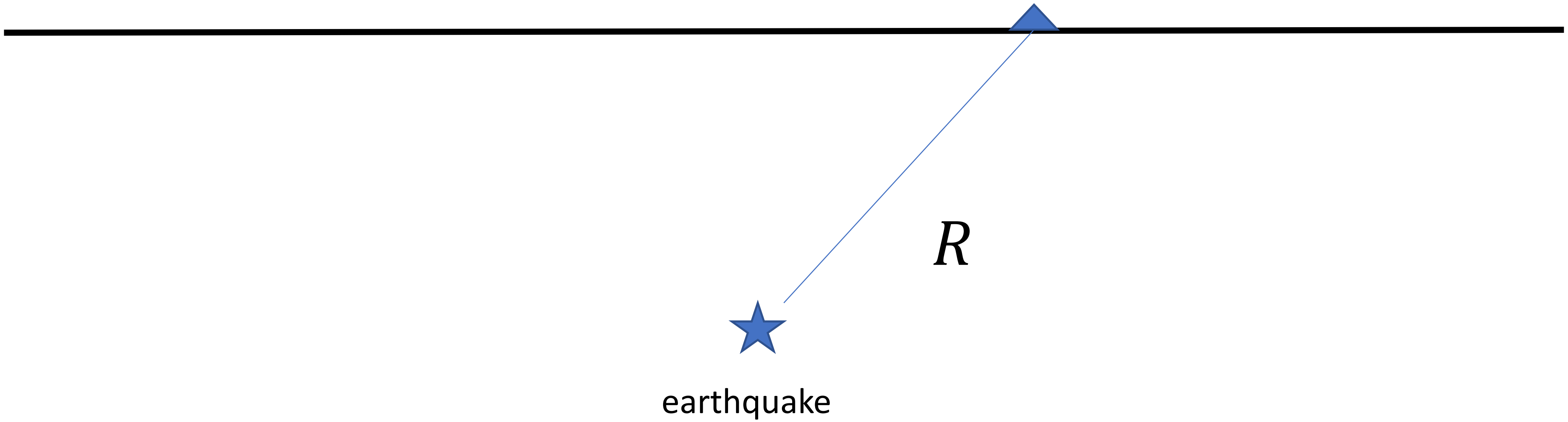
compressional wave speed

$$\alpha = \sqrt{\frac{k + 4\mu/3}{\rho}}$$

shear wave speed

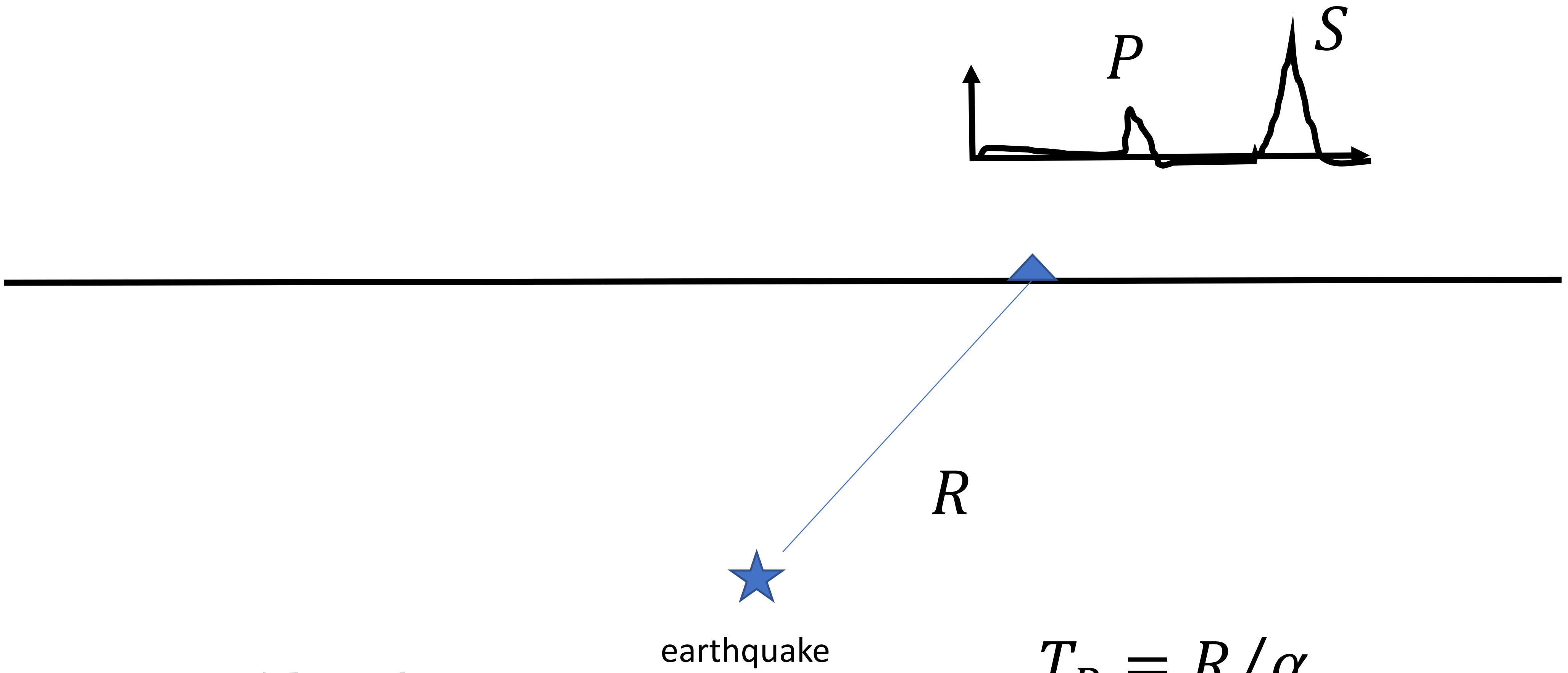
$$\beta = \sqrt{\frac{\mu}{\rho}}$$

for rocks $k \approx \frac{5}{3}\mu$ so $\frac{\alpha}{\beta} = \sqrt{3}$



$$\alpha = 6.5 \text{ km/s}$$

$$\beta = 3.75 \text{ km/s}$$

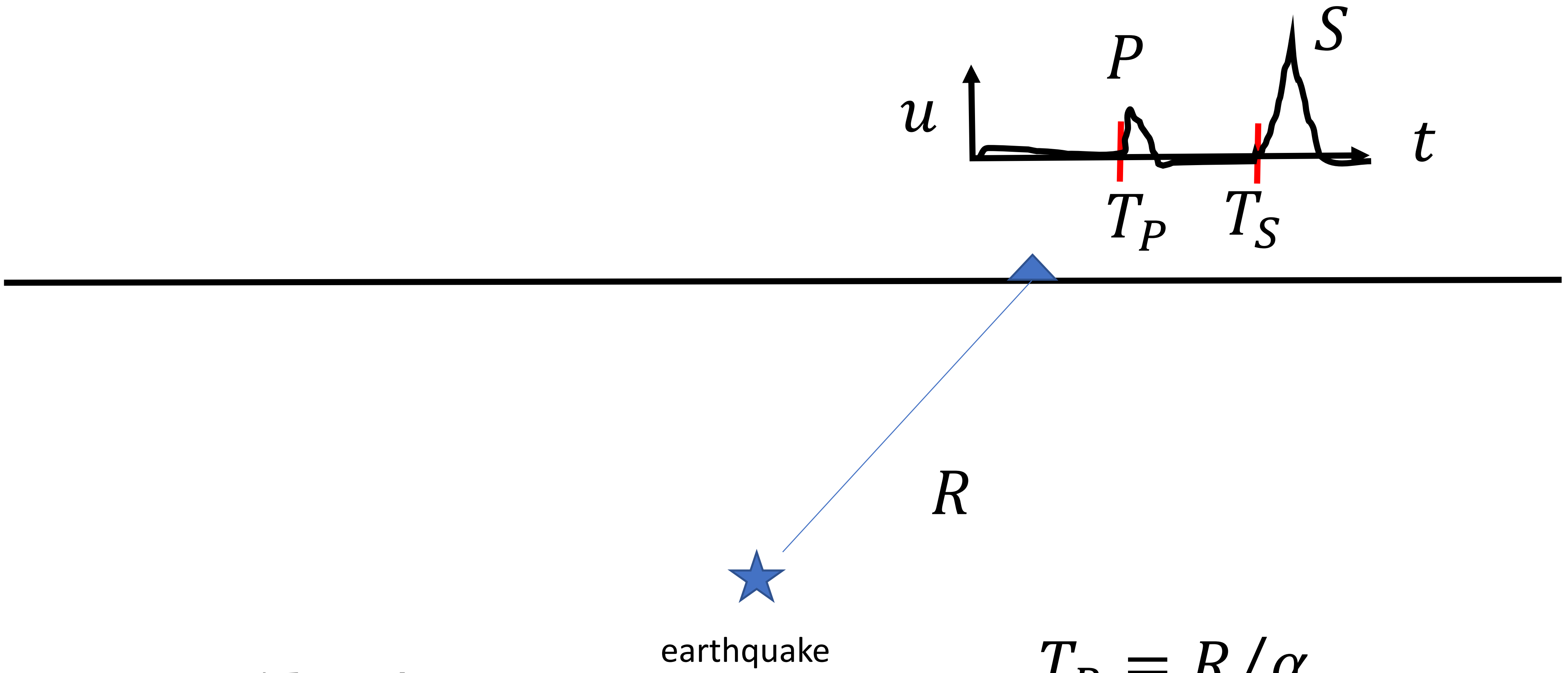


$\alpha = 6.5 \text{ km/s}$

$\beta = 3.75 \text{ km/s}$

