Solid Earth Dynamics

Bill Menke, Instructor

Lecture 17

1. P and S waves in the Earth

Compressional wave

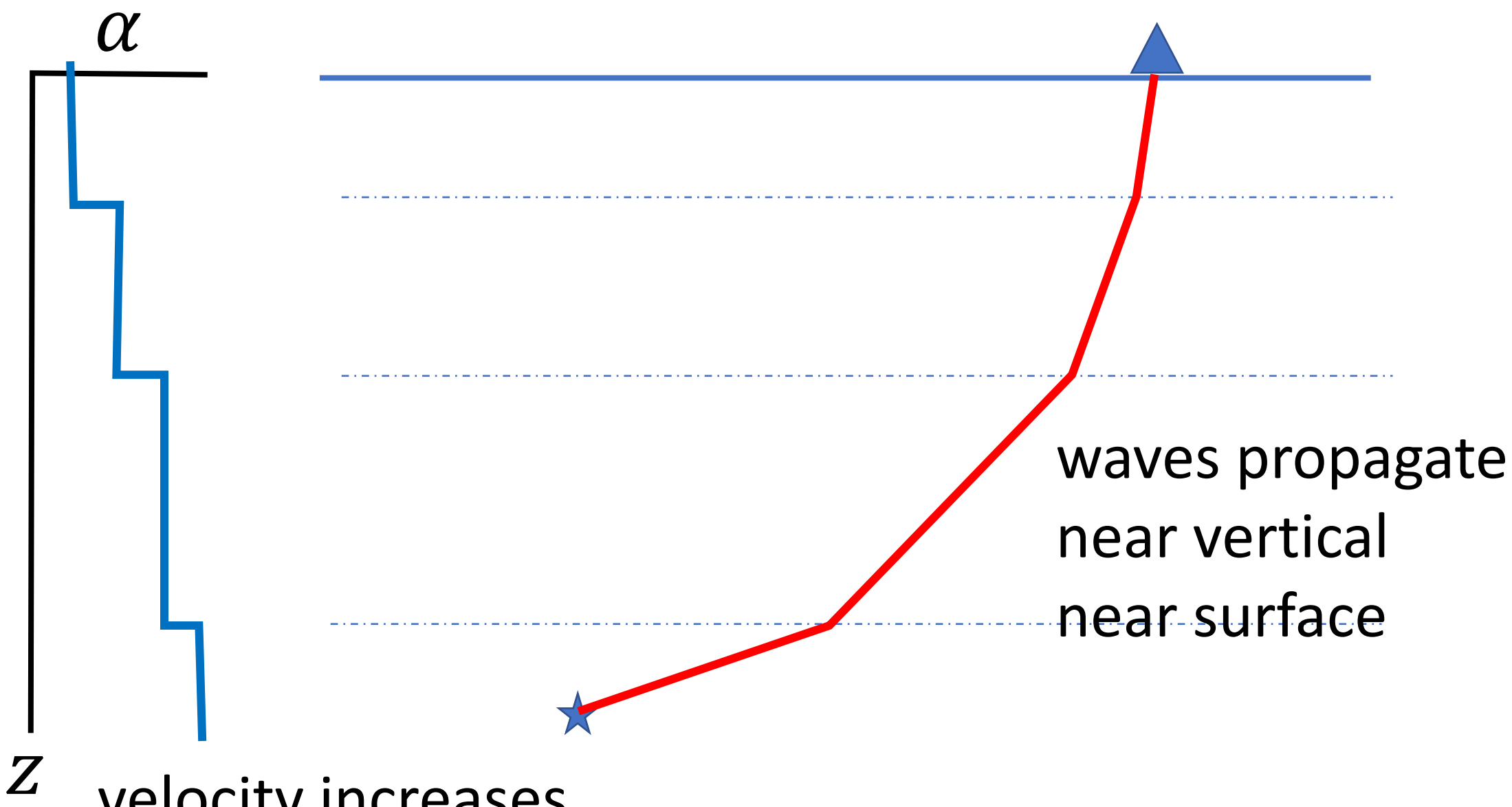
displacement parallel to propagation

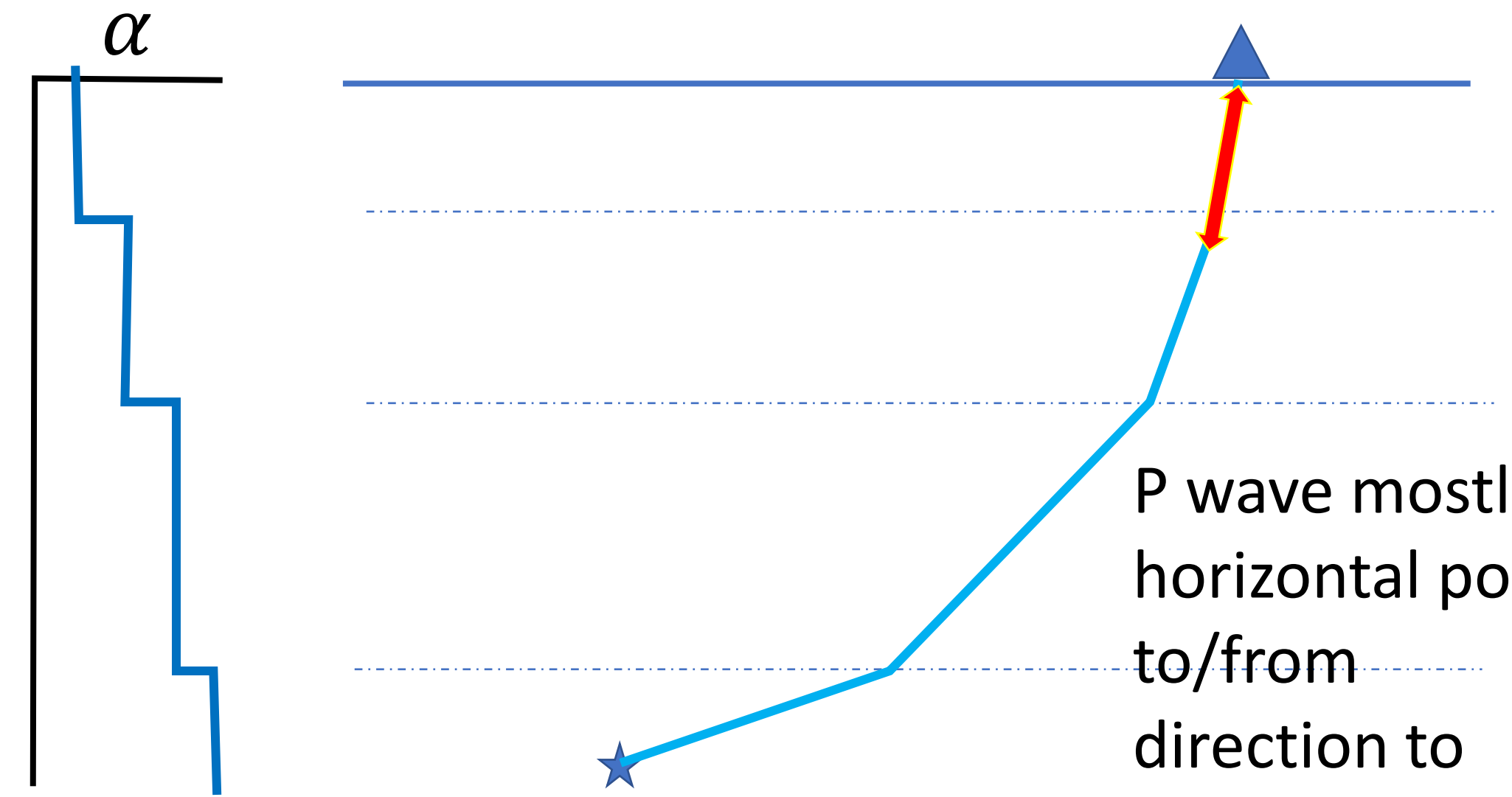
“longitudinally-polarized”

Shear wave

displacement perpendiculat to propagation

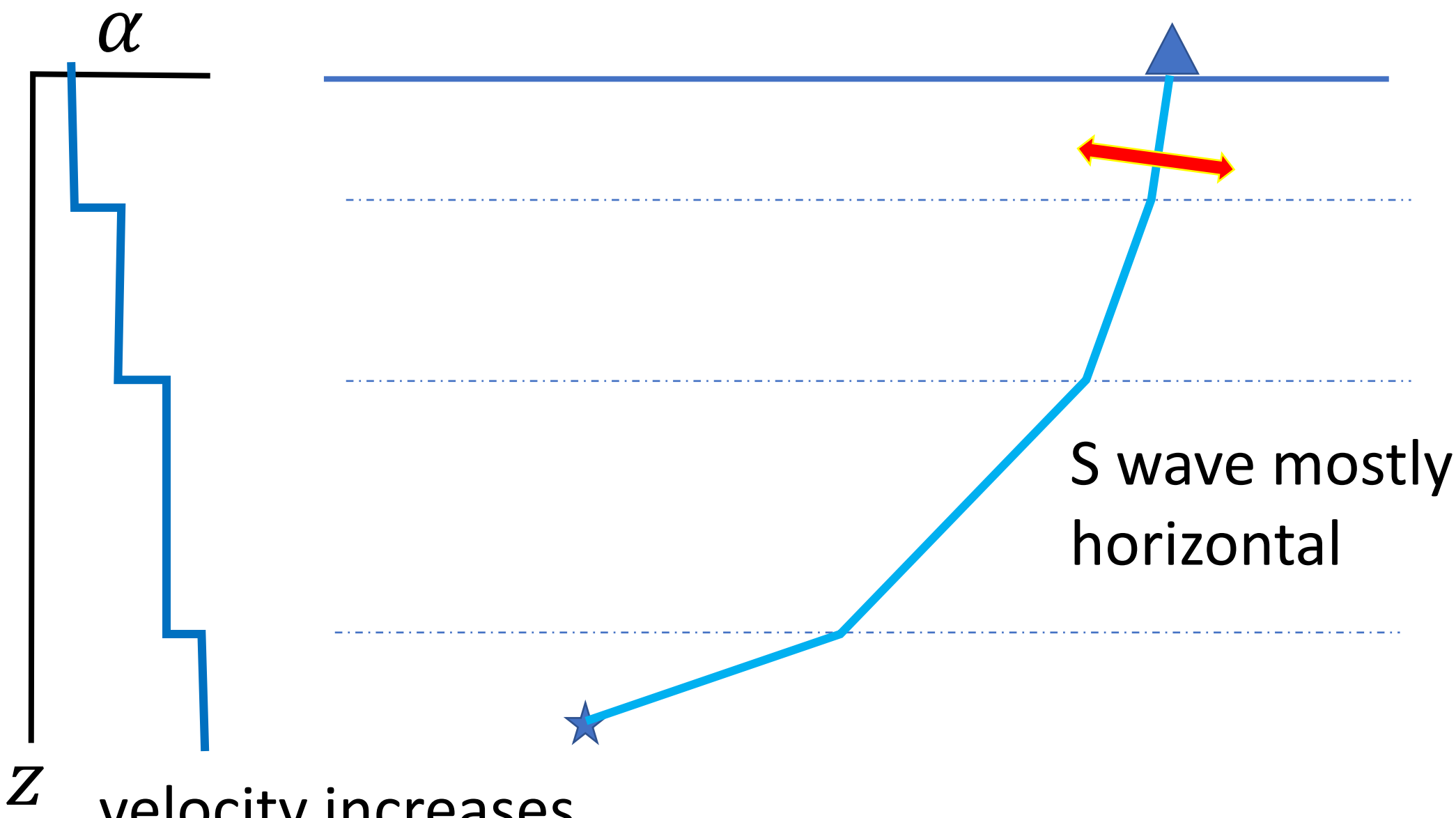
“transversely-polarized”

