Solid Earth Dynamics

Bill Menke, Instructor

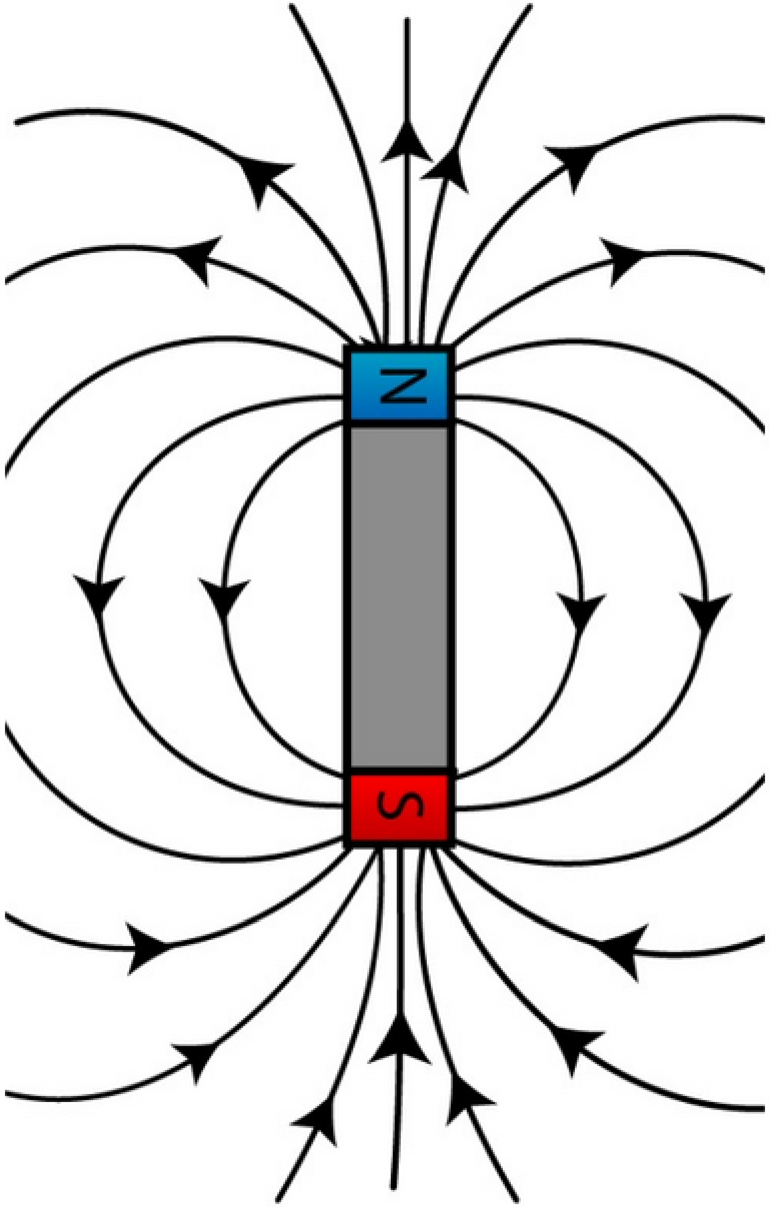
Lecture 22

Today:

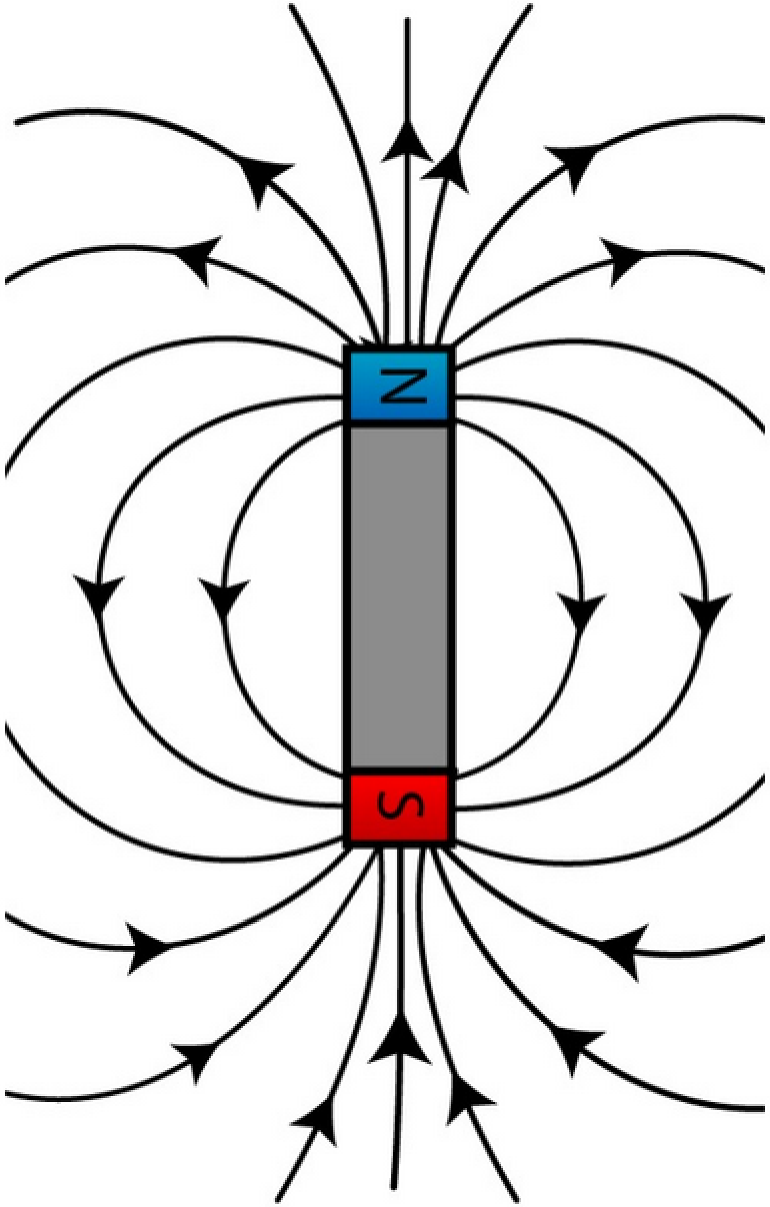
Geomagnetism:

- Paleomagnetism
- Return to the Dynamo

Paleomagnetism



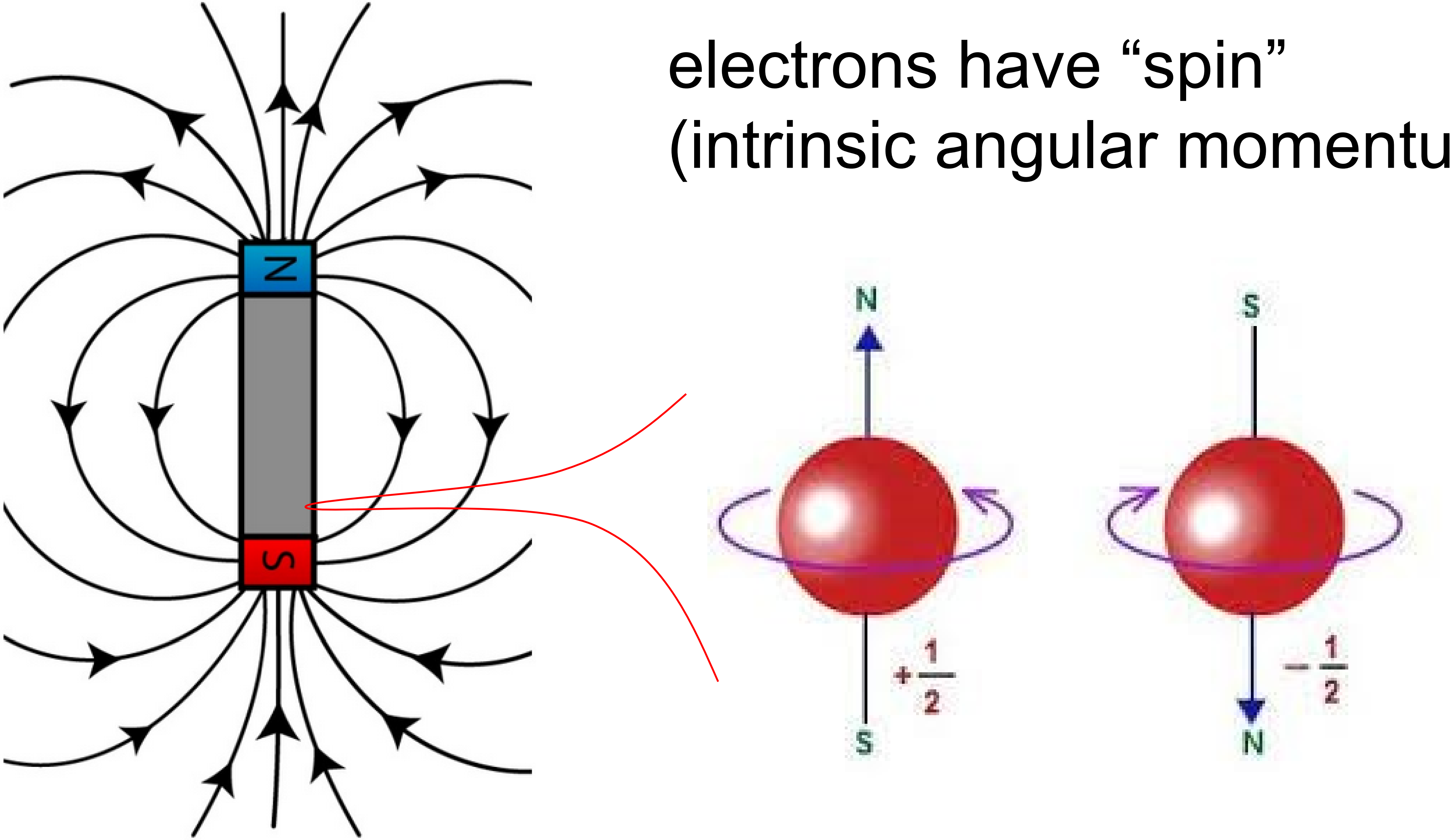
dipole field around a bar magnet

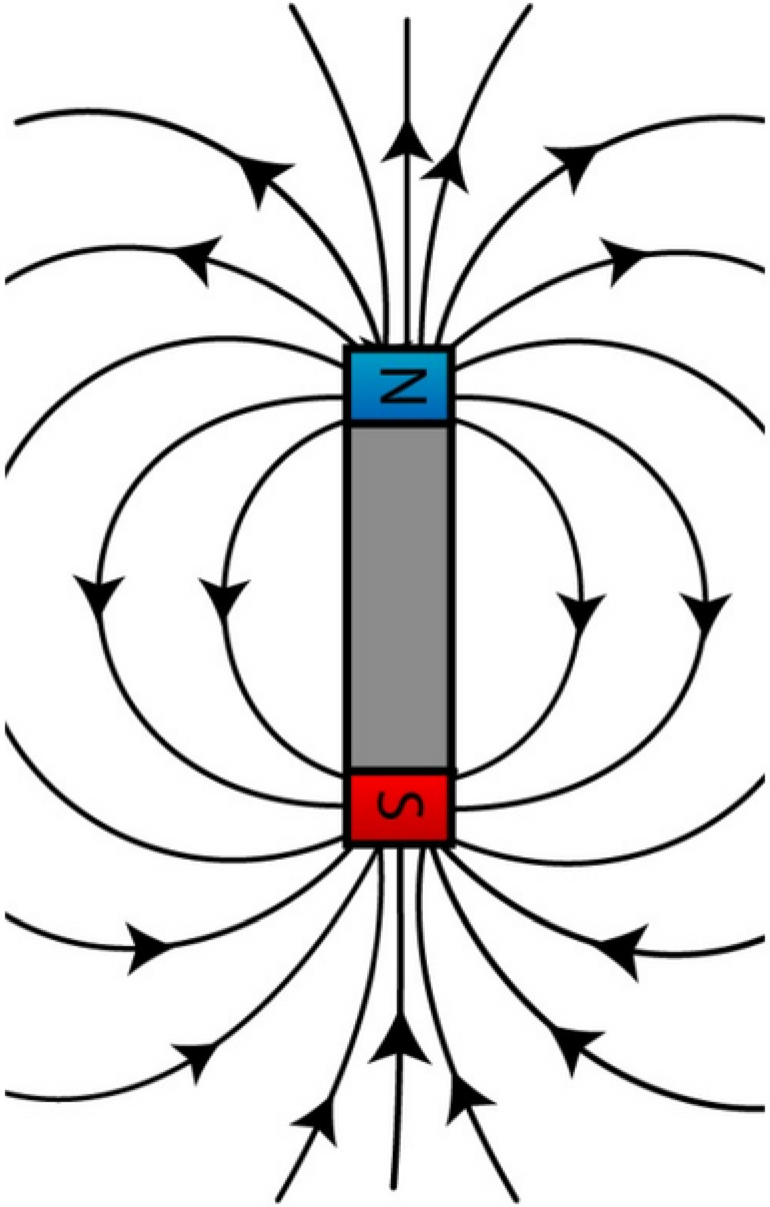


where does this magnetic field come from?

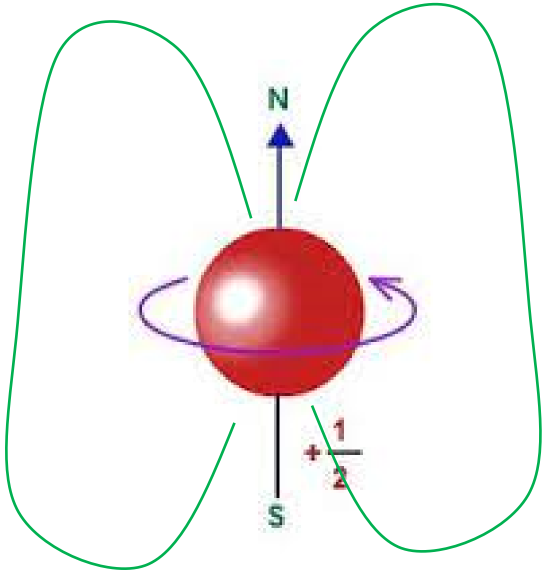
No (obvious) currents to make a solenoid

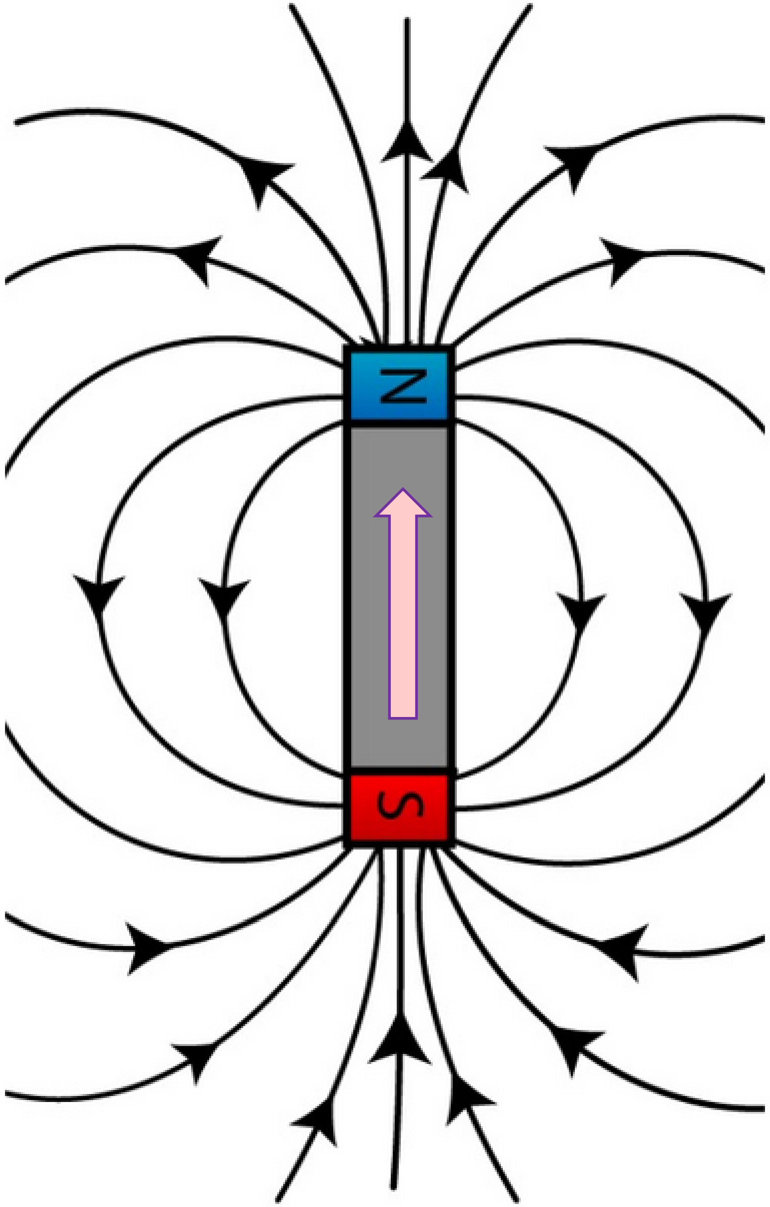
electrons have “spin” (intrinsic angular momentum)