$T_P = R/\alpha$

$T_S = R/\beta$



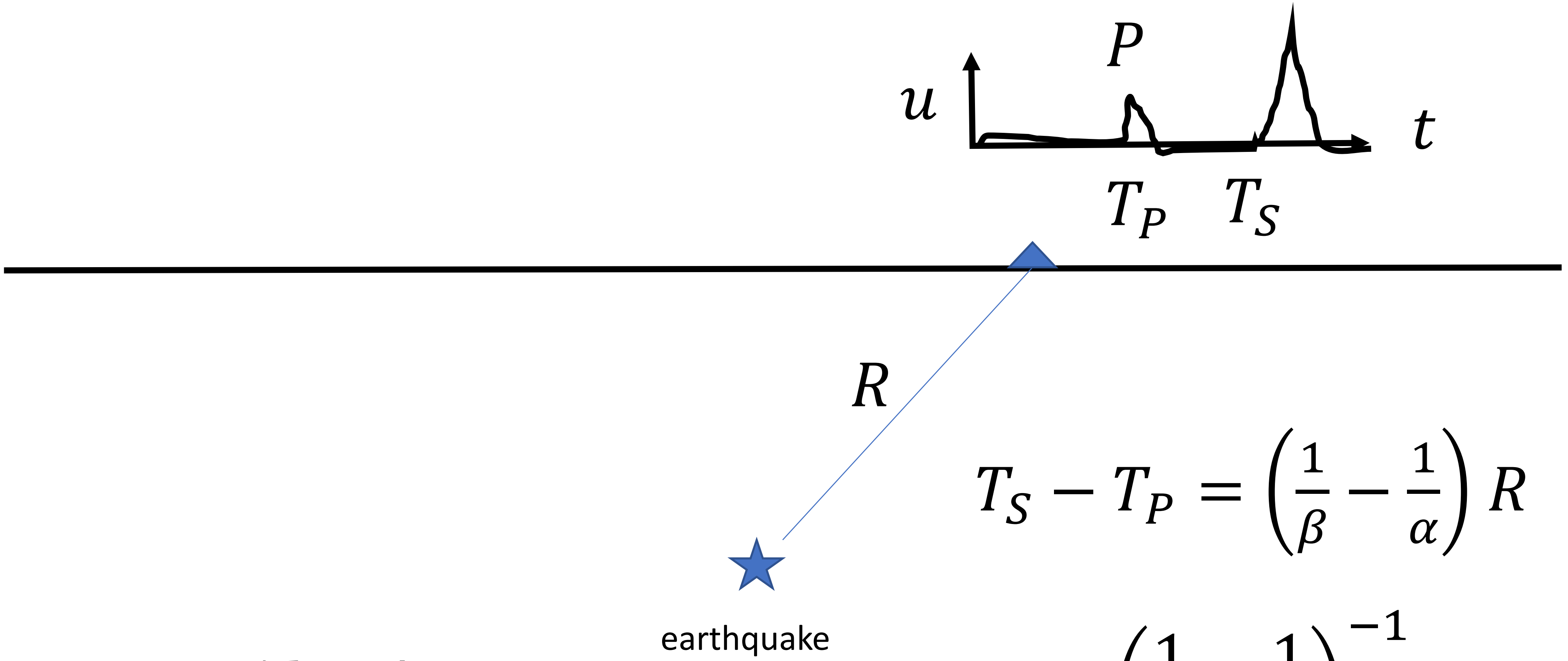
$\alpha = 6.5$ km/s

$\beta = 3.75$ km/s

$$T_P = R/\alpha$$

$$T_S = R/\beta$$

$$T_S - T_P = \left(\frac{1}{\beta} - \frac{1}{\alpha}\right) R$$



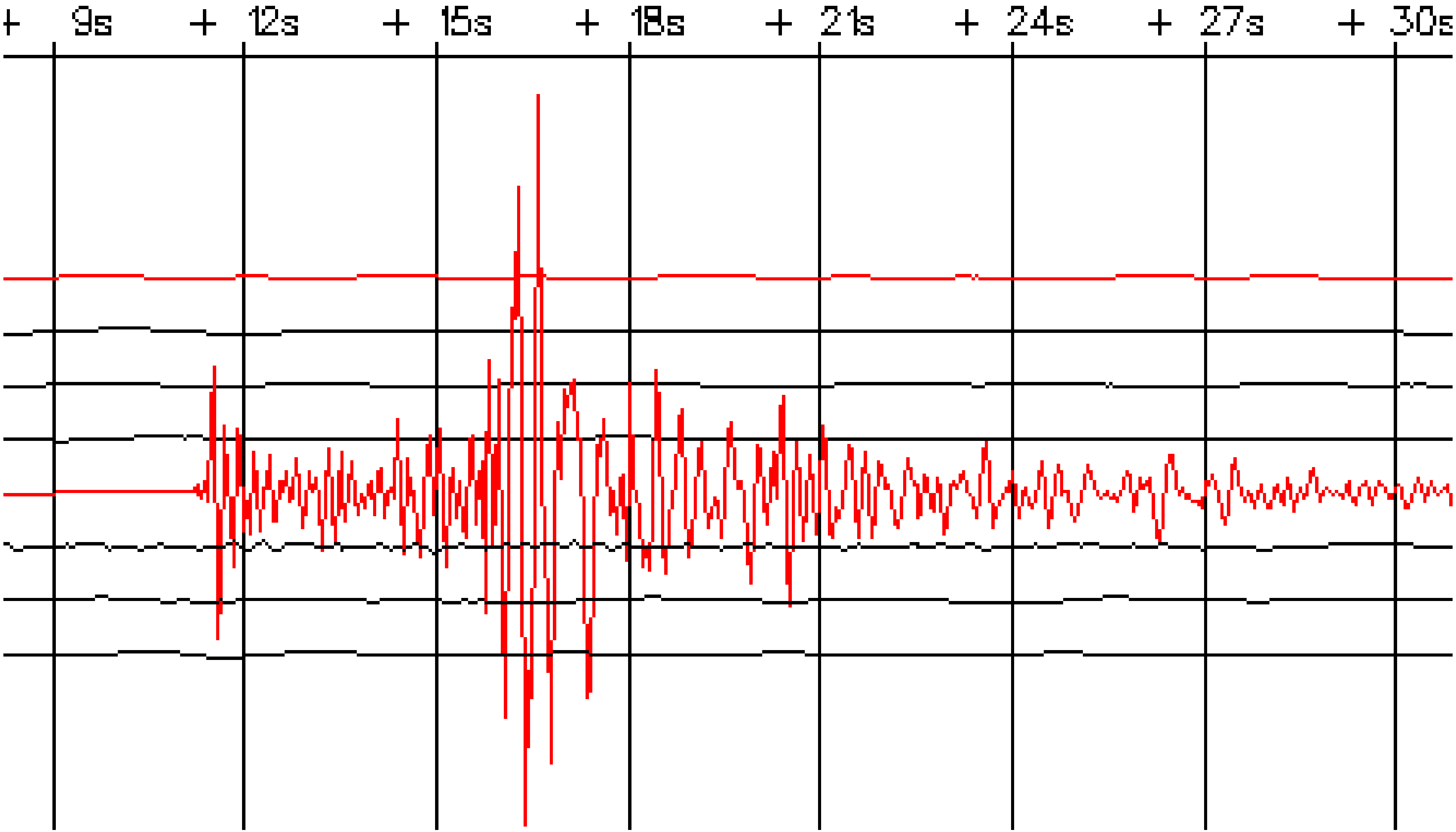
$$T_S - T_P = \left(\frac{1}{\beta} - \frac{1}{\alpha}\right) R$$

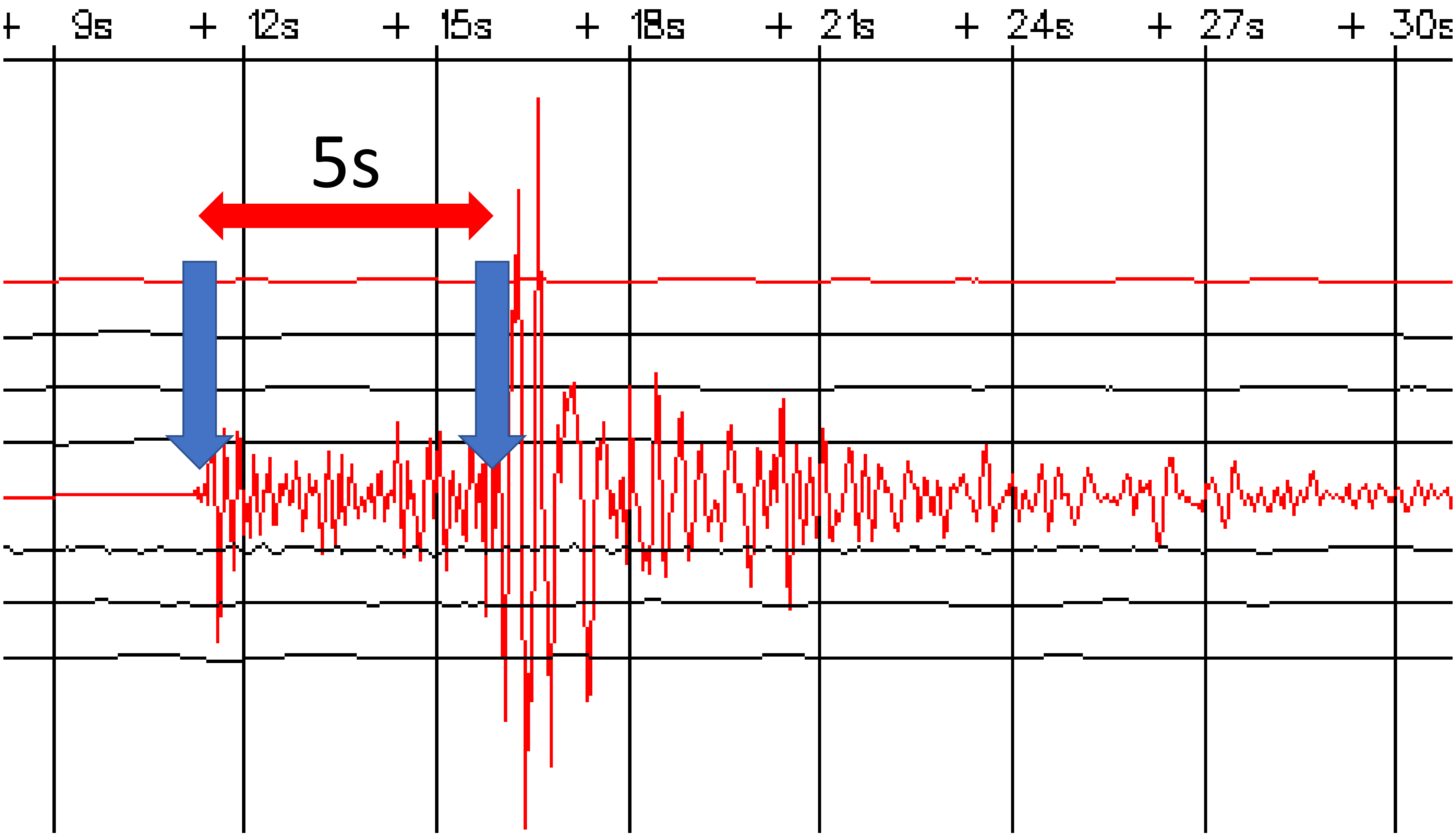
$$R = \left(\frac{1}{\beta} - \frac{1}{\alpha}\right)^{-1} (T_S - T_P)$$

