Solid Earth Dynamics

Bill Menke, Instructor

Lecture 23

Geomagnetism

Electromagnetic fields in matter

Induced magnetism

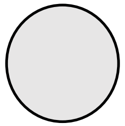
Electromagnetic waves

Part 1

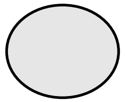
electric and magnetic fields inside matter

Part 1A

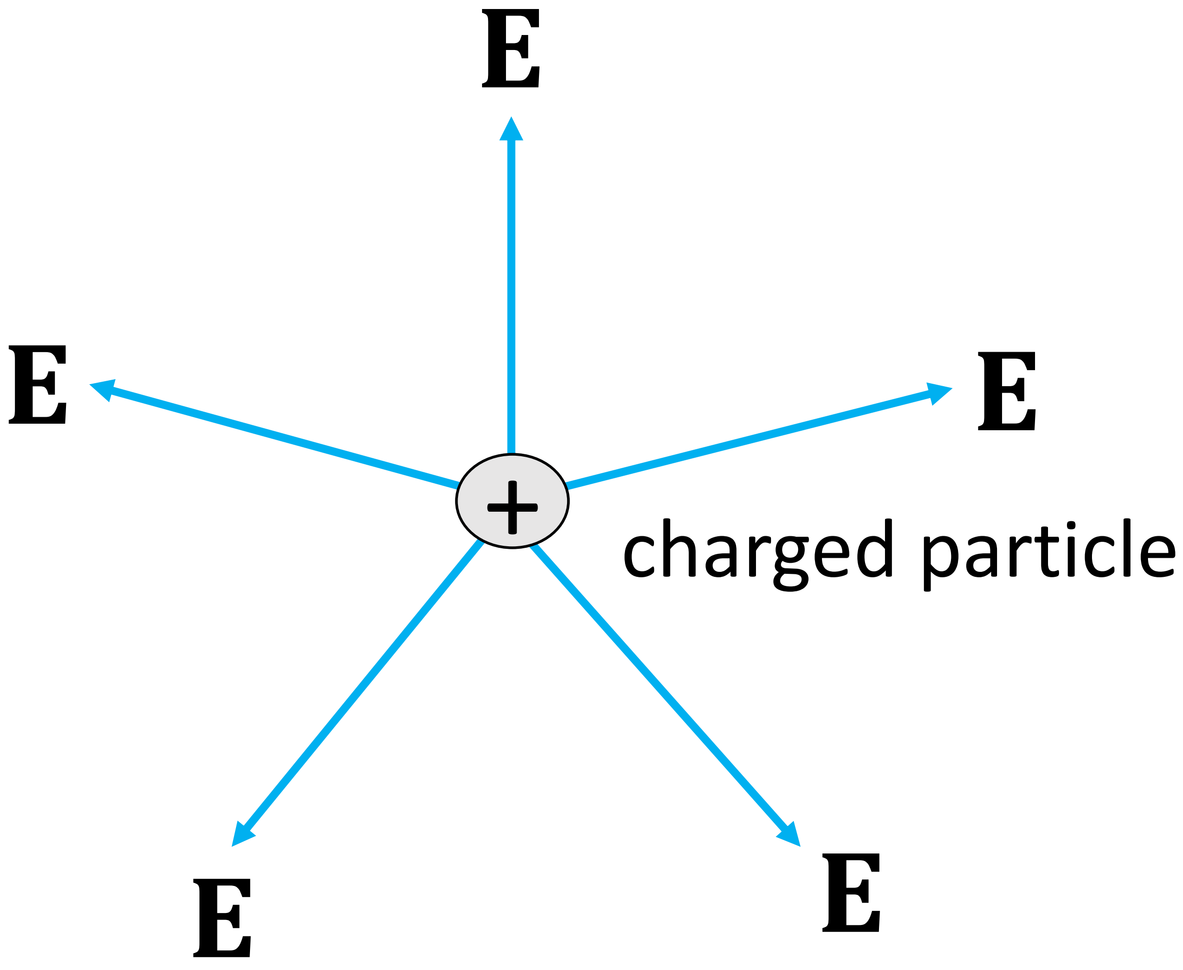
electric field inside matter



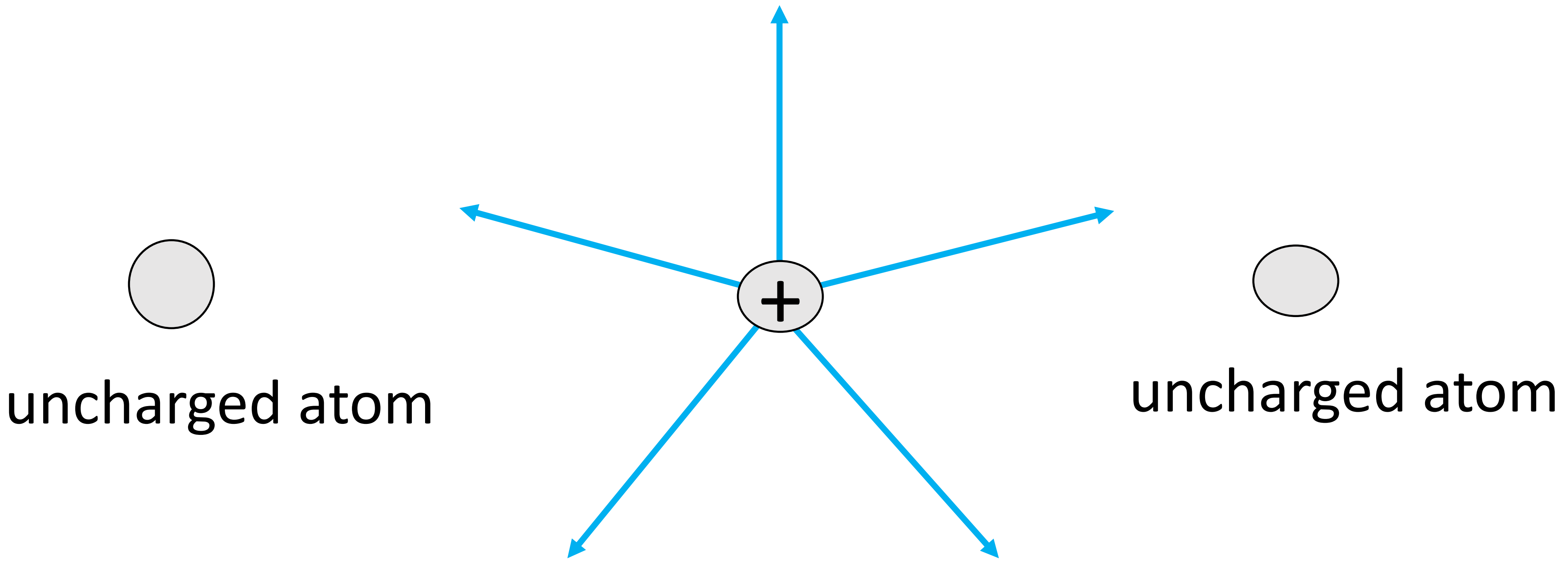
uncharged atom

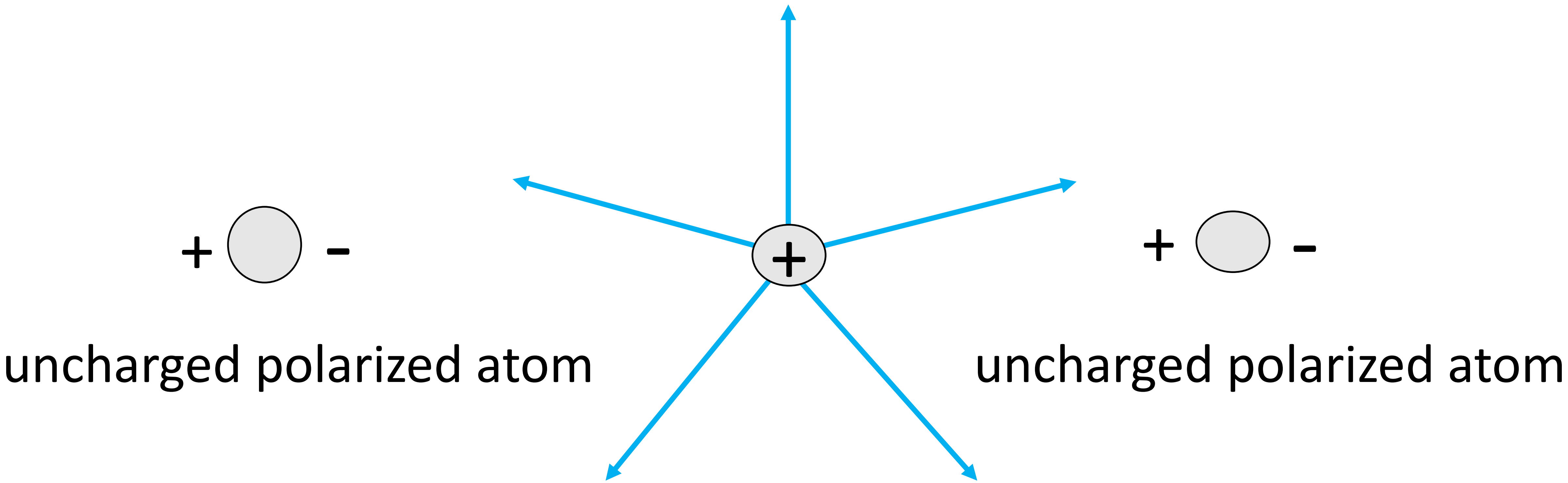


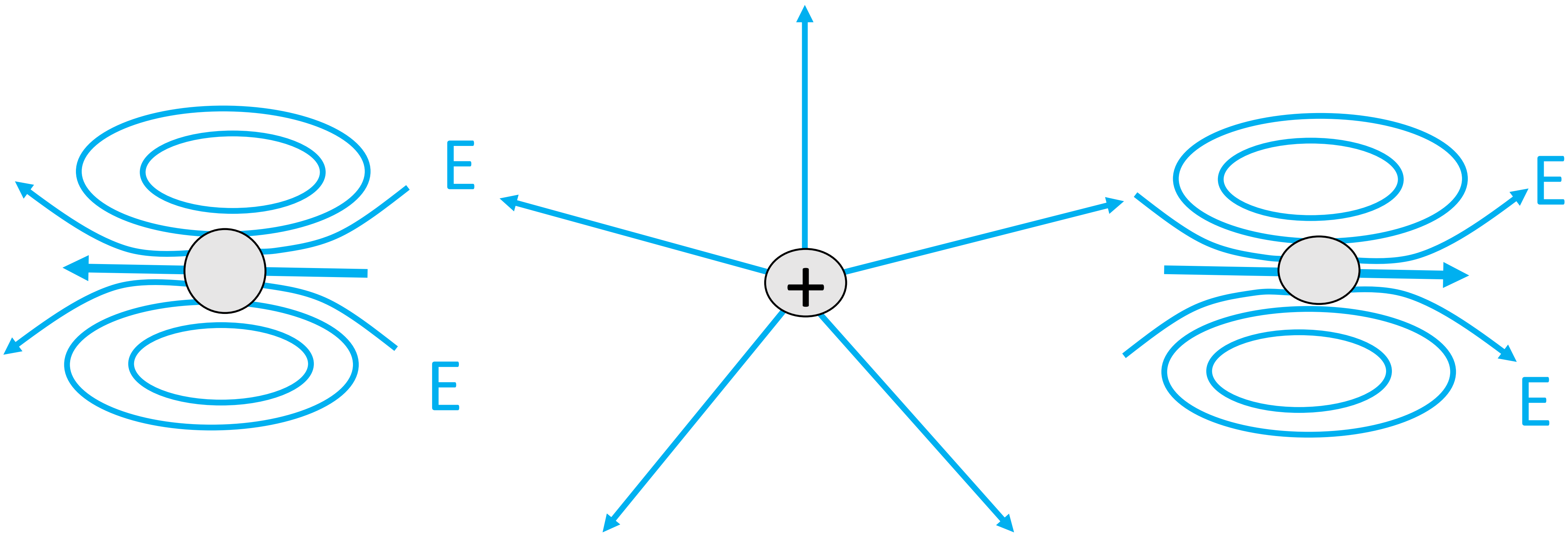
uncharged atom



$$|\mathbf{E}| = \frac{kq}{r^2}$$


uncharged atom
uncharged atom


+
-
uncharged polarized atom
+
+
-
uncharged polarized atom



Electric field of charged particle is amplified