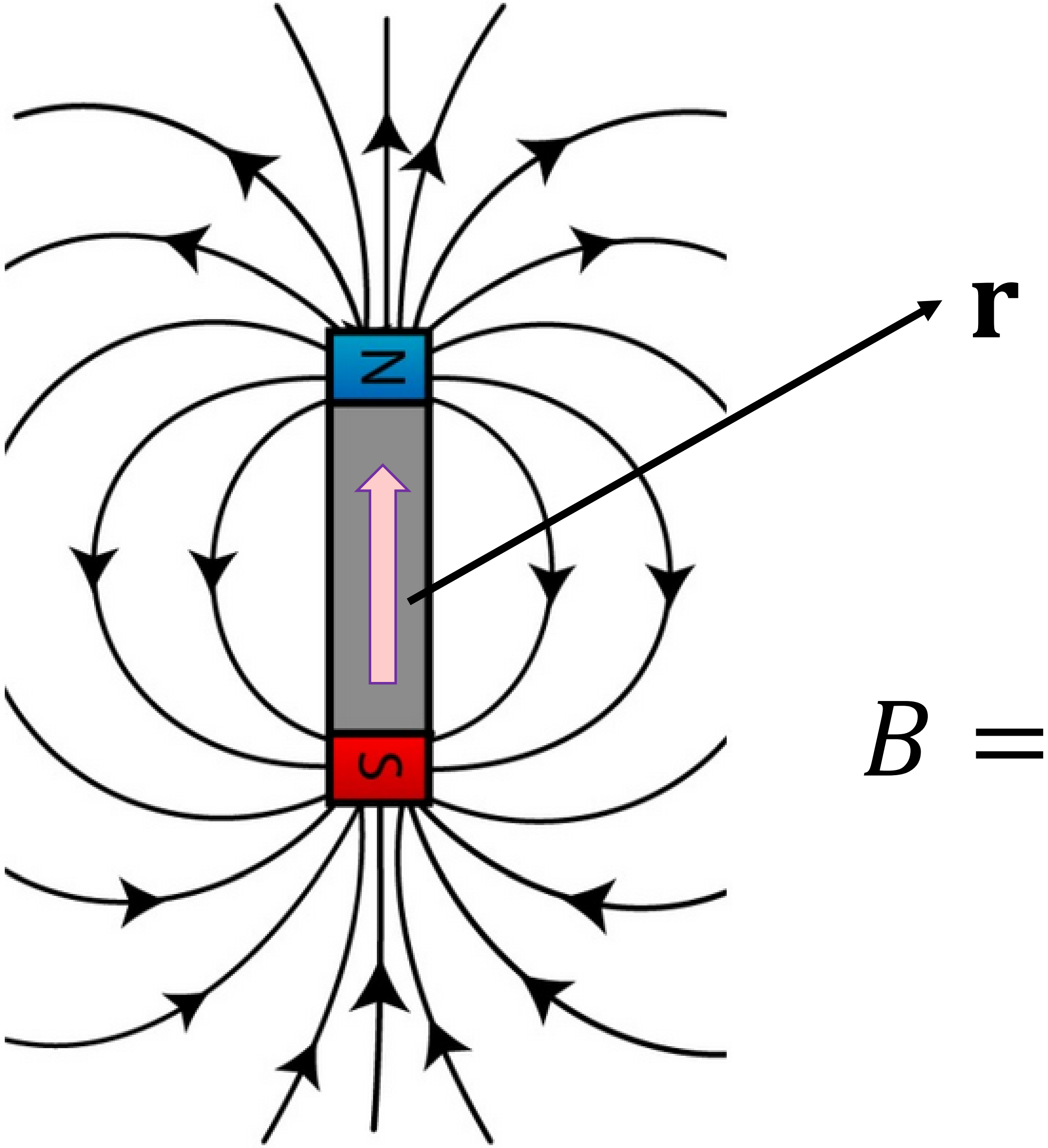
everything with spin generates a dipolar magnetic field



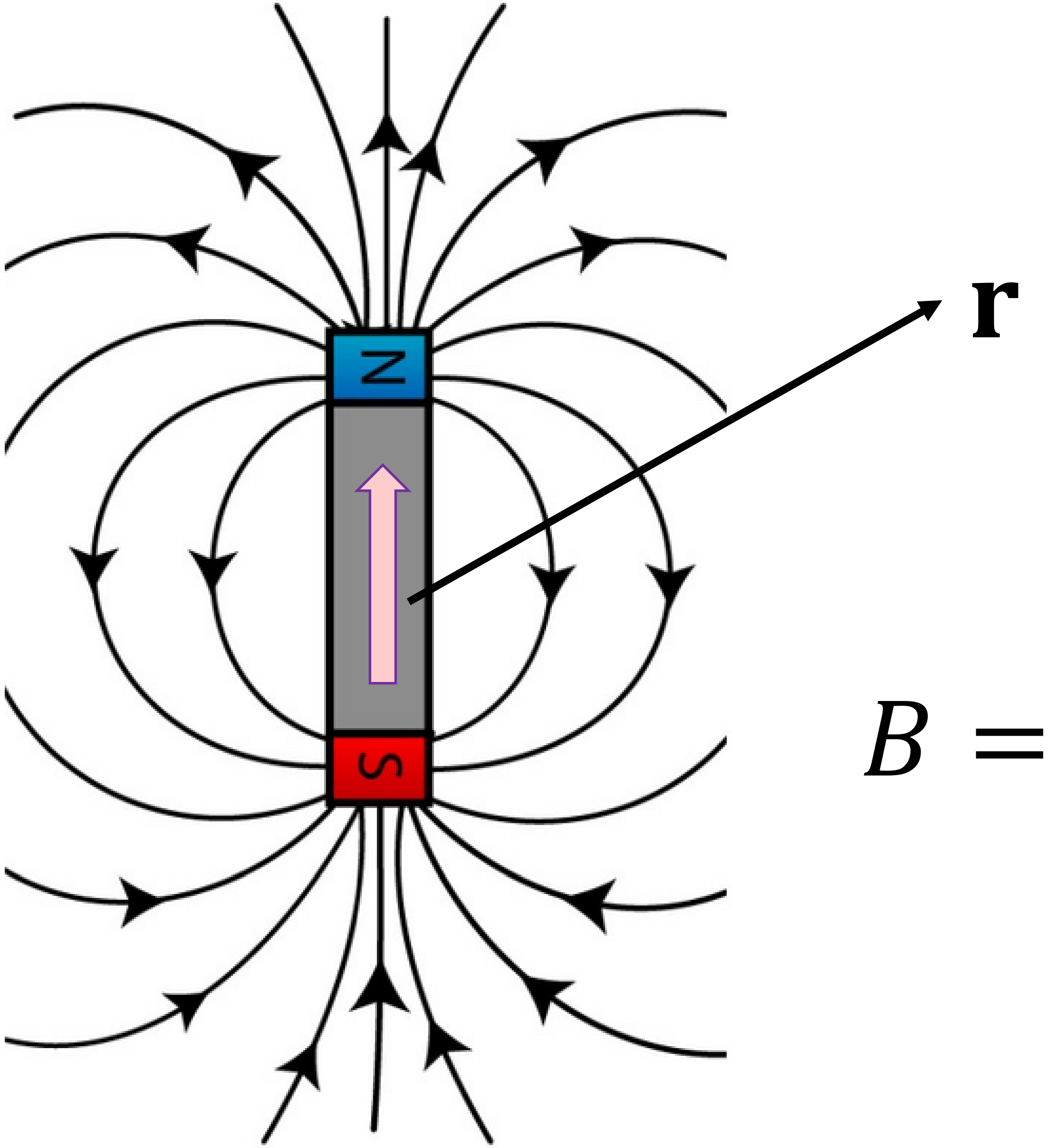


described by

magnetization vector $\mathbf{M}$ ⇧

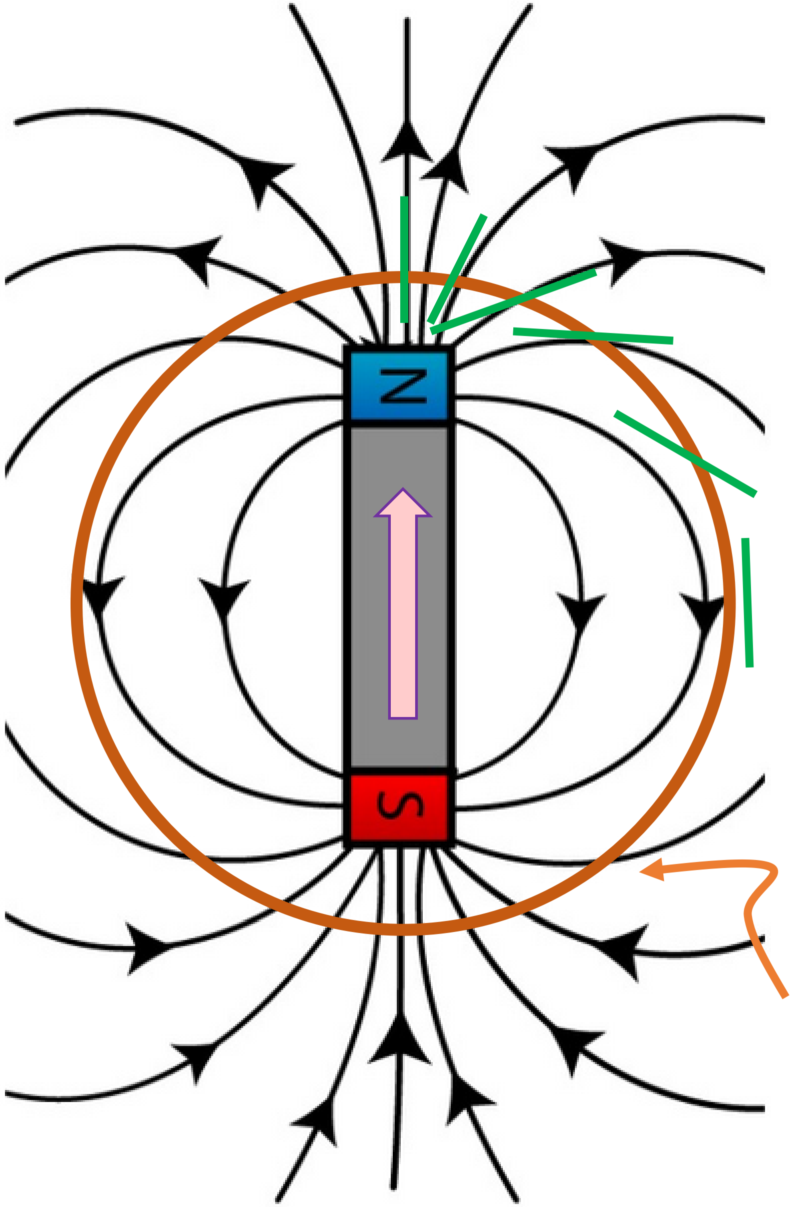


$$B = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{M}) - \mathbf{M}}{r^3}\right)$$



$$B = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{M})-\mathbf{M}}{r^3}\right)$$