$$R \approx 10\,(T_S - T_P)$$

$\alpha = 6.5$ km/s

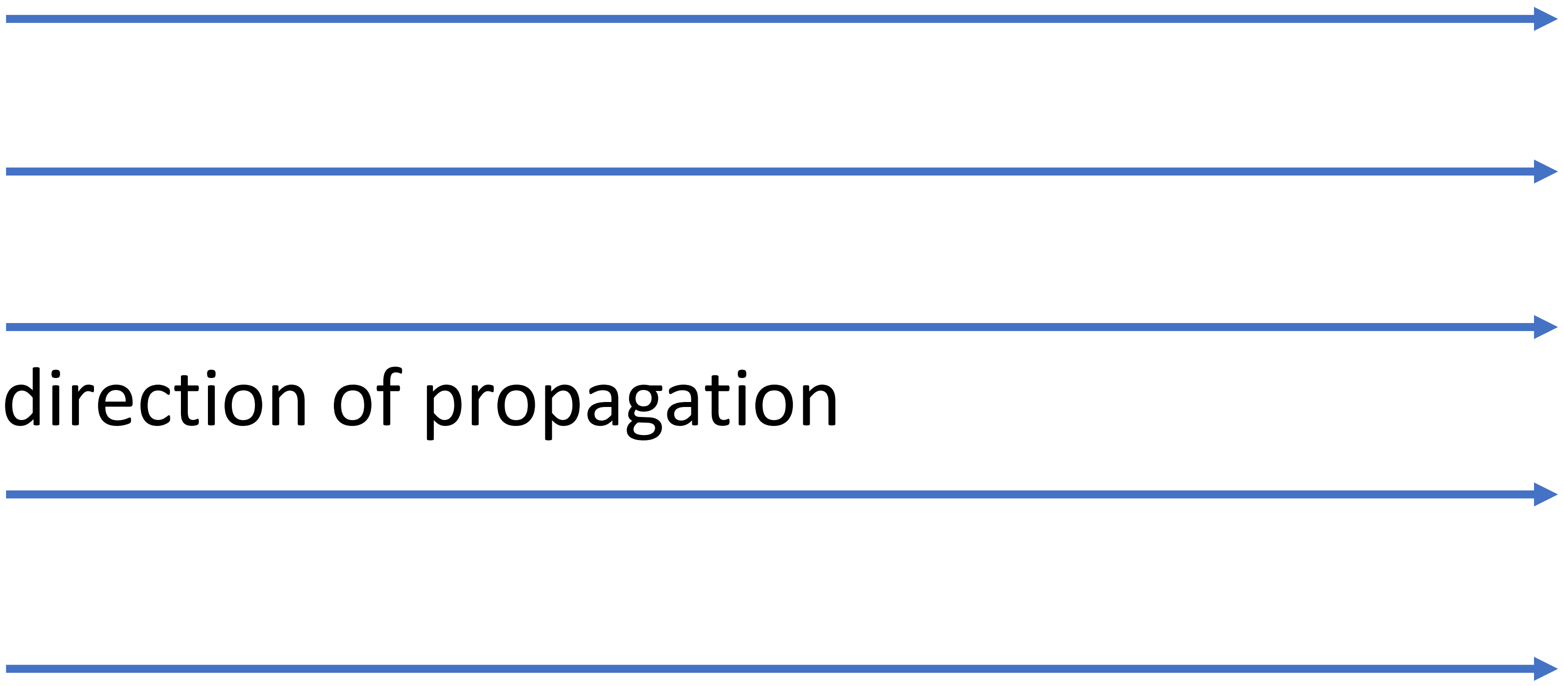
$\beta = 3.75$ km/s


9s
12s
15s
18s
21s
24s
27s
30s

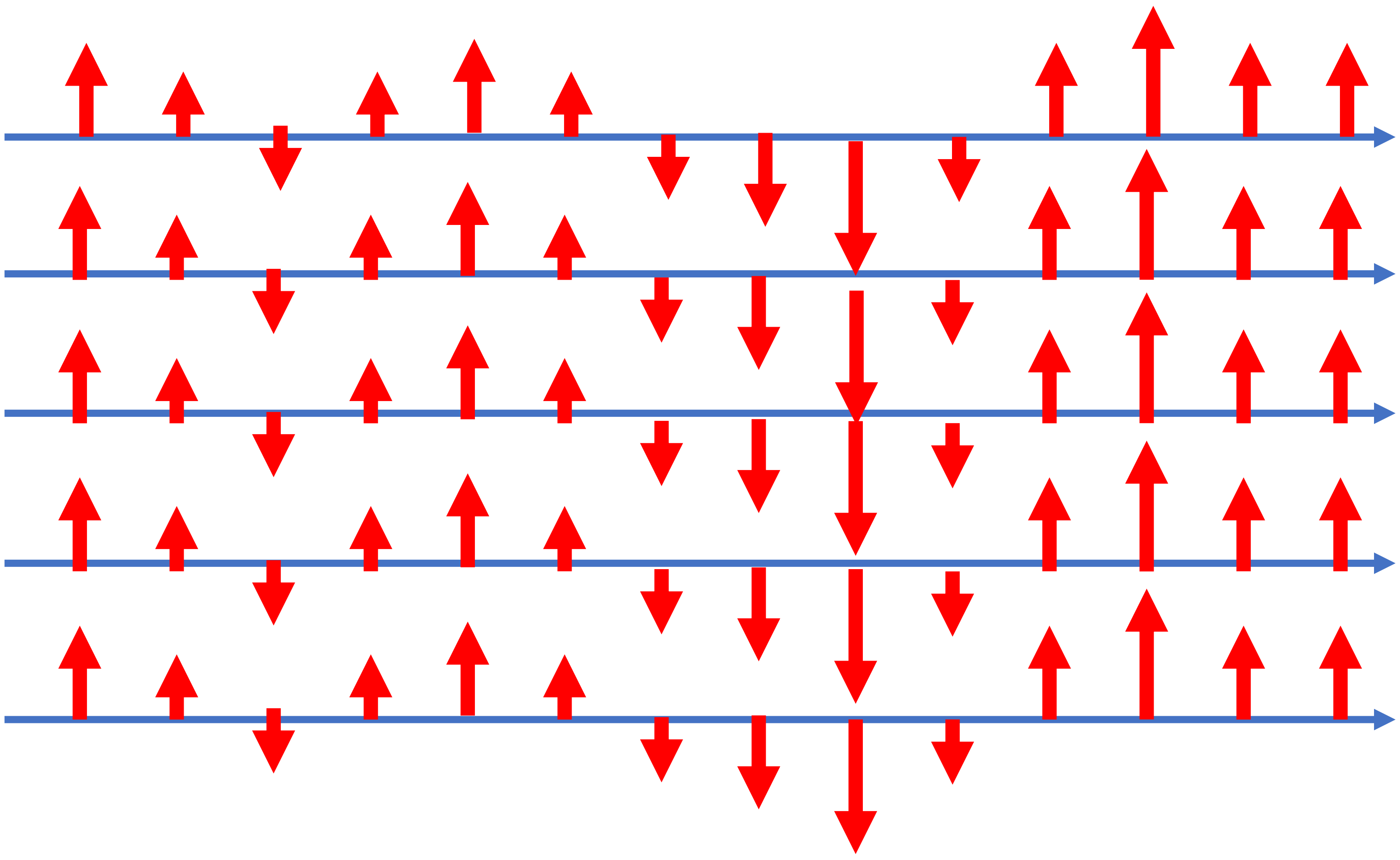


$$R \approx 10 \times 5 = 50\ km$$

shear waves in two dimensions

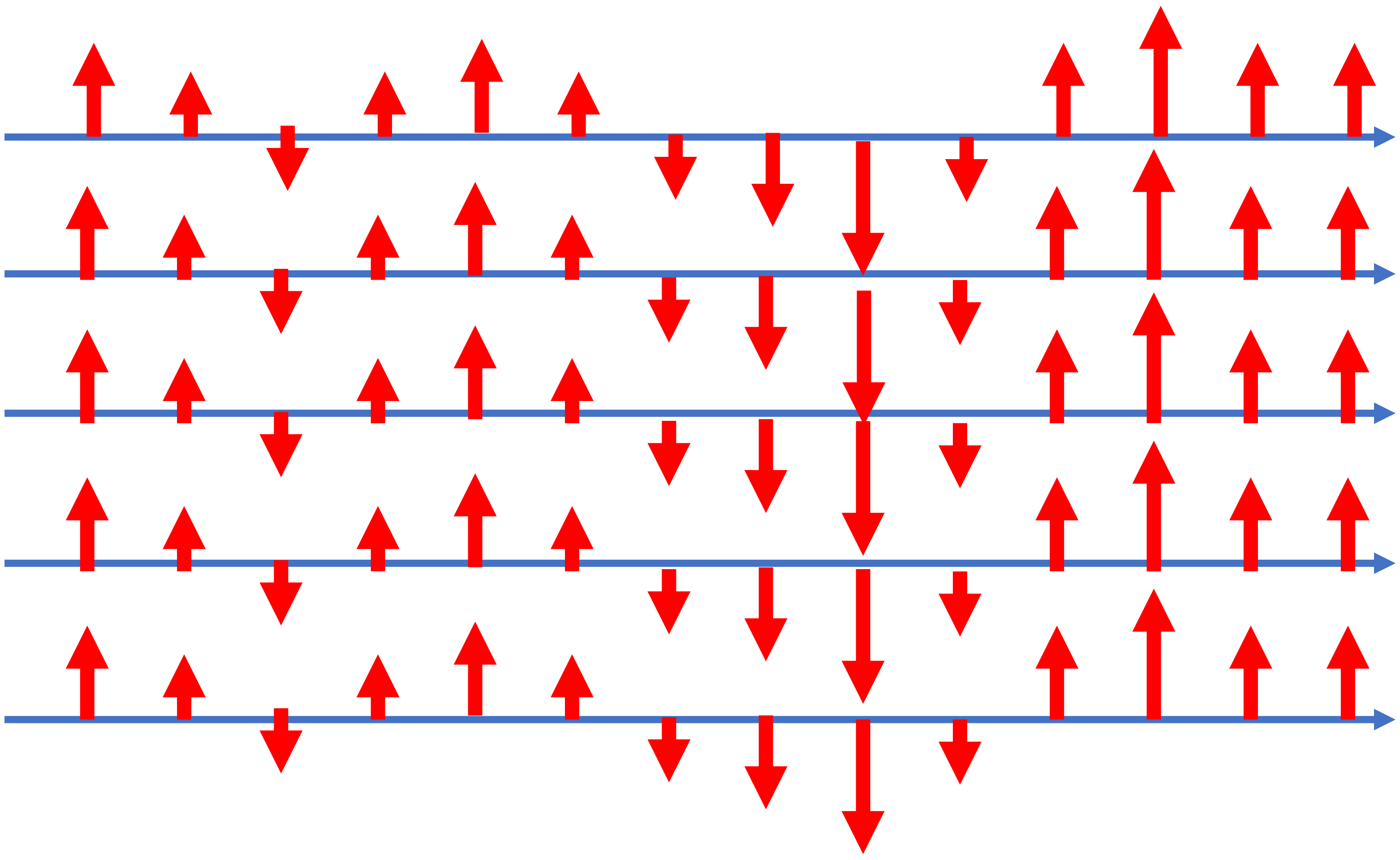


time t=0