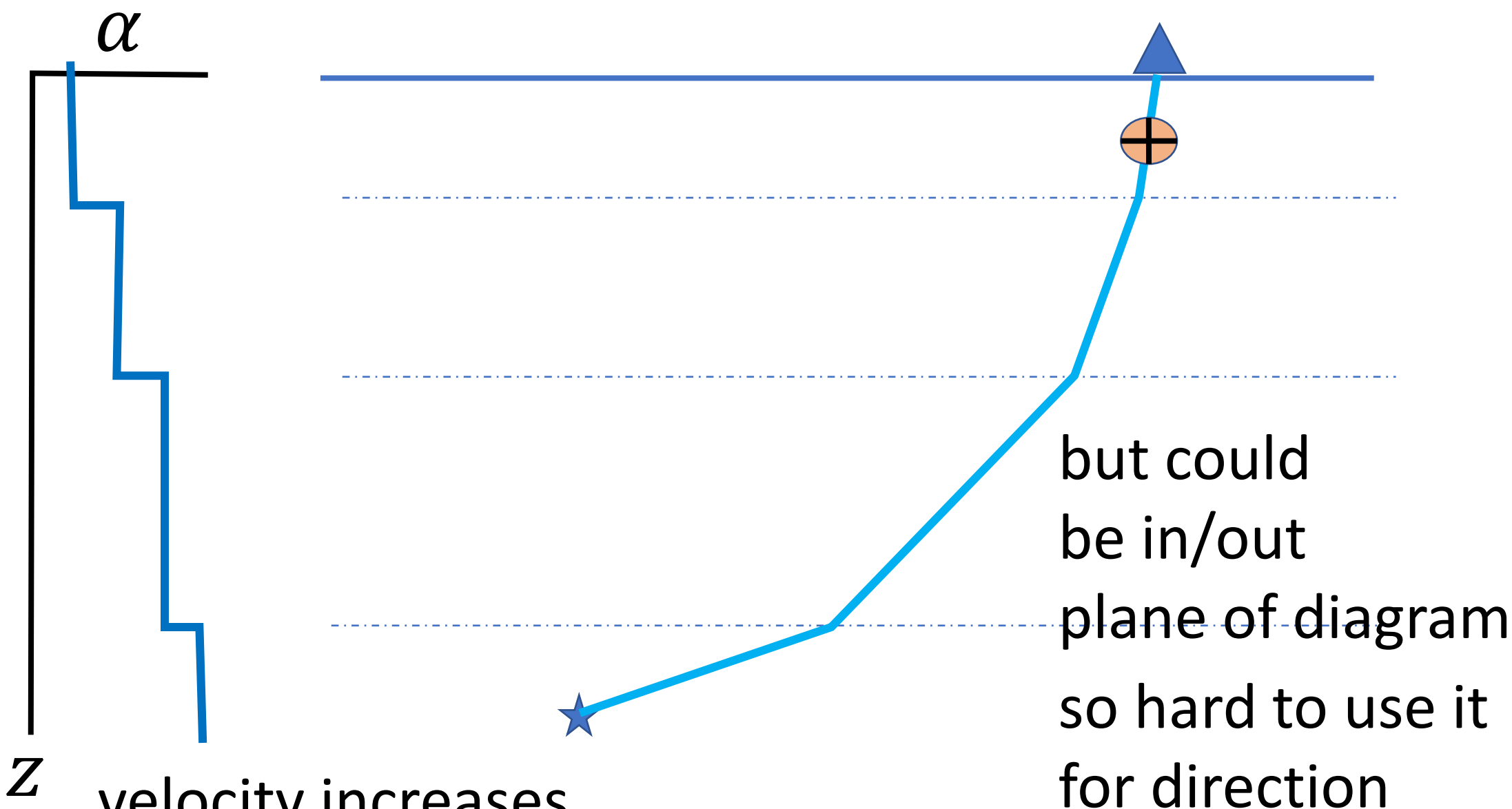

α
z
velocity increases
with depth
waves propagate
near vertical
near surface

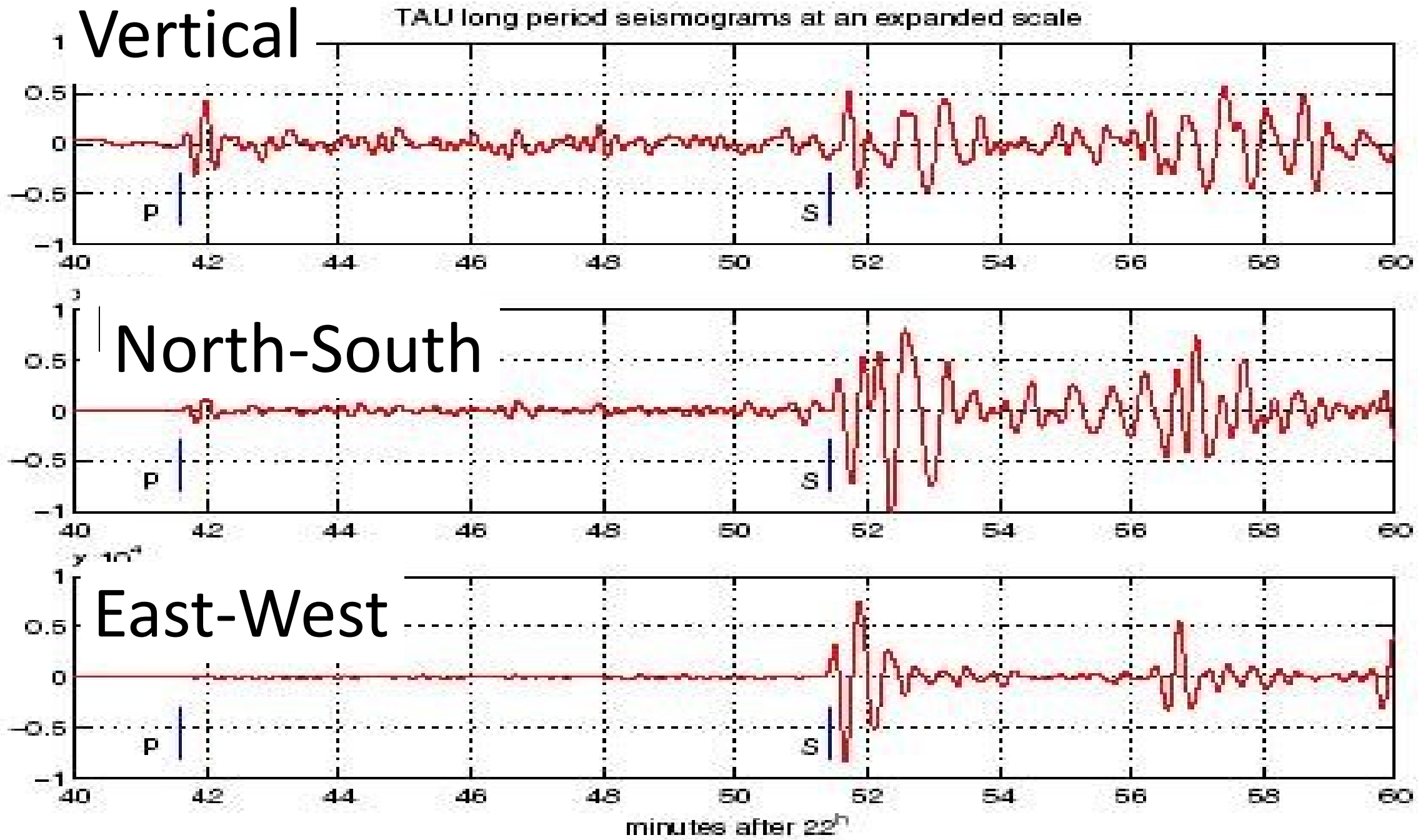


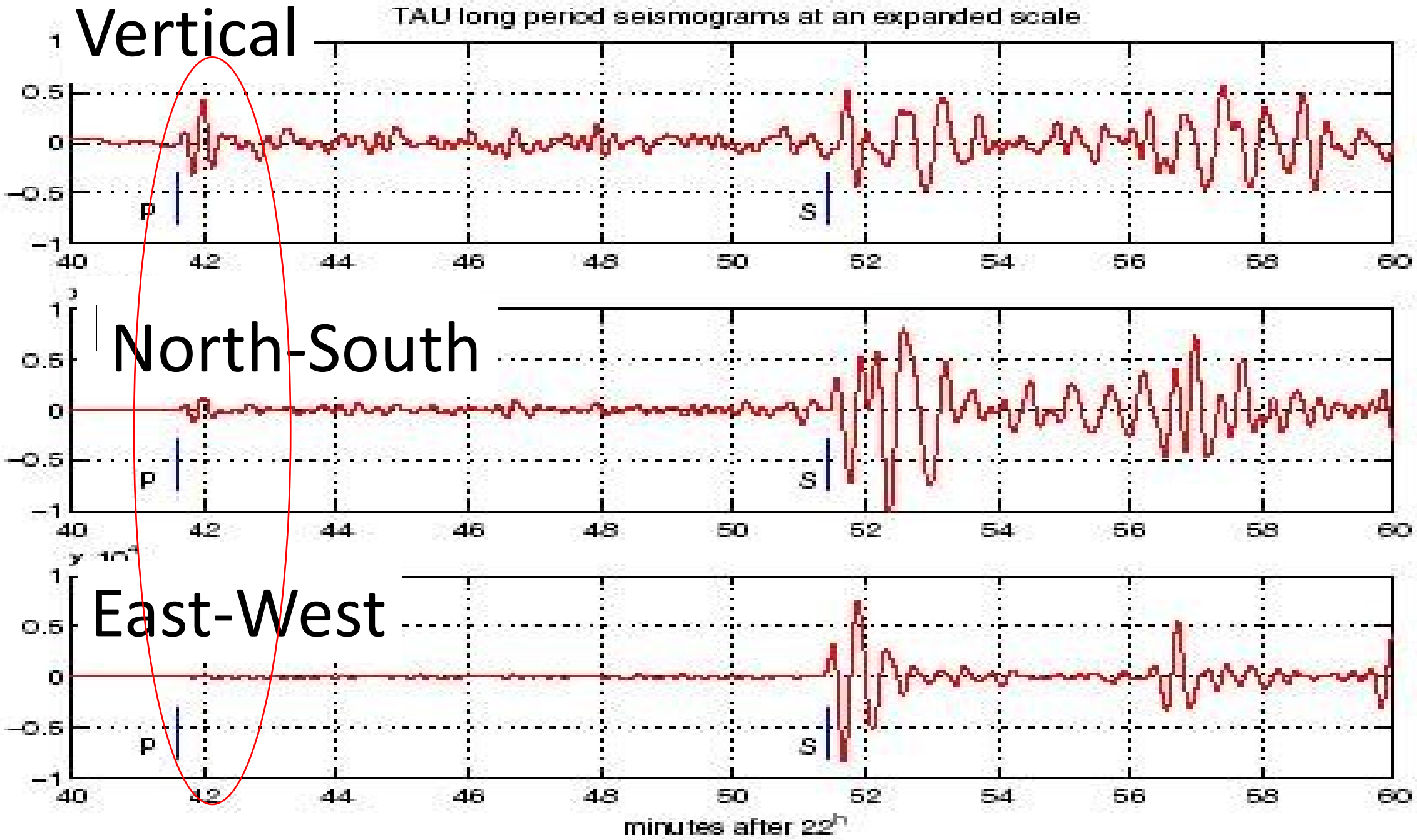
velocity increases with depth

P wave mostly vertical horizontal points to/from direction to earthquake

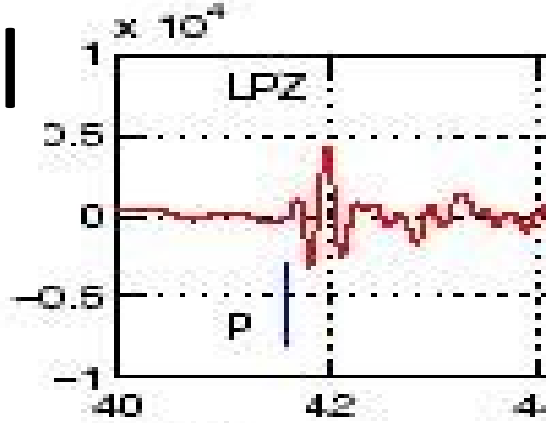

α
z
S wave mostly horizontal
velocity increases with depth


α
z
velocity increases
with depth
but could
be in/out
plane of diagram
so hard to use it
for direction

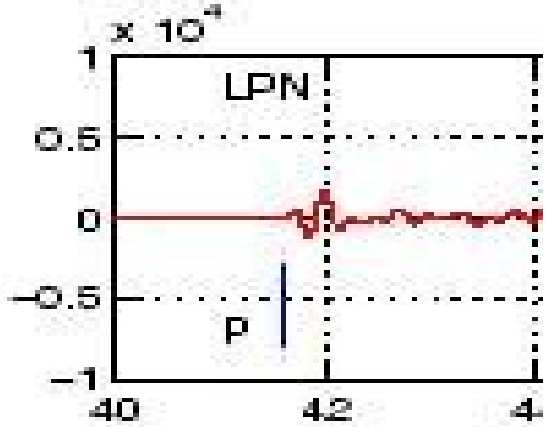

TAU long period seismograms at an expanded scale
Vertical
North-South
East-West
P
S
P
S
P
S
40
42
44
46
48
50
52
54
56
58
60
minutes after 22h


Vertical
North-South
East-West
TAU long period seismograms at an expanded scale
P
S
minutes after 22h