In material

$$E^{total} = E^{particle} + E^{induced}$$

In material

$$\mathbf{E}^{total} = \mathbf{E}^{particle} + \mathbf{E}^{induced}$$

but

$$\mathbf{E}^{induced} \propto \mathbf{E}^{particle}$$

so

$$\mathbf{E}^{total} = \varepsilon\mathbf{E}^{particle}$$

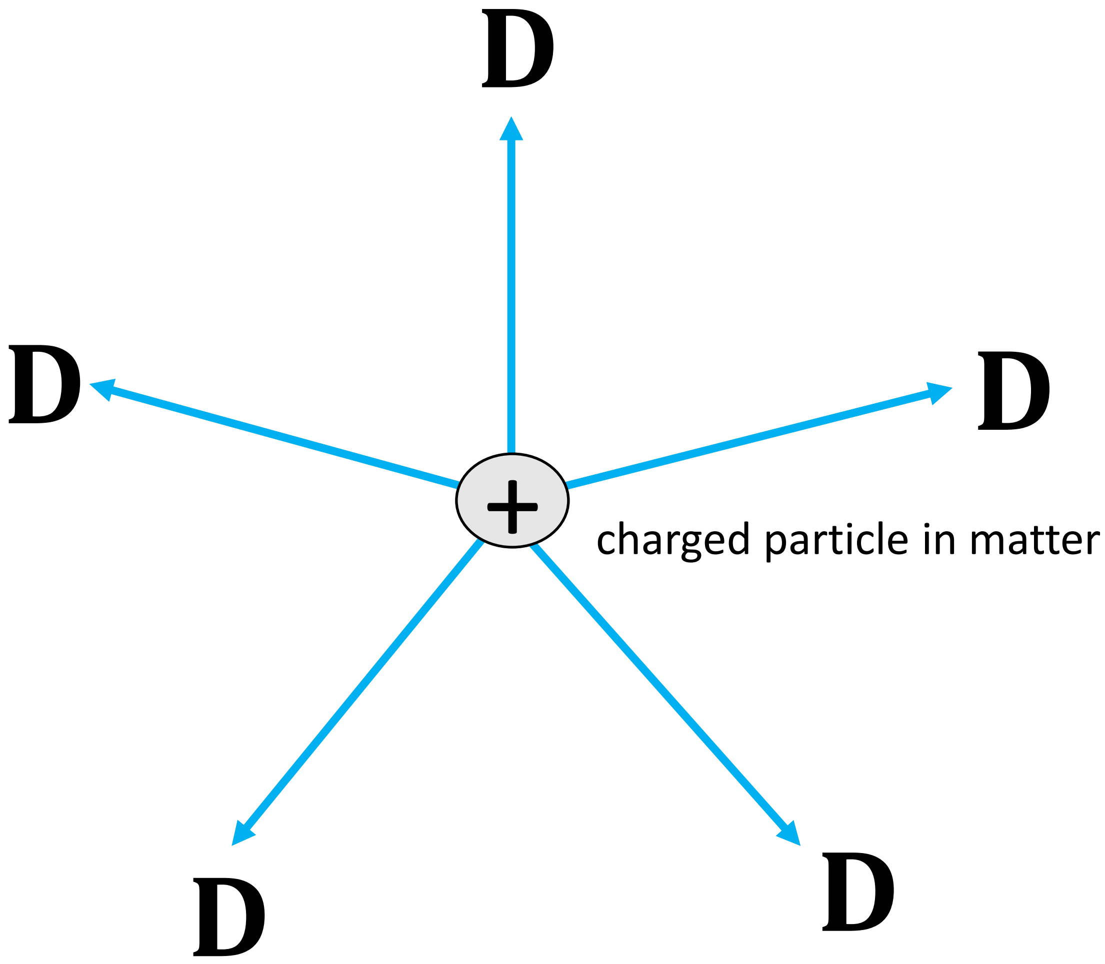
electrical permittivity ε

new name

$$\mathbf{D} = \varepsilon\mathbf{E}^{particle}$$

electric displacement

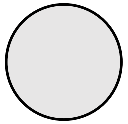
coulomb's law inside matter



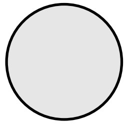
$$|\mathbf{D}| = \frac{kq}{r^2}$$

Part 1B

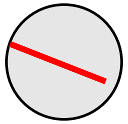
magnetic induction inside matter