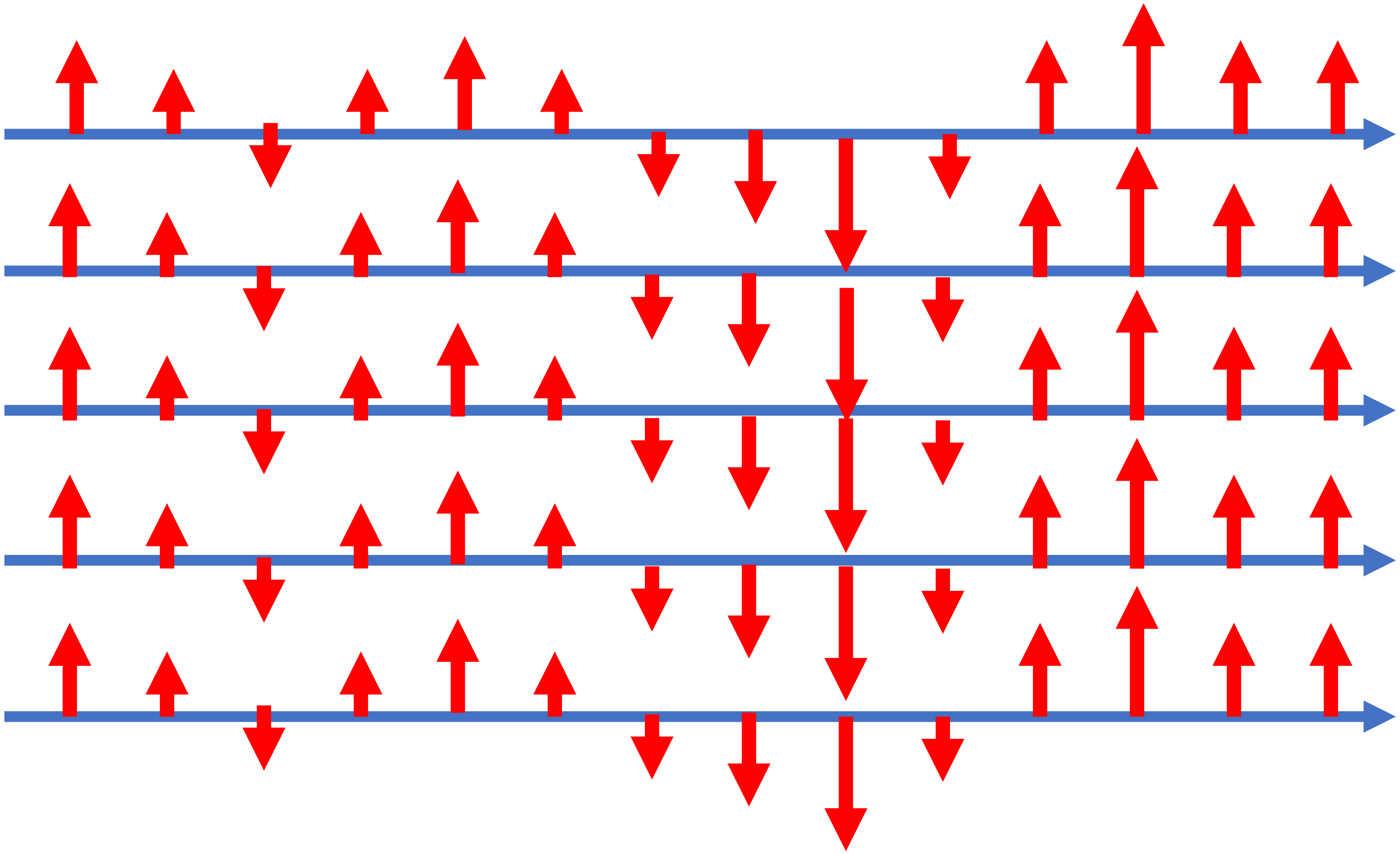
direction of displacement

time t=1



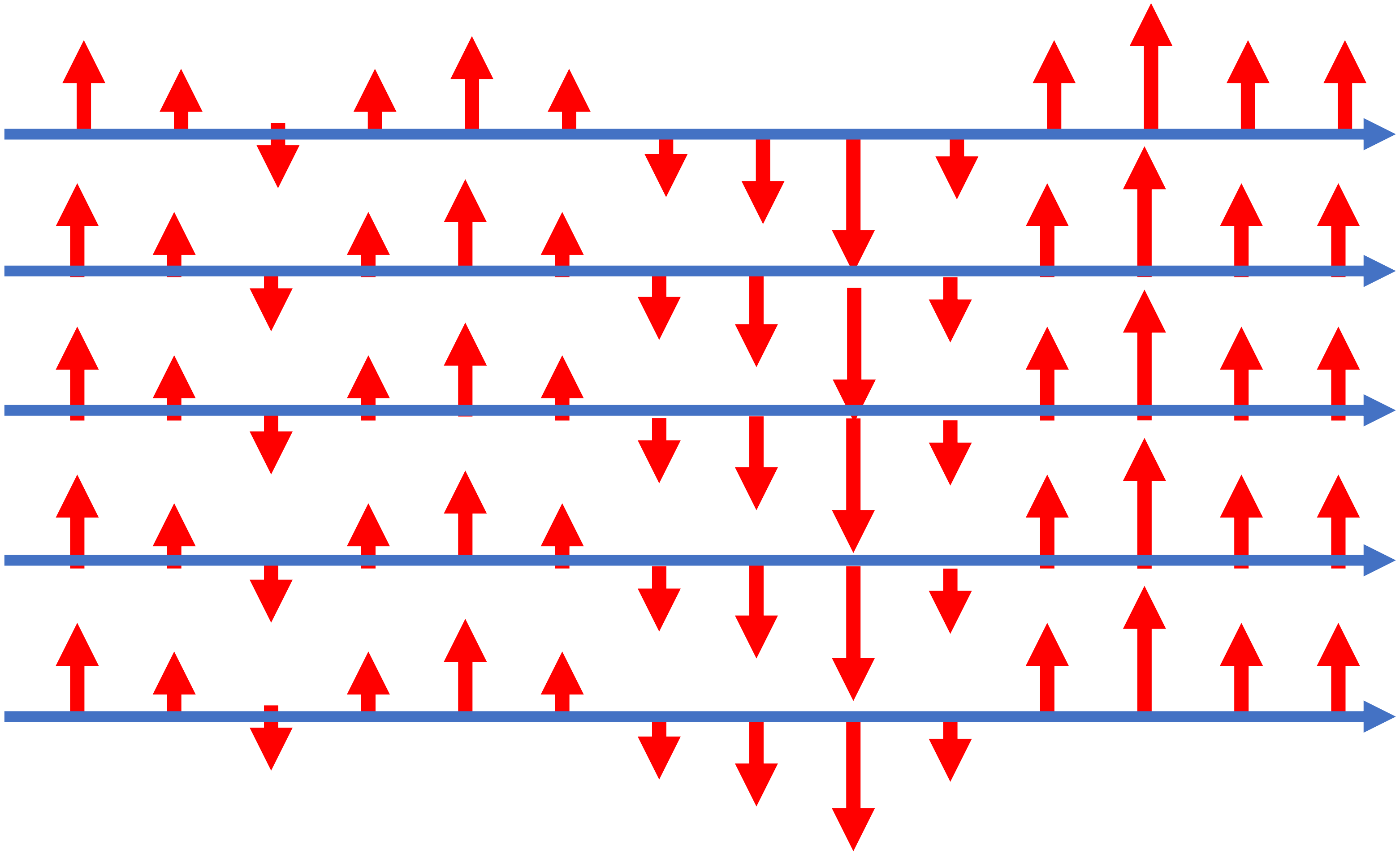
moving at shear velocity

time t=2



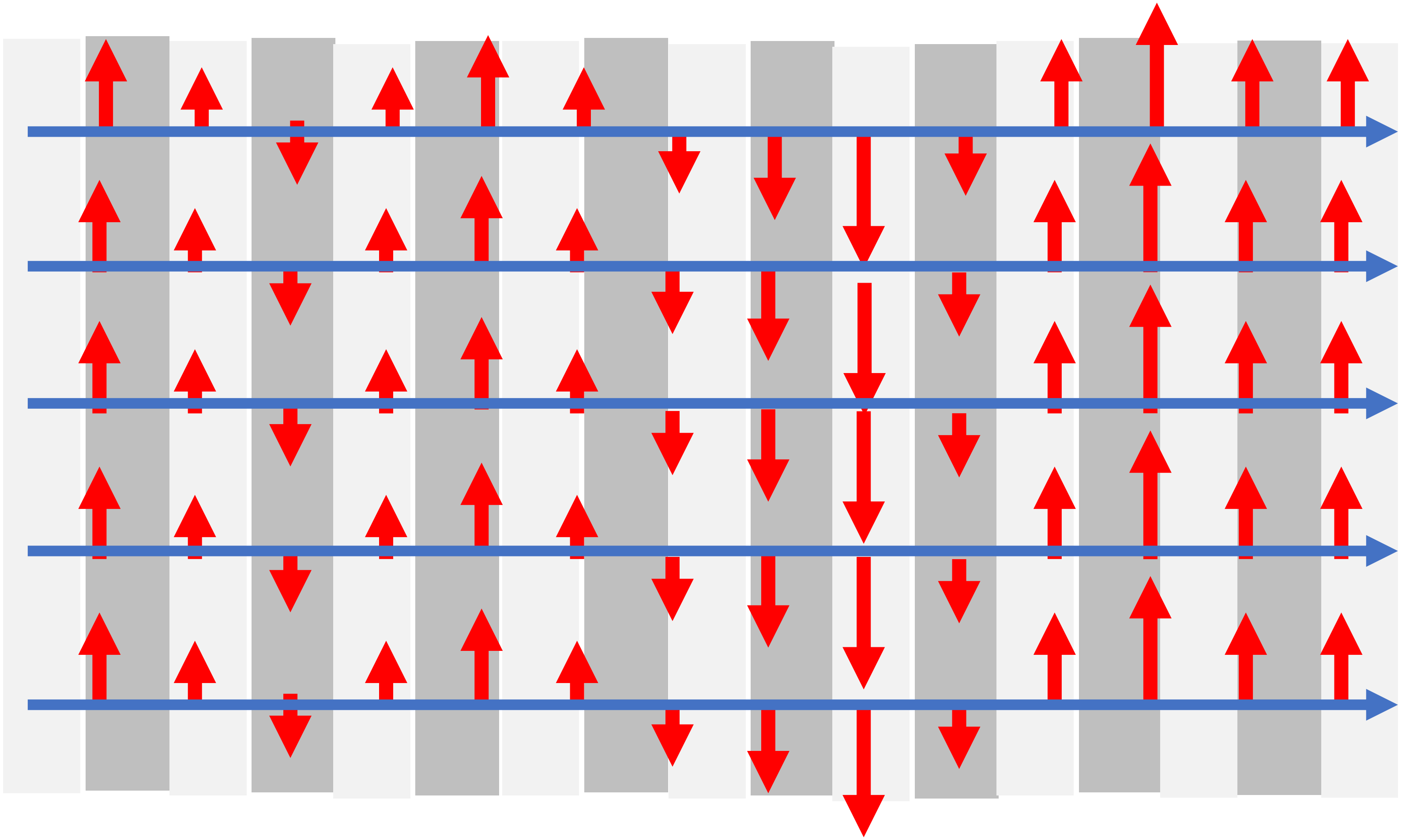
moving at shear velocity

time t=3



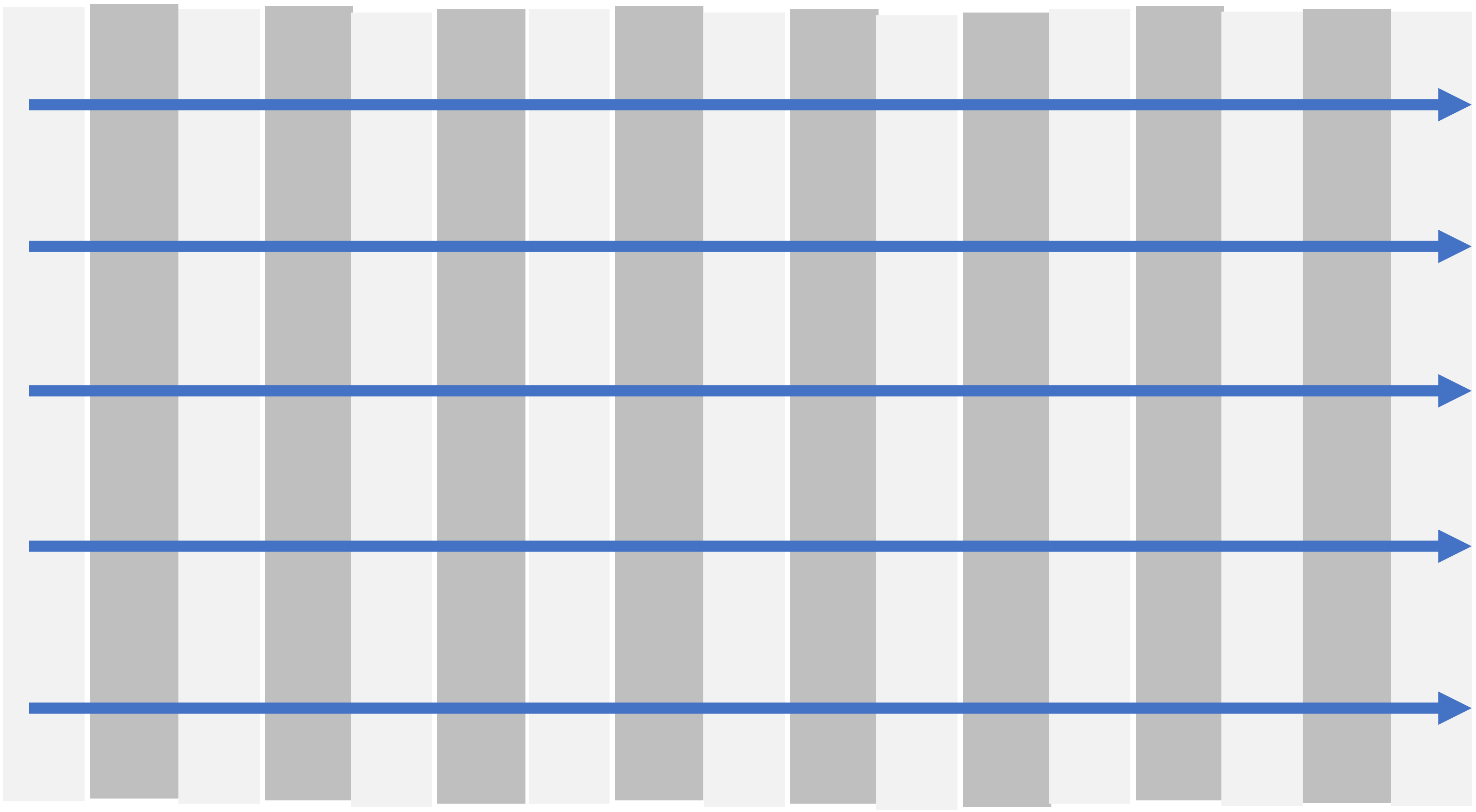
moving at shear velocity

wavefronts