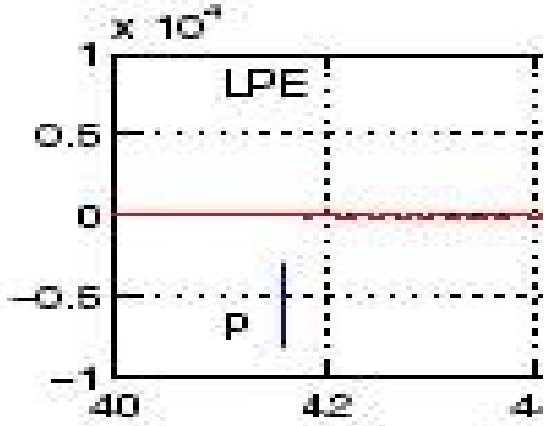
Pl



Vertical

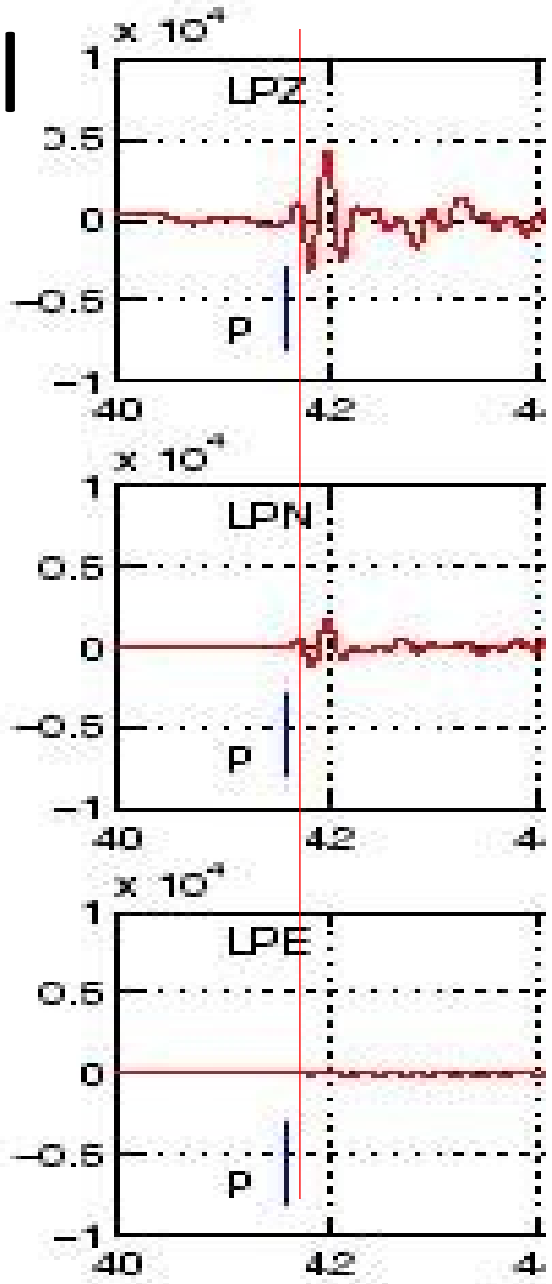


North-South



East-West

Pl



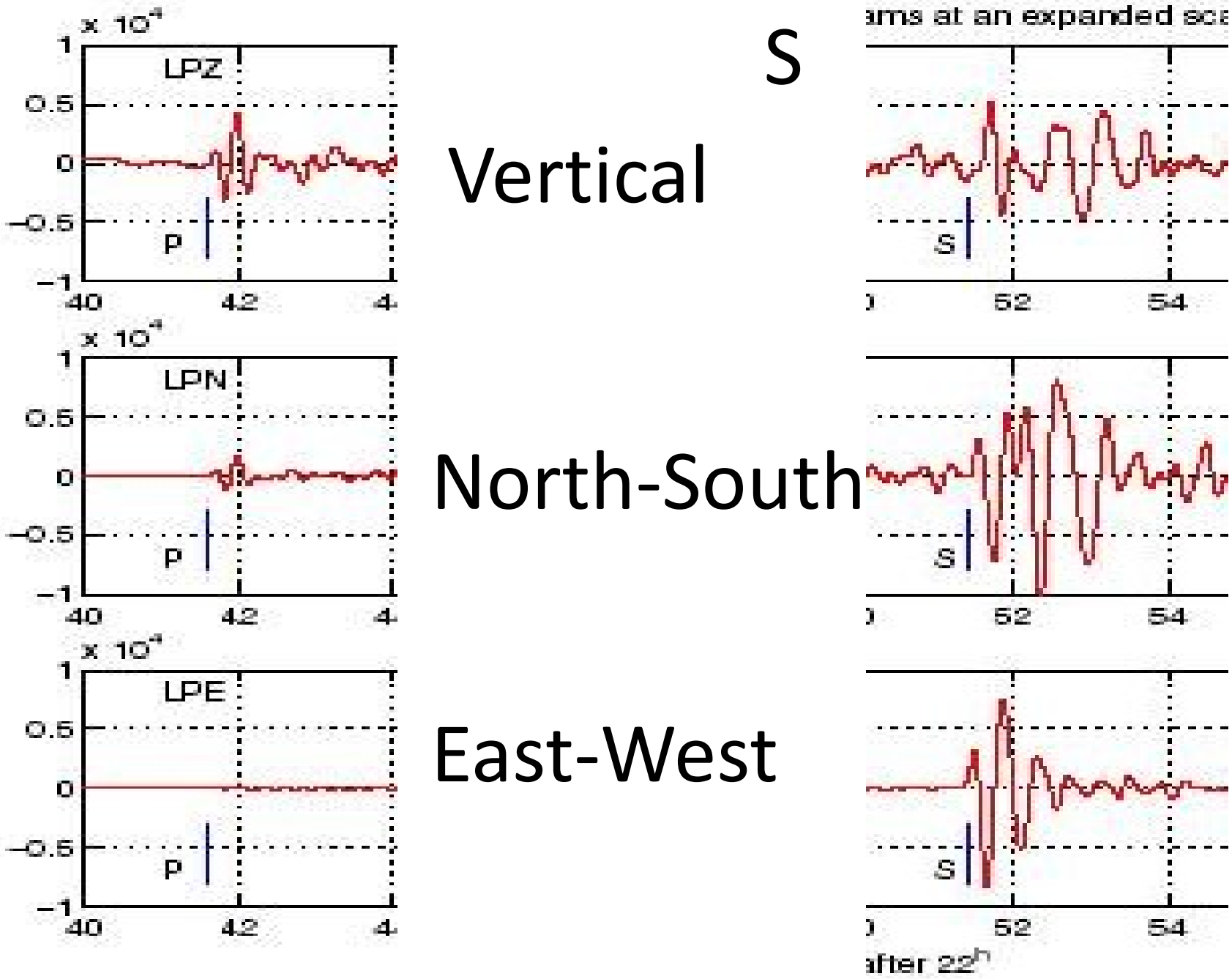
UP

North

south

North

from south


x 10^-4
LPZ
P
S
Vertical
LPN
North-South
LPE
East-West
40
42
52
54
after 22h

Rule of thumb for Earth

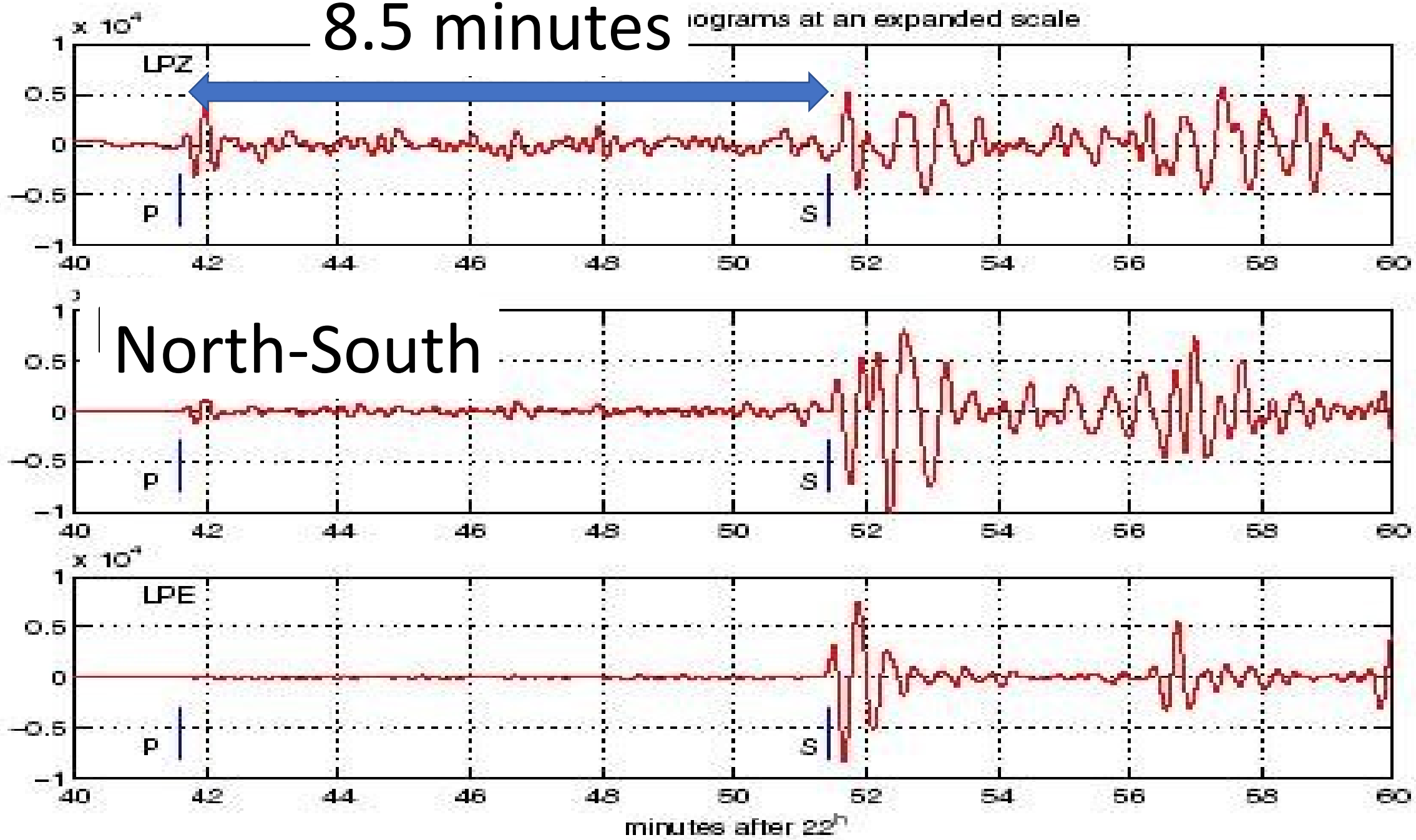
S minus P time in minutes

minus two

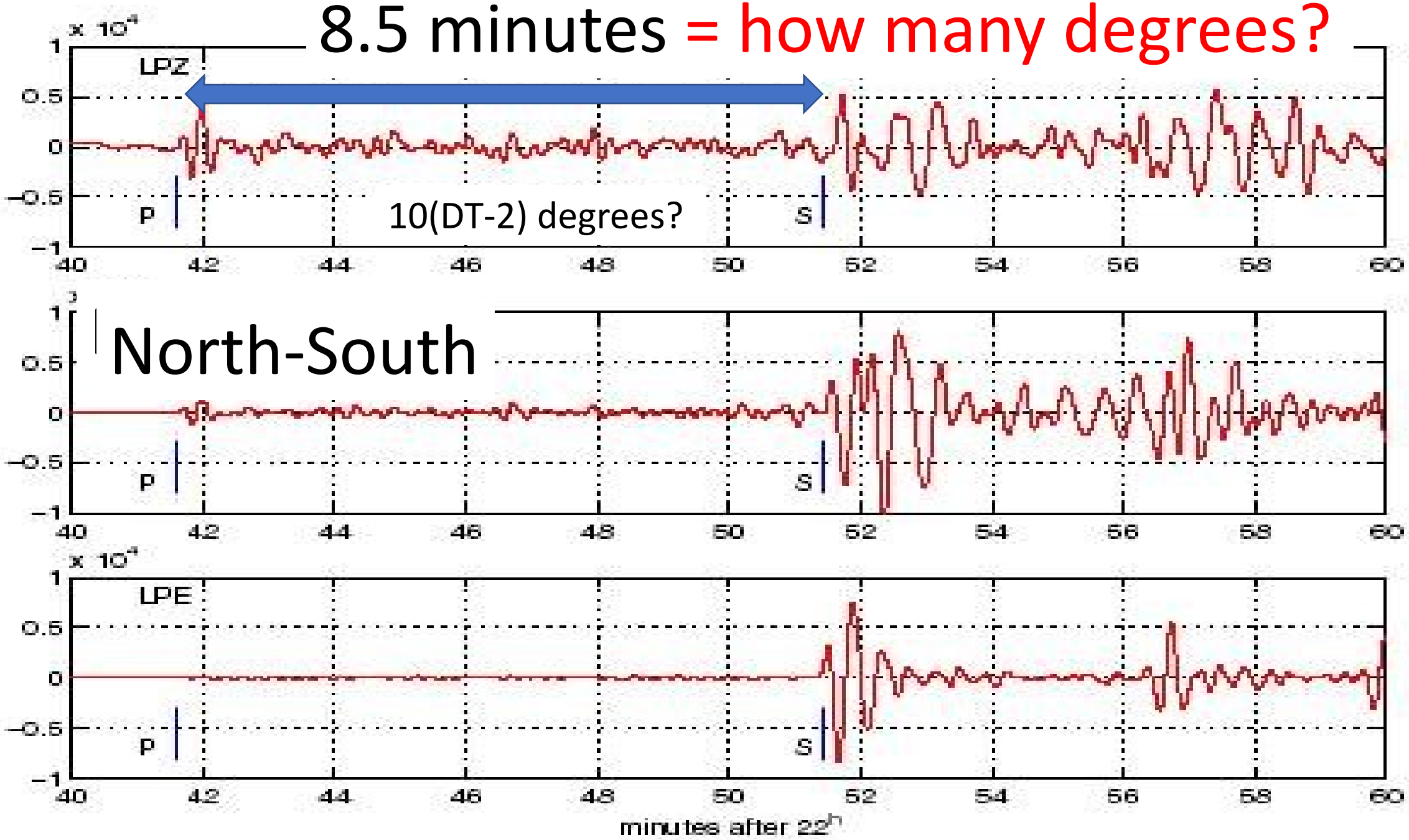
time ten

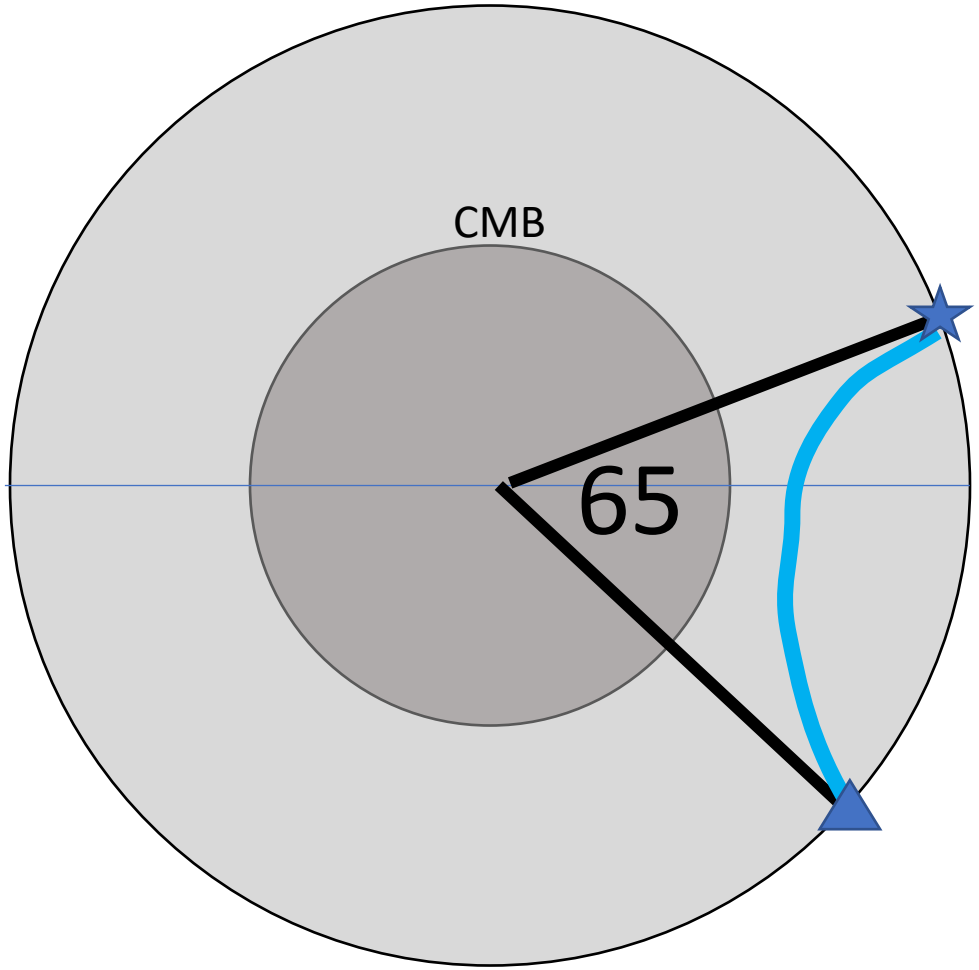
= distance in degrees

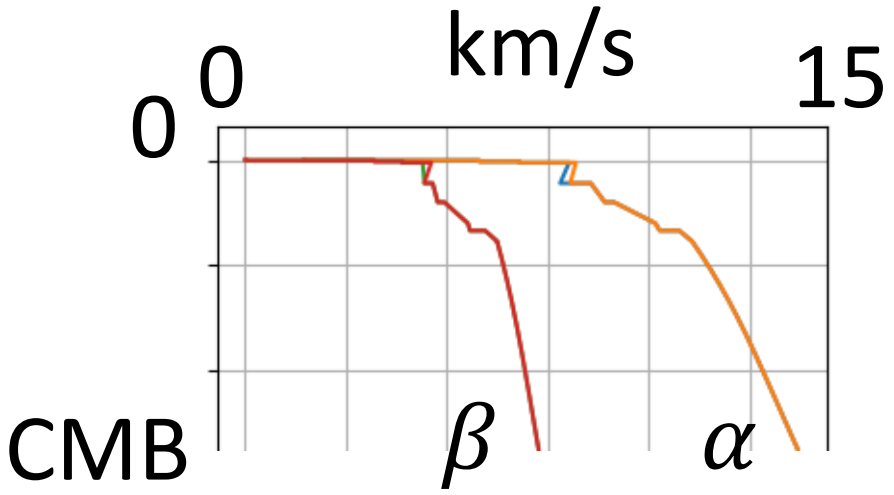
(1 deg = 111 km)