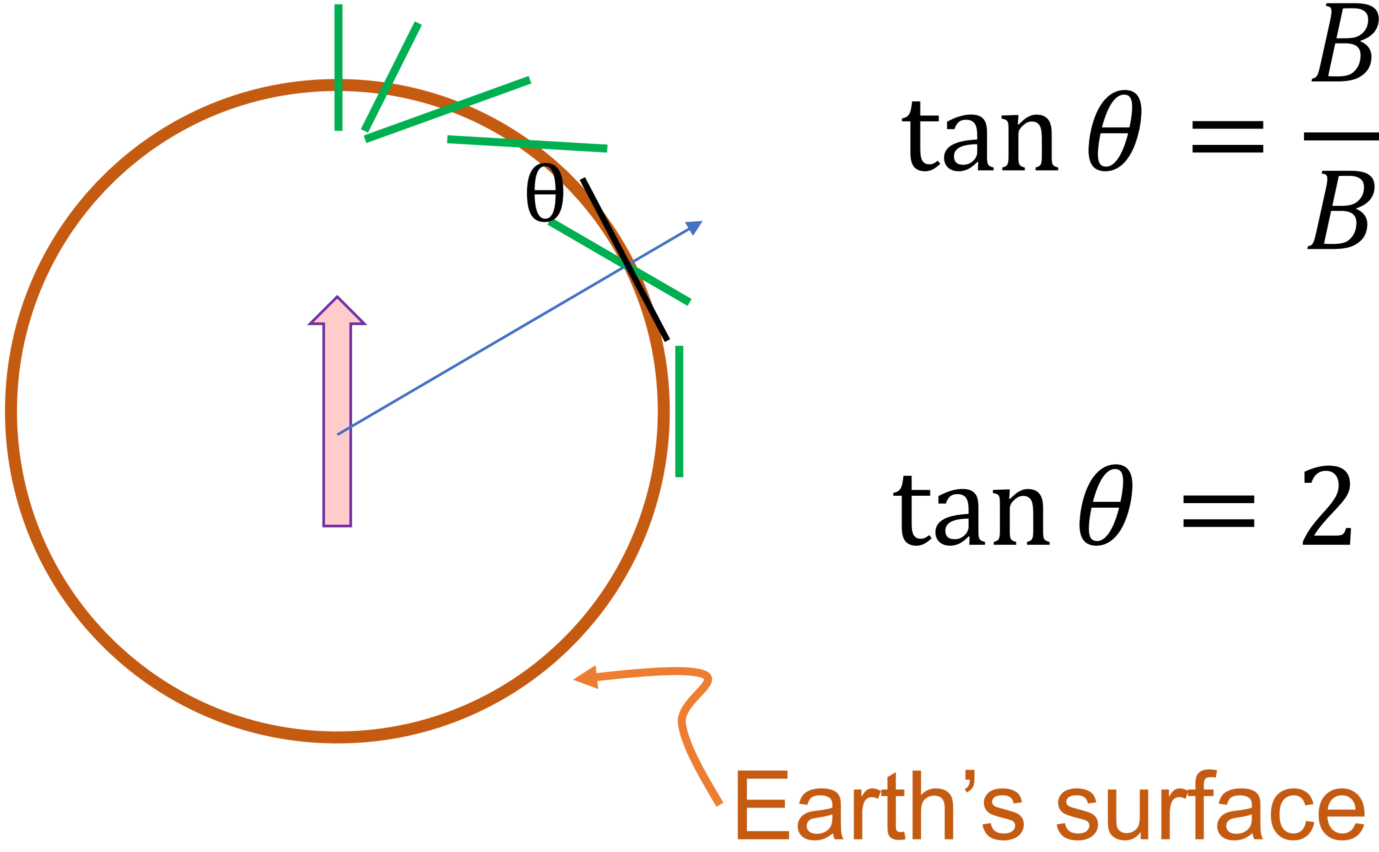
inverse cubed law



magnetic field
not horizontal

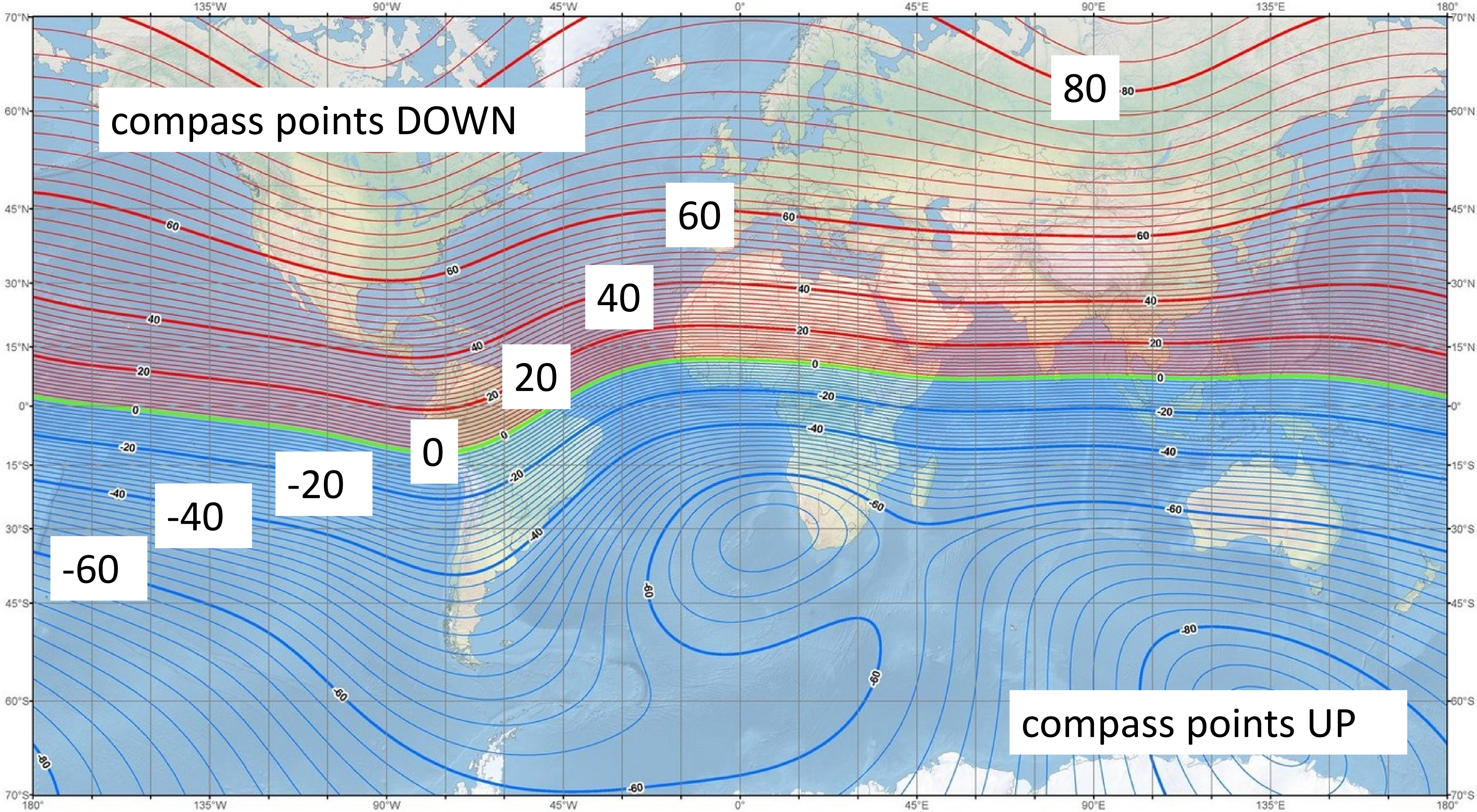
Earth’s surface

formula for inclination



$$\tan\theta = \frac{B_V}{B_H}$$

$$\tan\theta = 2\tan(\text{latitude})$$



compass points DOWN

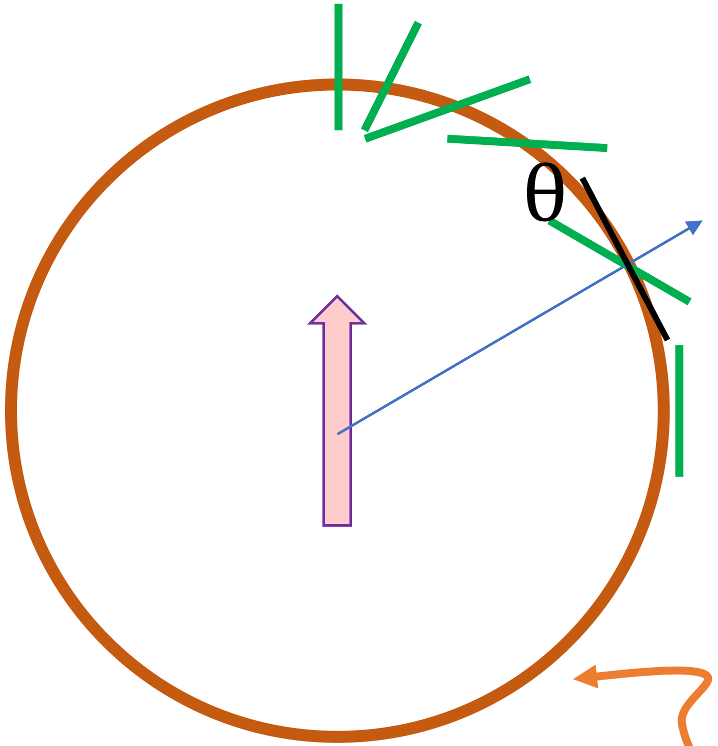
compass points UP

turn formula around

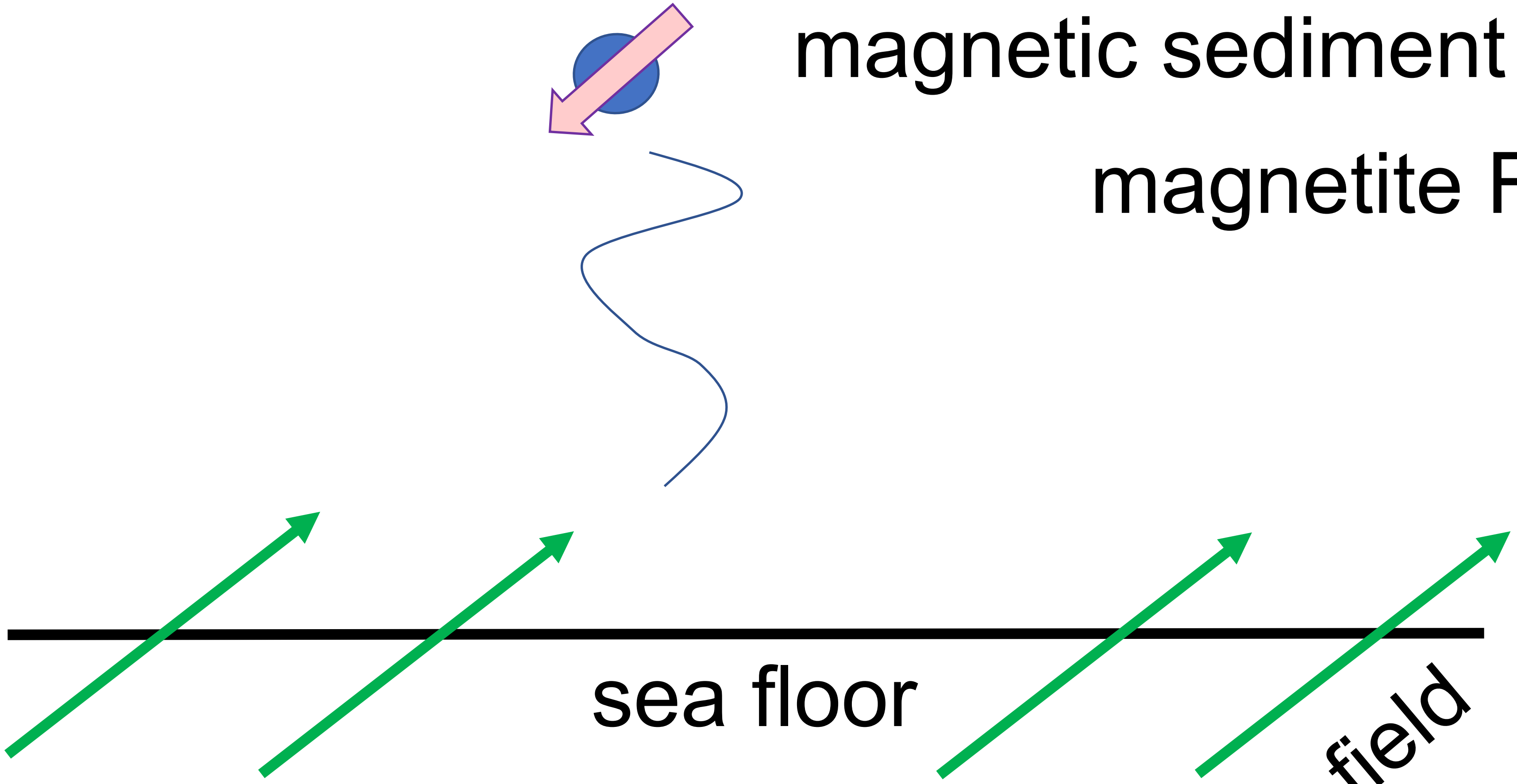
$$\tan(\text{latitude}) = \tfrac{1}{2}\tan\theta$$

measure inclination

infer latitude



Earth’s surface



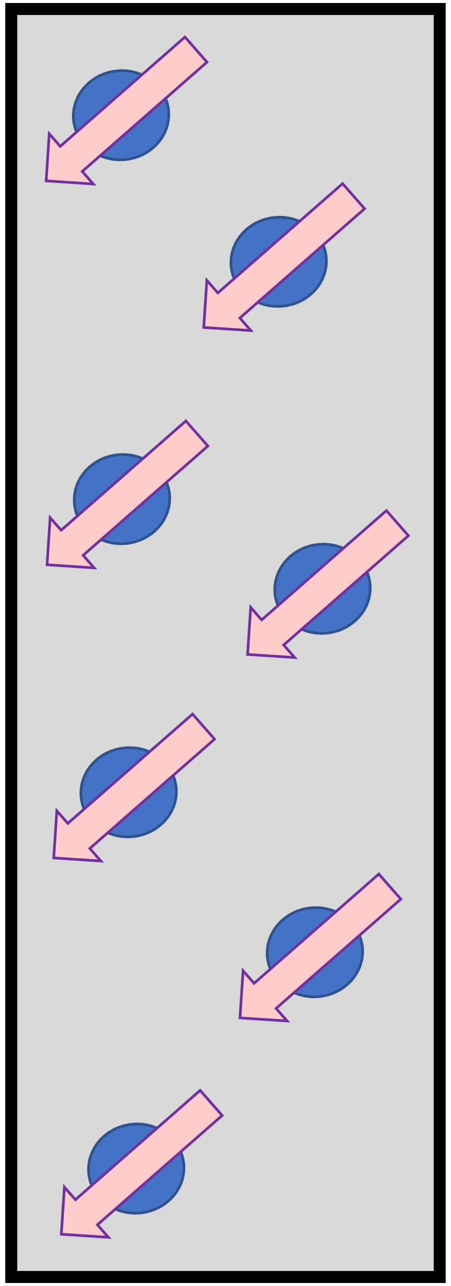
magnetic sediment grain

magnetite Fe_3O_4

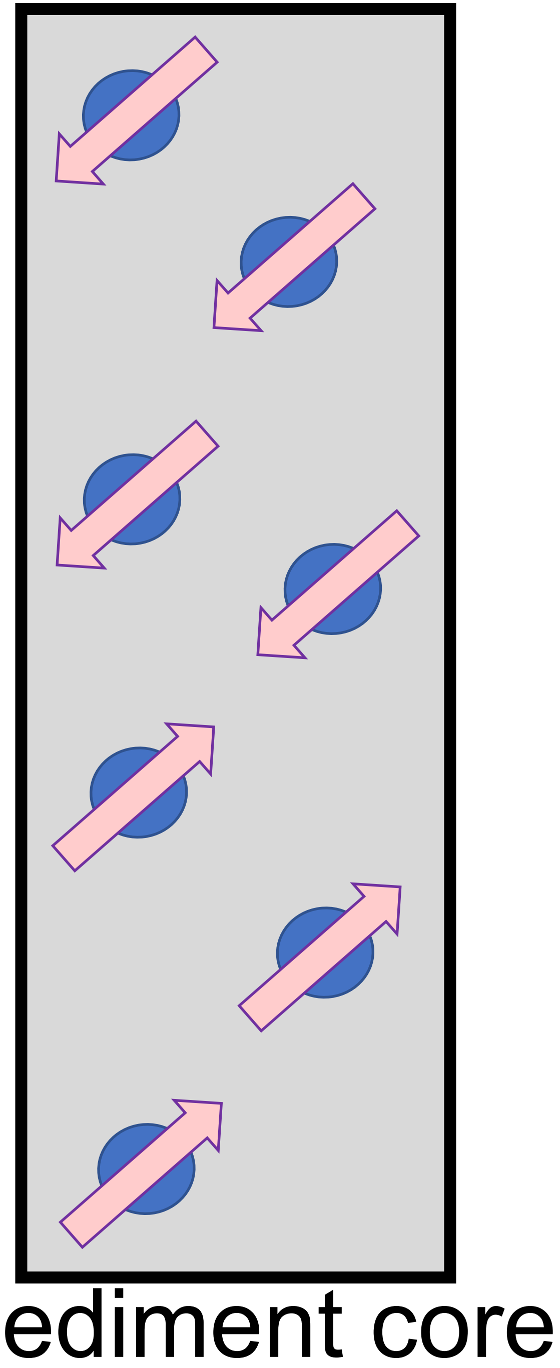
sea floor

earth's field

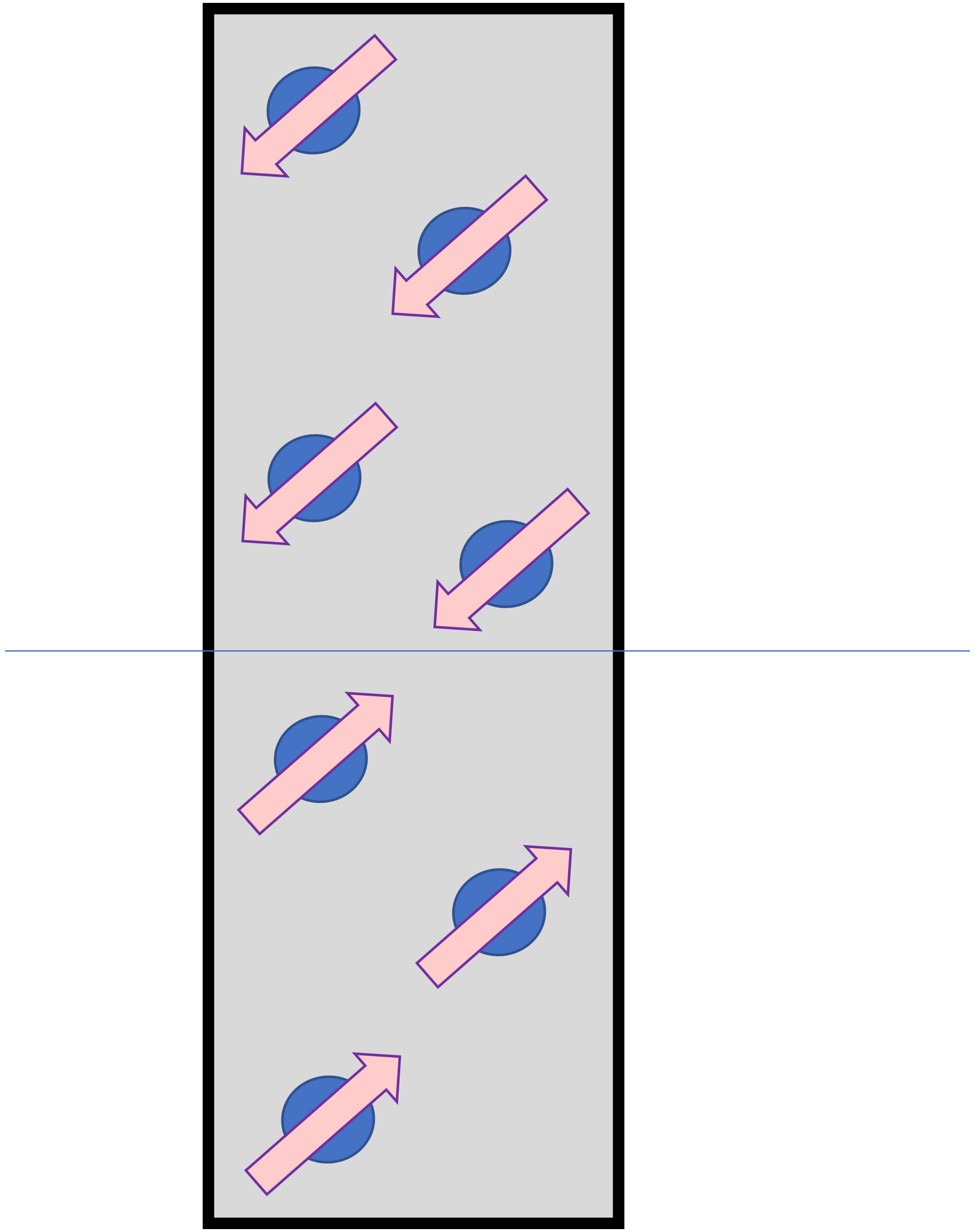
northern hemisphere
field points up
compass (sediment grain) points down