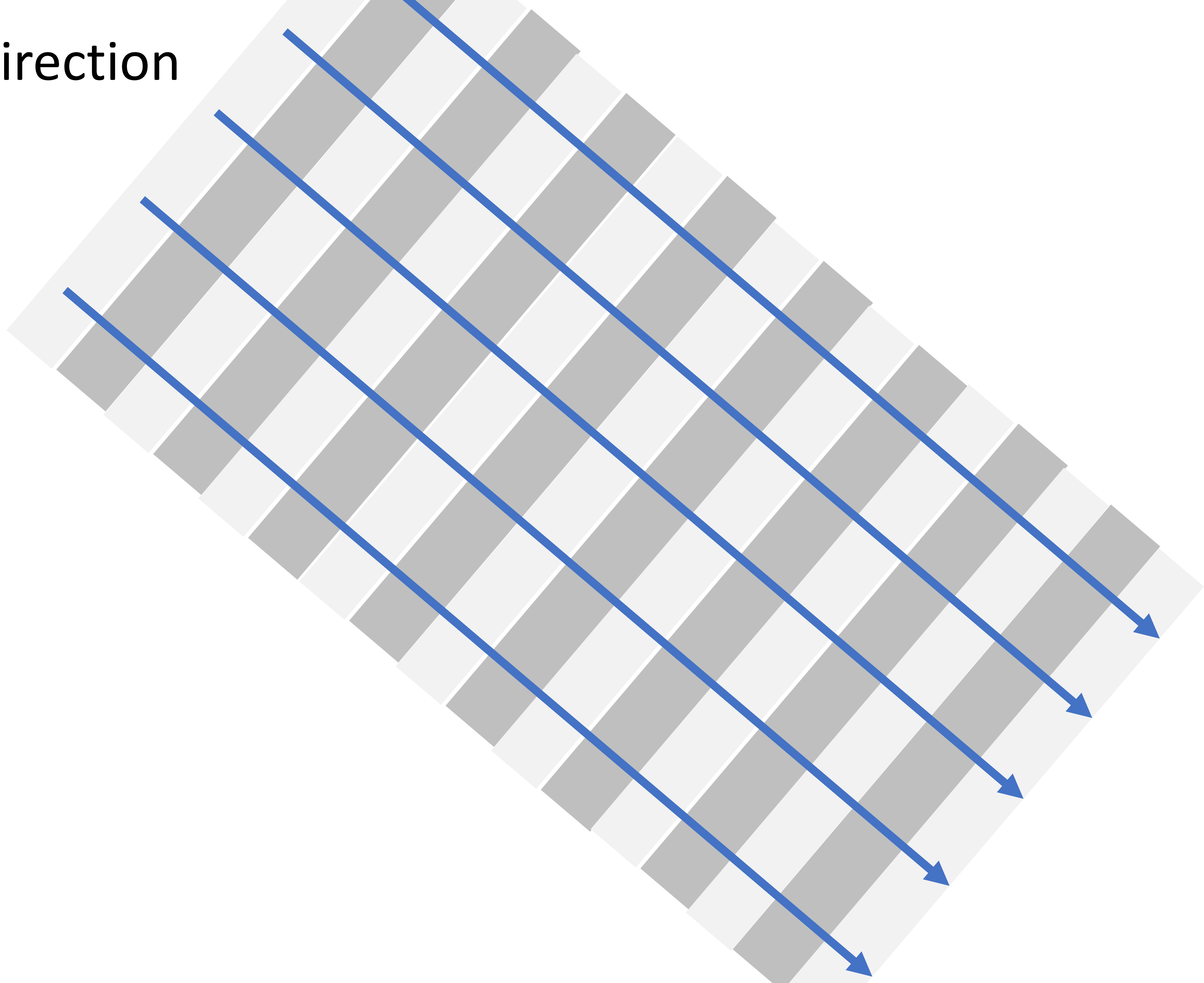


displacement on all points on a wavefront the same

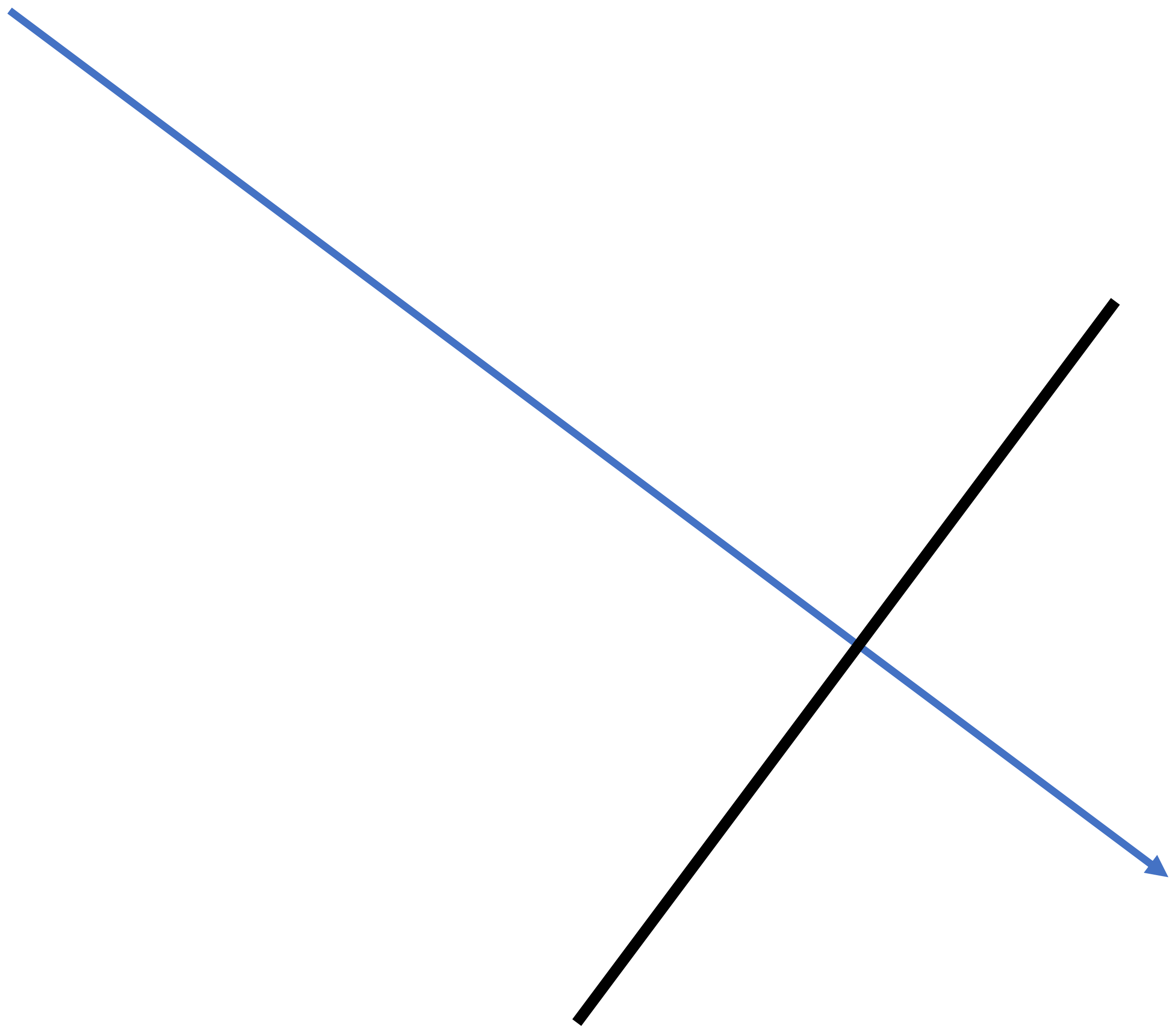
wavefronts



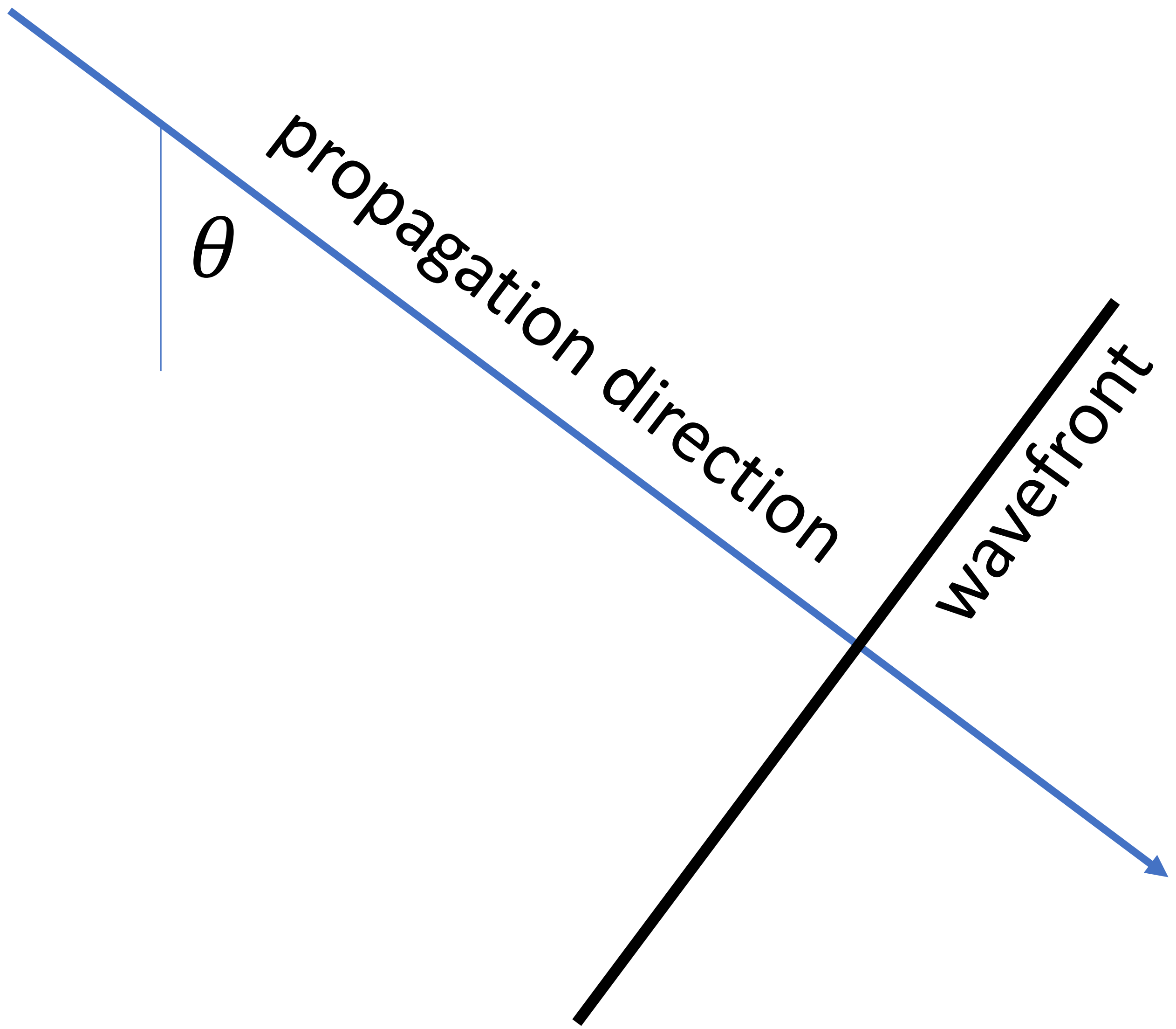
different direction



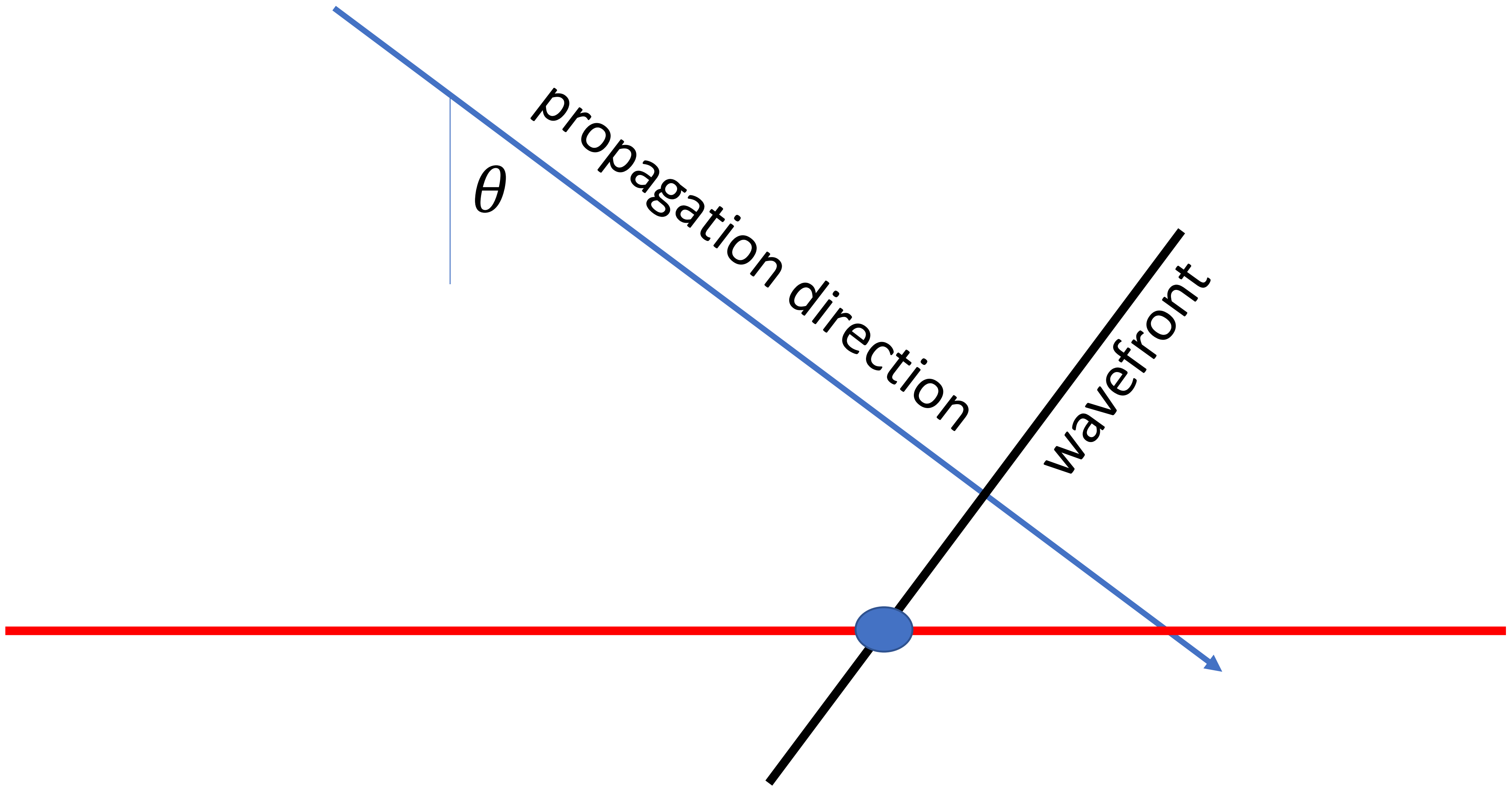
simplified



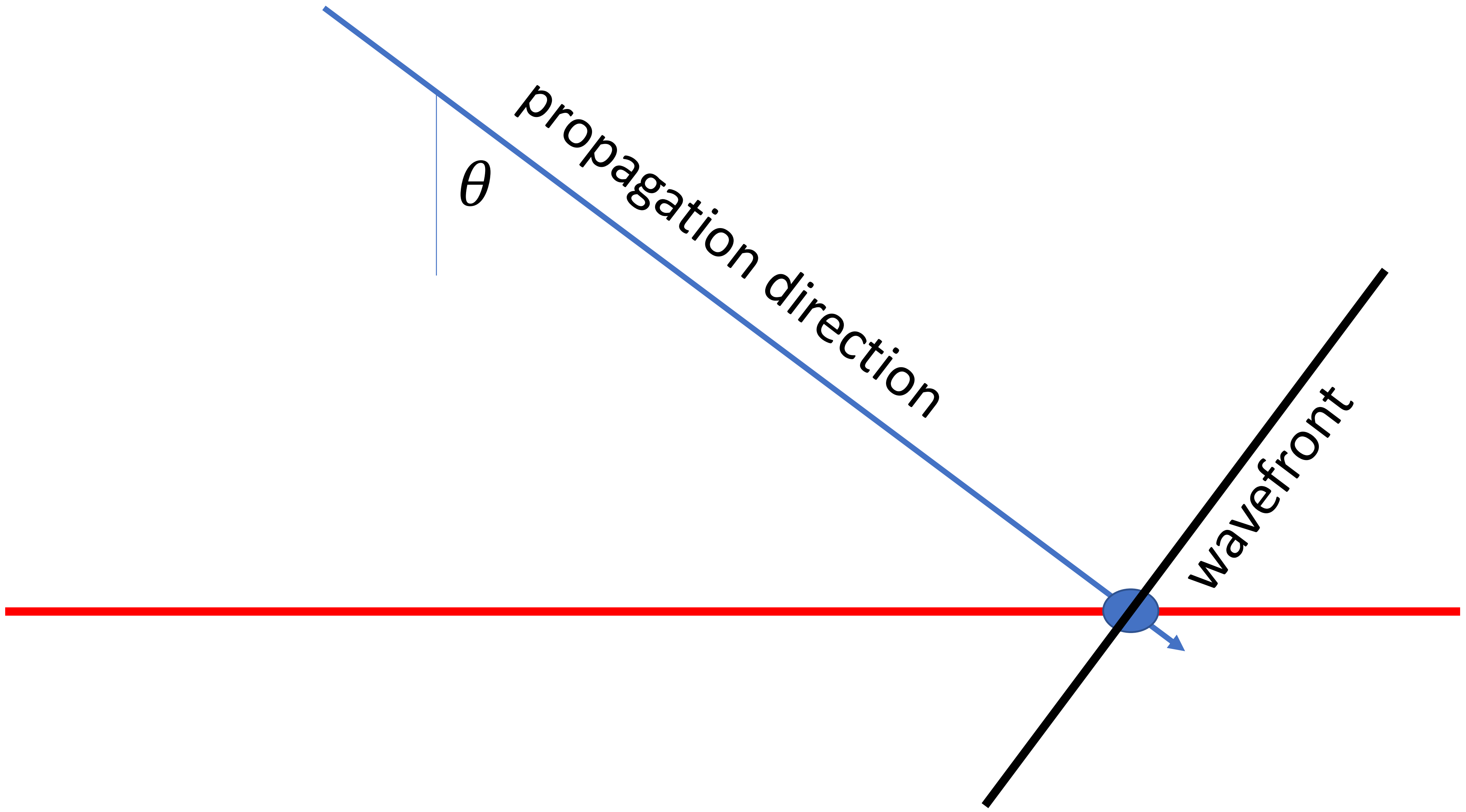
angle of incidence