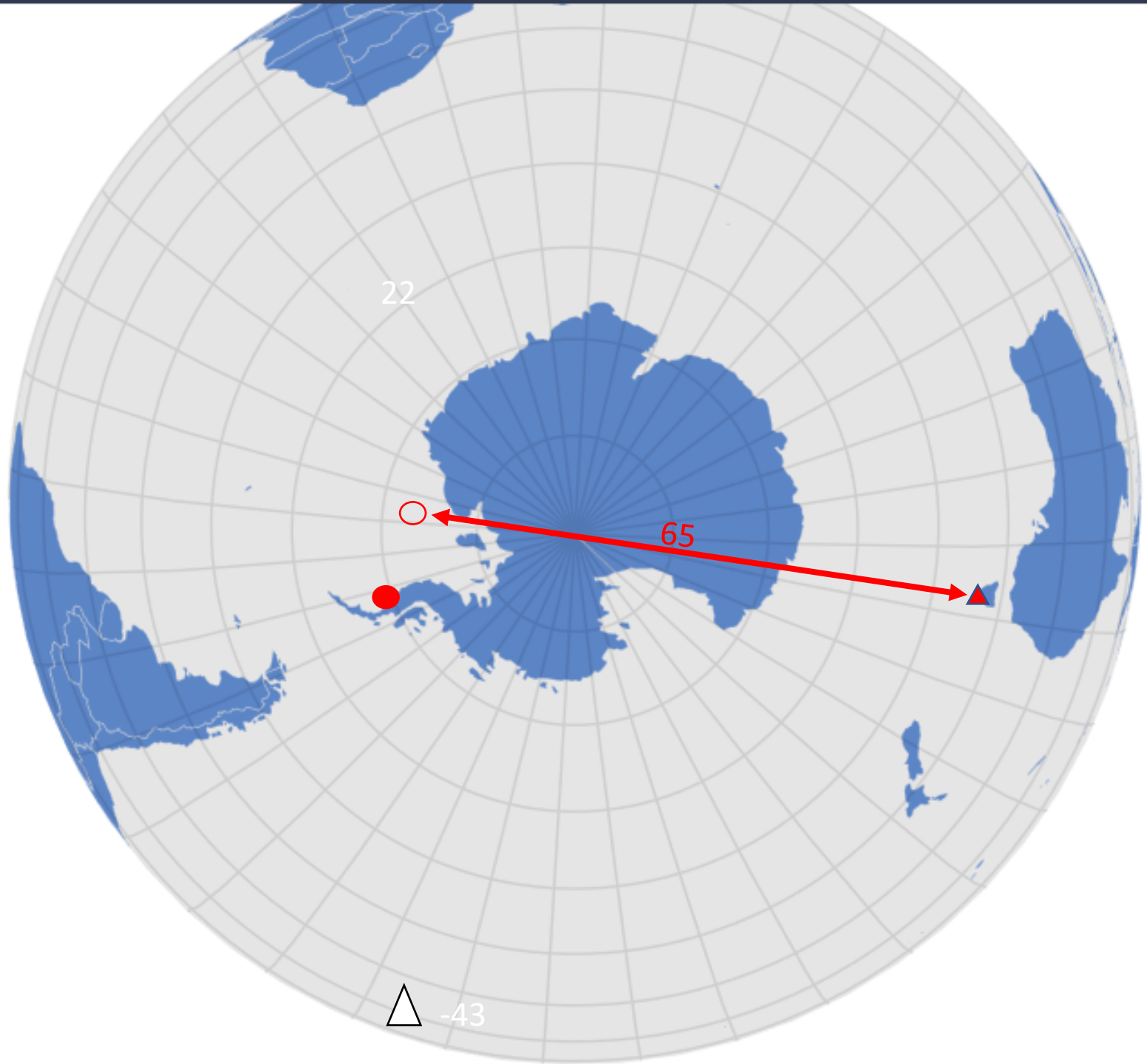

8.5 minutes
iograms at an expanded scale
x 10^-4
LPZ
P
S
North-South
LPE
minutes after 22h

8.5 minutes = how many degrees?