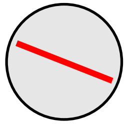
atom with no net current



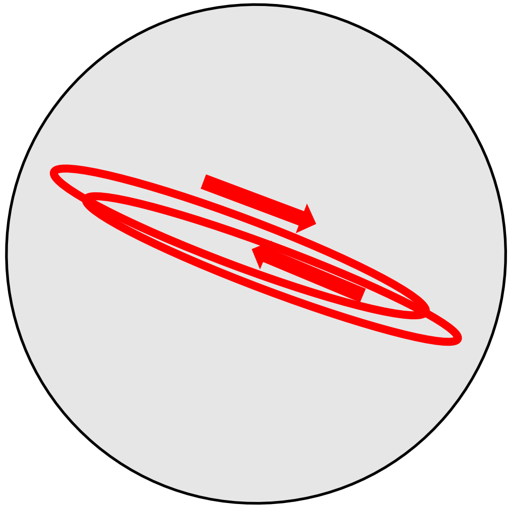
atom with no net current



atom with no net current

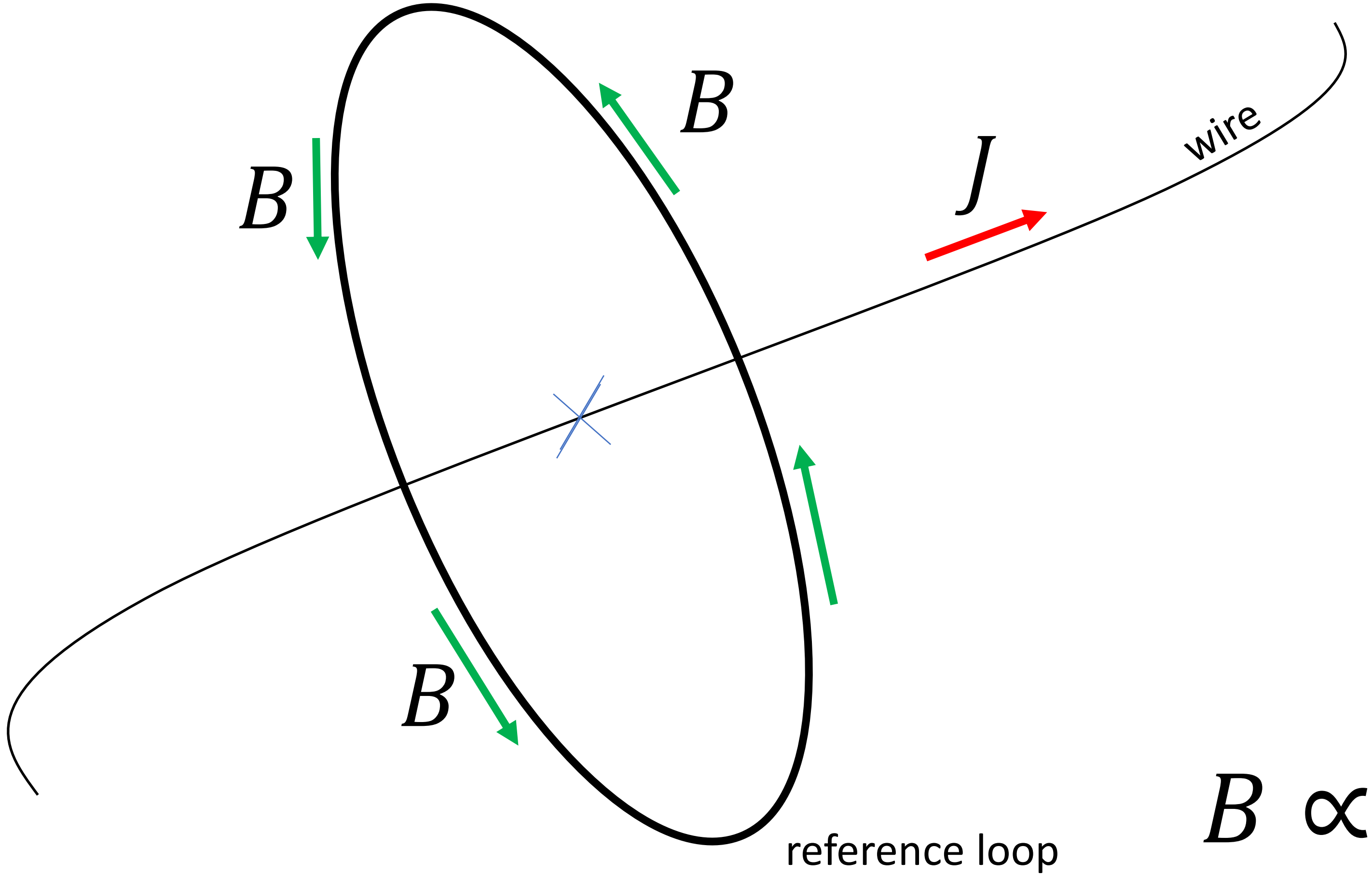


atom with no net current



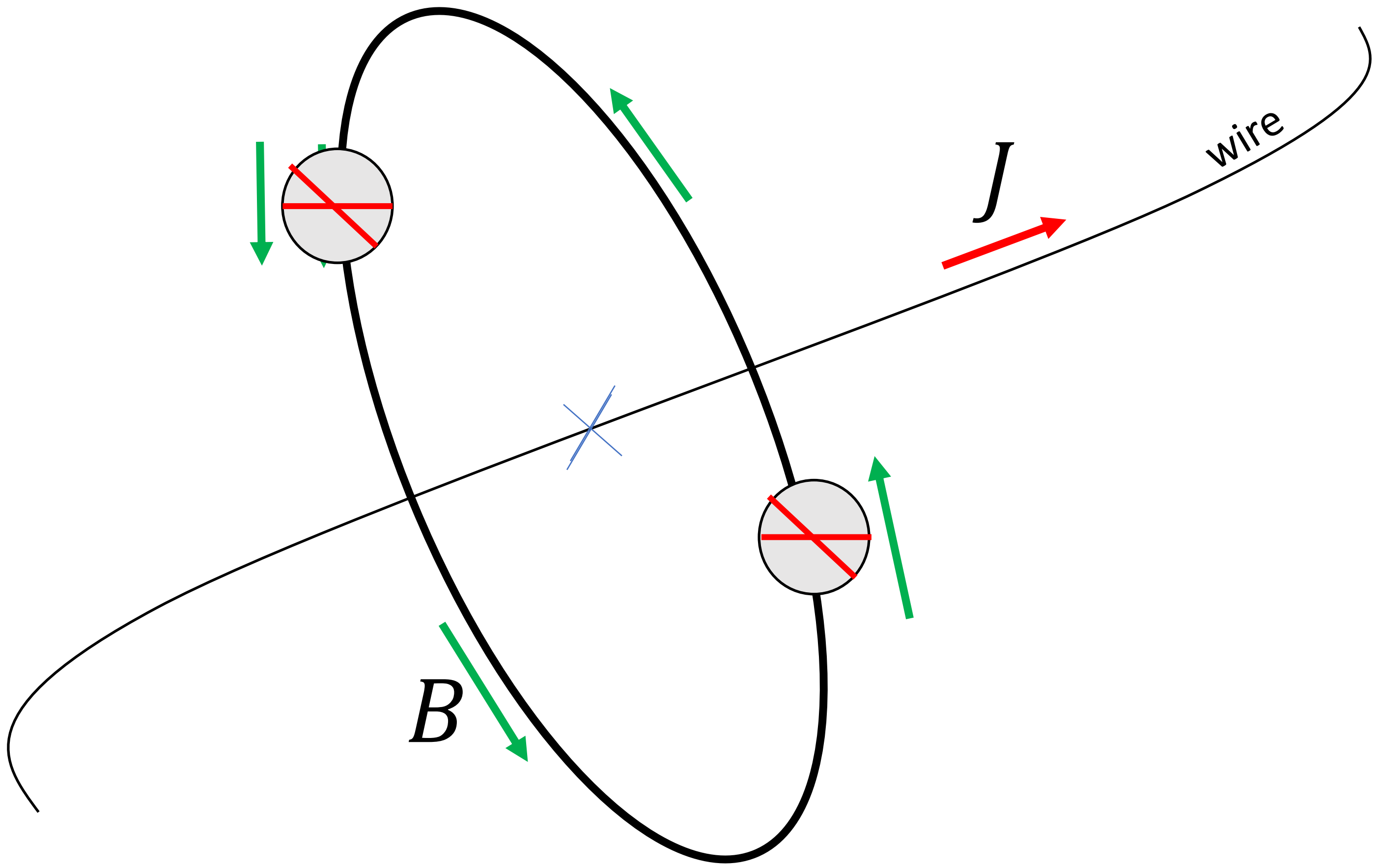
two opposing current loops,
on top of each other

Solenoid equation

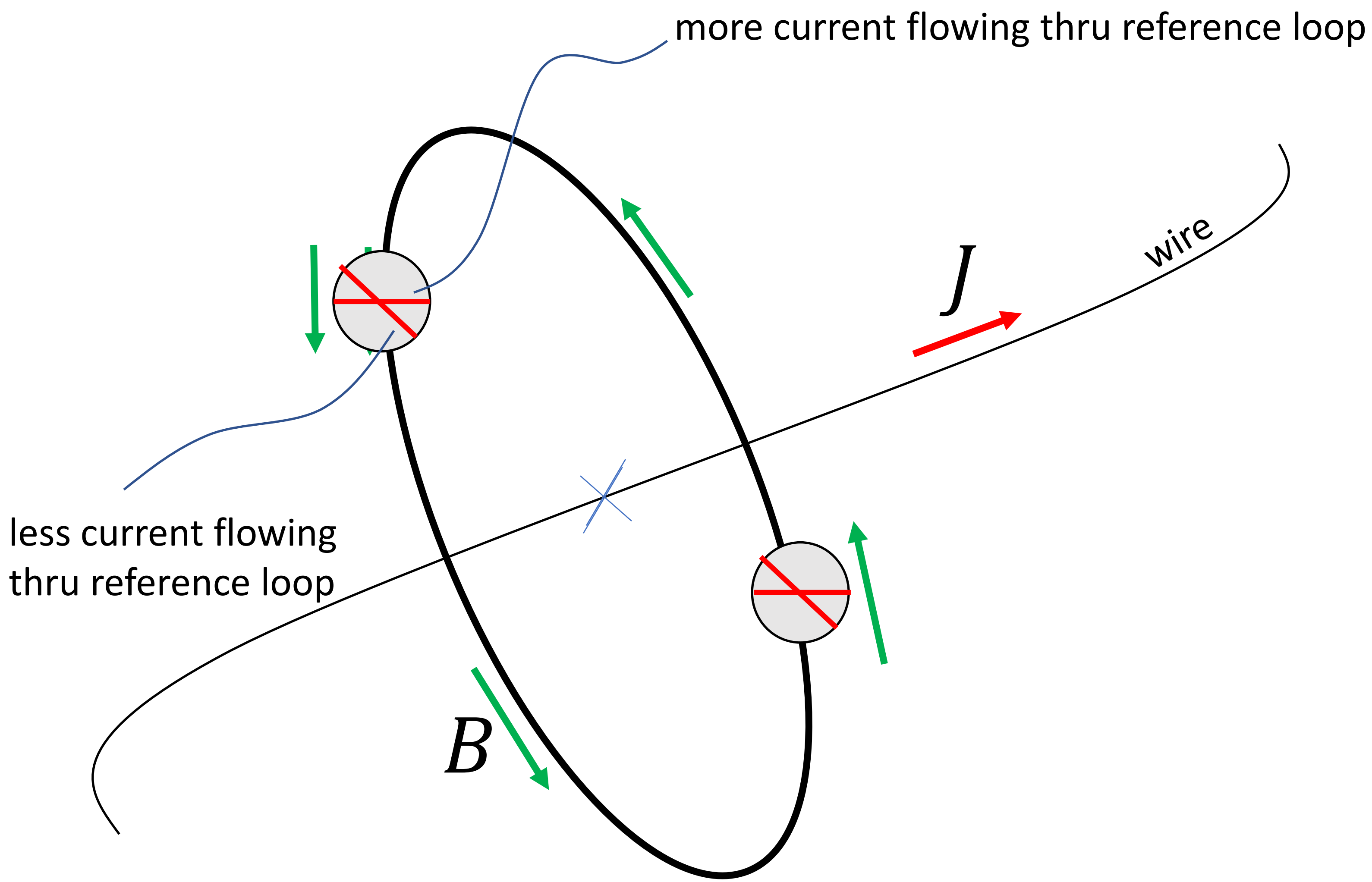


$$B \propto \mu_0 J$$

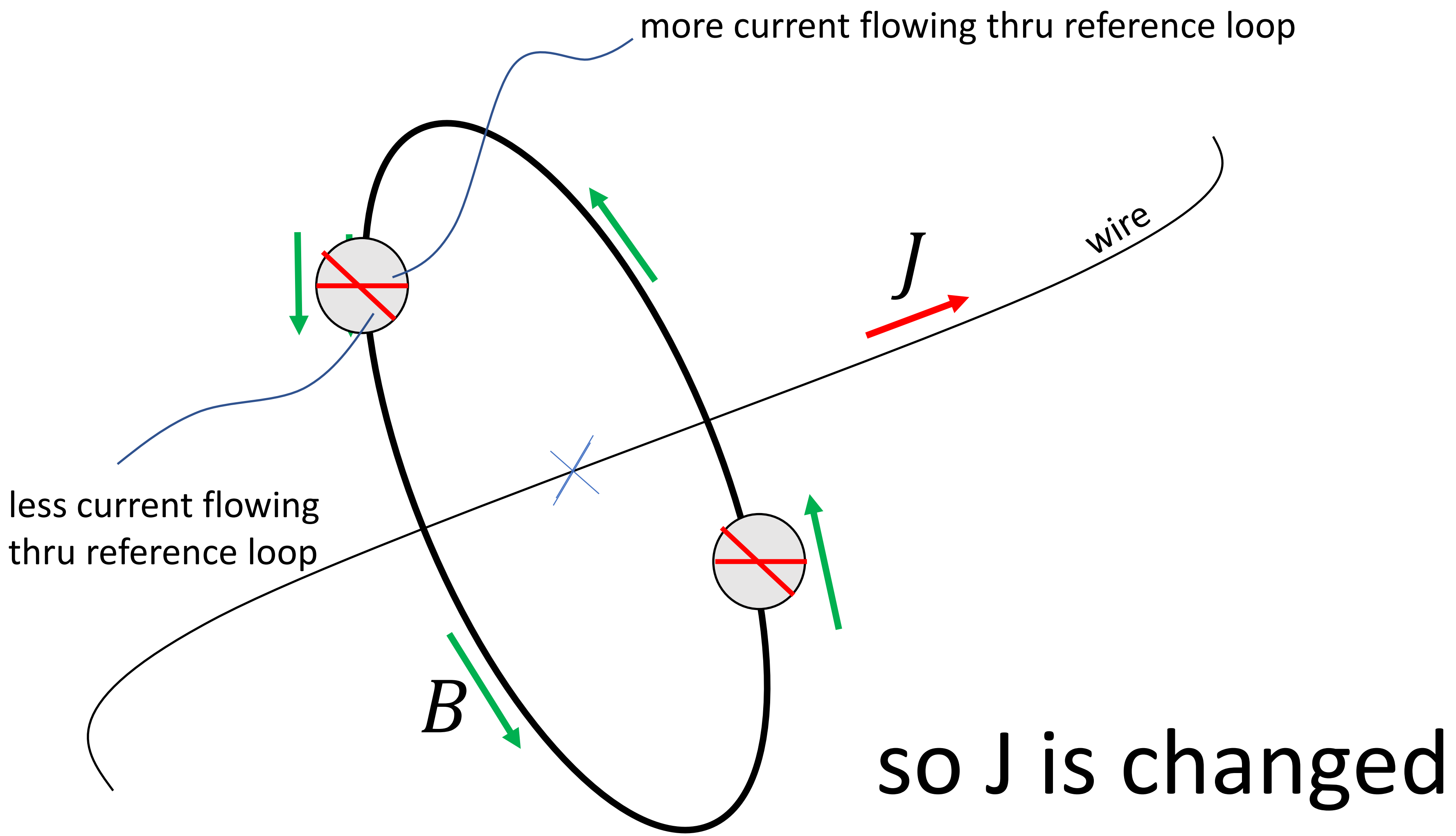
$$\frac{B}{\mu_0} \propto J$$



wire
J
B



more current flowing thru reference loop
wire
J
less current flowing
thru reference loop
B


more current flowing thru reference loop
wire
J
less current flowing
thru reference loop
B
so J is changed