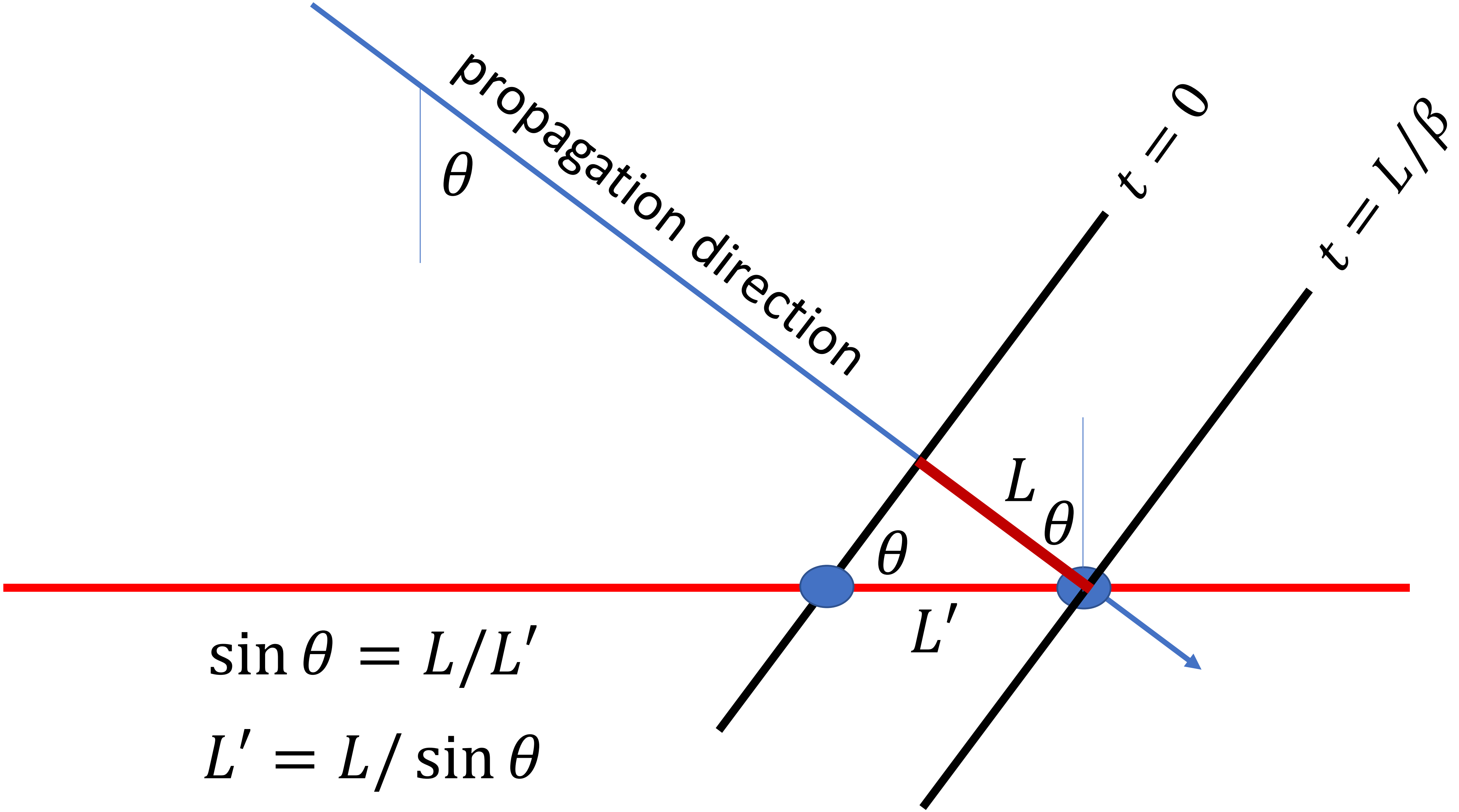
intersection of wavefront with horizontal surface at time=0



intersection of wavefront with horizontal surface at time=t



apparent velocity: speed of intersection



$$\sin\theta = L/L'$$

$$L' = L/\sin\theta$$

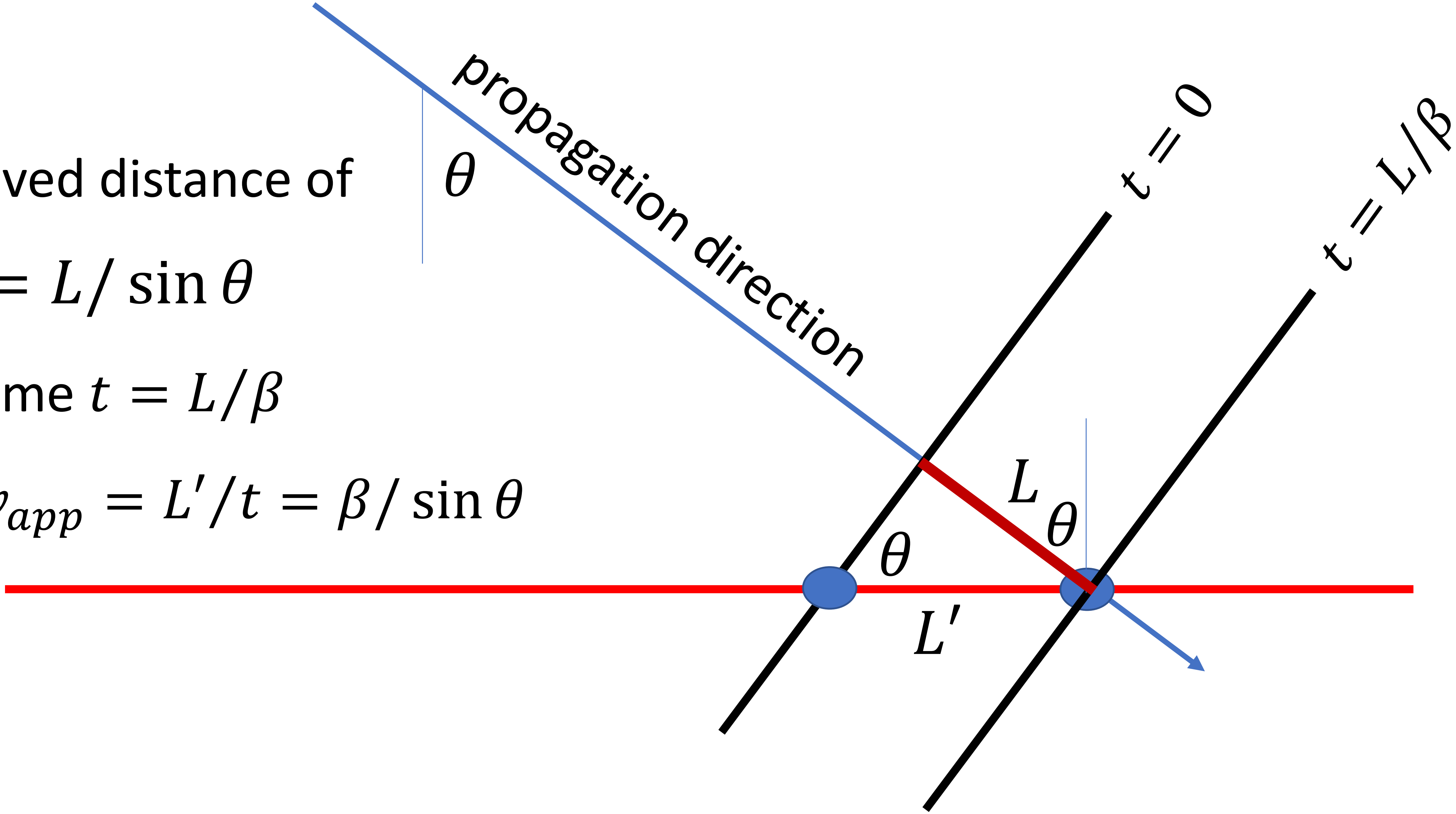
apparent velocity: speed of intersection