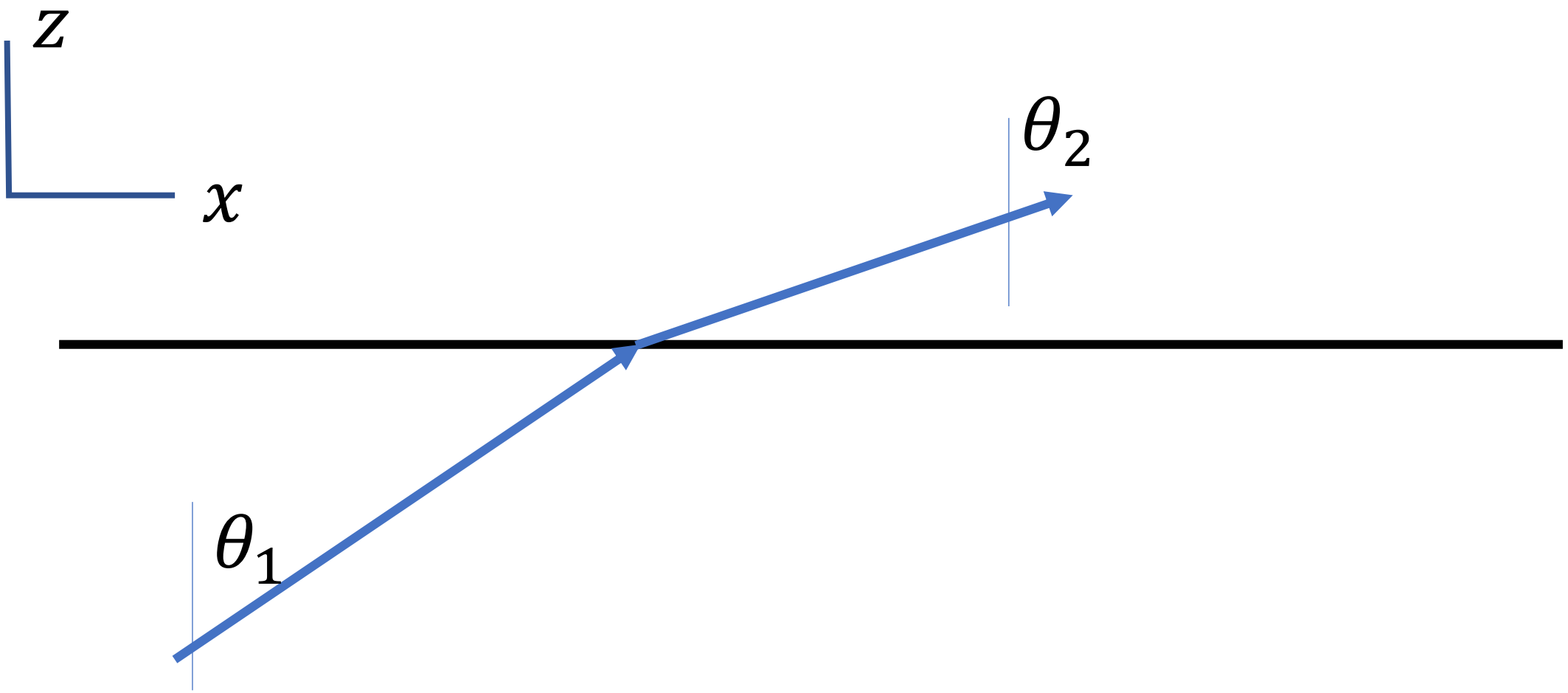

CMB
65

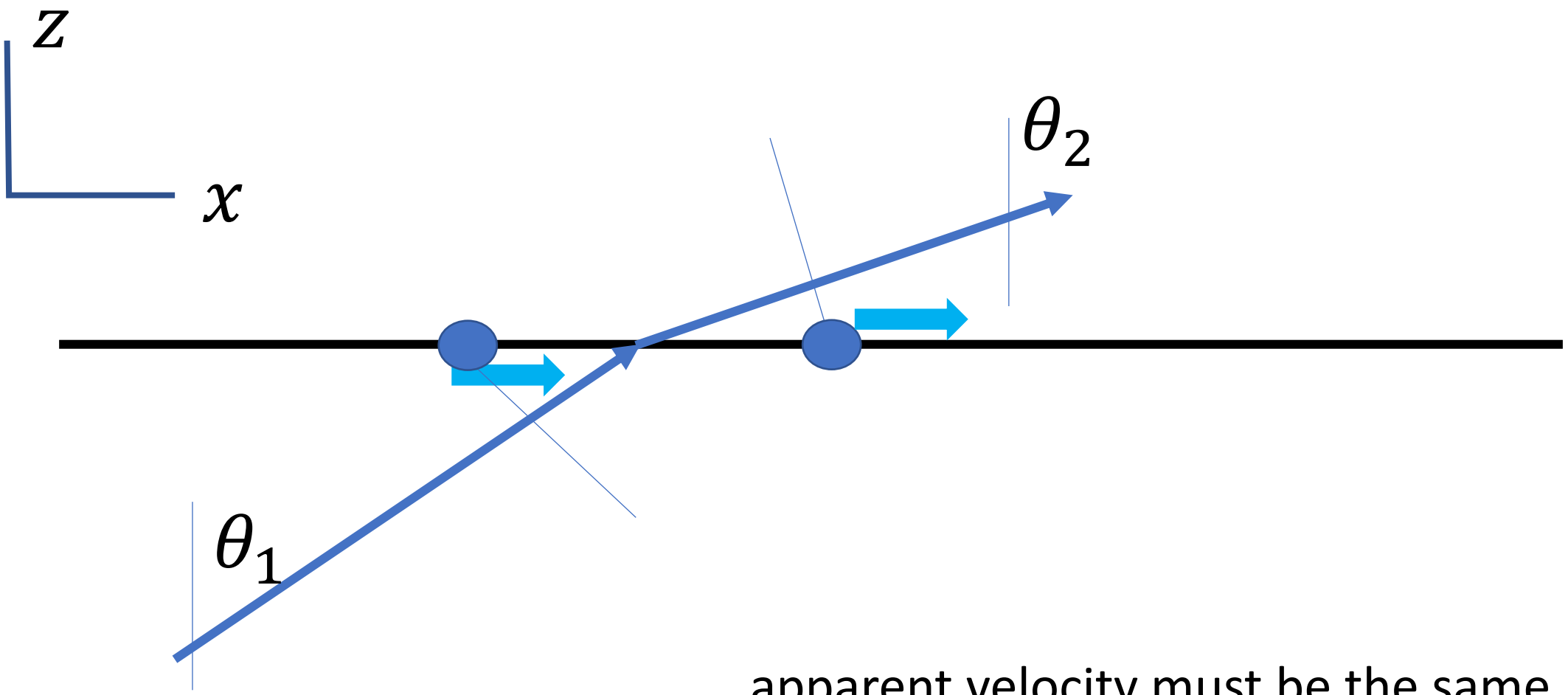

km/s
0
0
15
CMB
β
α



-43

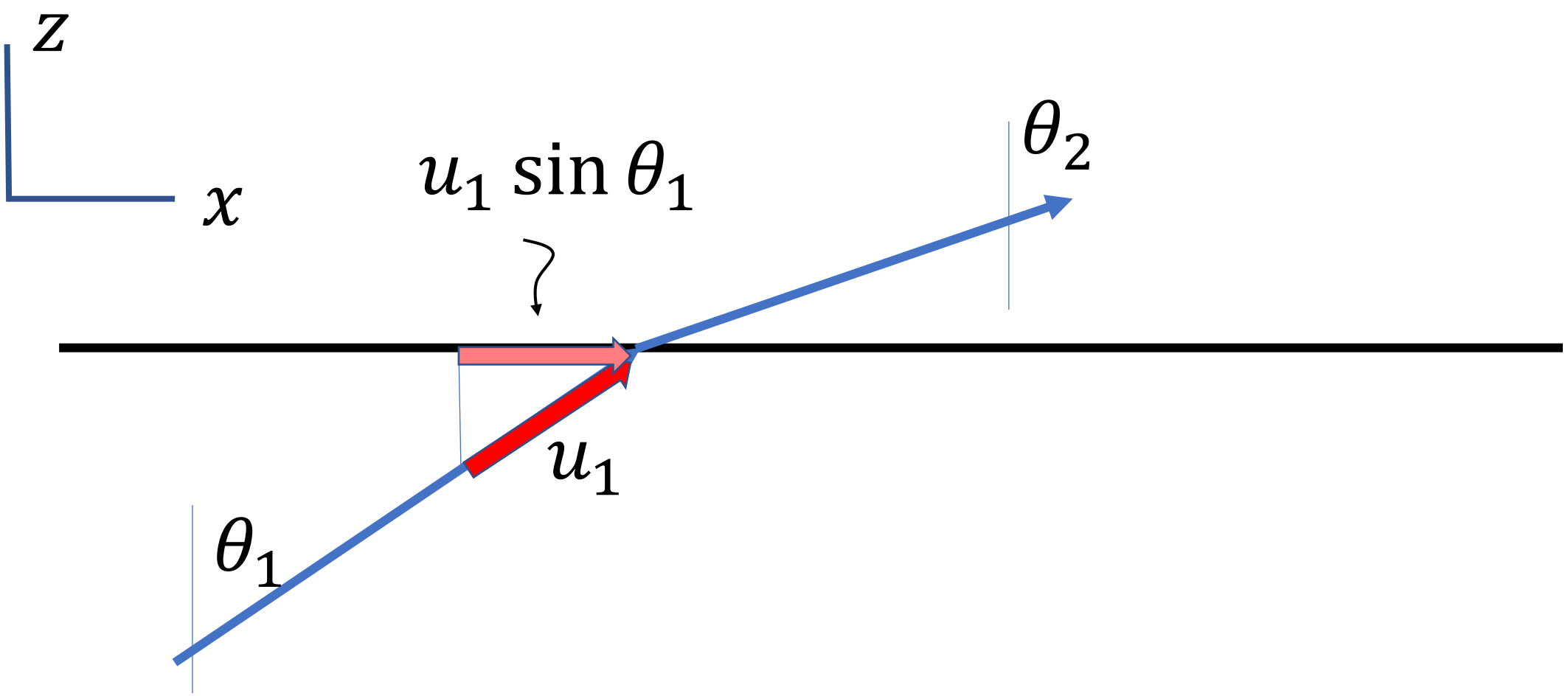
2. Reflected waves

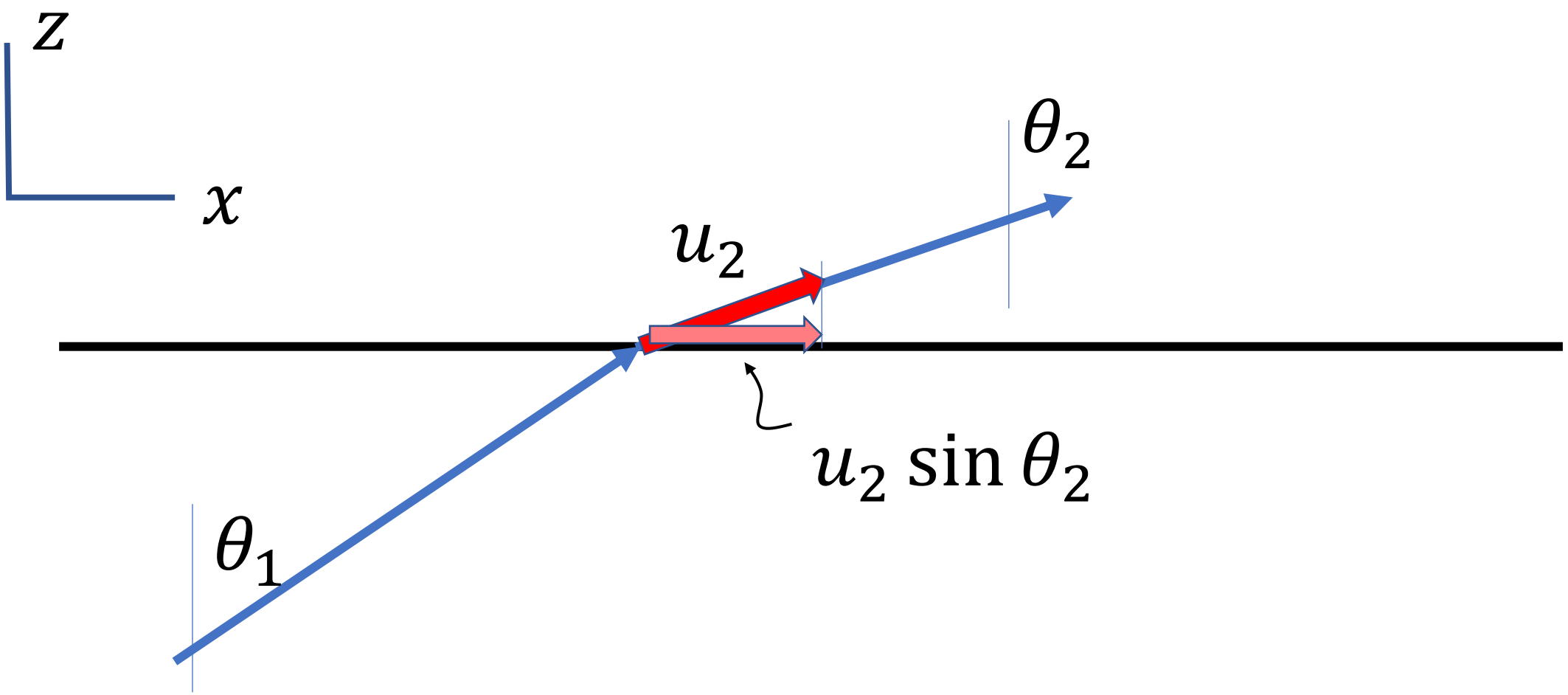

z
x
θ_2
θ_1

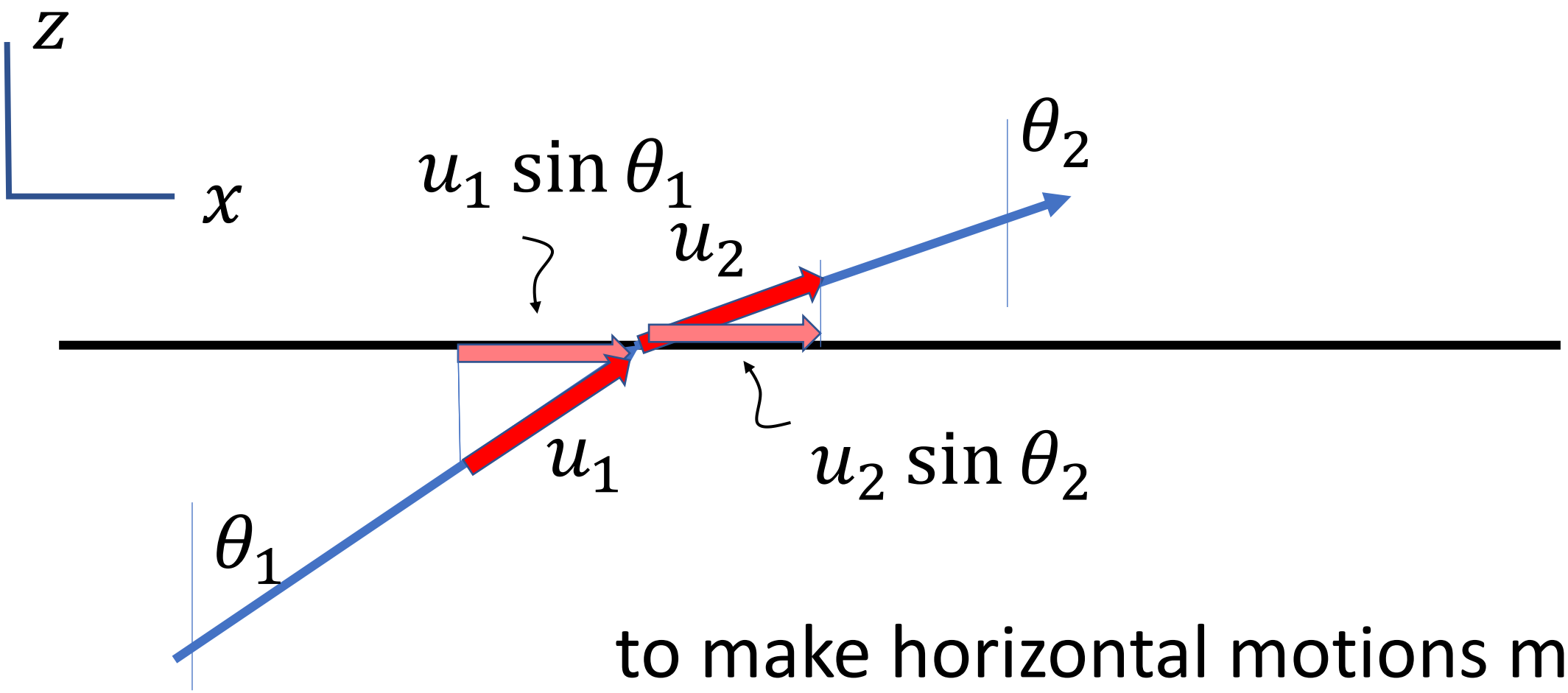


apparent velocity must be the same

implies Snell's law $\frac{\sin\theta_1}{\alpha_1} = \frac{\sin\theta_2}{\alpha_2}$


z
x
$u_1 \sin \theta_1$
θ_2
u_1
θ_1

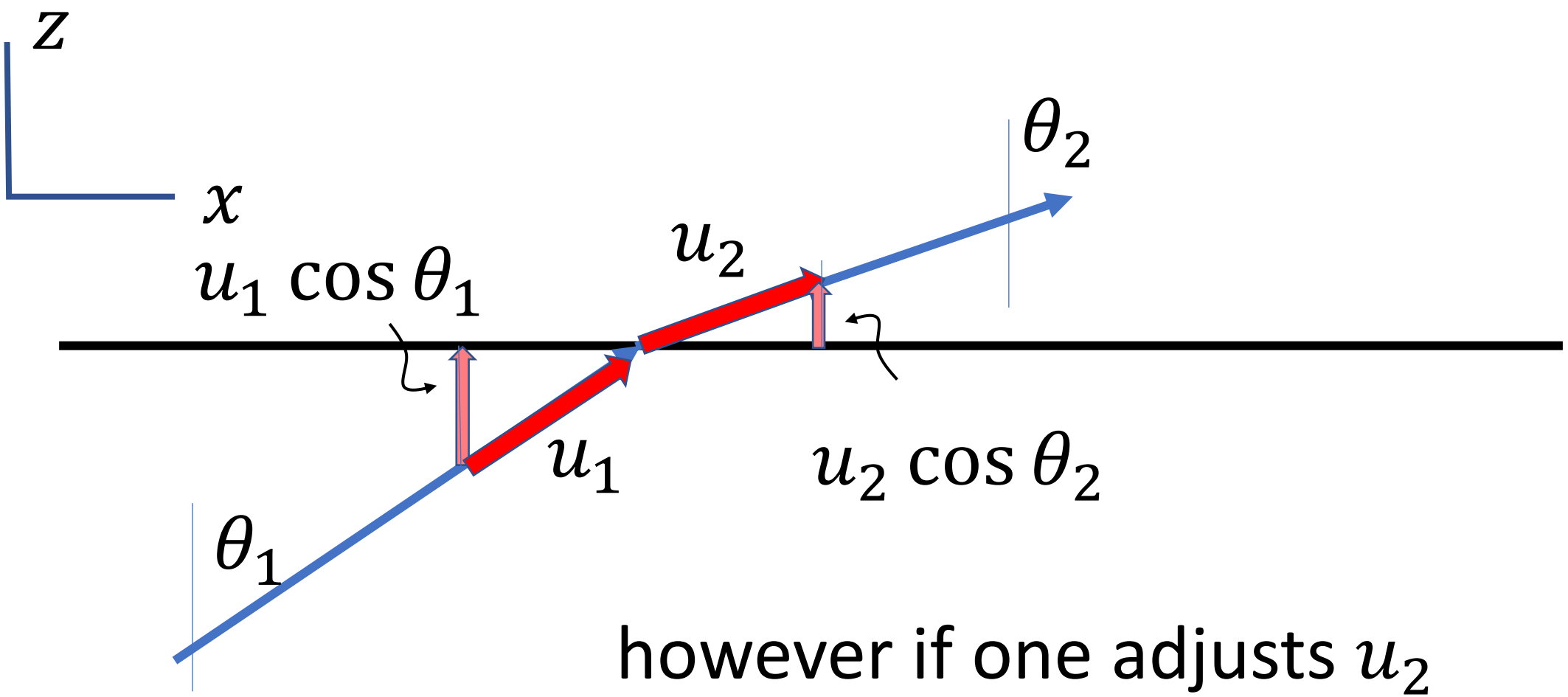

z
x
θ_2
u_2
$u_2 \sin \theta_2$
θ_1



to make horizontal motions match

$$u_1 \sin \theta_1 = u_2 \sin \theta_2$$

so $u_1 \neq u_2$



however if one adjusts u_2
so horizontal motions match ...

vertical motions won't match

$$u_2 \sin\theta_2 = u_1 \sin\theta_1$$

$$u_2 = u_1 \frac{\sin\theta_1}{\sin\theta_2} = u_1 \frac{\alpha_1}{\alpha_2}$$

by Snell's law $\frac{\sin\theta_1}{\alpha_1} = \frac{\sin\theta_2}{\alpha_2}$

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\alpha_1}{\alpha_2}$

$$u_2 \cos\theta_2 = u_1 \cos\theta_1$$

$$u_1 \frac{\alpha_1}{\alpha_2} \cos\theta_2 \stackrel{?}{=} u_1 \cos\theta_1$$

$$\frac{\cos\theta_2}{\alpha_2} \stackrel{?}{=} \frac{\cos\theta_1}{\alpha_1}$$

$$\frac{\cos\theta_2}{\alpha_2} \stackrel{?}{=} \frac{\cos\theta_1}{\alpha_1}$$

$$\frac{\sqrt{1-\sin^2\theta_2}}{\alpha_2} \stackrel{?}{=} \frac{\sqrt{1-\sin^2\theta_1}}{\alpha_1}$$

$$\frac{1-\sin^2\theta_2}{\alpha_2^2} \stackrel{?}{=} \frac{1-\sin^2\theta_1}{\alpha_1^2}$$

by Snell's law

$$\frac{\sin\theta_1}{\alpha_1} = \frac{\sin\theta_2}{\alpha_2}$$

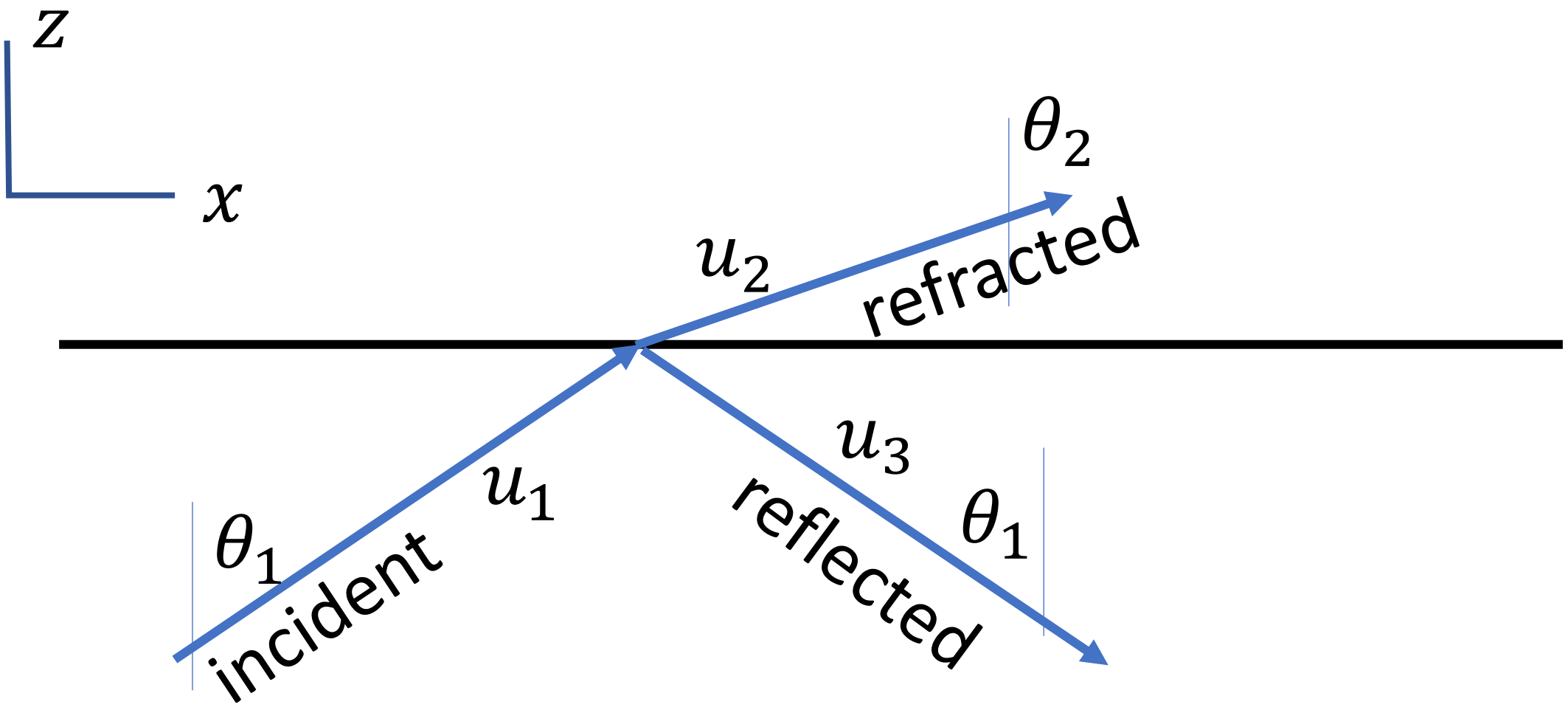
$$\alpha_2 \sin\theta_1 = \alpha_1 \sin\theta_2$$