sediment core

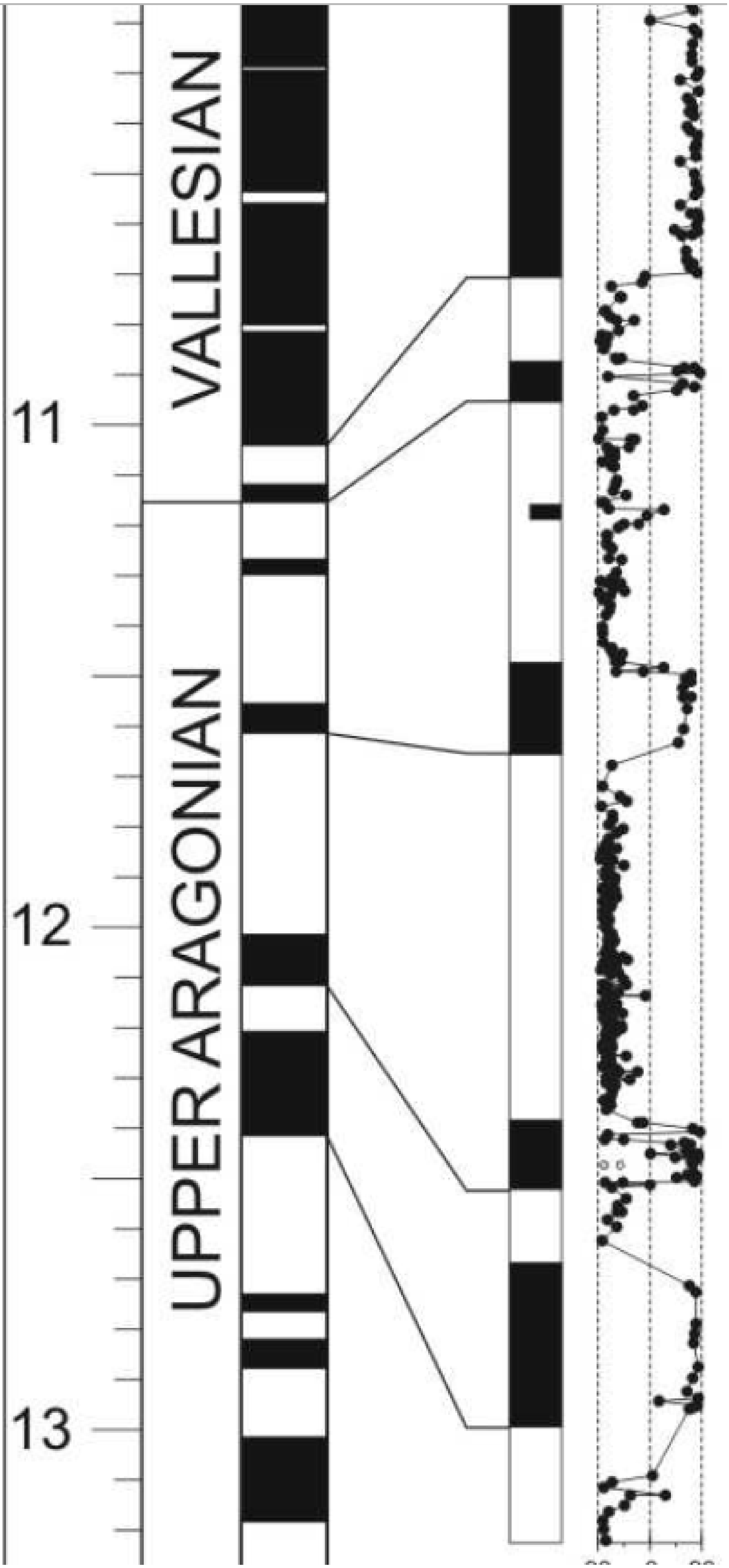


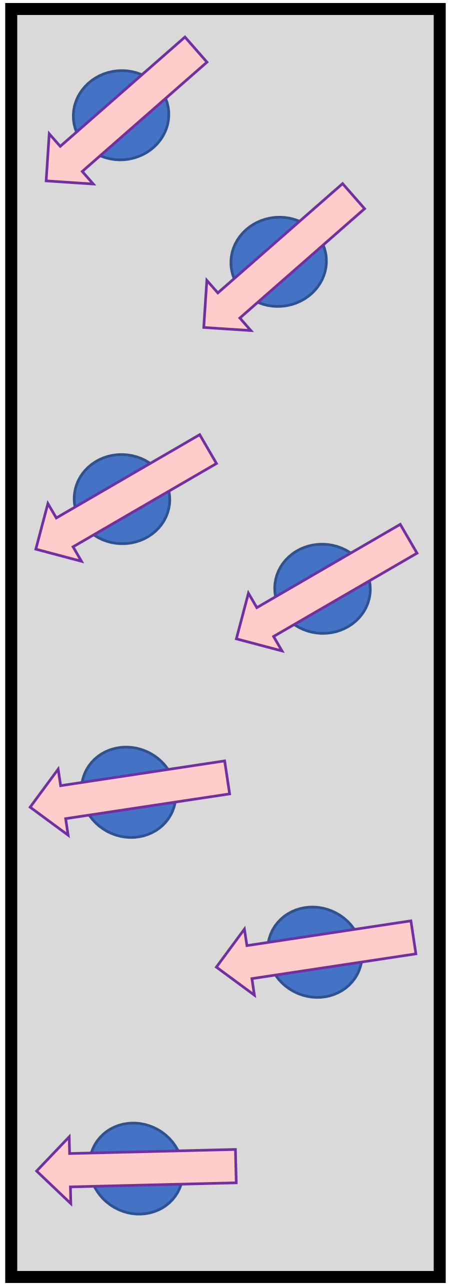
what’s happened here?



reversal

sediment core

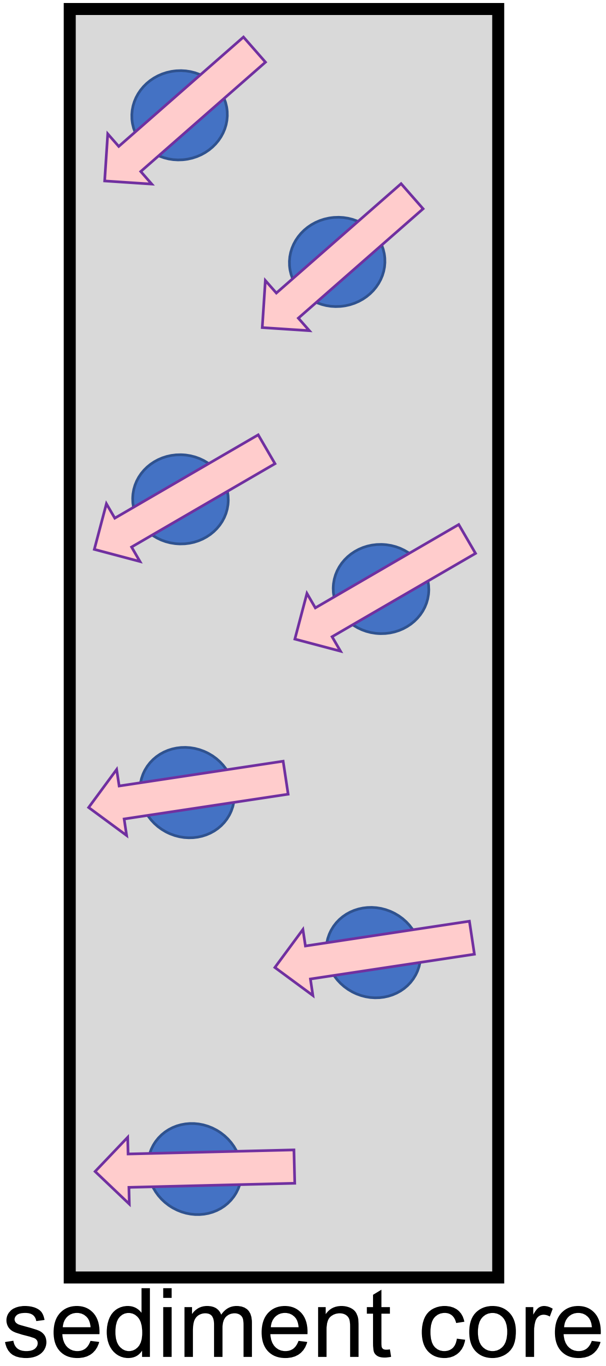

11
12
13
VALLESIAN
UPPER ARAGONIAN

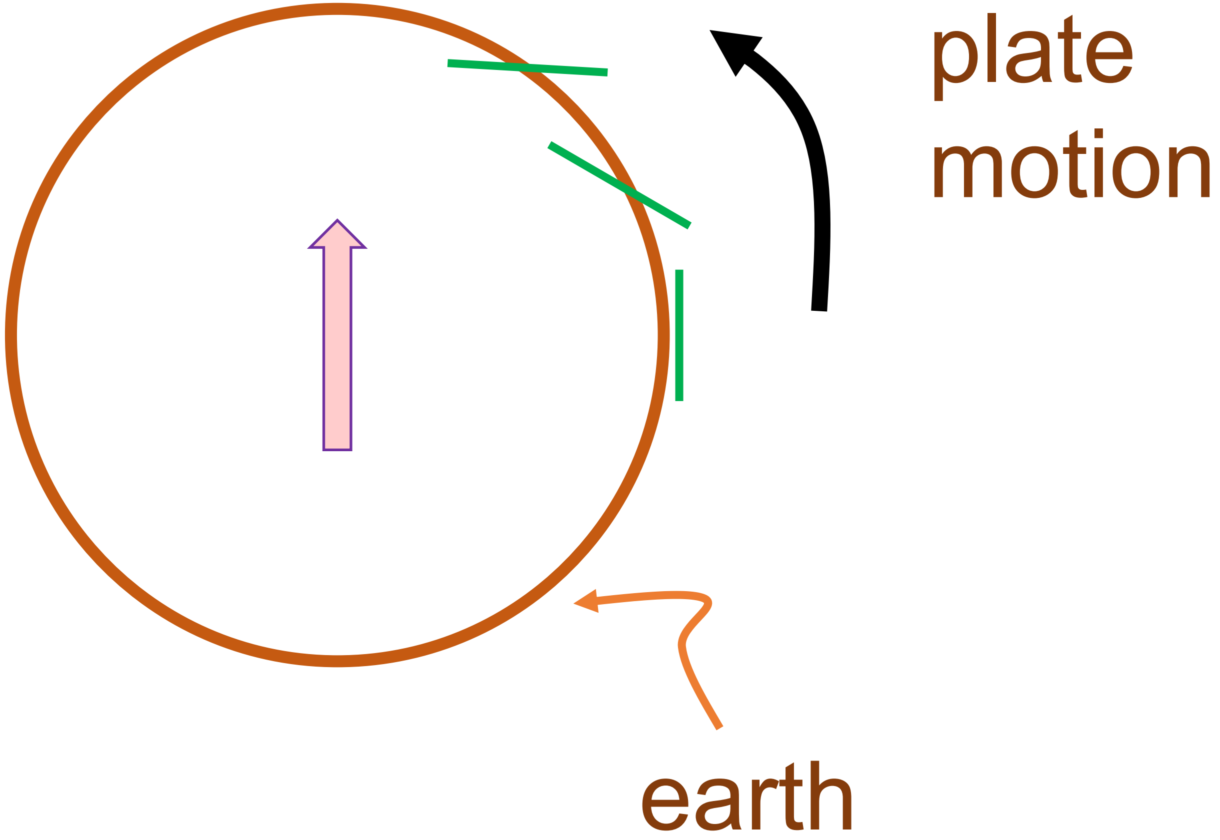


sediment core

what’s happened here?