moved distance of

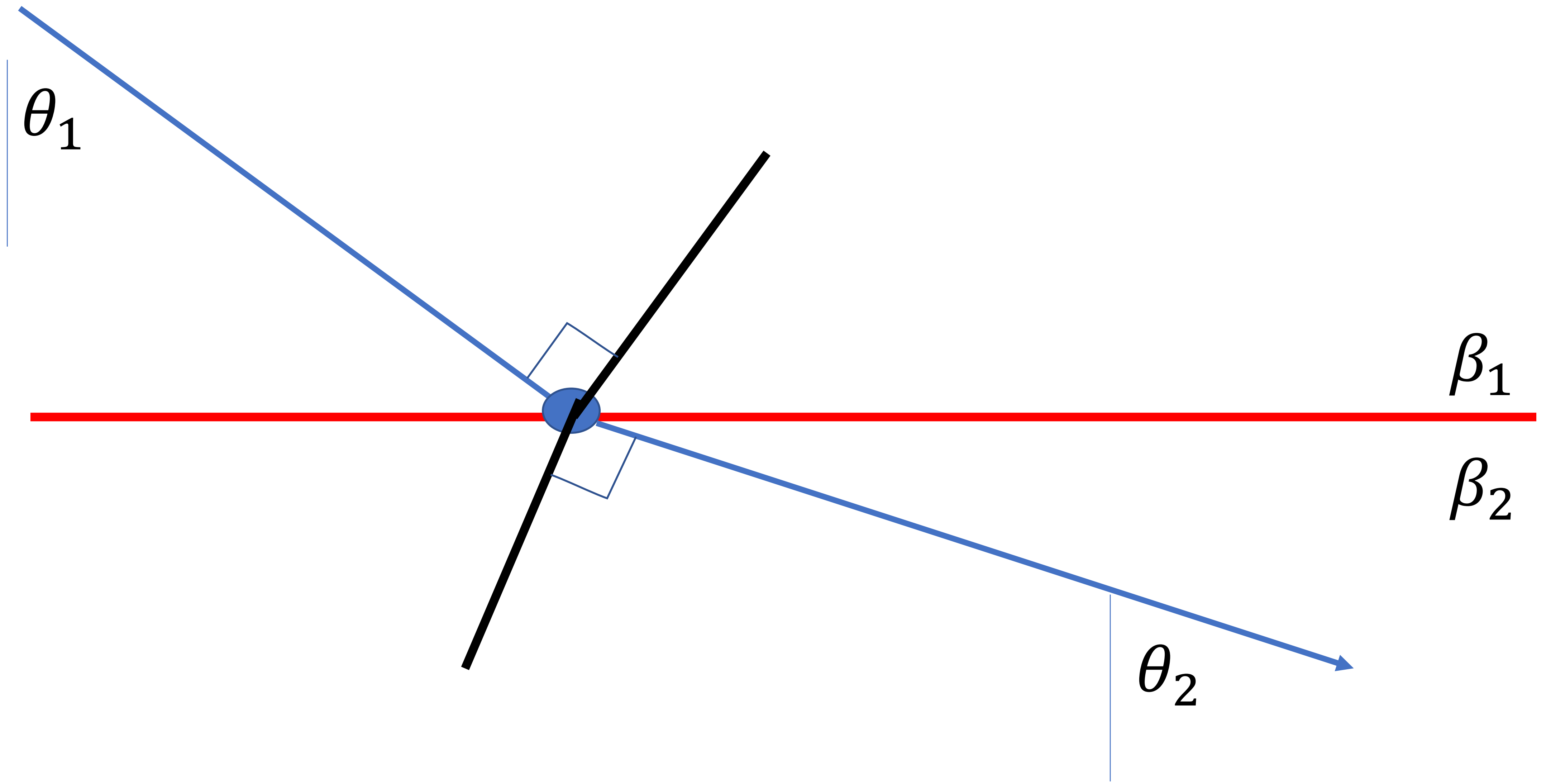
$L' = L/\sin\theta$

in time $t = L/\beta$

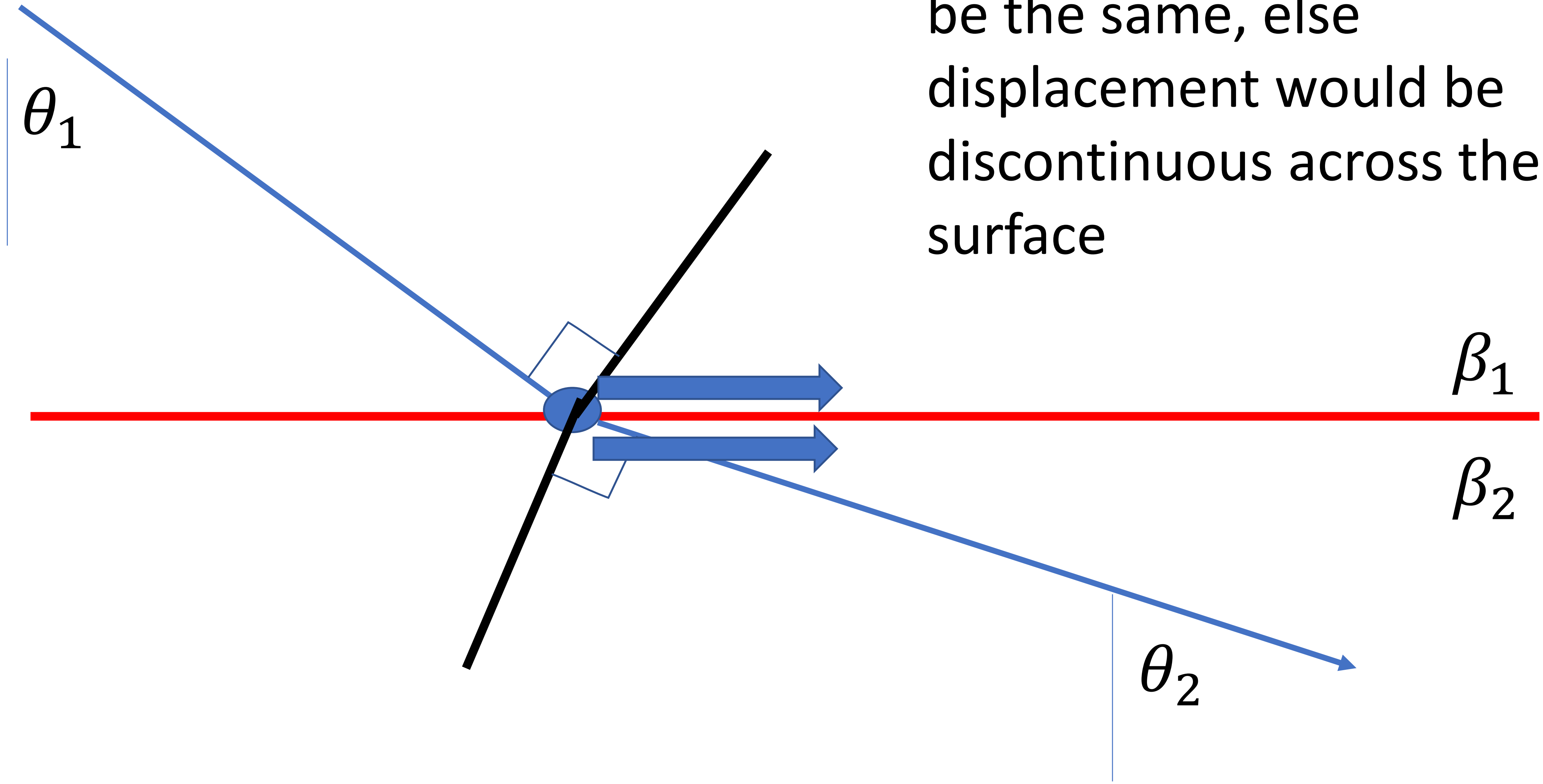
so $v_{app} = L'/t = \beta/\sin\theta$



wave moving from one rock to another

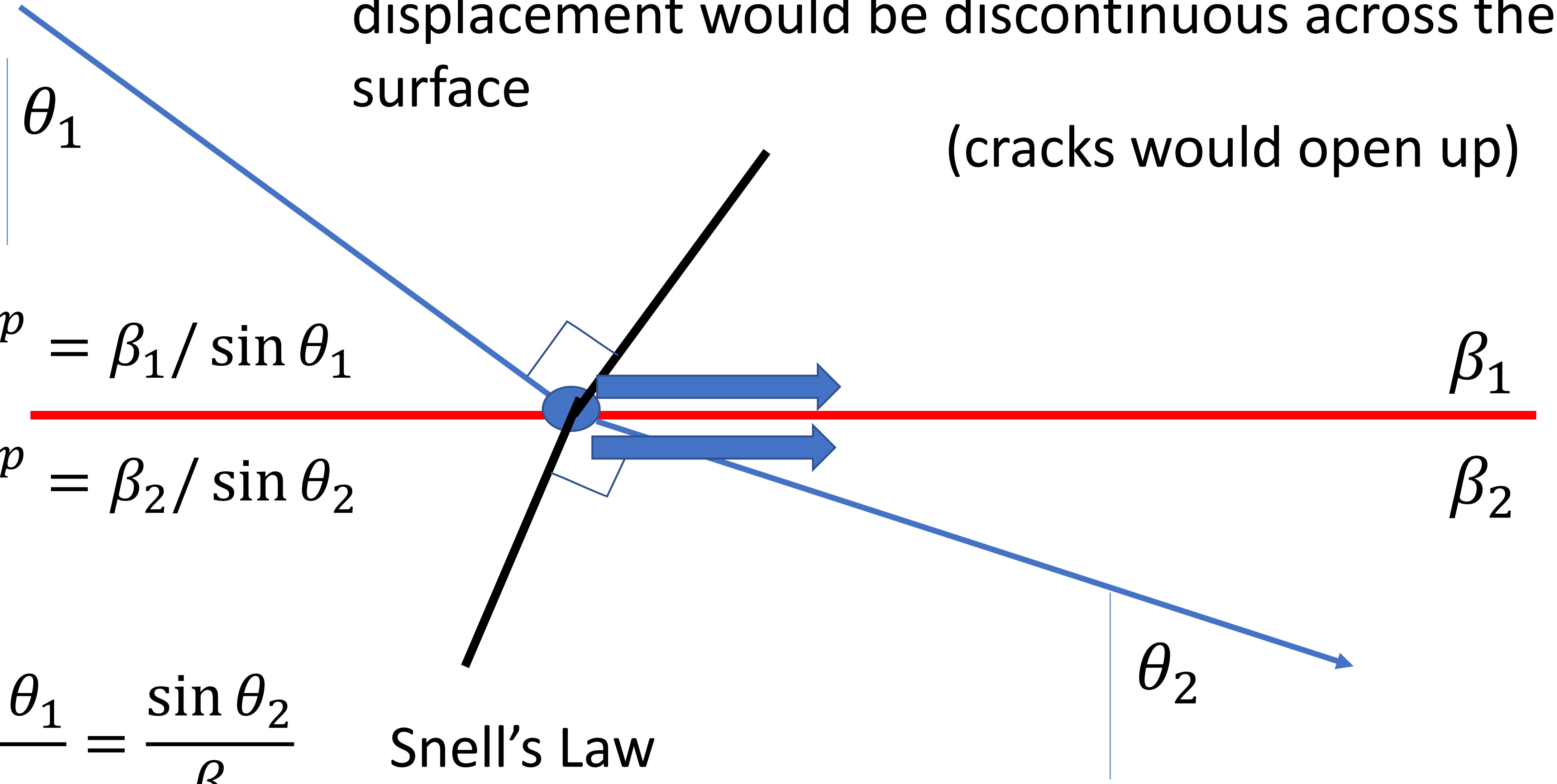


apparent velocities must be the same, else displacement would be discontinuous across the surface



apparent velocities must be the same, else displacement would be discontinuous across the surface

(cracks would open up)