$$\mathbf{J}^{total} \propto \mathbf{J}^{wire} + \mathbf{J}^{induced}$$

$$\frac{B}{\mu_0} \propto J^{total} = J^{wire} + J^{induced}$$

$$\frac{B}{\mu_0} - J^{induced} \propto J^{wire}$$

$$H \propto J^{wire}$$

with $H = \dfrac{B}{\mu_0} - J^{induced}$

magnetic field

$$H \propto J^{wire}$$ with $$H = \frac{B}{\mu_0} - J^{induced}$$

assume linear law

χ magnetic susceptibility

$$H = \frac{B}{\mu_0} - \chi H$$

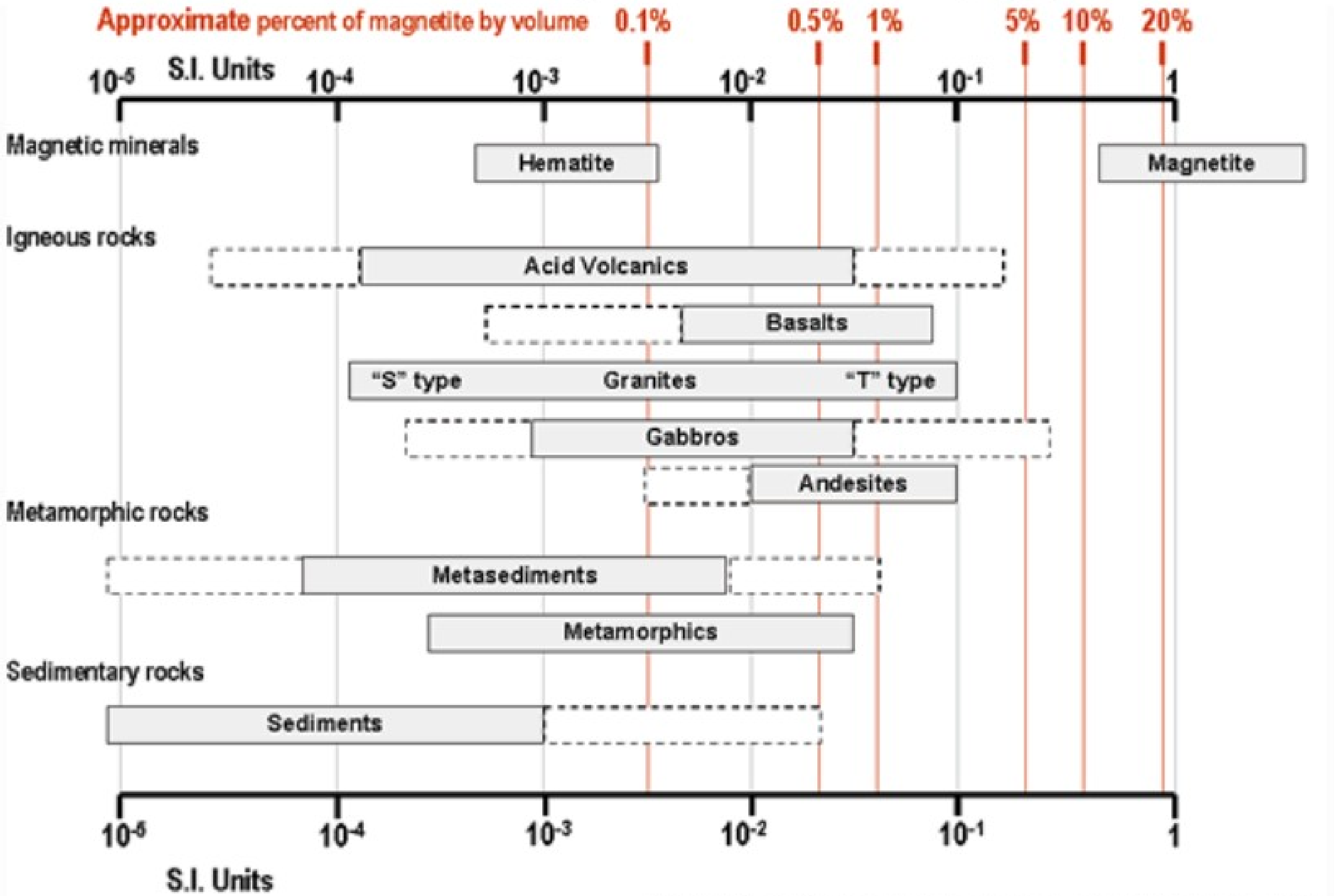
$$H + \chi H = \frac{B}{\mu_0}$$

μ magnetic permeability

$$\mu_0 (1 + \chi) H = B$$

$$\mu H = B$$

Intrinsic Magnetic Susceptibility [SI]



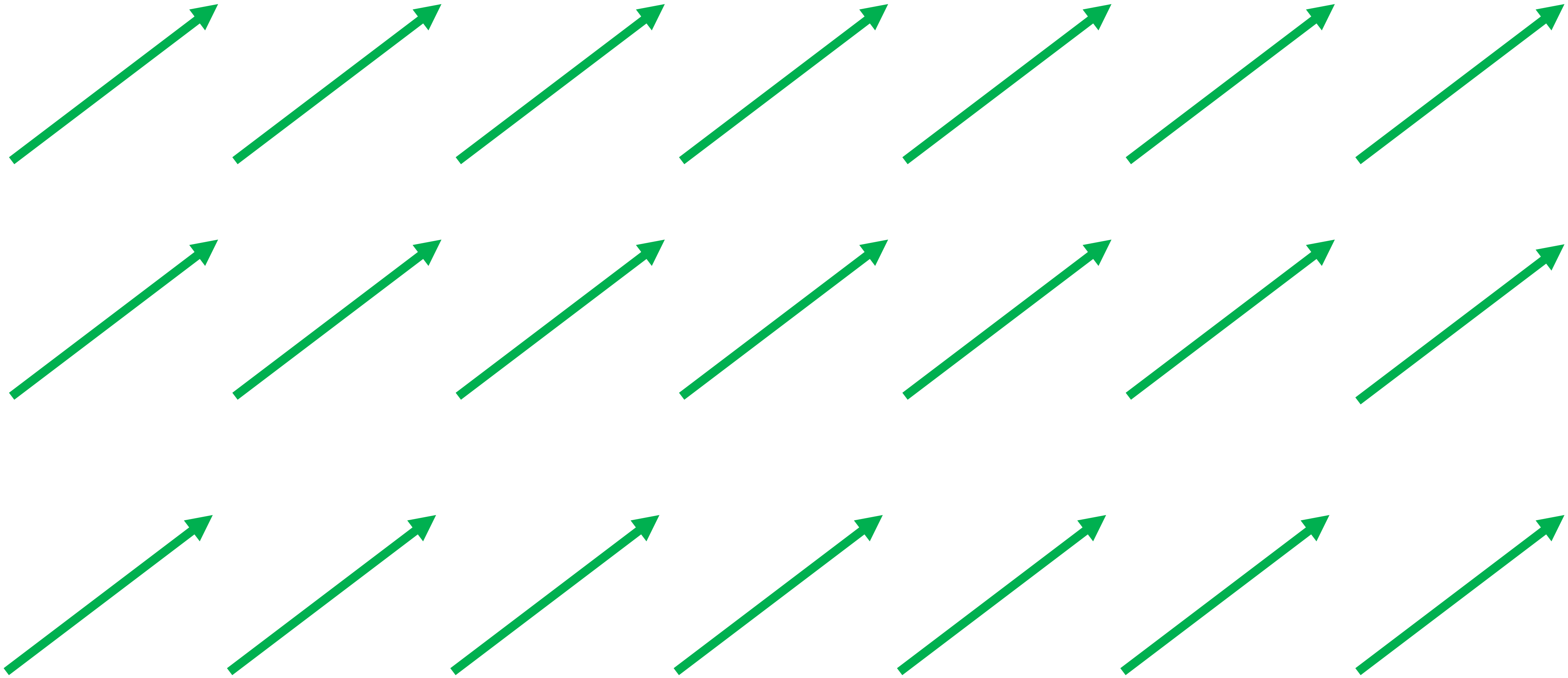
Adapted from Clark and Emerson, Exploration Geophysics, 1991