plate is moving away from the equator





Site at Oregon – California Border

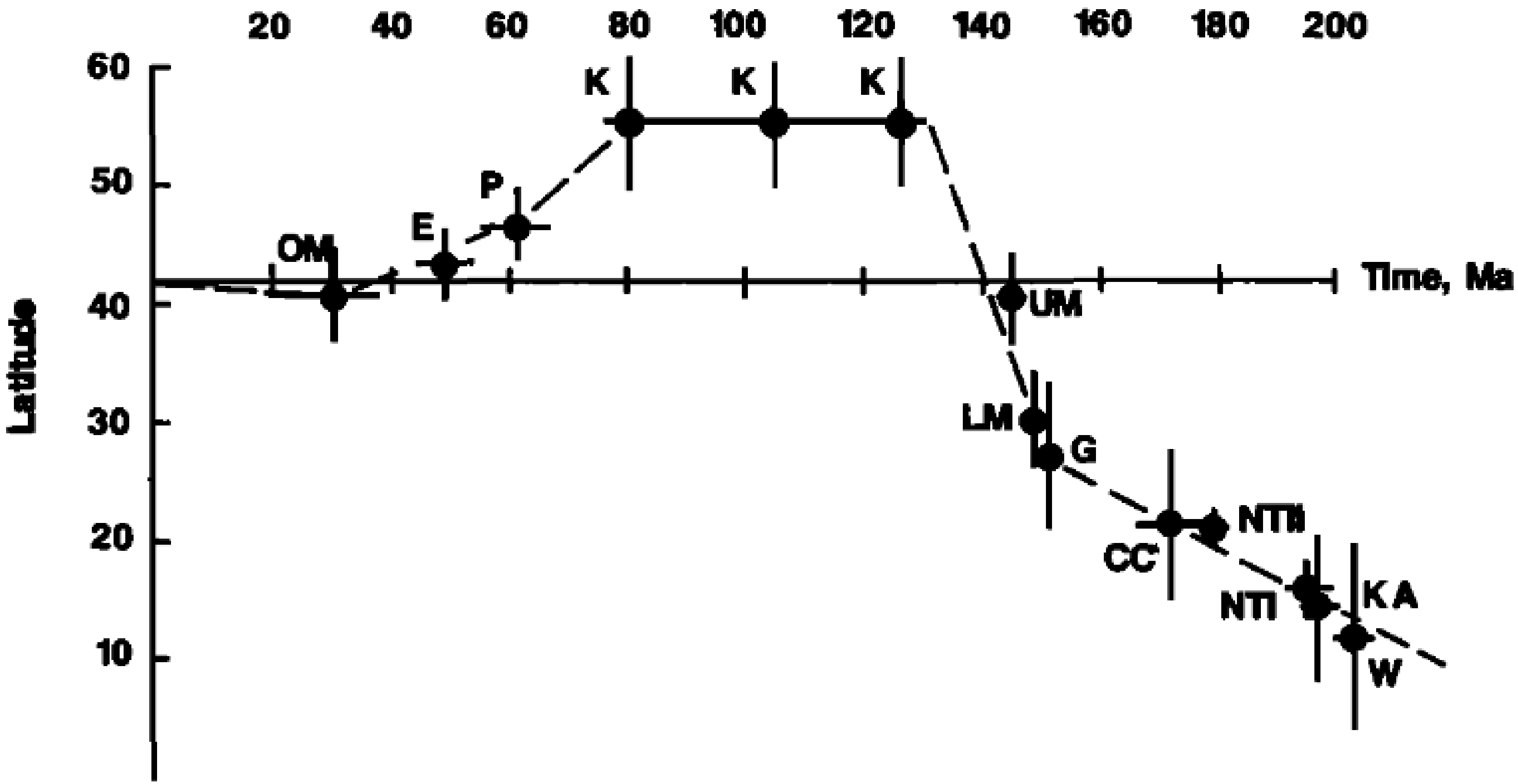
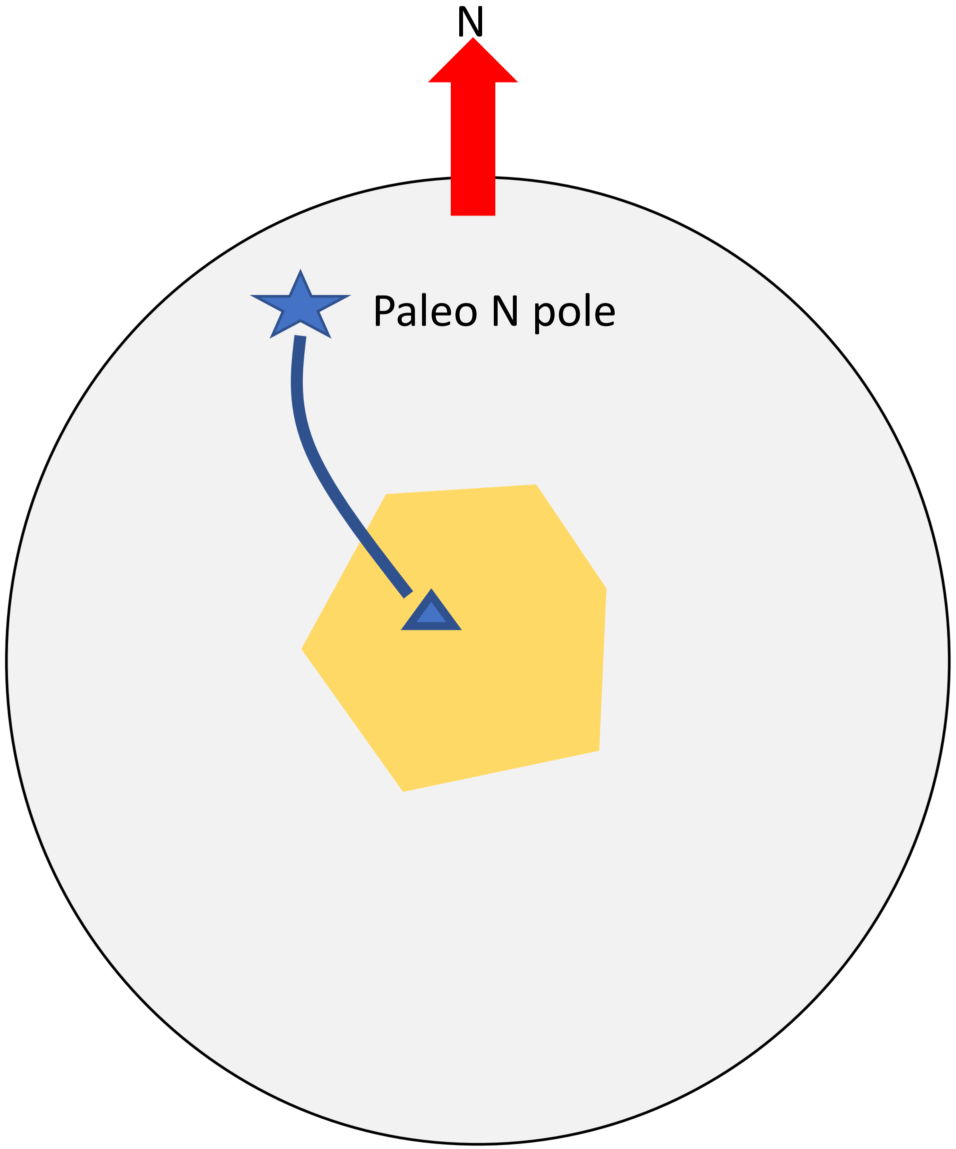
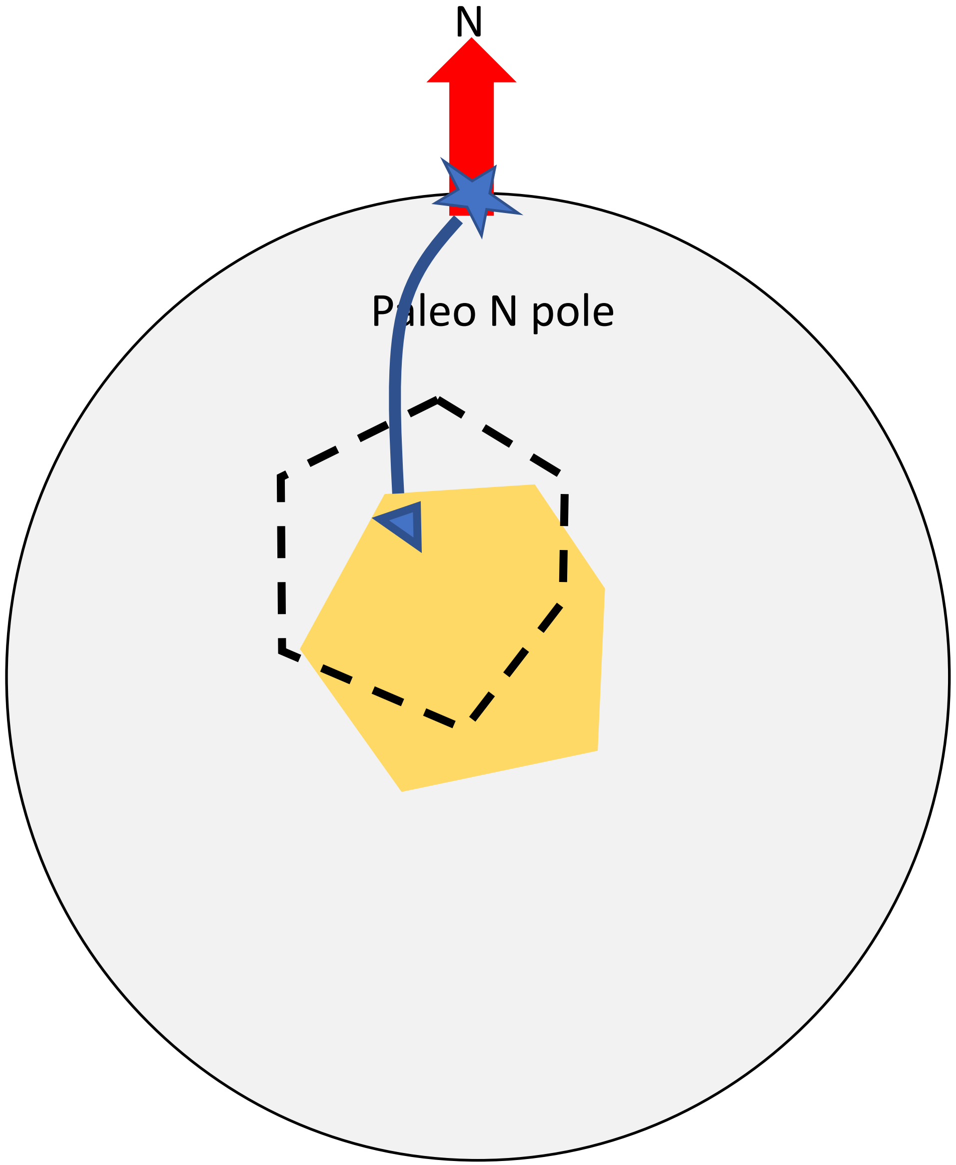


Fig. 2. Paleolatitude of "observation site" (42°N, 124°W) as a function of time, calculated from reference poles shown in Figure 1. Note the rapid increase in paleolatitude during the Late Jurassic–Early Cretaceous. Symbols as in Figure 1.



200 my ago
According to a site on the plate
The pole was here



200 my ago
According to a site on the plate
The pole was here

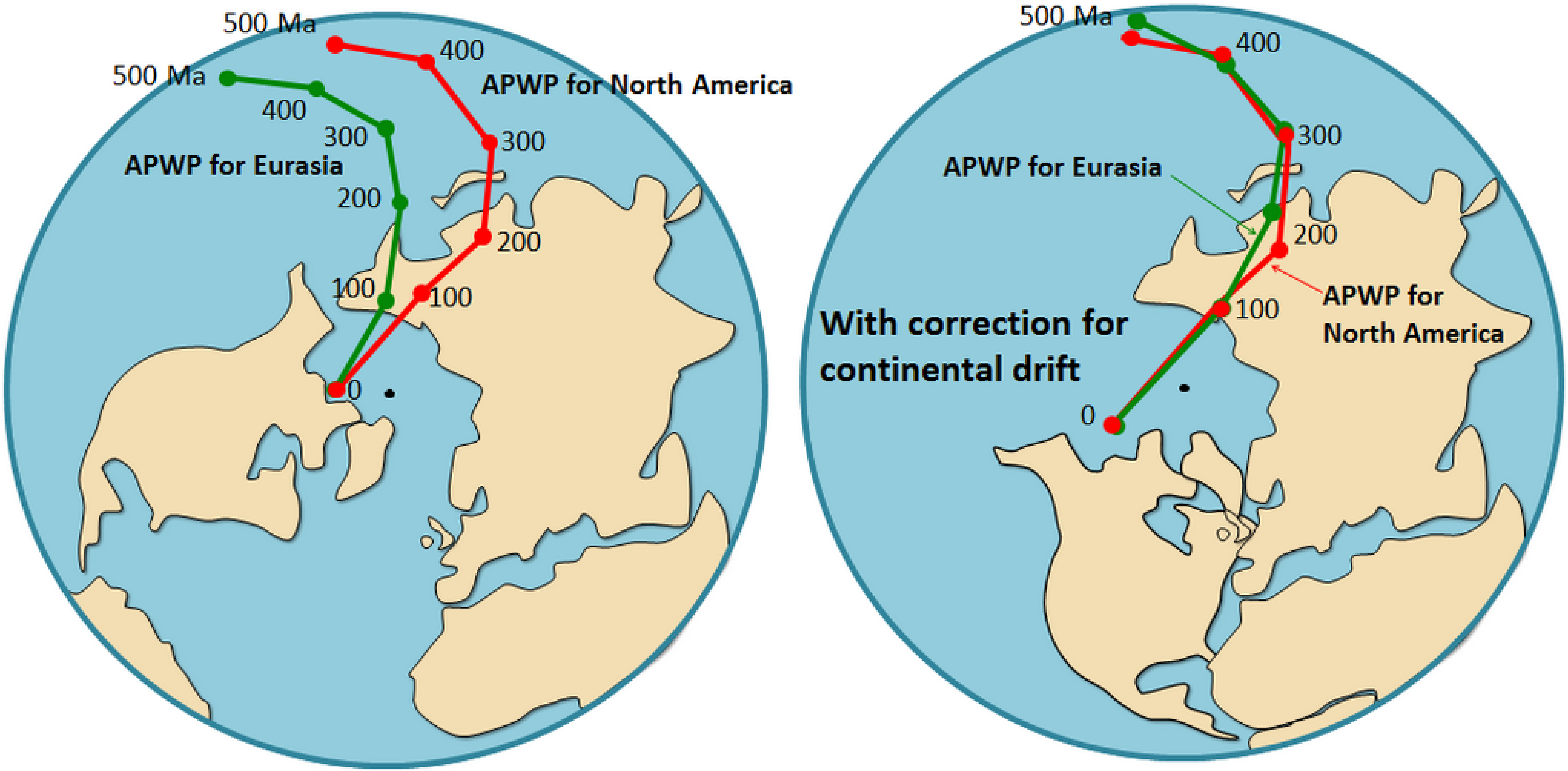


Figure 4.2.2 Polar wandering curves. Curves from Eurasia and North America seem to show that the north magnetic pole was located in two places simultaneously throughout history (left). However, if the continents are rearranged into Pangaea, the two curves overlap, showing that it is the continents than have moved, not the pole (right) (Steven Earle, "Physical Geology").

Dynamo

(a) multipole expansion

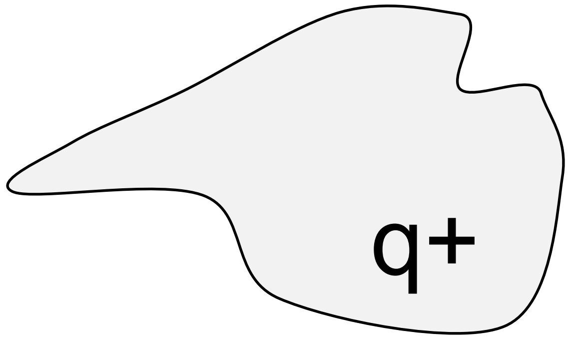
(b) frozen flux approximation

(c) magnetic diffusion

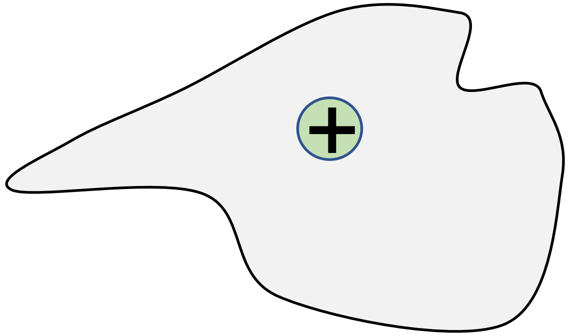
Dynamo

(a) multipole expansion

field of complicated objects

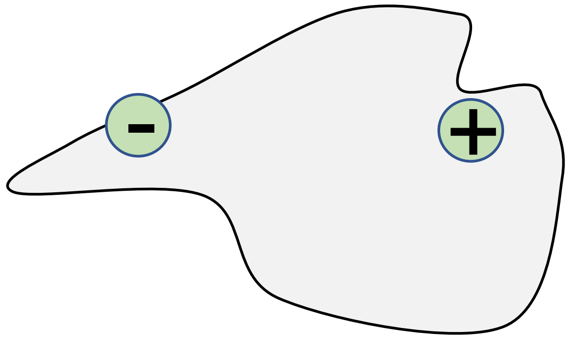


field of complicated objects, made by summing



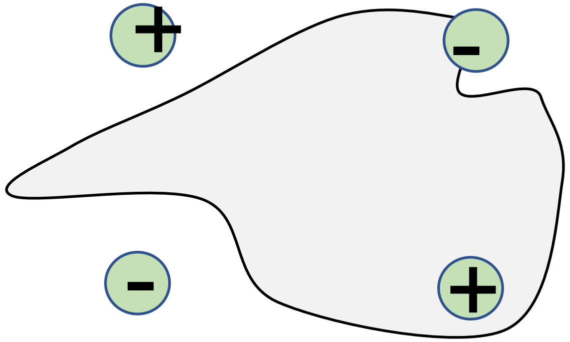
monopole
(point charge)

plus



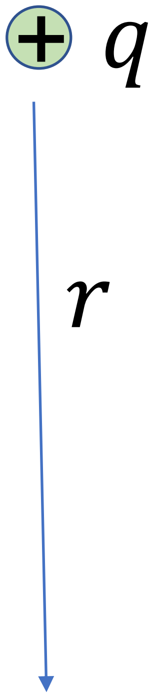
dipole

plus



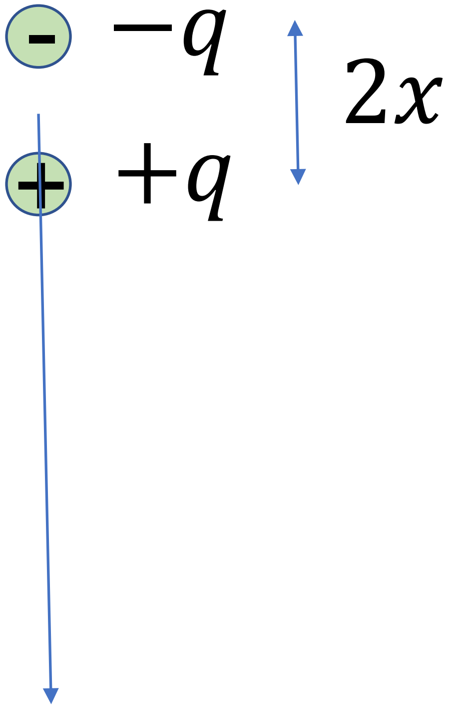
quadrupole

monopole



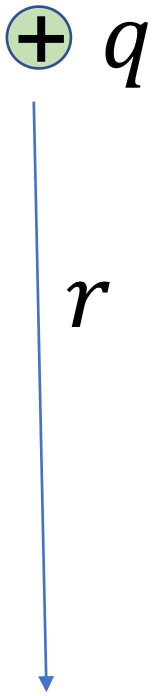
$$-\frac{q}{-r^2}$$

dipole



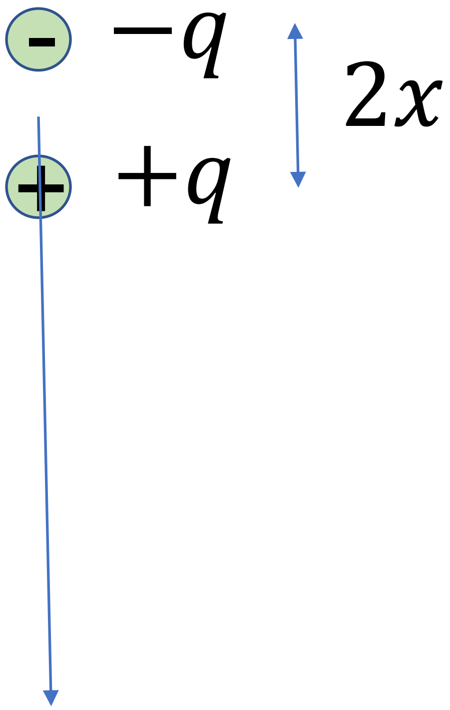
$$\frac{q}{(r-x)^2}-\frac{q}{(r+x)^2}$$

monopole



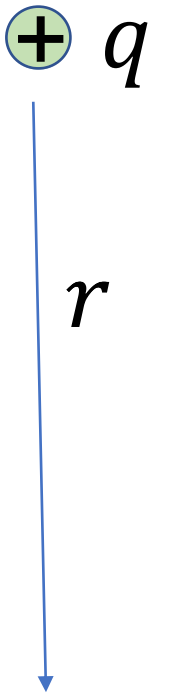
$$-\frac{q}{r^2}$$

dipole



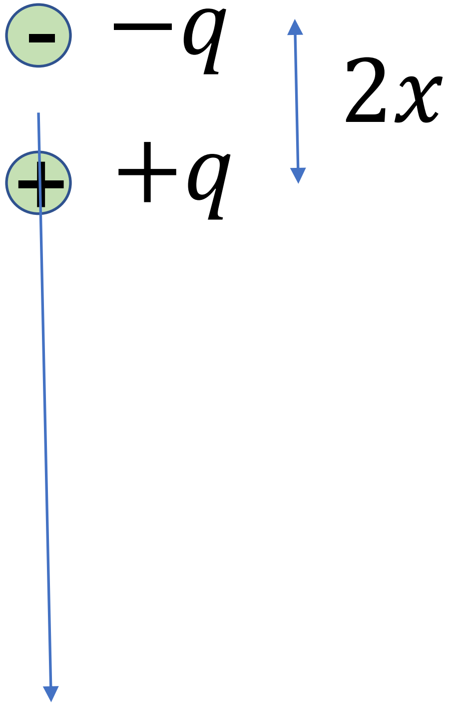
$$\frac{q}{r^2(1-x/r)^2}-\frac{q}{r^2(1+x/r)^2}$$