$$\alpha_1^2 - \alpha_1^2 \sin^2\theta_2 \stackrel{?}{=} \alpha_2^2 - \alpha_2^2 \sin^2\theta_1$$

$$\alpha_1^2 - \alpha_1^2 \sin^2 \theta_2 \stackrel{?}{=} \alpha_2^2 - \alpha_2^2 \sin^2 \theta_1$$

$$\alpha_1^2 \neq \alpha_2^2$$ Not equal

To make them match, there must be a reflected wave



need a reflected wave
with angle θ_1

$$u_2 \sin\theta_2 = u_1 \sin\theta_1 + u_3 \sin\theta_1$$

$$u_2 \cos\theta_2 = u_1 \cos\theta_2 - u_3 \cos\theta_1$$

$$u_2 \sin\theta_2 - u_3 \sin\theta_1 = u_1 \sin\theta_1$$

$$u_2 \cos\theta_2 + u_3 \cos\theta_1 = u_1 \cos\theta_1$$

$$\begin{bmatrix} \sin\theta_2 & -\sin\theta_1 \\ \cos\theta_2 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = u_1 \begin{bmatrix} \sin\theta_1 \\ \cos\theta_1 \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\cos\theta_2 & \sin\theta_2 \end{bmatrix} \begin{bmatrix} \sin\theta_1 \\ \cos\theta_1 \end{bmatrix} u_1$$

$$D = \sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2$$

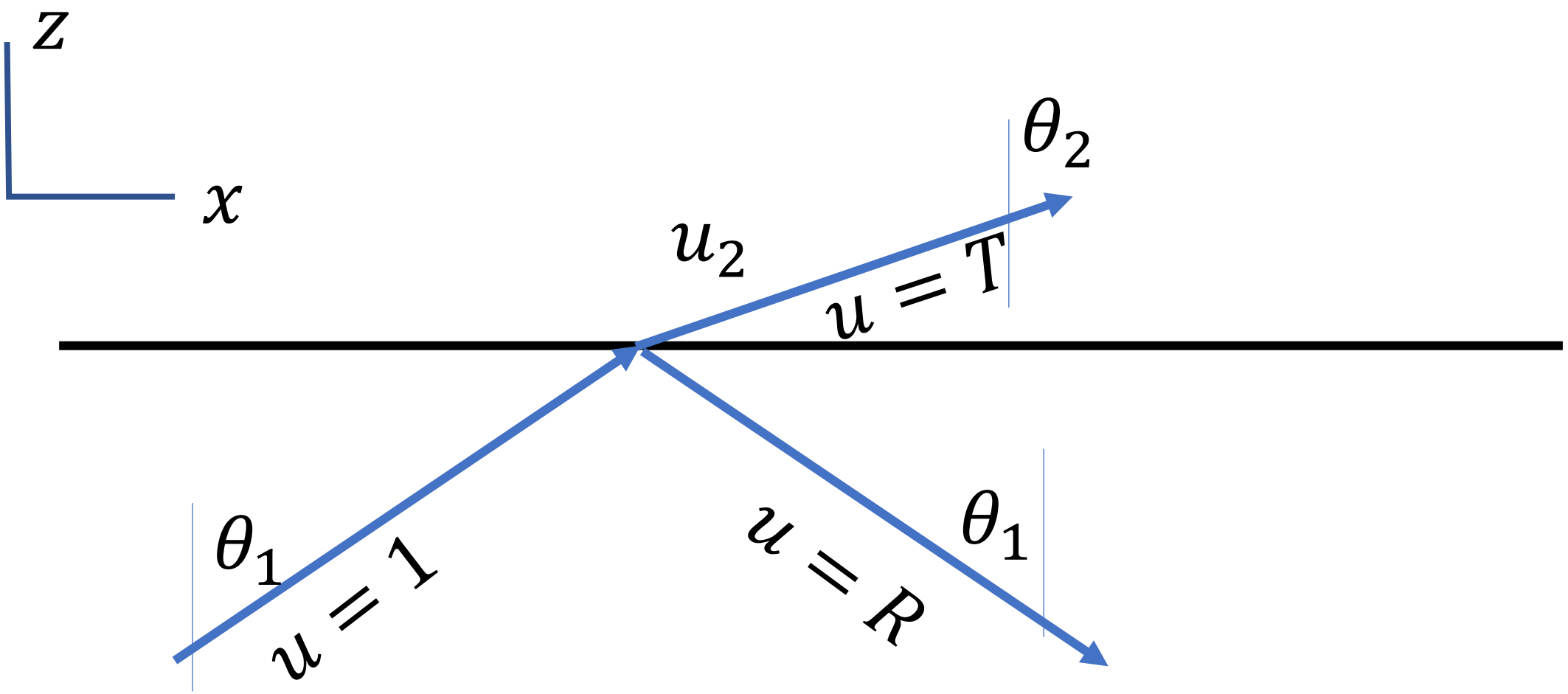
$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} 2\cos\theta_1 \sin\theta_1 \\ \sin\theta_2 \cos\theta_1 - -\cos\theta_2 \sin\theta_1 \end{bmatrix} u_1$$

$$u_3 = \frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2} u_1 =$$

$$u_3 = \frac{+\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1}{\sin\theta_2\cos\theta_1 + \sin\theta_1\cos\theta_2}u_1$$

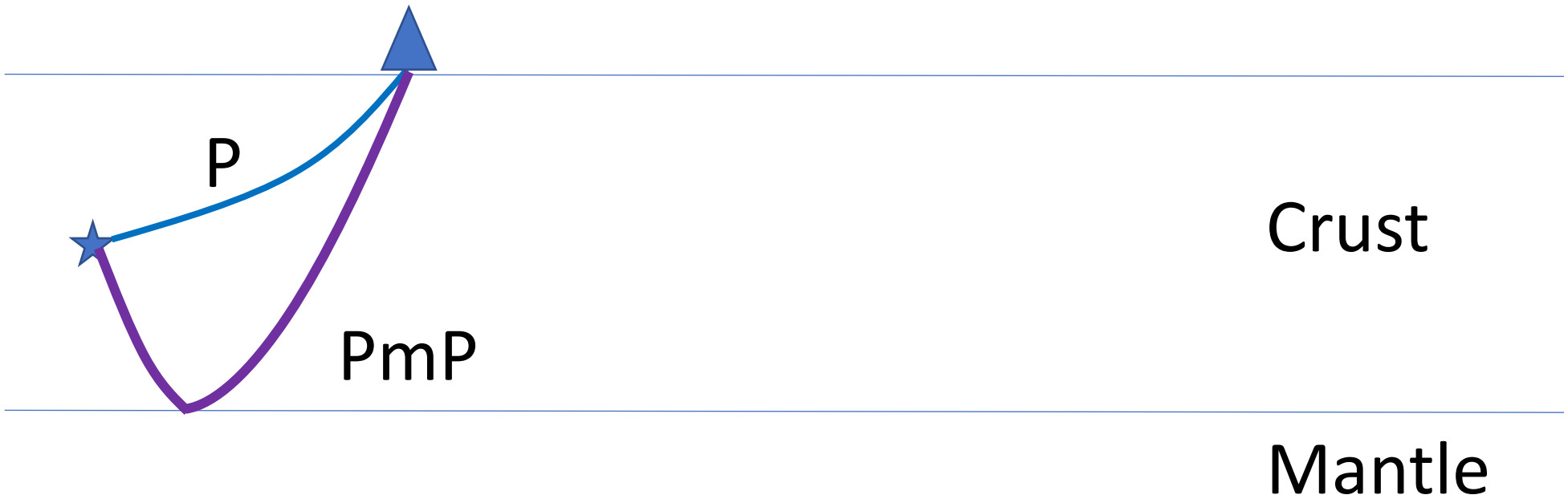
$$u_3 = u_1\frac{\frac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 - \cos\theta_2}{\frac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 + \cos\theta_2}$$

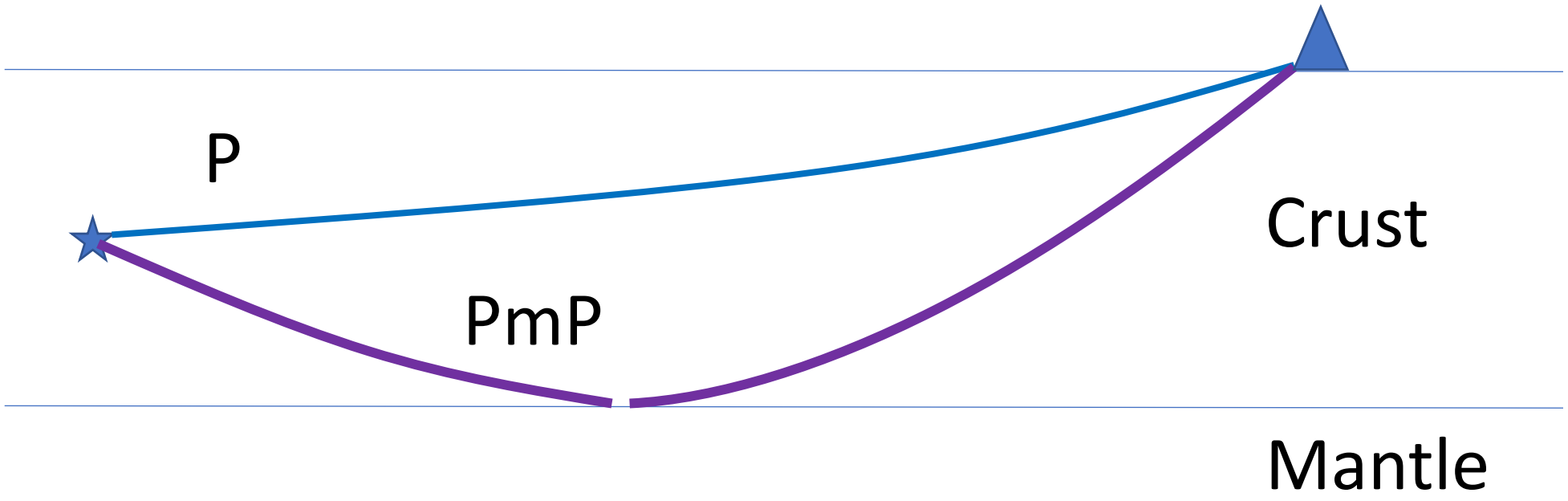
$$u_3 = \frac{\frac{\alpha_2}{\alpha_1}\cos\theta_1 - \cos\theta_2}{\frac{\alpha_2}{\alpha_1}\cos\theta_1 + \cos\theta_2}u_1$$