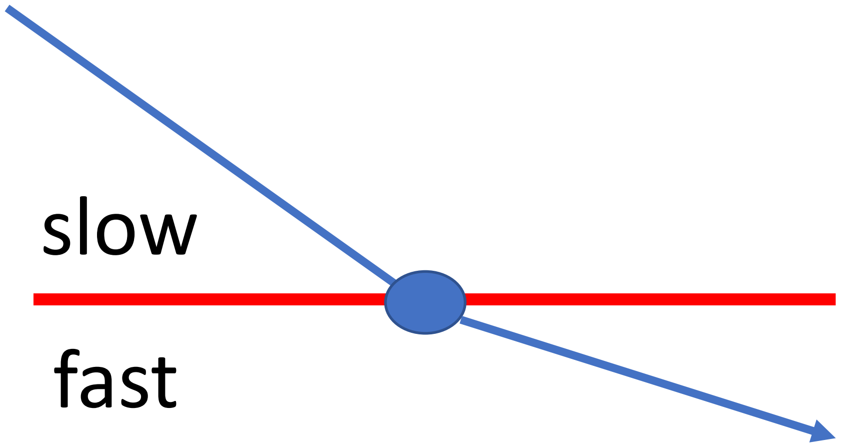


$$v_1^{app} = \beta_1 / \sin\theta_1$$

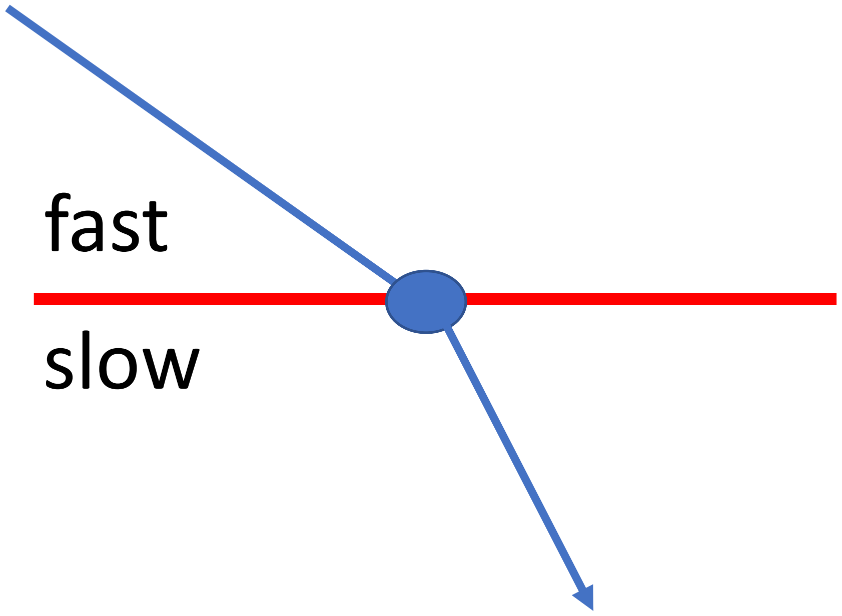
$$v_2^{app} = \beta_2 / \sin\theta_2$$

$$\frac{\sin\theta_1}{\beta_1} = \frac{\sin\theta_2}{\beta_2}$$

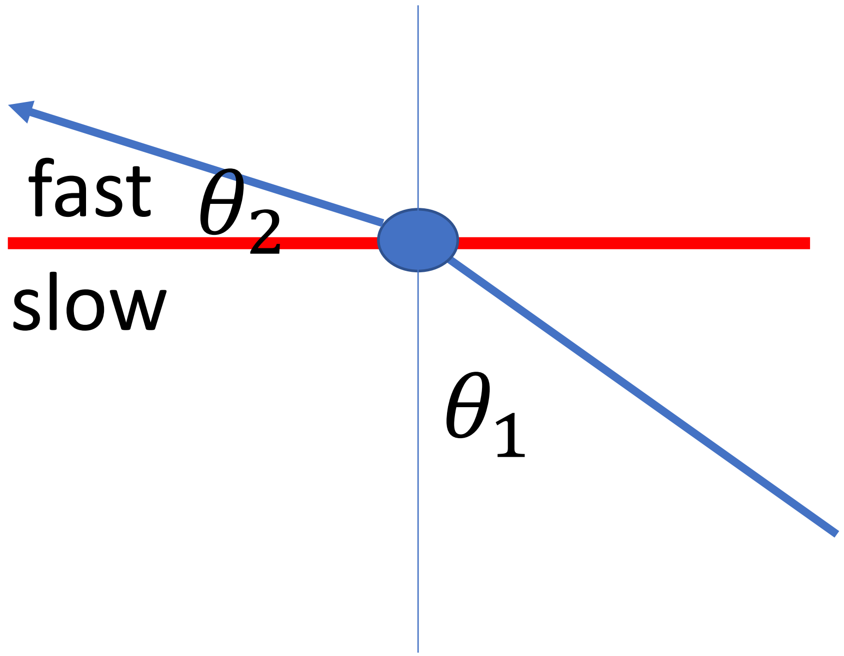
Snell's Law



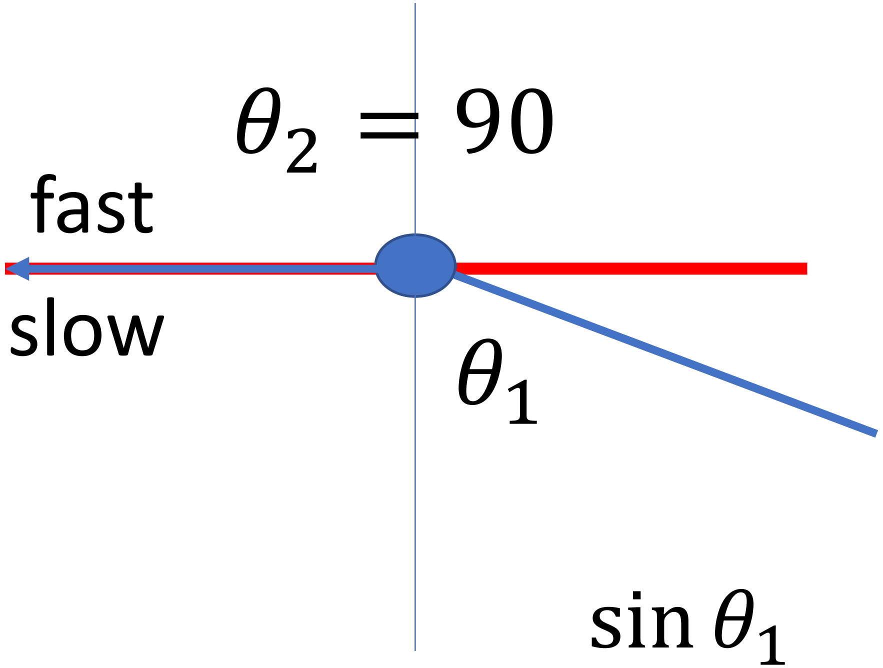
refracts towards horizontal



refracts towards vertical



refracts towards horizontal

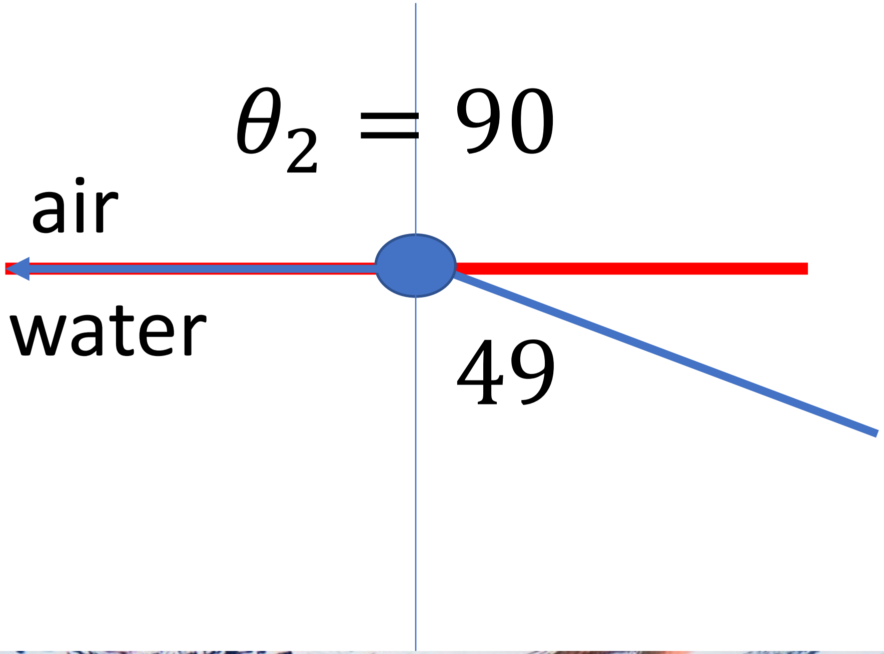


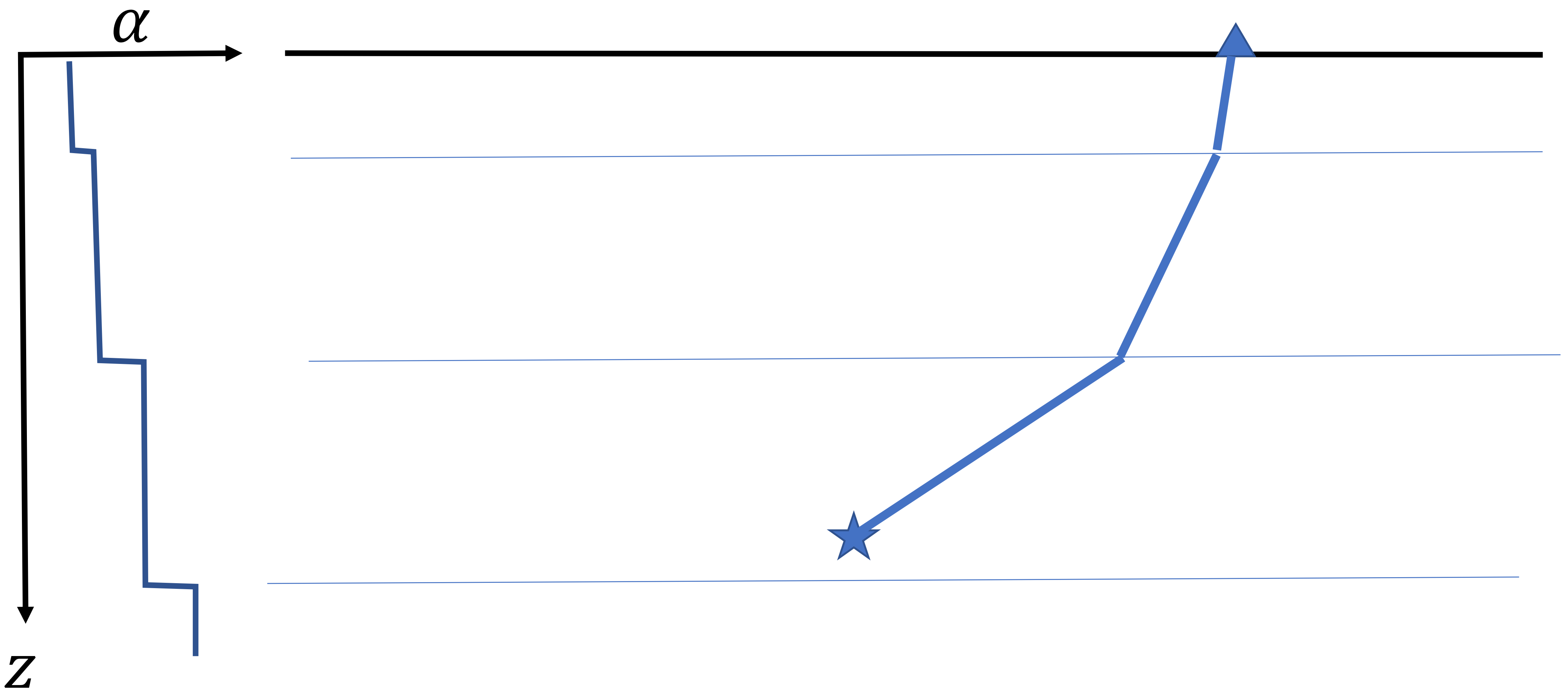
$$\frac{\sin\theta_1}{\beta_1} = \frac{\sin 90}{\beta_2}$$

$$\frac{\sin\theta_1}{\beta_1} = \frac{\sin 90}{\beta_2}$$

$$\theta_1 = \sin^{-1}\frac{\beta_1}{\beta_2}$$

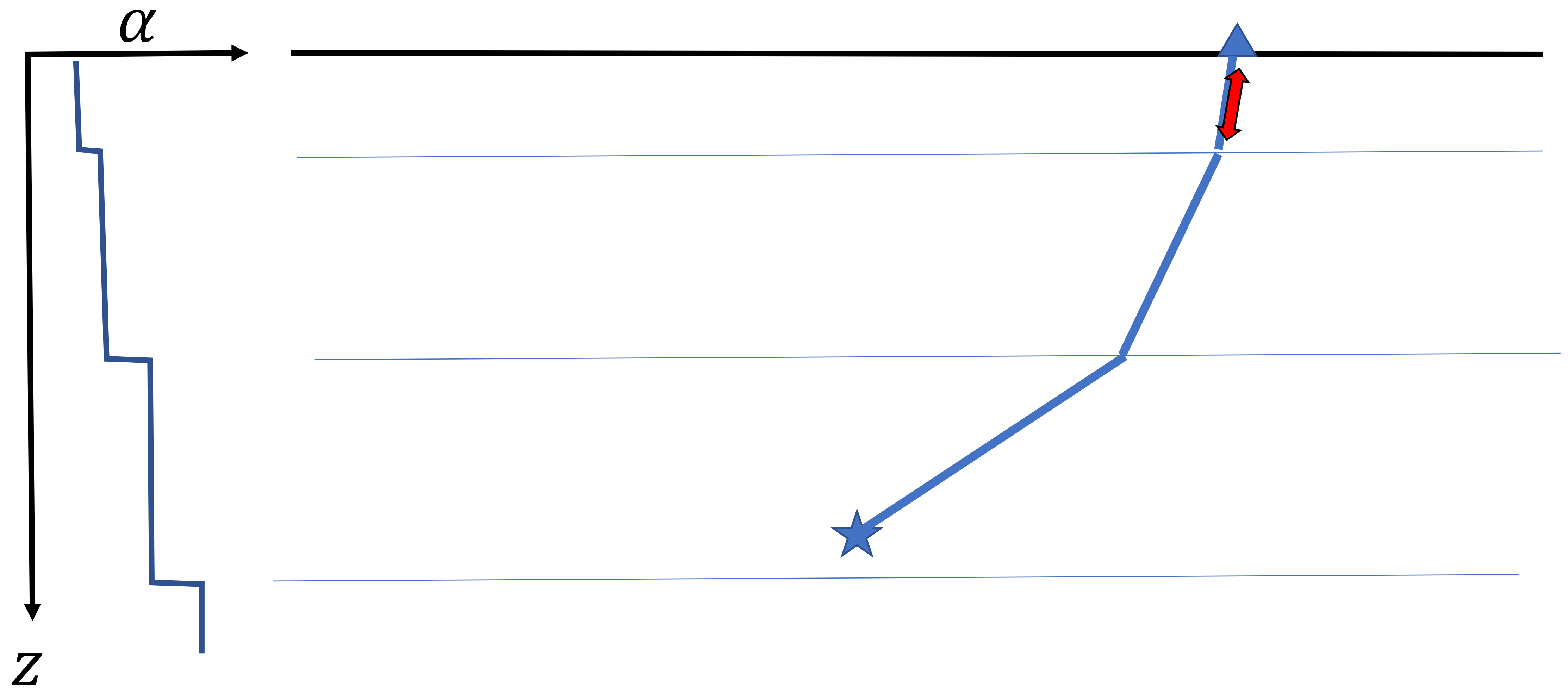
$$\theta_1 = \sin^{-1}\frac{1}{1.33} = 49\ \text{deg}$$



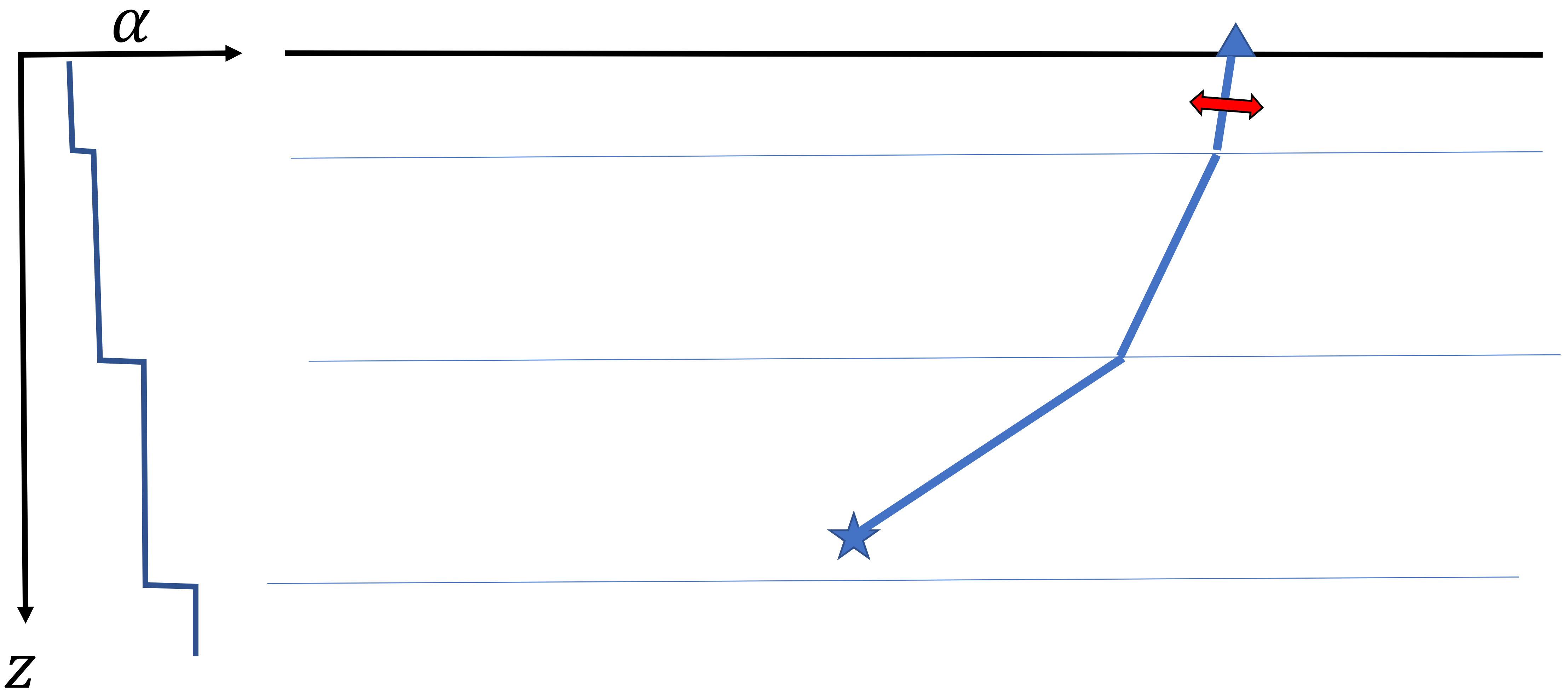


velocity increases with depth

path gets steeper as wave nears surface



P wave motion nearly vertical



S wave motion nearly horizontal