monopole



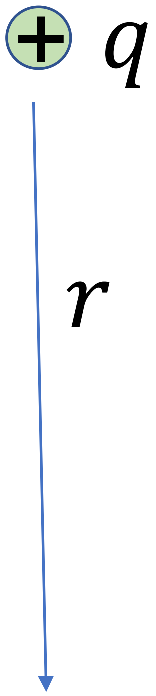
$$-\frac{q}{r^2}$$

dipole



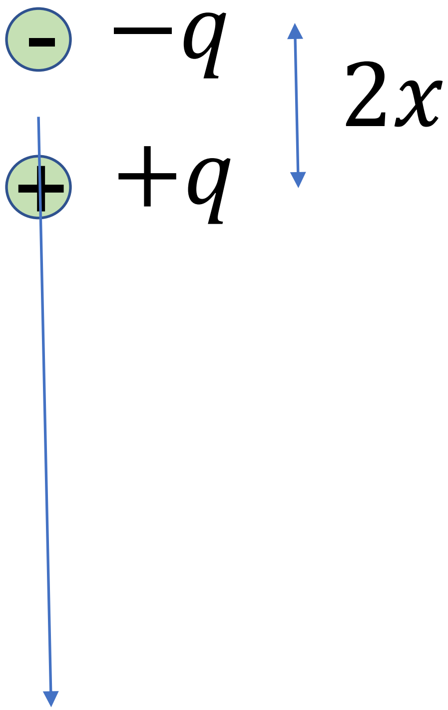
$$\frac{q(1-2x/r)}{r^2}-\frac{q(1+2x/r)}{r^2}$$

monopole



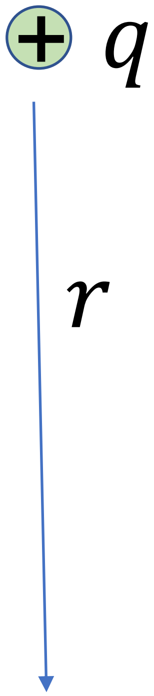
$$-\frac{q}{r^2}$$

dipole



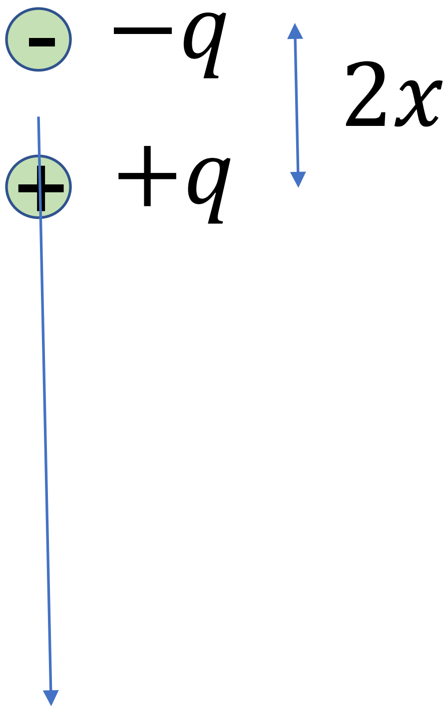
$$-\frac{q(2x/r)}{r^2}-\frac{q(2x/r)}{r^2}$$

monopole



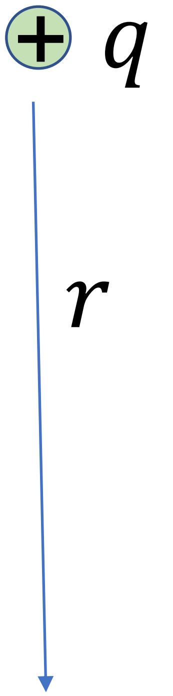
$$-\frac{q}{r^2}$$

dipole



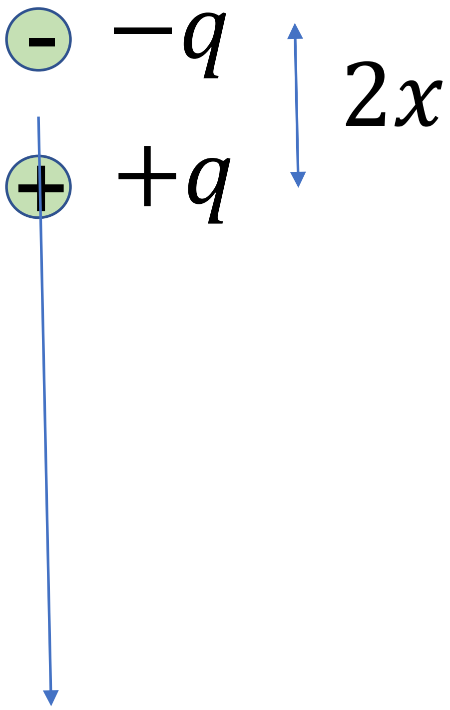
$$-\frac{4qx}{r^3}$$

monopole



$$-\frac{q}{r^2}$$

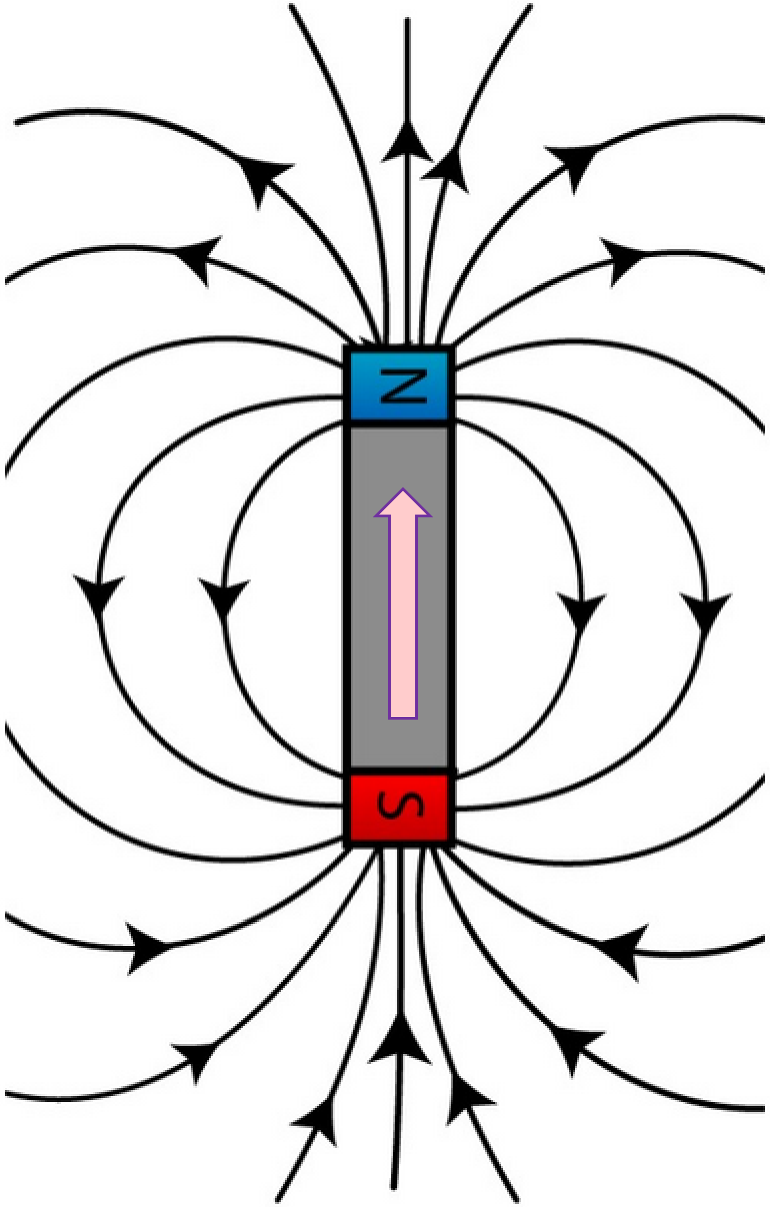
dipole



$$-\frac{4\boxed{qx}}{r^3}$$

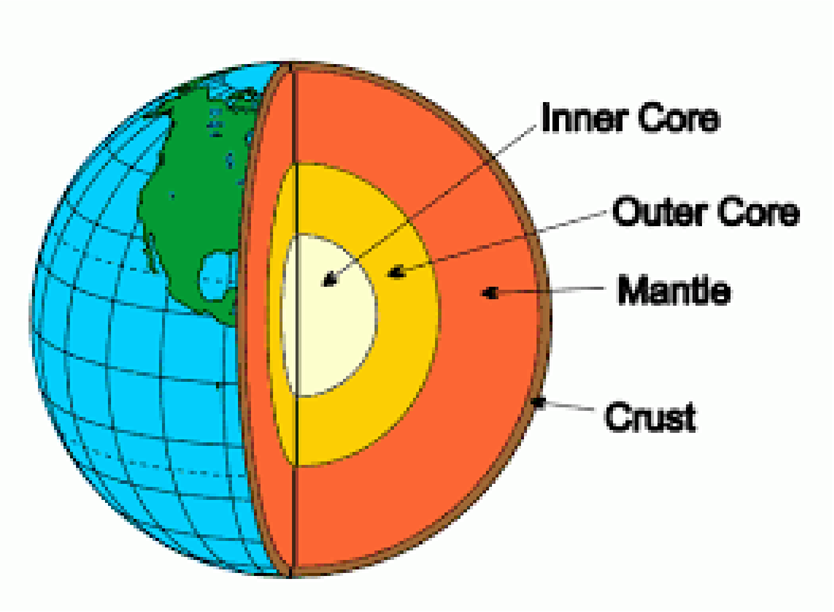
dipole moment m_2

faster falloff



magnetization vector $\mathbf{M}$ ⇧

is just the overall dipole moment of the object



Lesson

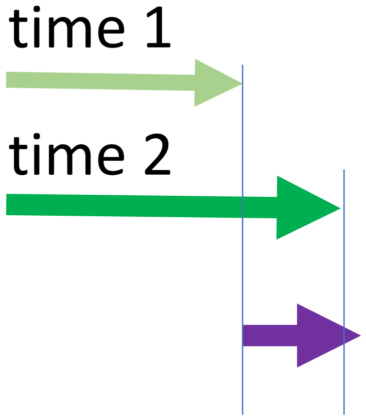
field does not have to be perfect dipole in core