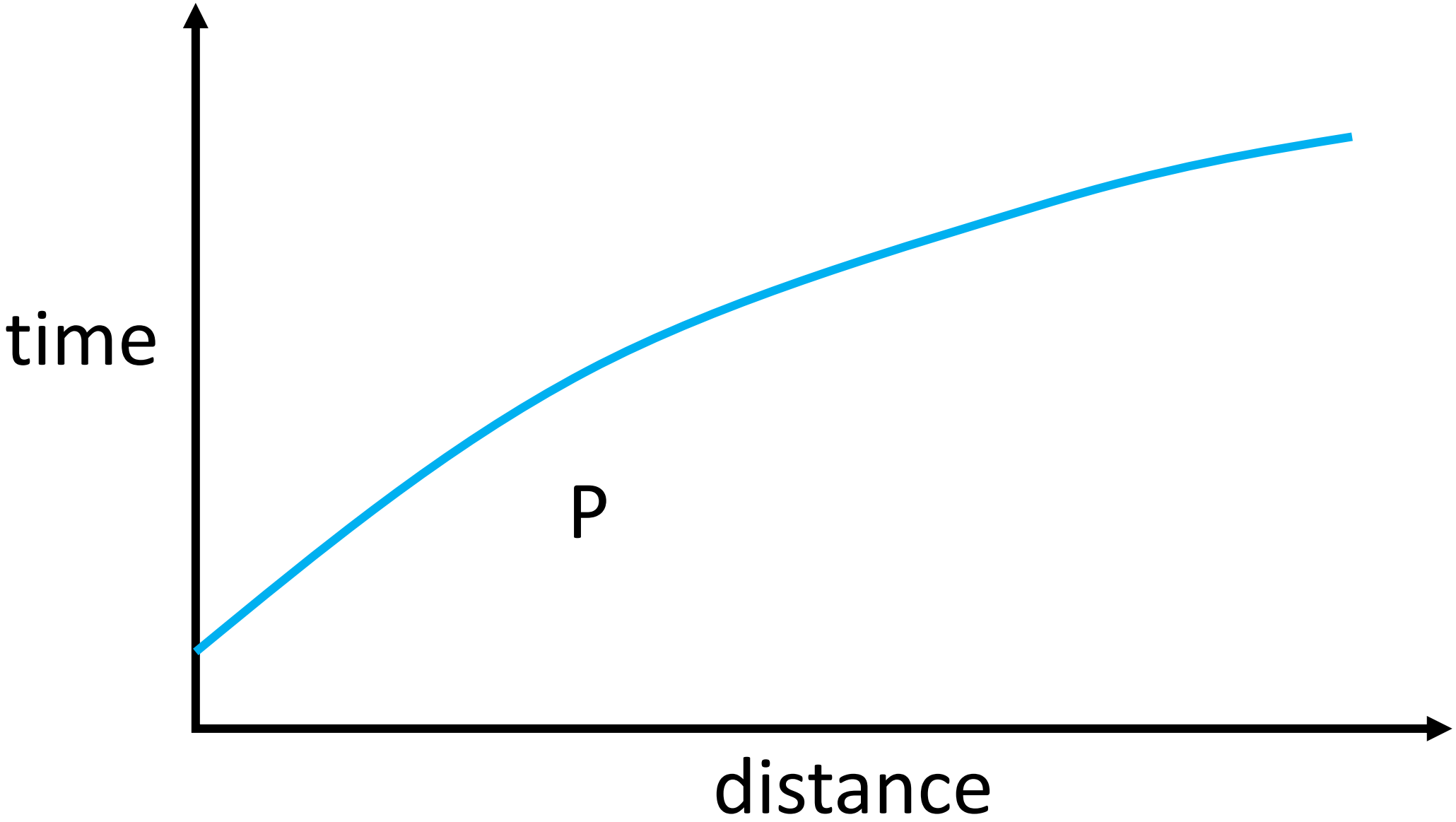


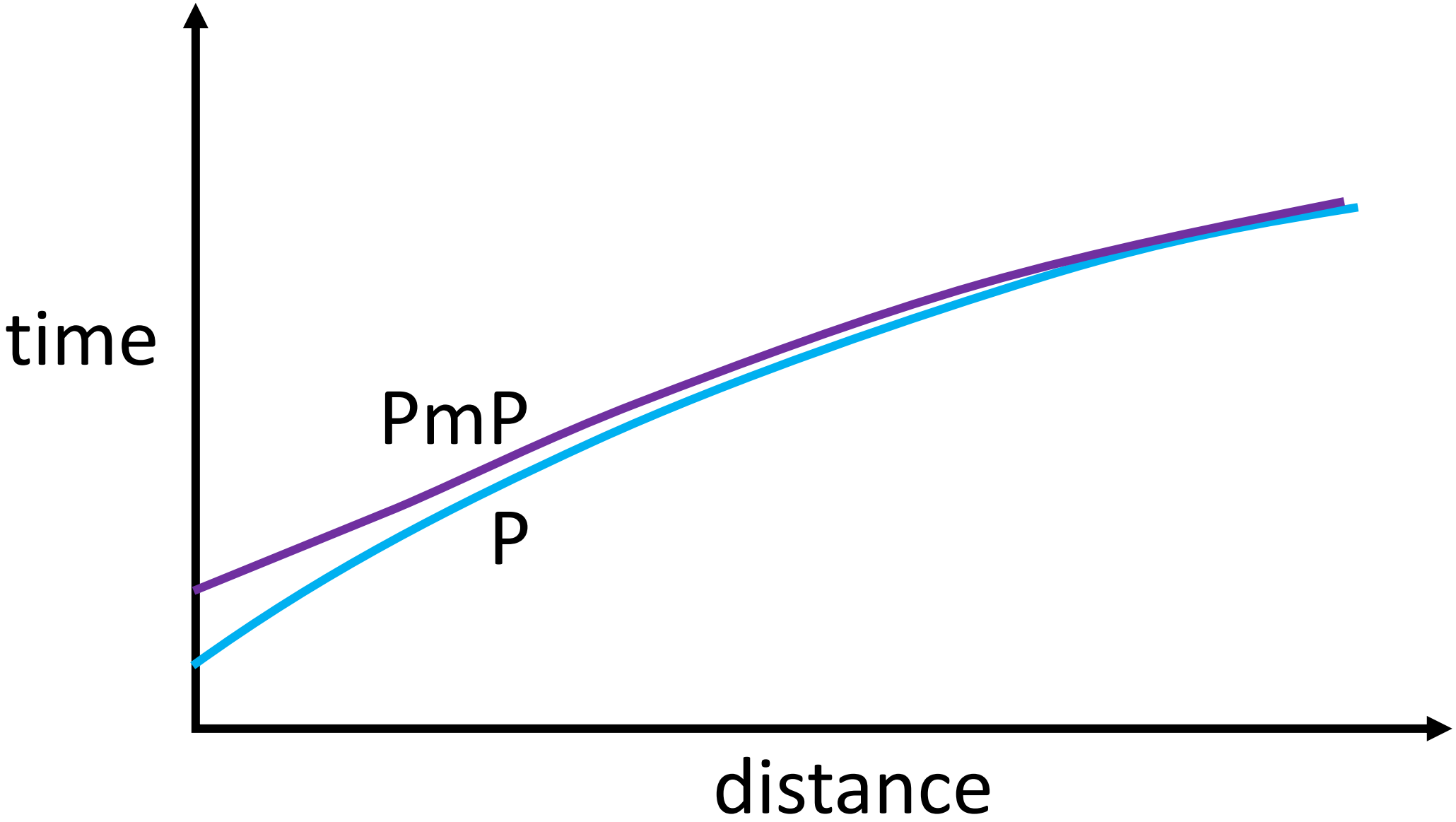
reflection coefficient R

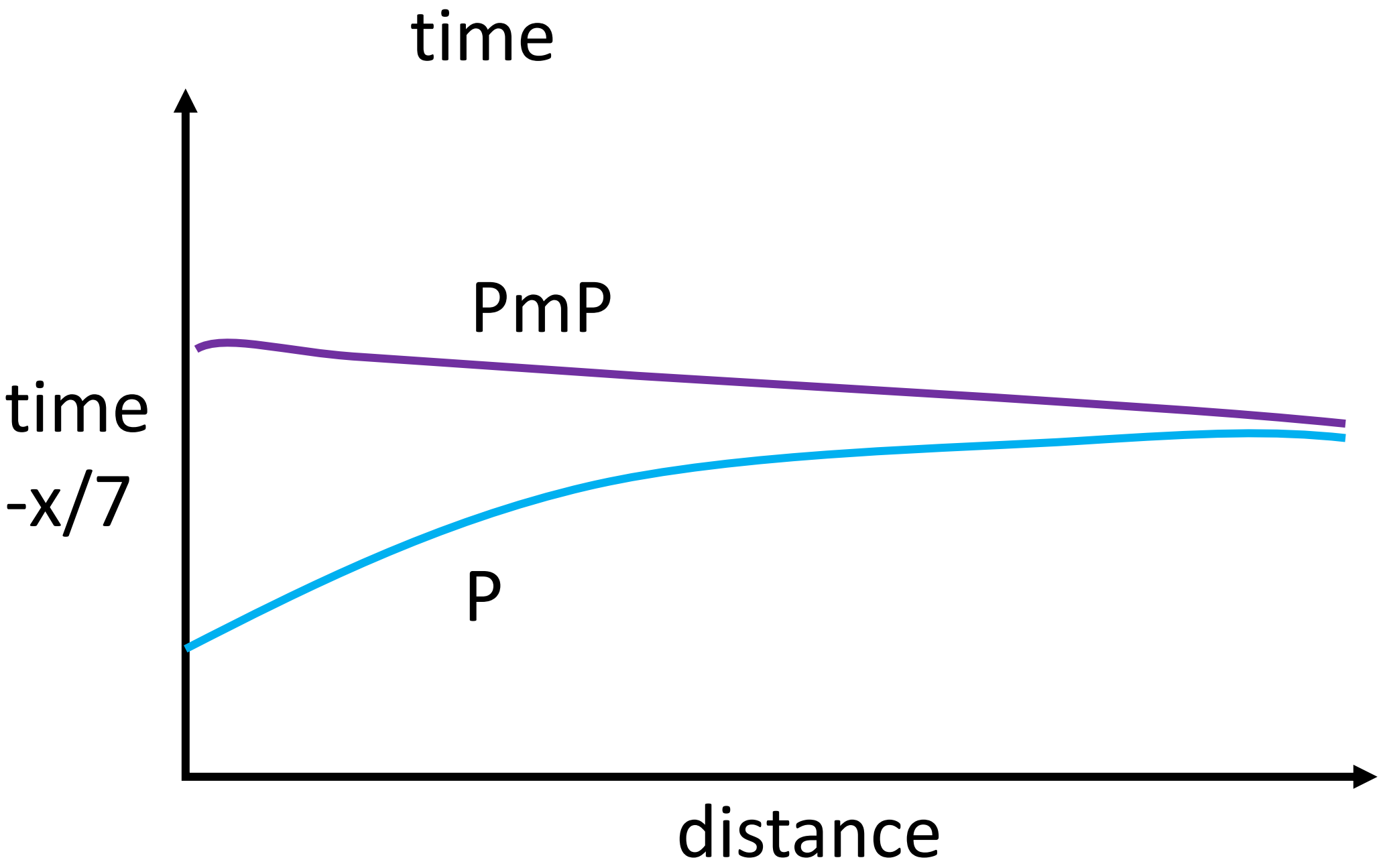
transmission coefficient T


P
PmP
Crust
Mantle


P
PmP
Crust
Mantle


time
P
distance

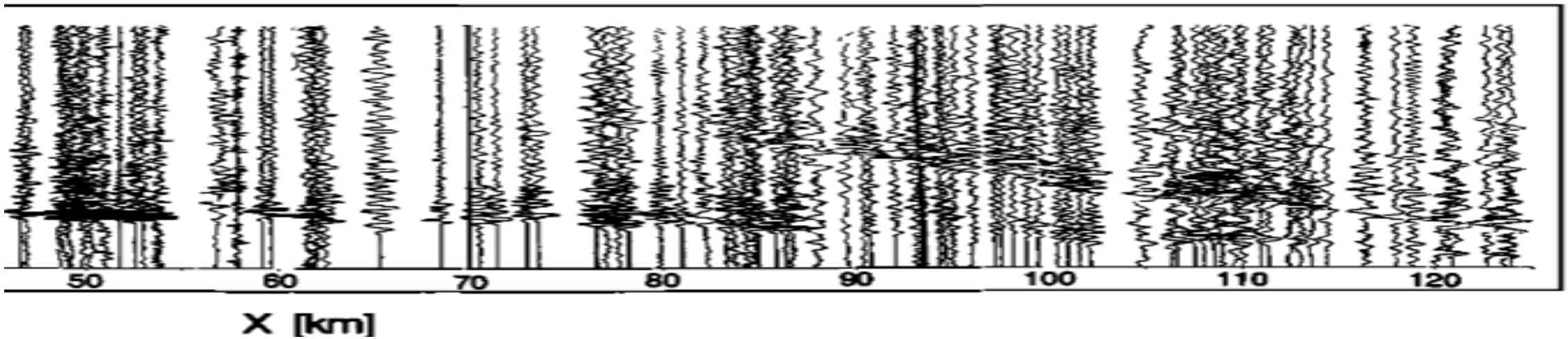

time
PmP
P
distance


time
PmP
time
-x/7
P
distance

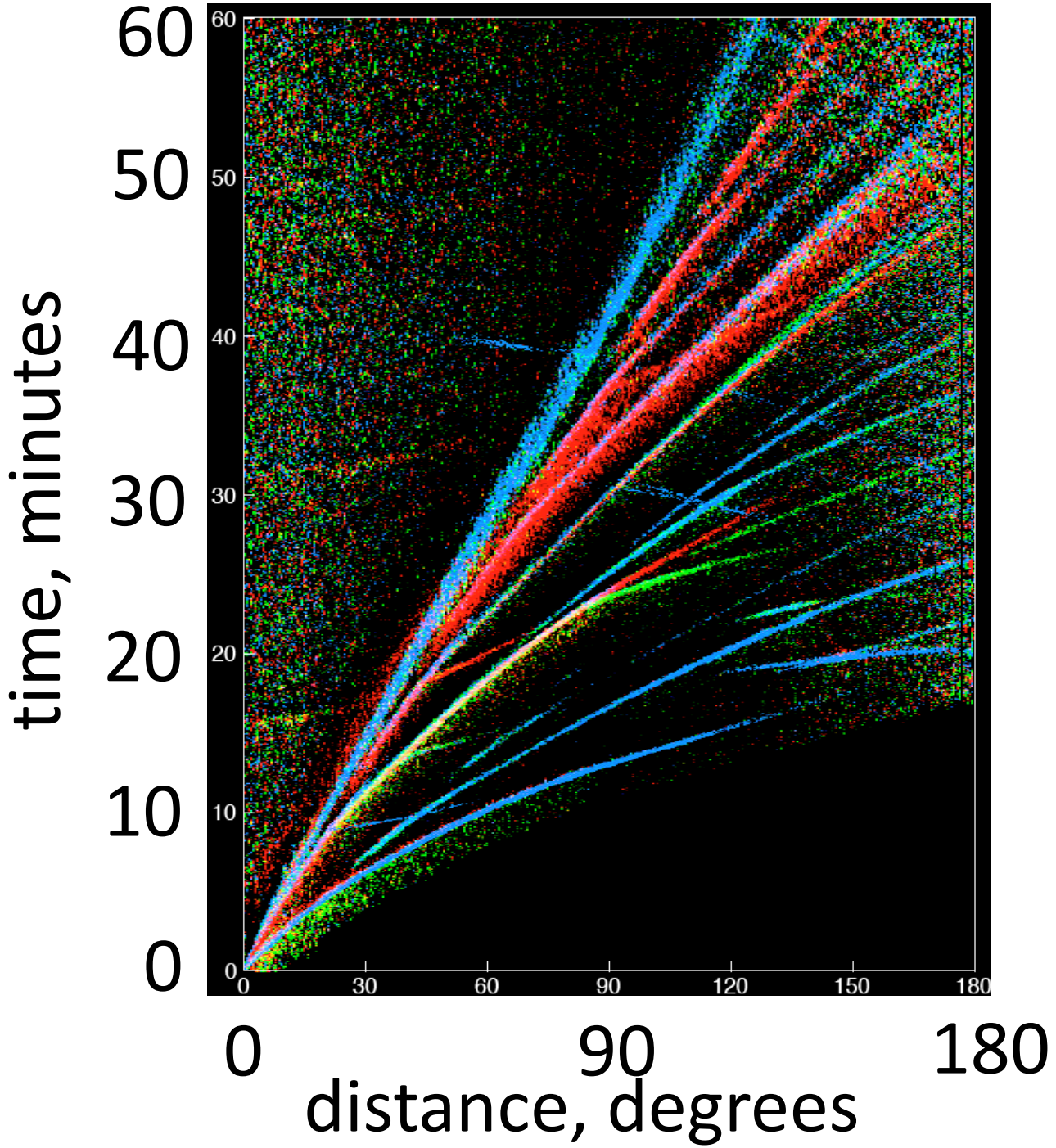
Solid Earth Dynamics

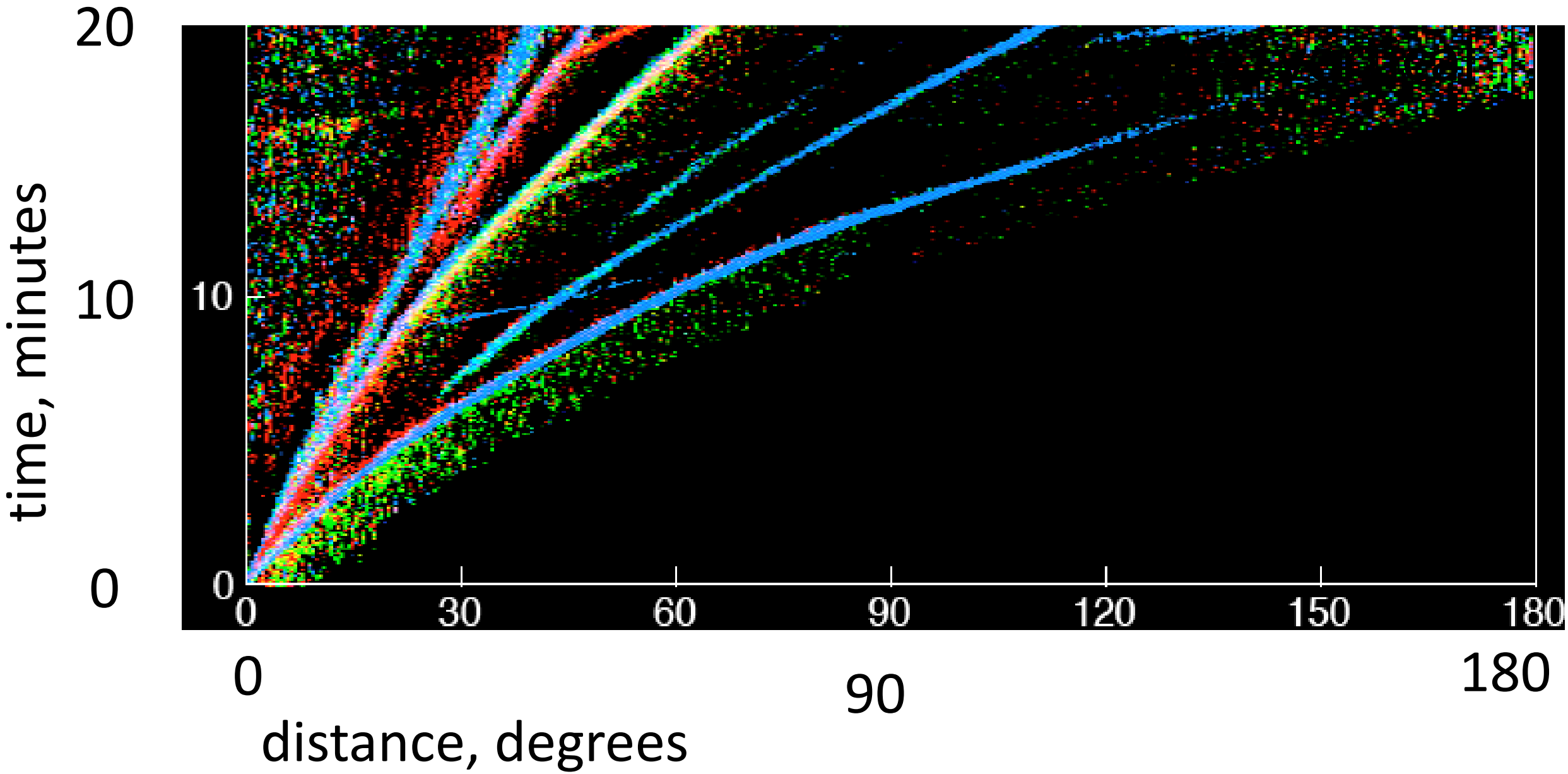
Bill Menke, Instructor