Part 2

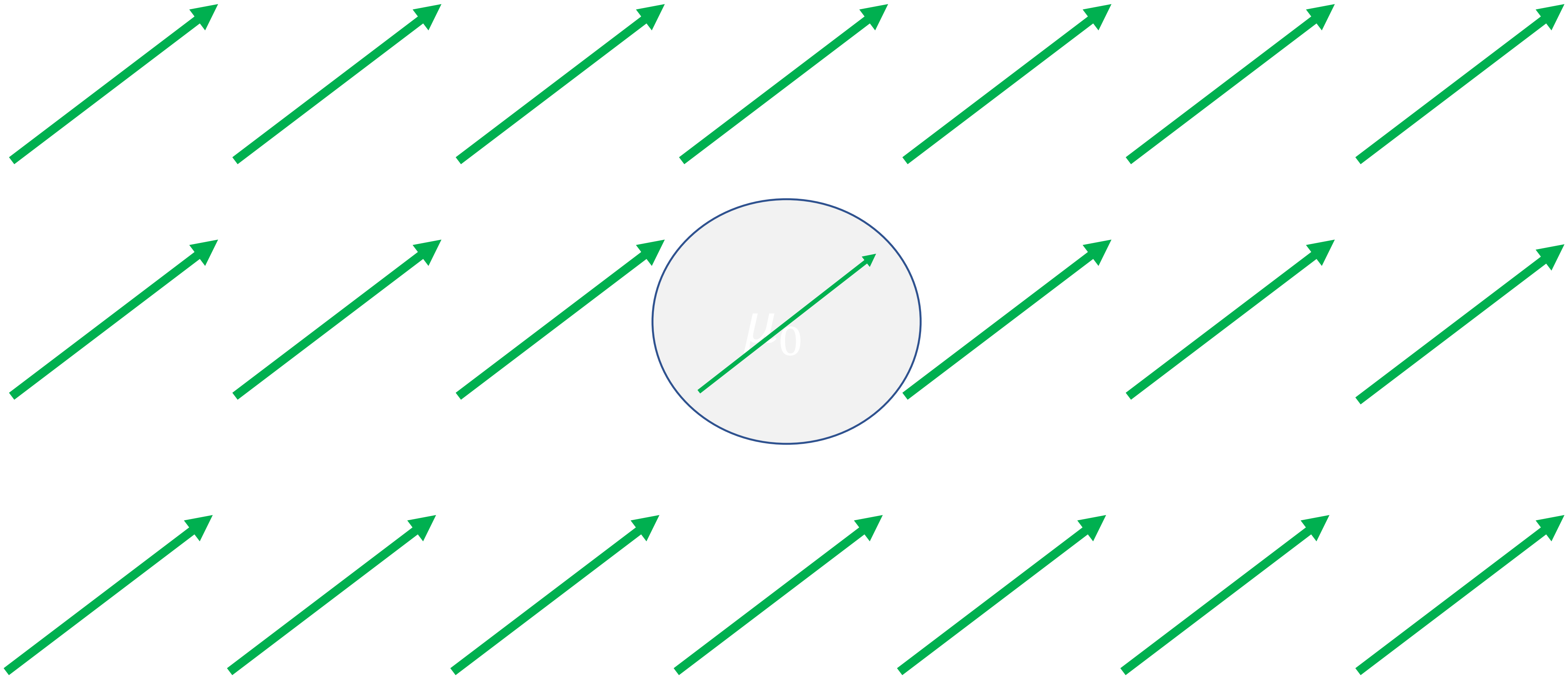
induced magnetic field

no magnetic field

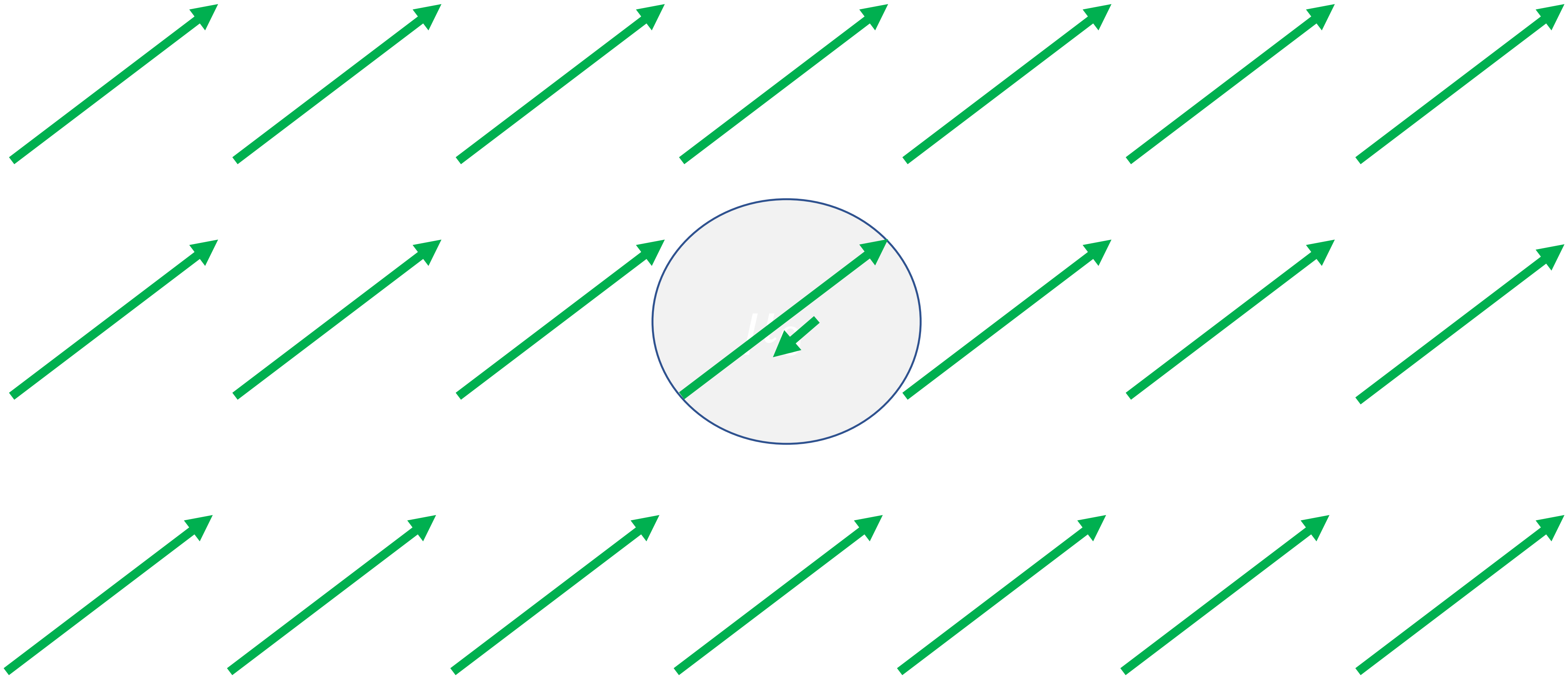
magnetic permeativity $\mu > \mu_0$



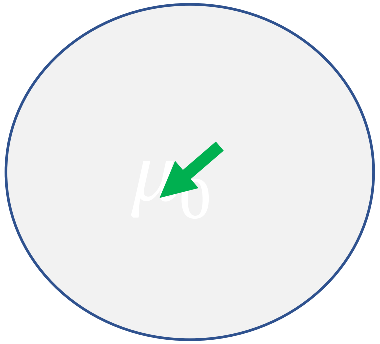
uniform magnetic field $\mathbf{H} = \mathbf{B}/\mu_0$



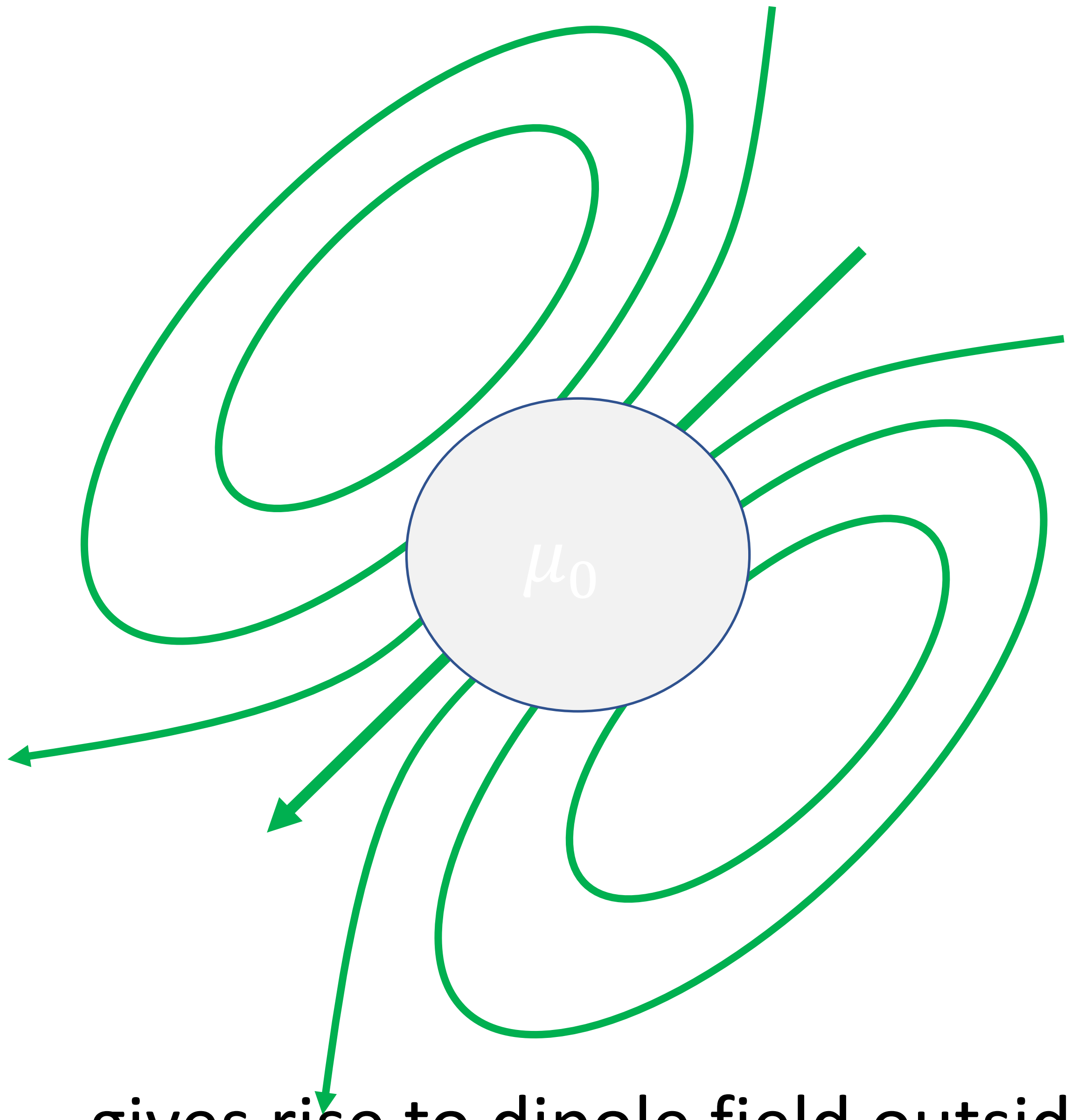
smaller $\mathbf{H}$ inside since $\mathbf{H} = \mathbf{B}/\mu$