to be reasonably dipolar at the surface of the Earth

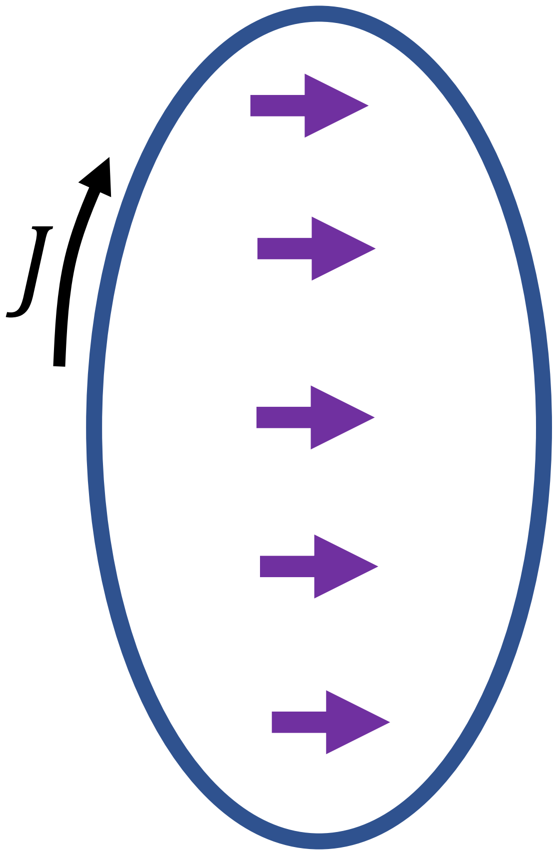
Dynamo

(b) frozen flux approximation

Generator principle

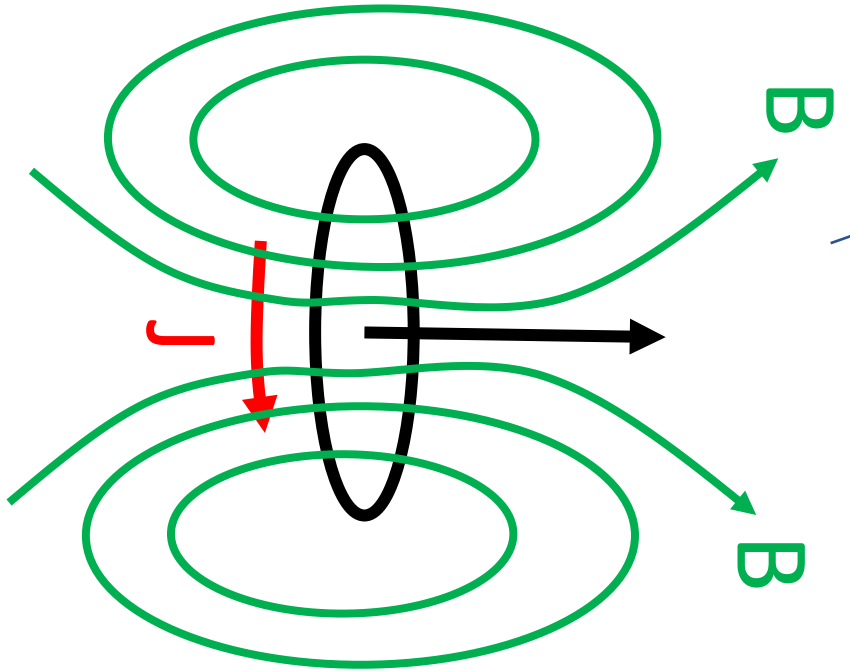


$$\frac{d\mathbf{B}}{dt}$$



$$J \propto -\frac{d\mathbf{B}}{dt}$$

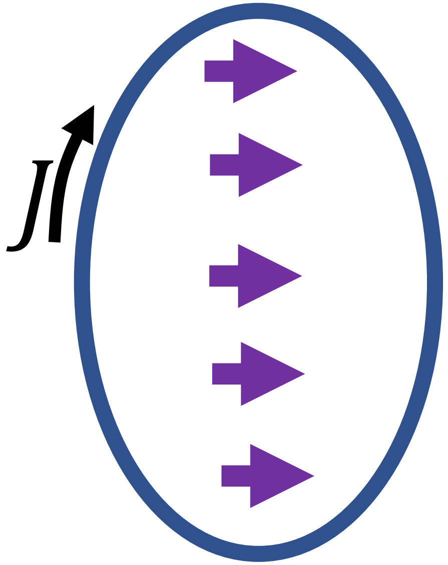
Solenoid principle



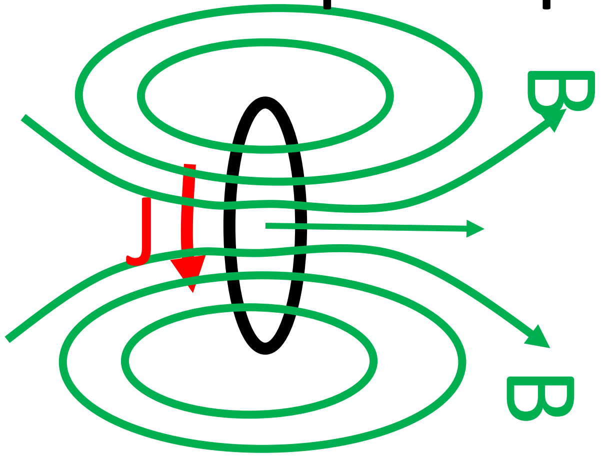
inside loop

$$\mathbf{B} \propto \mu_0 J$$

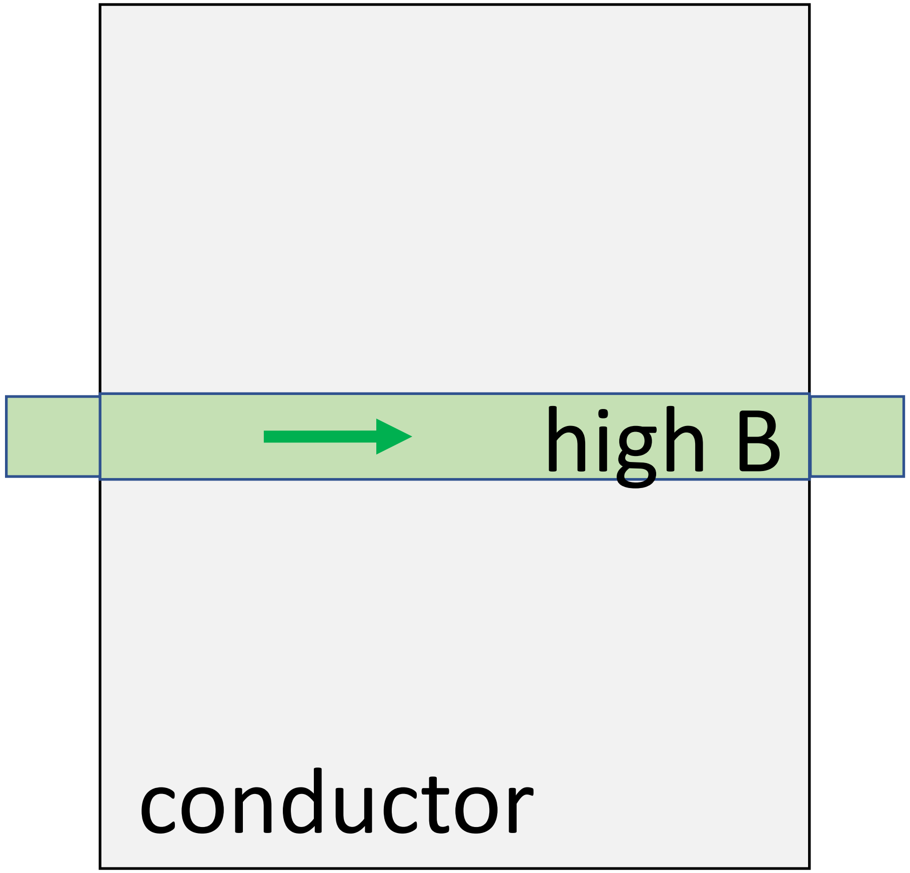
Generator principle



Solenoid principle

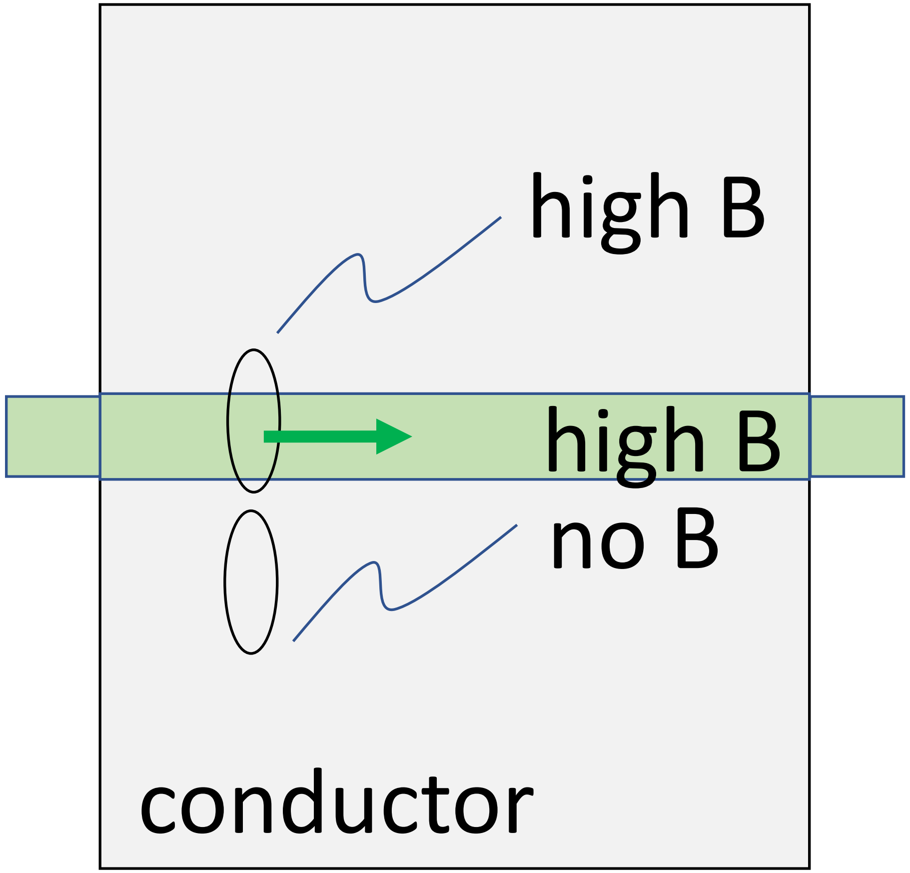


$$\left.\begin{array}{l} J \propto -\dfrac{d\mathbf{B}}{dt} \\[6em] \mathbf{B} \propto \mu_0 J \end{array}\right\} \quad \mathbf{B} \propto -\mu_0 \dfrac{d\mathbf{B}}{dt}$$

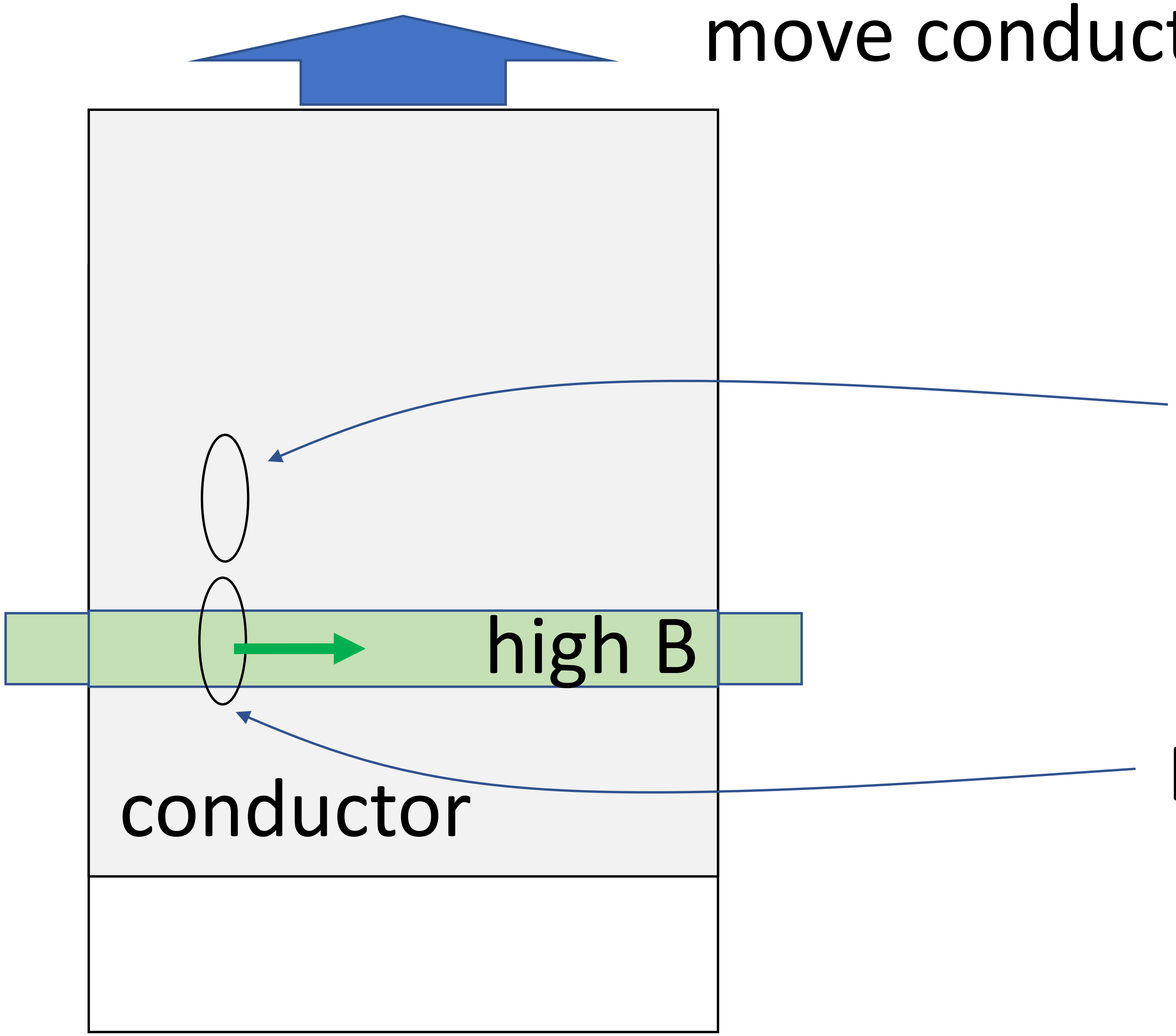


high B

conductor



imaginary loops



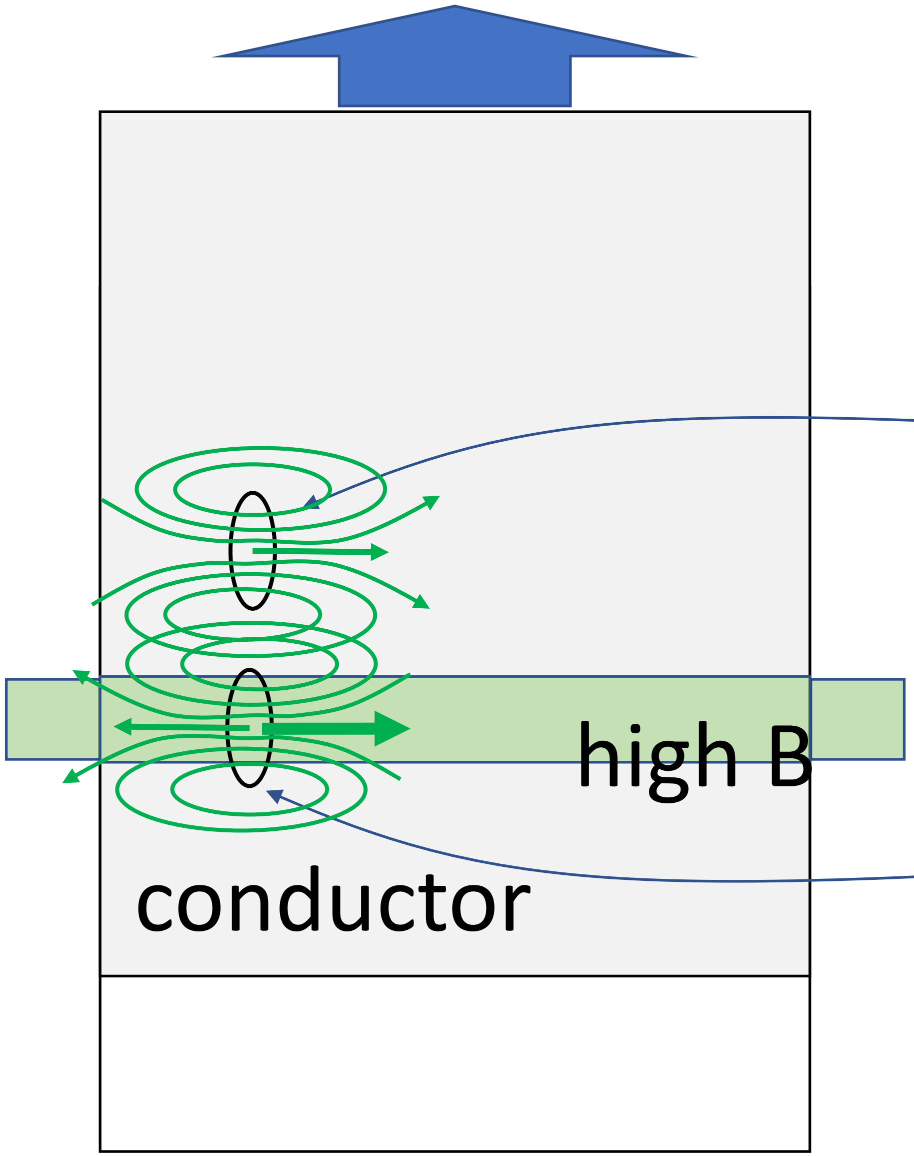
move conductor

high B

conductor

B went down $\frac{d\mathbf{B}}{dt}$ $-$

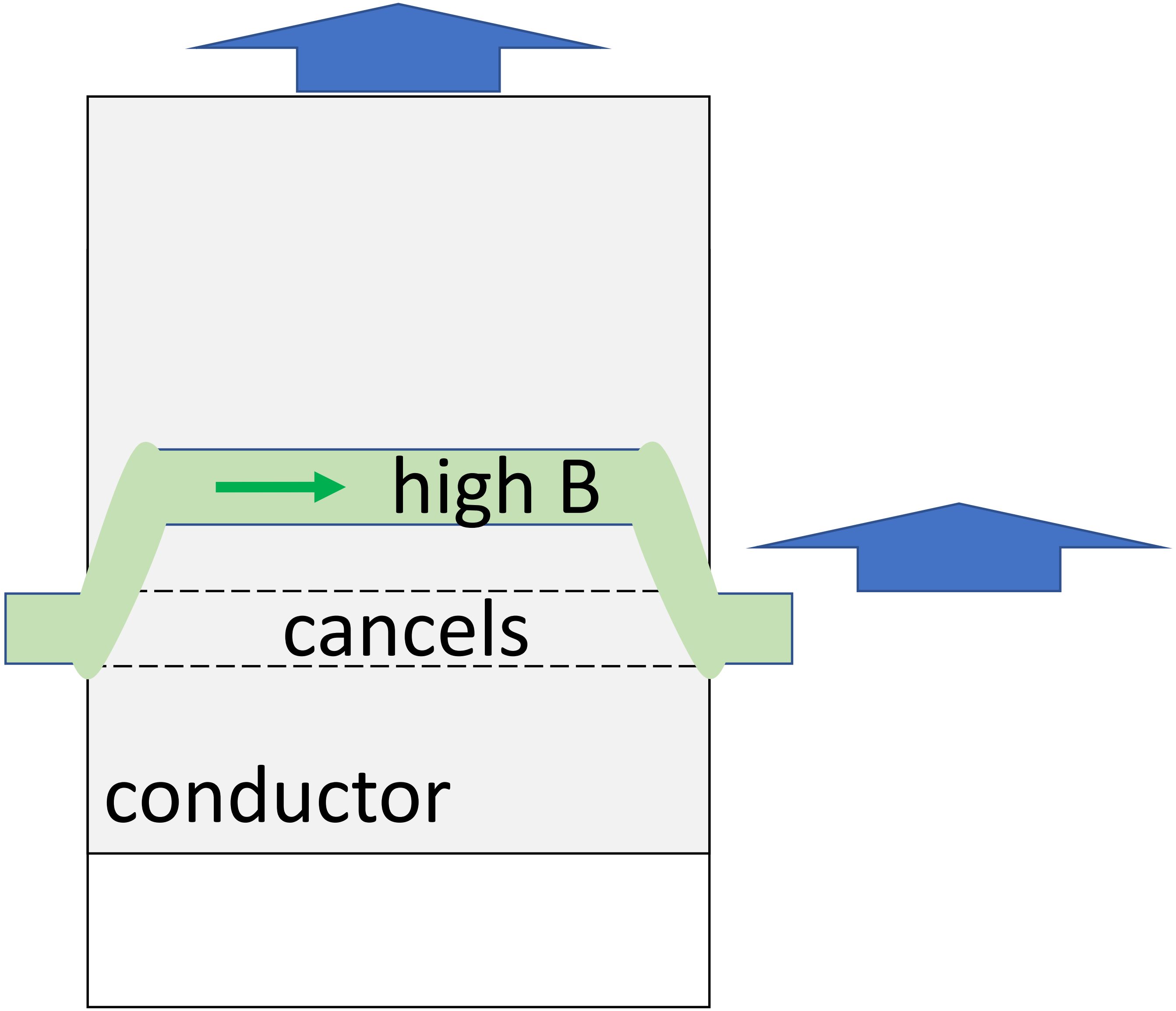
B went up $\frac{d\mathbf{B}}{dt}$ $+$



$$\mathbf{B} \propto -\mu_0 \frac{d\mathbf{B}}{dt}$$

B+ because $\frac{d\mathbf{B}}{dt}\ -$

B- because $\frac{d\mathbf{B}}{dt}\ +$



net results
magnetic field
acts like it is “frozen in”