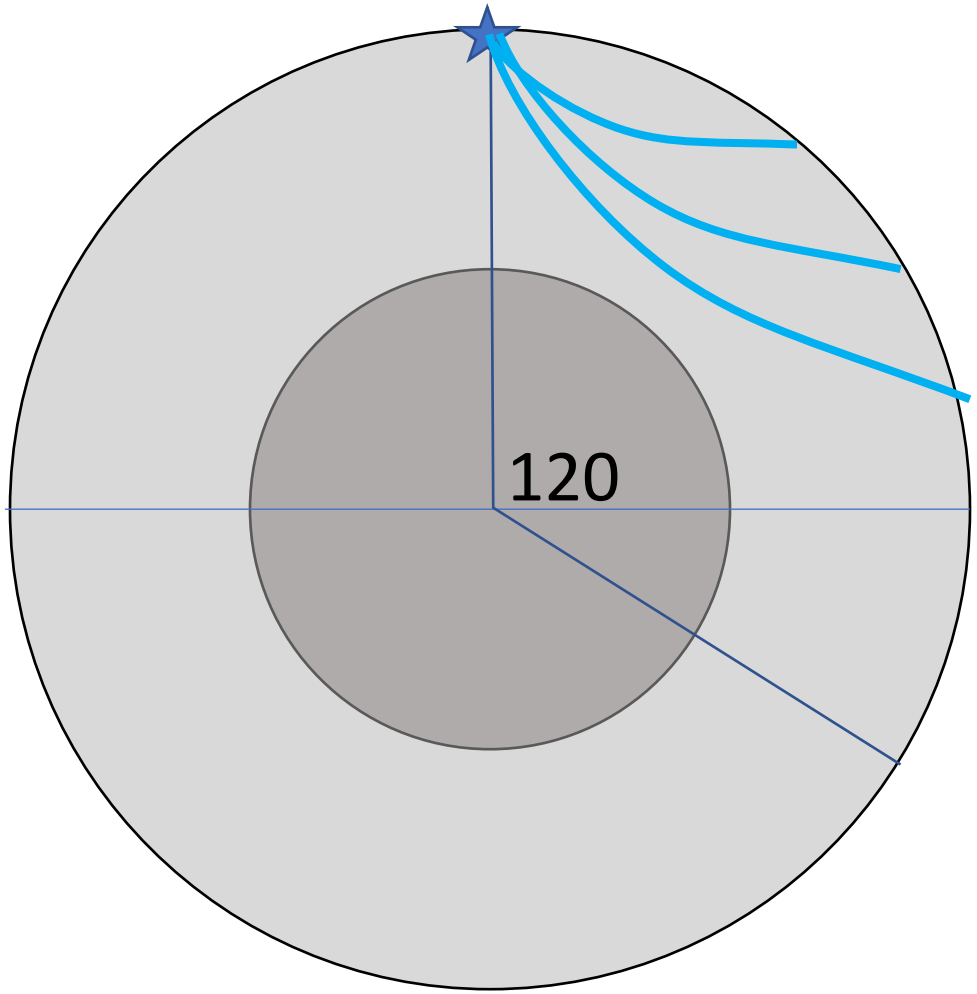
Lecture 16

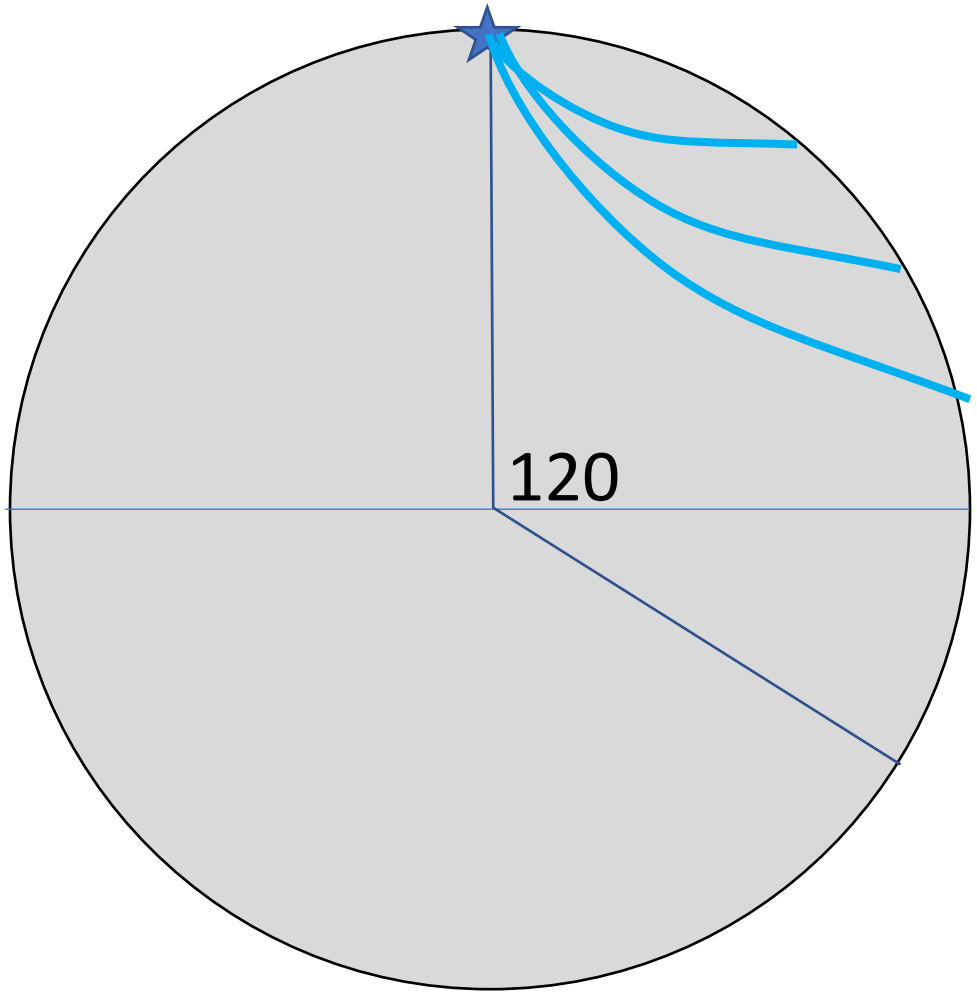

50
60
70
80
90
100
110
120
X [km]

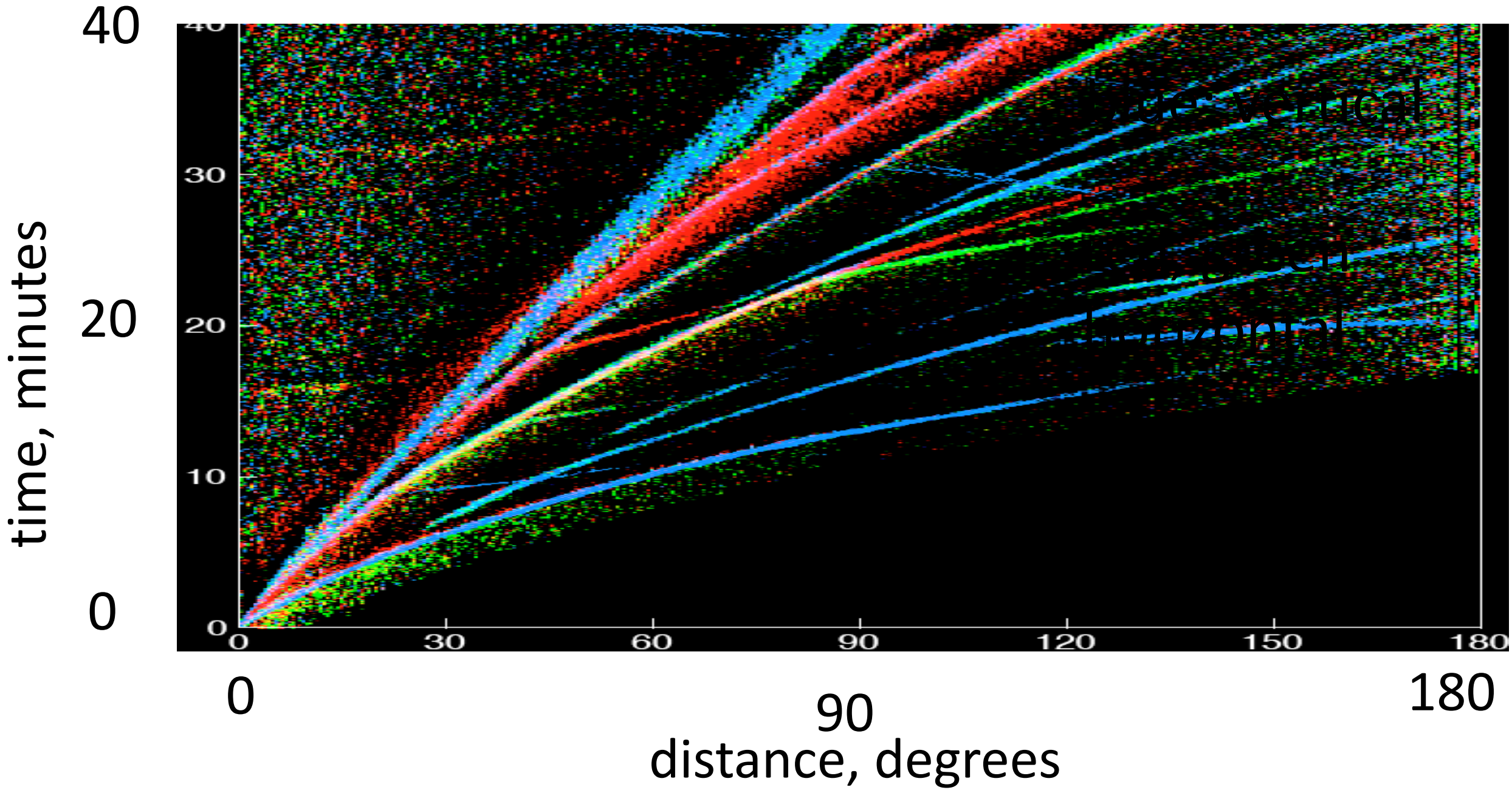
3. Seismic Record Section for the Earth as a whole


time, minutes
60
50
40
30
20
10
0
0
90
180
distance, degrees
blue: vertical
red, green
horizontal


20
10
0
time, minutes
10
0
0
30
60
90
120
150
180
0
90
180
distance, degrees


120


120



40
20
0
time, minutes
0
30
60
90
120
150
180
0
90
180
distance, degrees