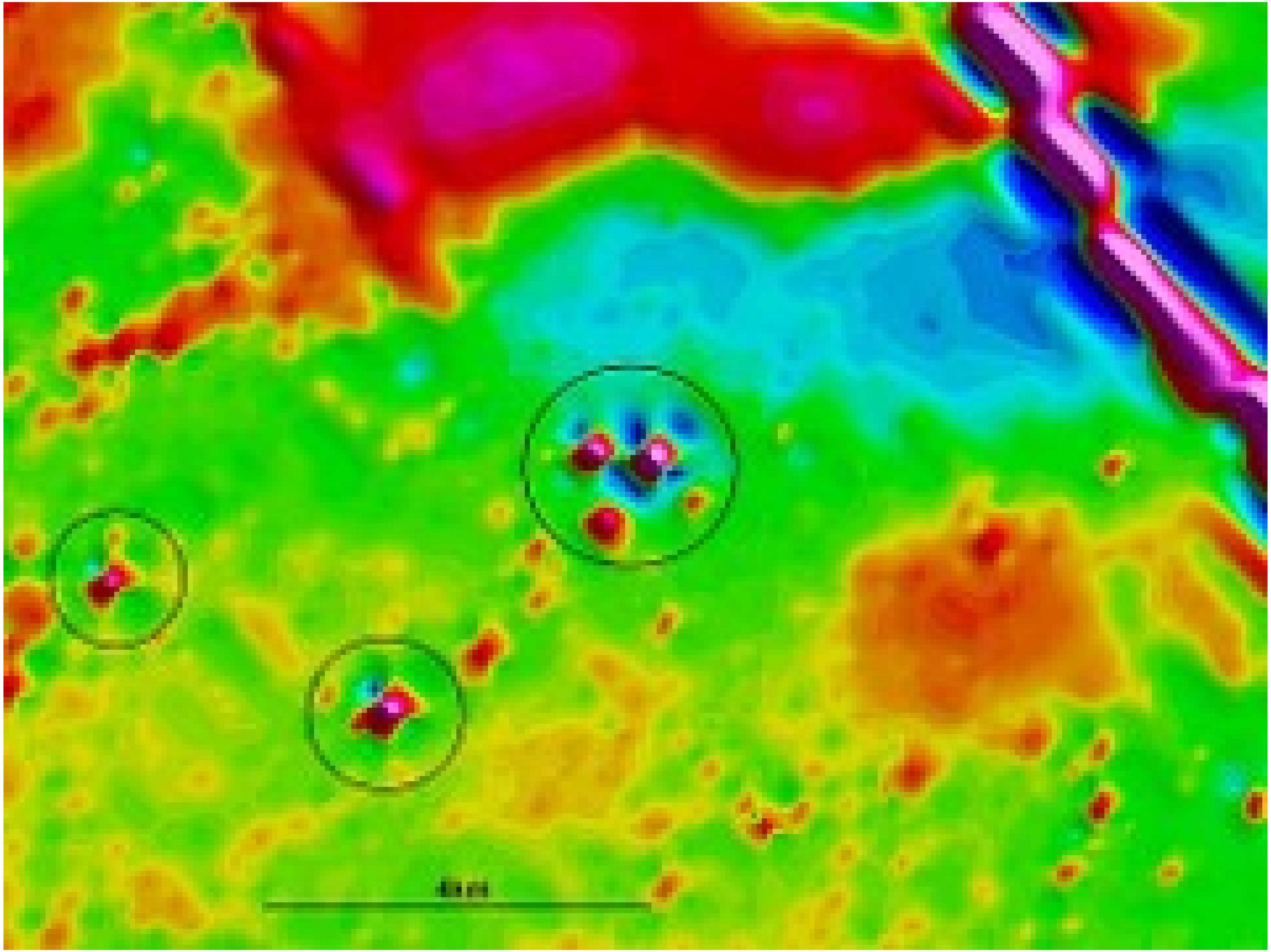
represent as original plus deviation



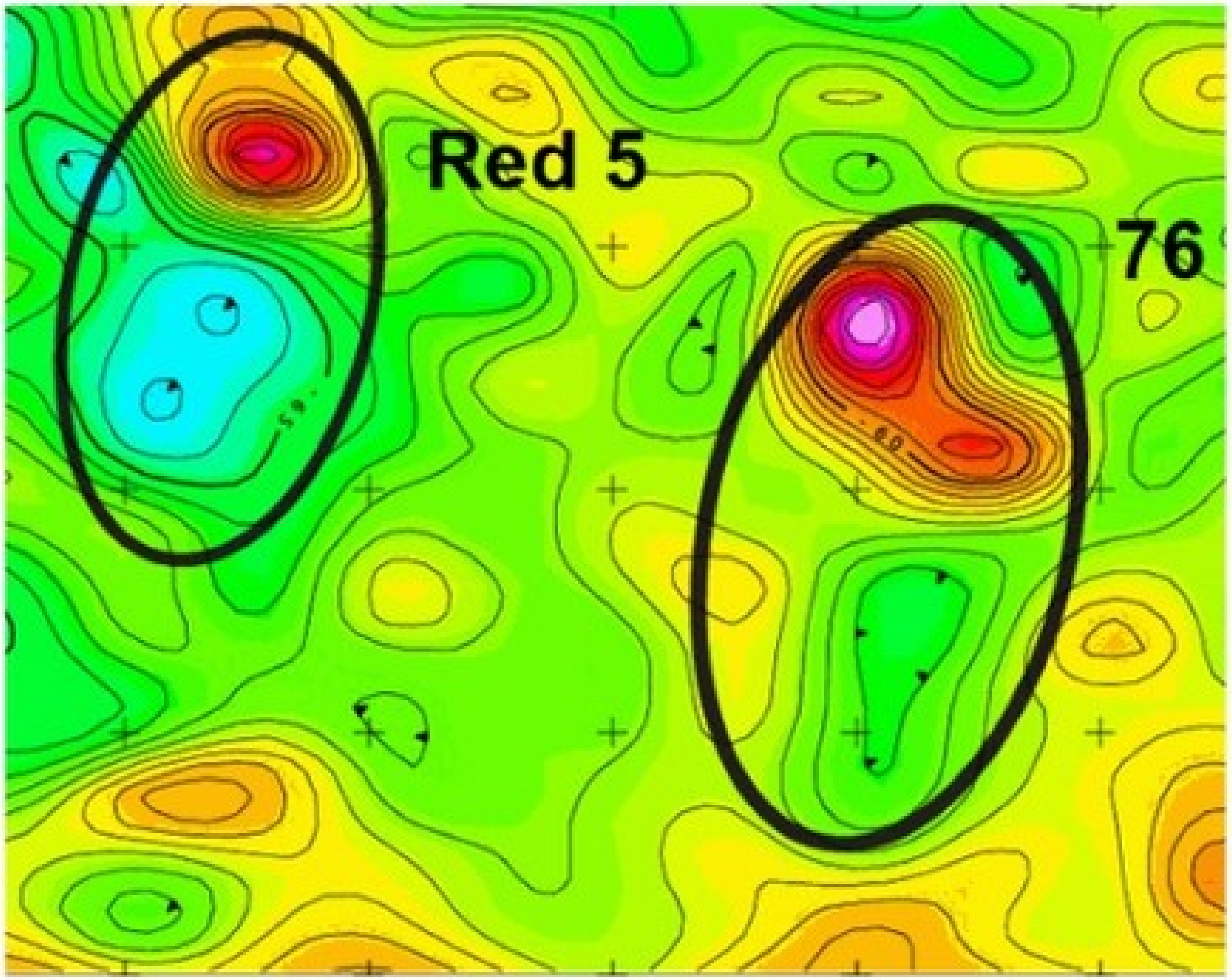
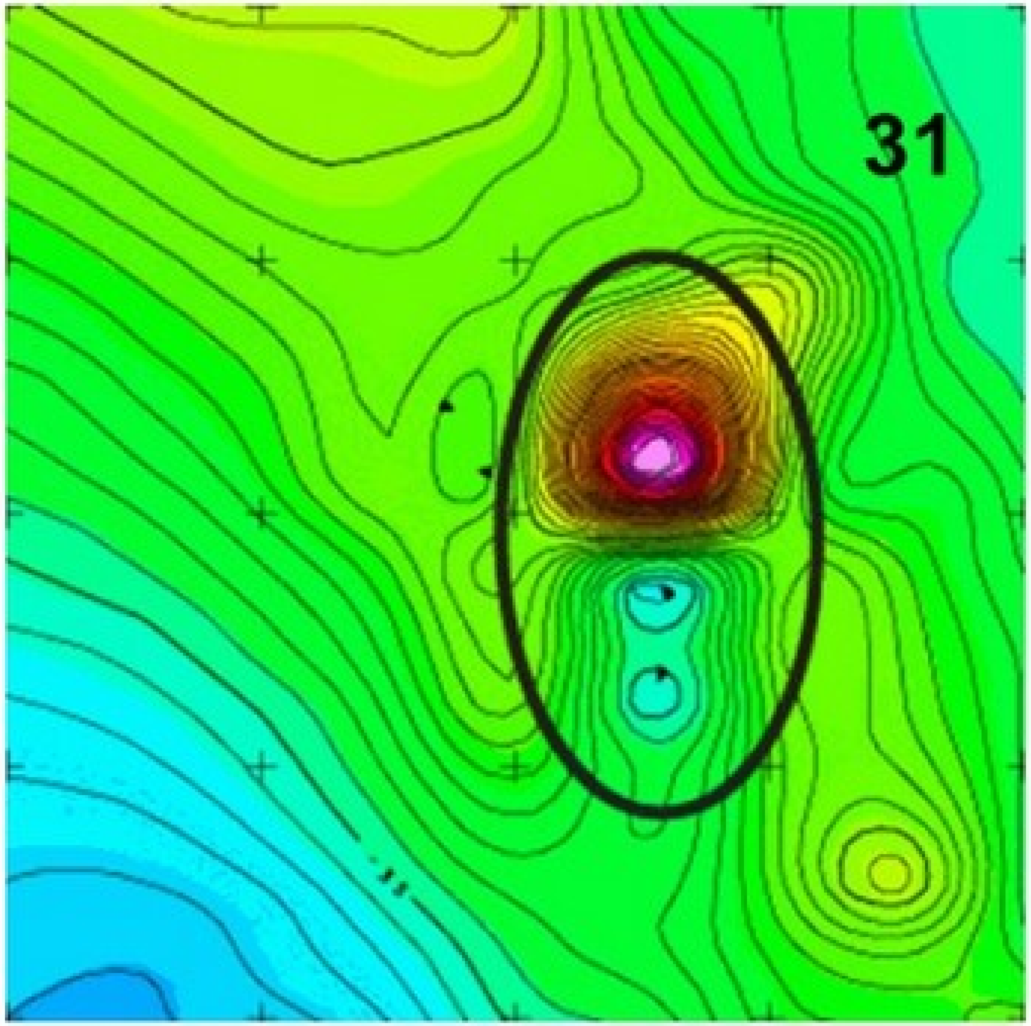
ignore original