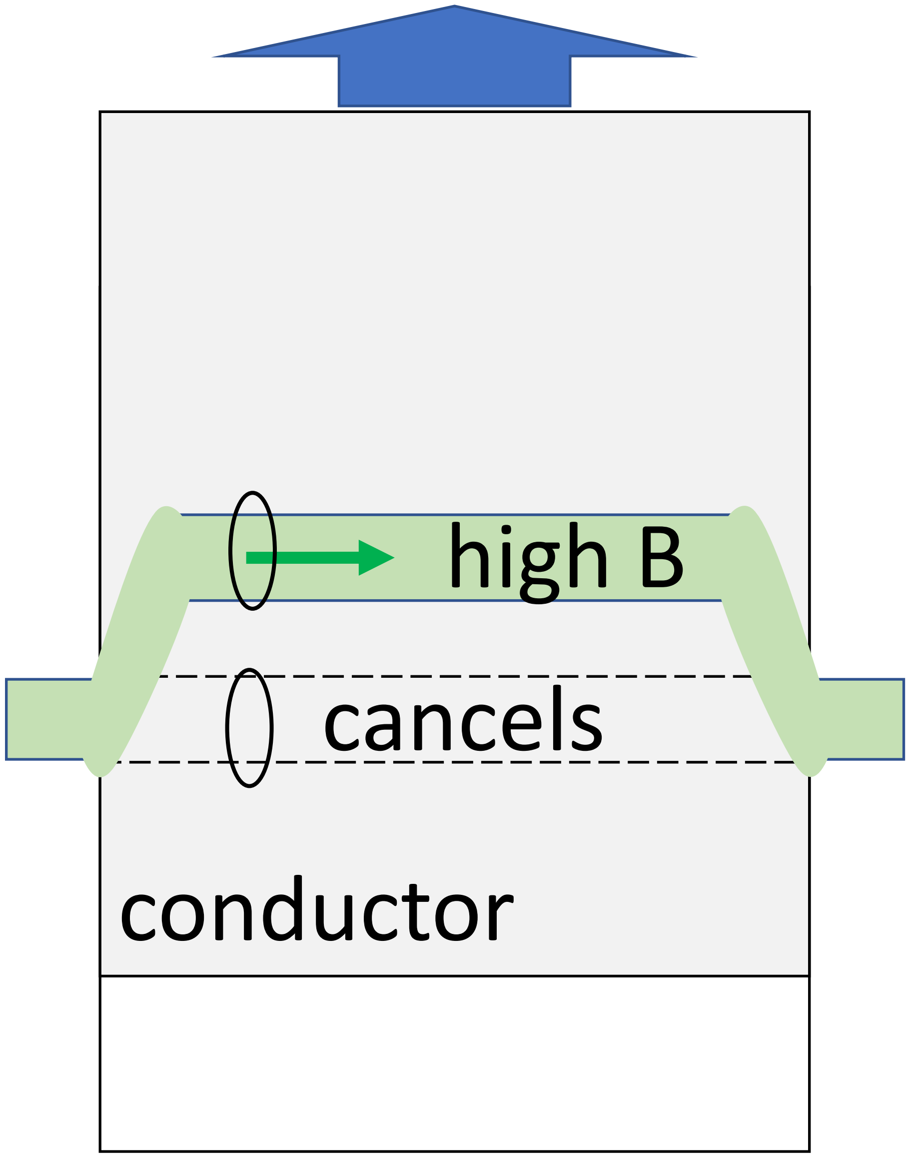
Lesson

Convection drags the magnetic field with it

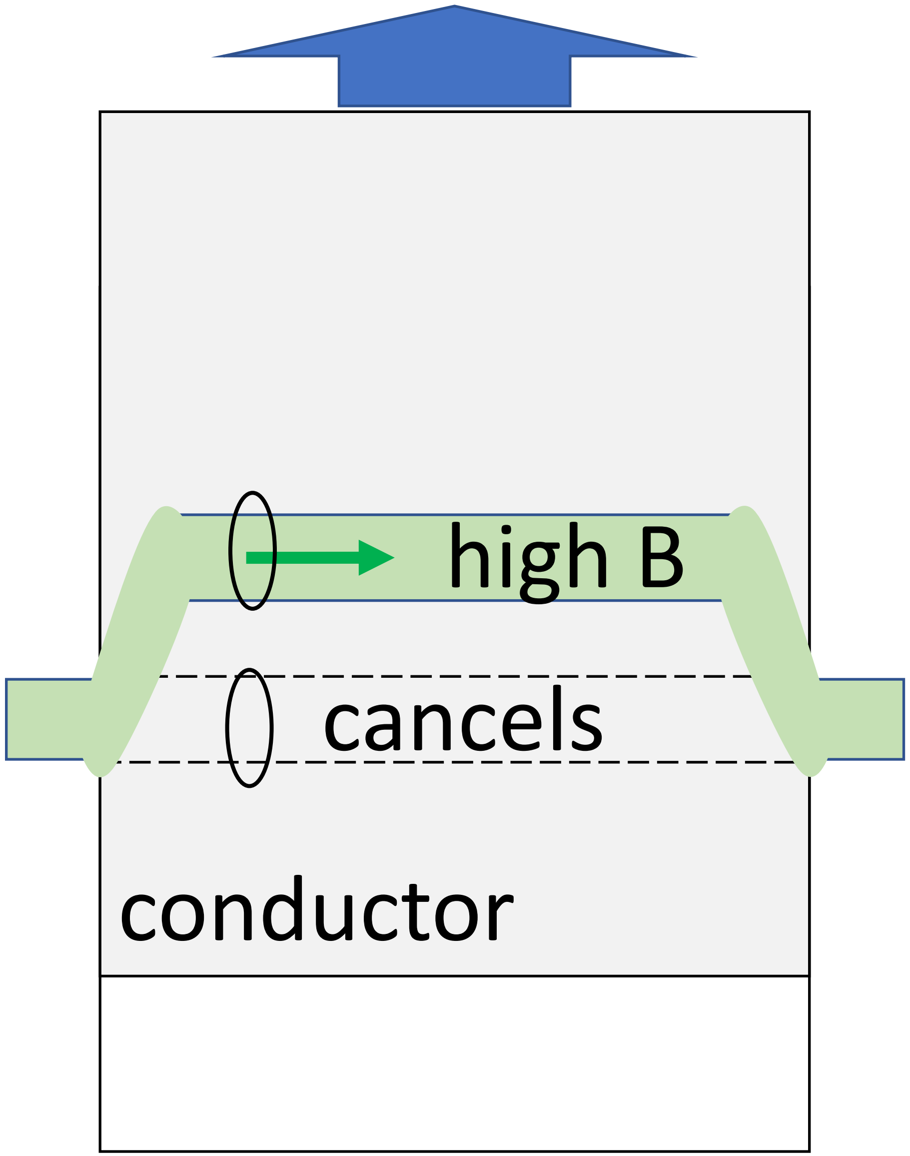
creates a field that changes with time

Dynamo

(c) magnetic diffusion

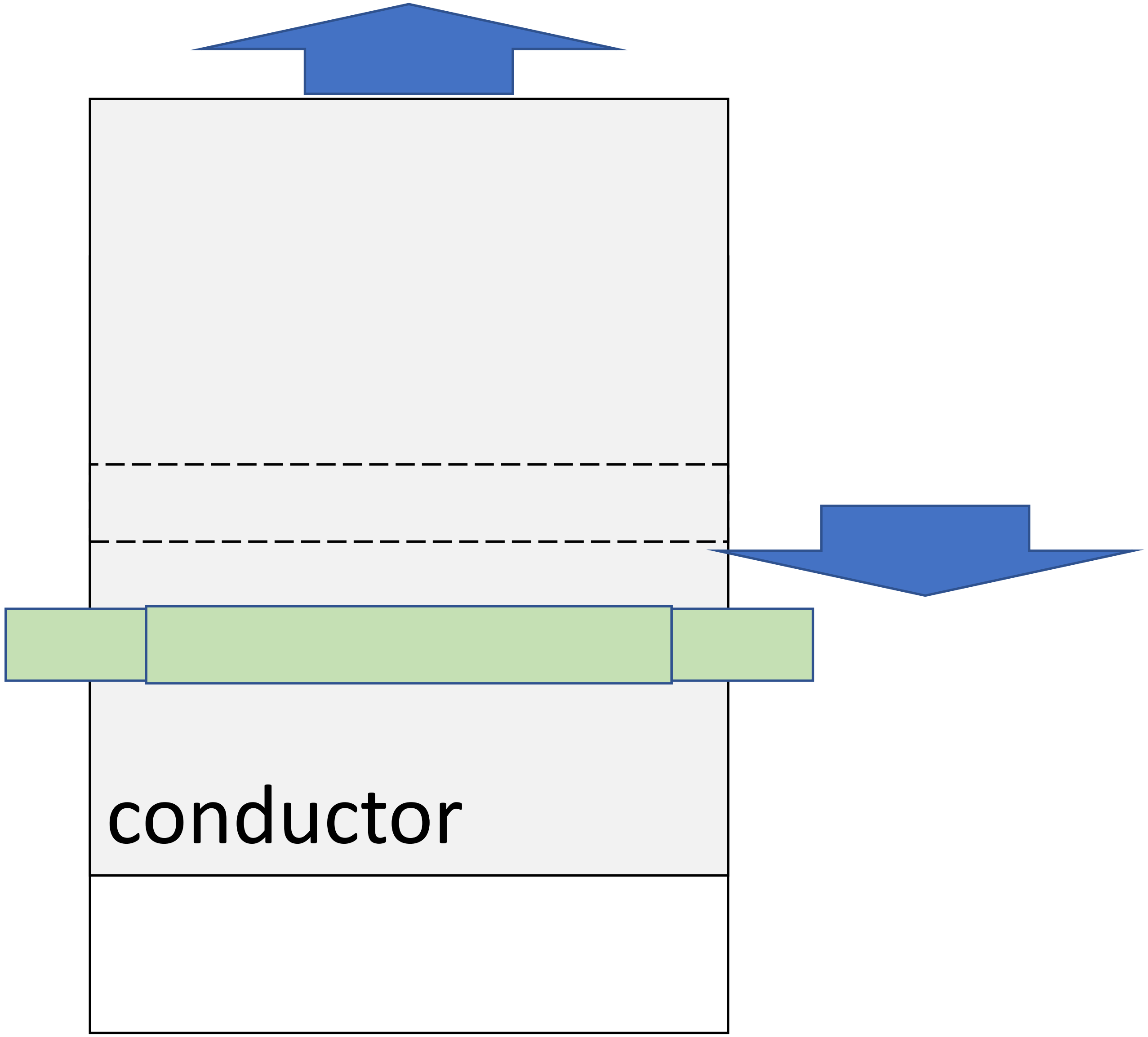


shape being sustained
by currents in the loops



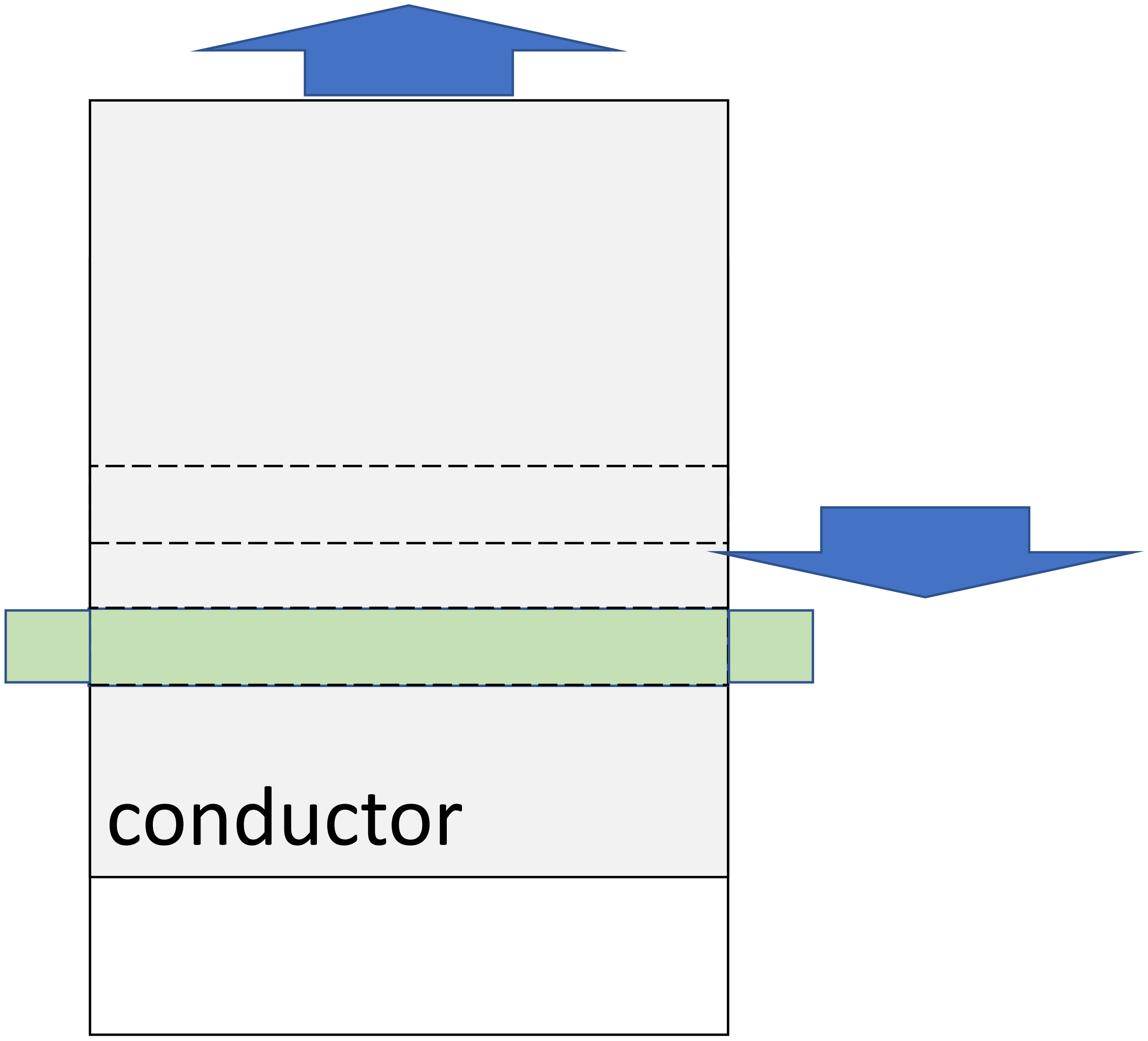
if not perfect conductor
loops lose energy

acts like heat flow with
a diffusivity of $\kappa = \frac{1}{\sigma\mu_0}$



Important question

what is the time scale of diffusion



Temperature solution

$$\operatorname{erf}\left(\frac{z}{\sqrt{4\kappa t}}\right)$$

$$\frac{z}{\sqrt{4\kappa t}} = 1$$

$$t = \frac{z^2}{4\kappa} = \frac{z^2}{4\sigma\mu_0}$$

z = length scale

$\sigma = 10^5$ S/m

$z = 400$ km   t = 10,000 years

Lesson

field not completely “frozen in”

so field in core cannot become absurdly complicated