gives rise to dipole field outside



diamond bearing kimberlite pipes inn Canada

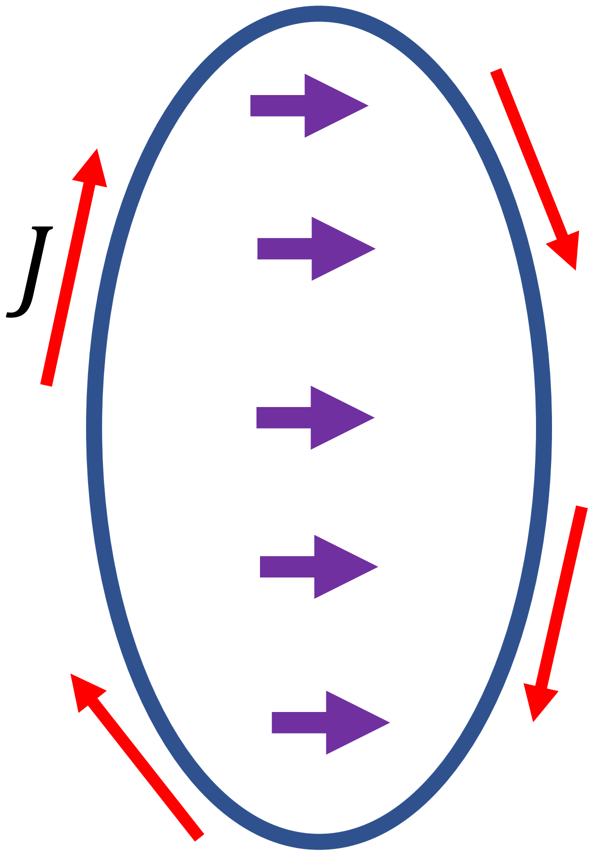

Red 5
76

31

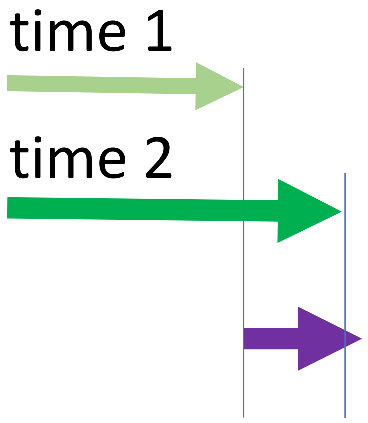
Part 3

Electromagnetic waves

Generator Equation

The more dB/dt crossing the plane of the loop, the bigger the current J



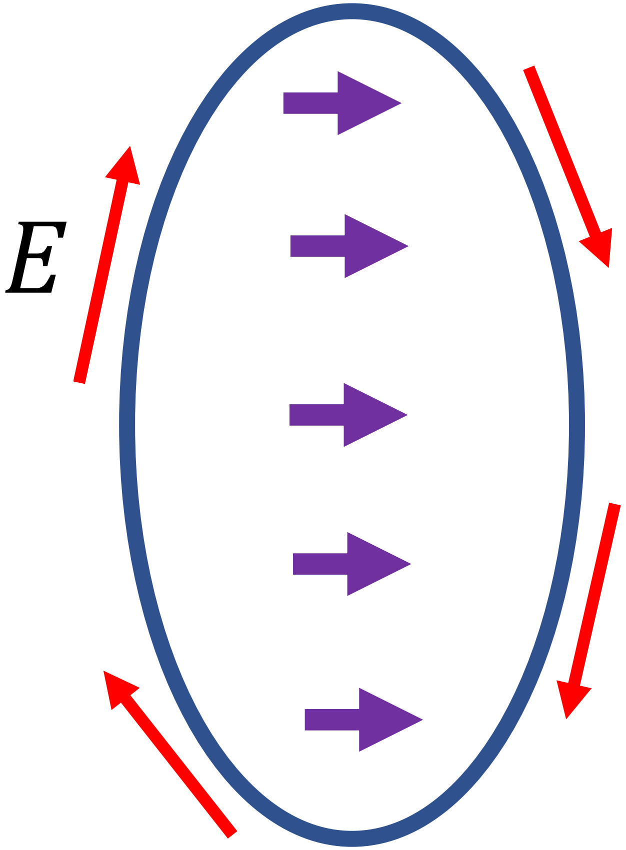


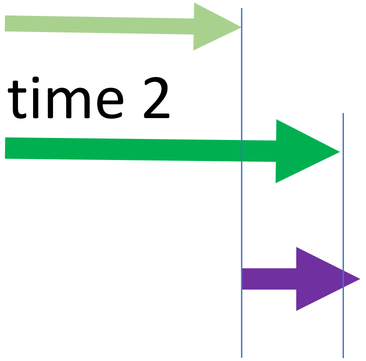
$$\frac{d\mathbf{B}}{dt}$$

$$J \propto -\frac{d\mathbf{B}}{dt}$$

Generator Equation

since $J = \sigma E$

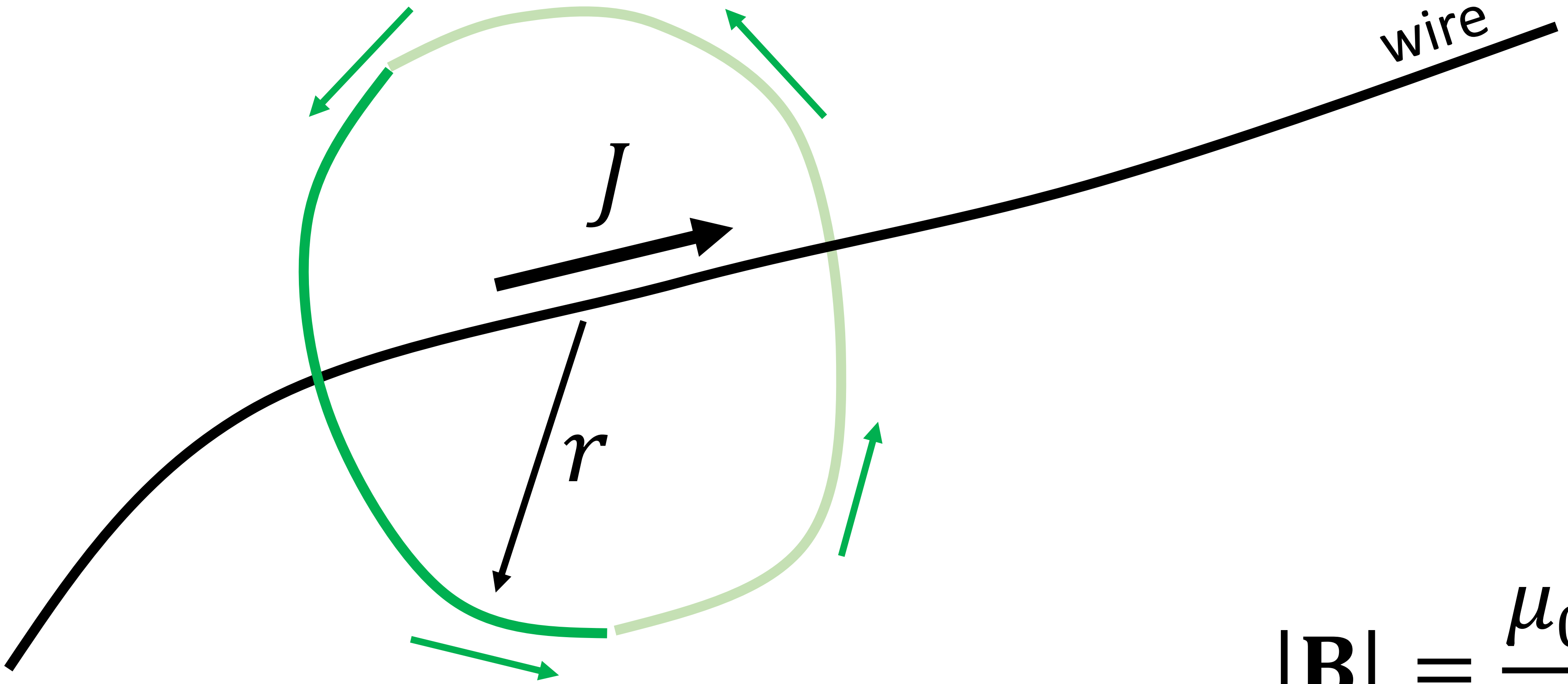




$$\frac{d\mathbf{B}}{dt}$$

$$E \propto -\frac{d\mathbf{B}}{dt}$$

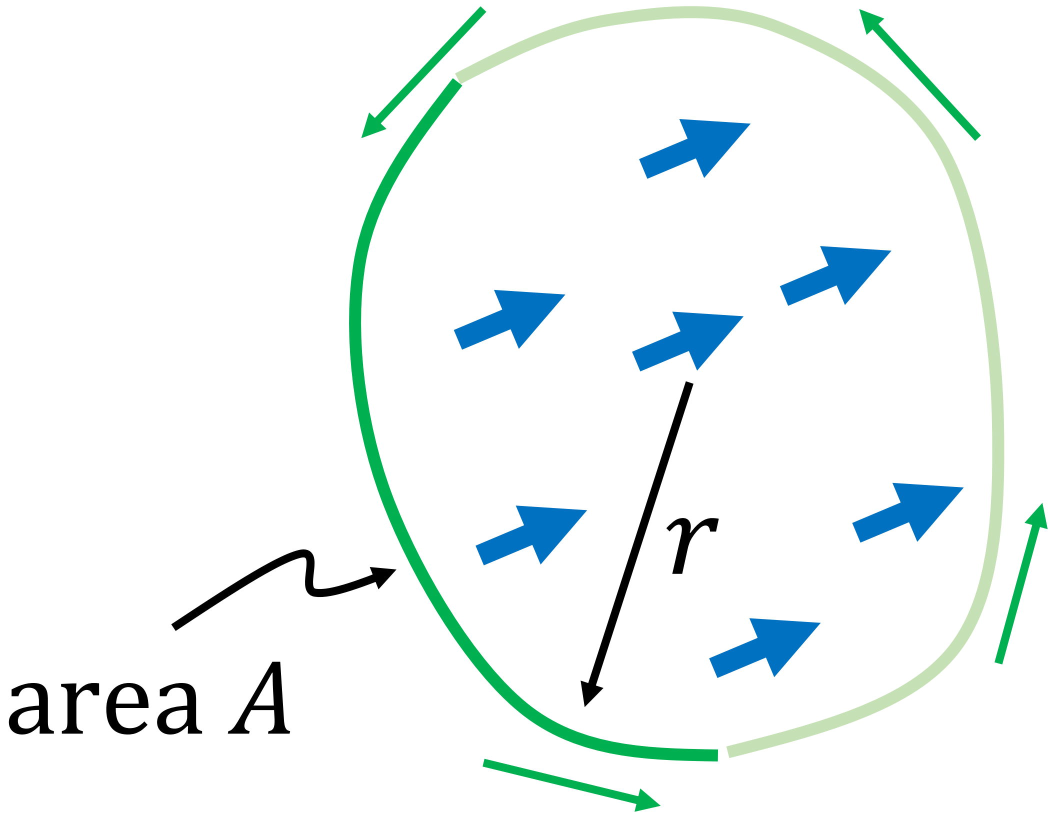
Solenoid Equation: Moving charge makes an electric field

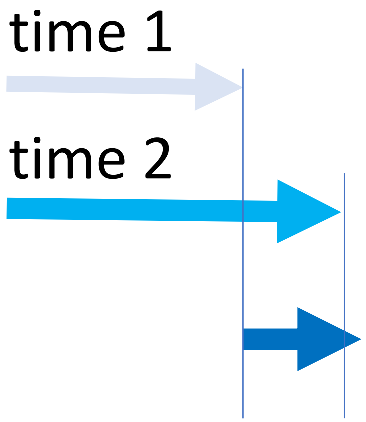


$$|\mathbf{B}| = \frac{\mu_0 J}{2\pi r}$$

μ_0 magnetic permeability

Changing electric field also makes a magnetic field, too



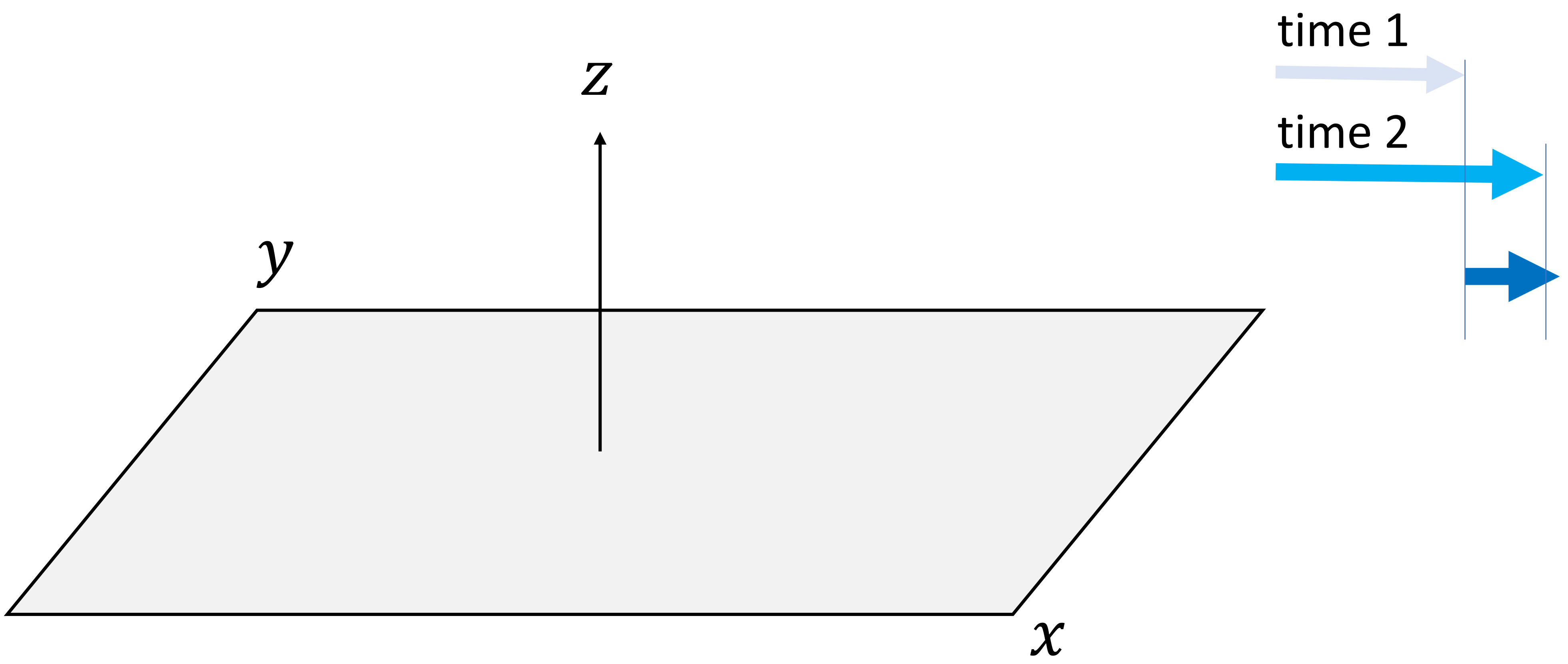


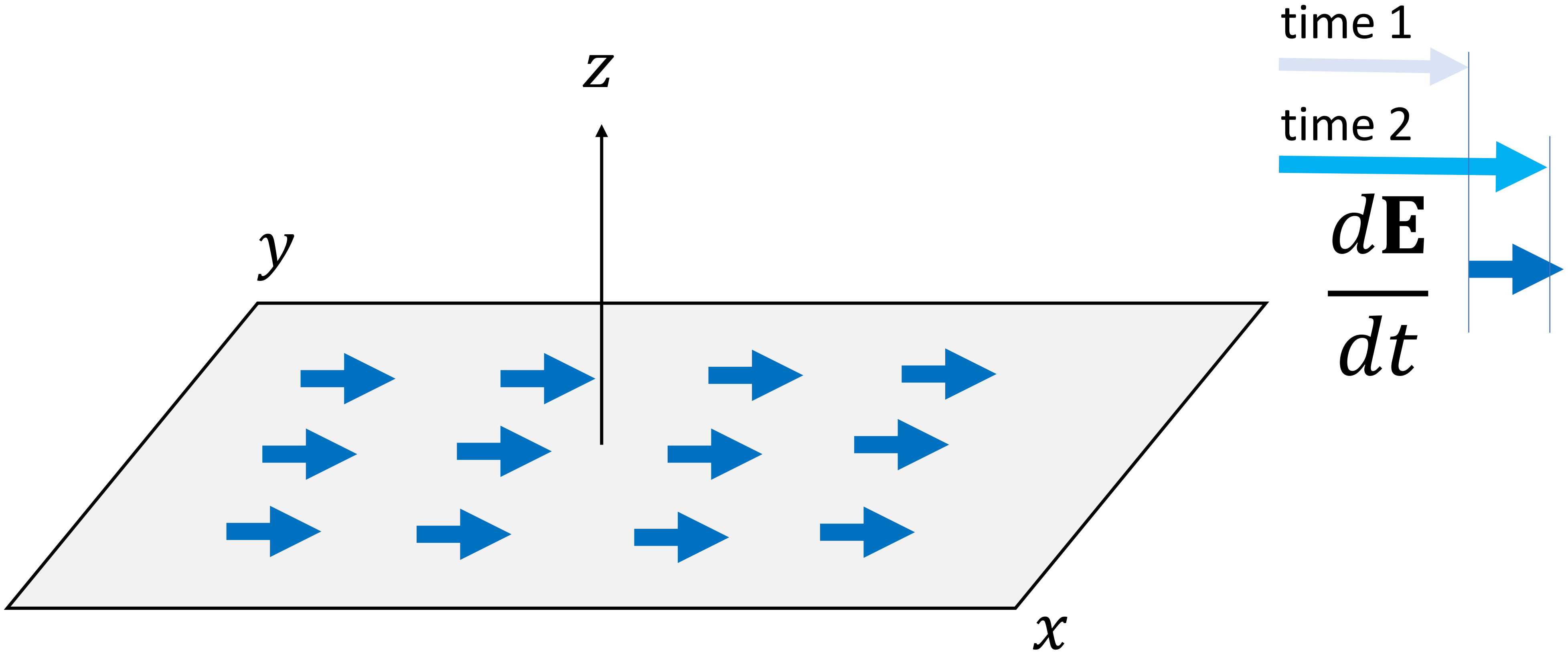
$$|\mathbf{B}| = \frac{\varepsilon_0\, \mu_0\, A}{2\pi r} \frac{d\mathbf{E}}{dt}$$

μ_0 magnetic permeability

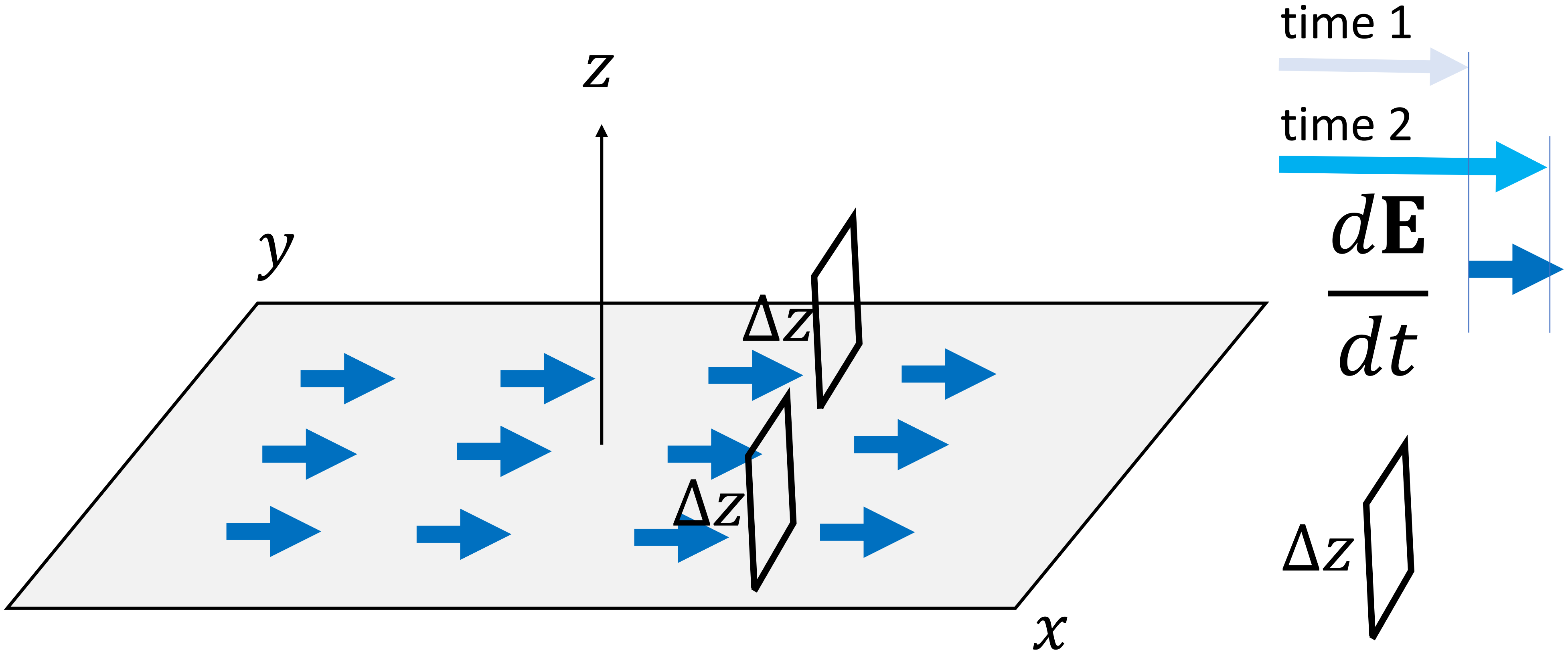
ε_0 permittivity of free space

Generator Equation

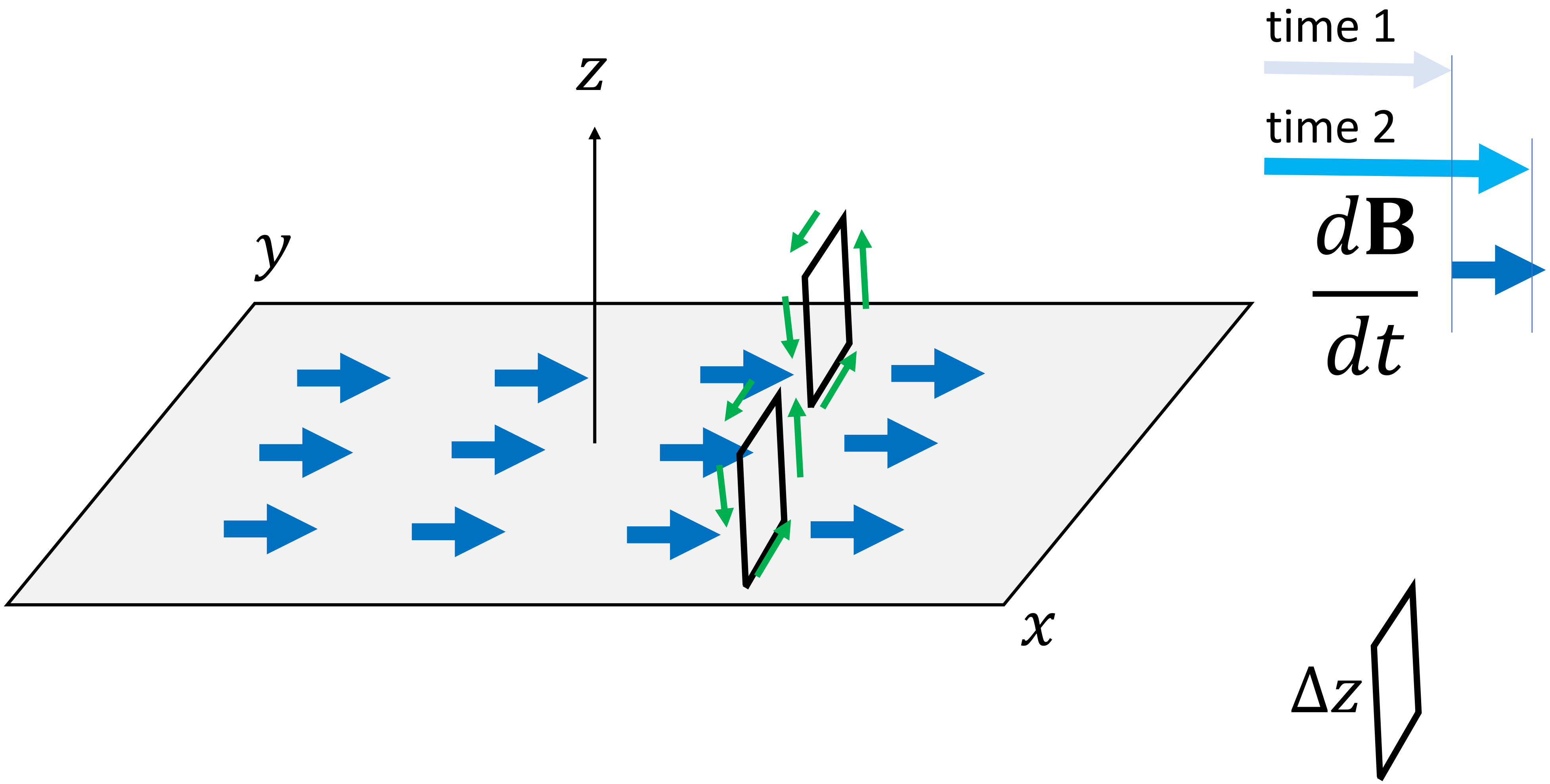




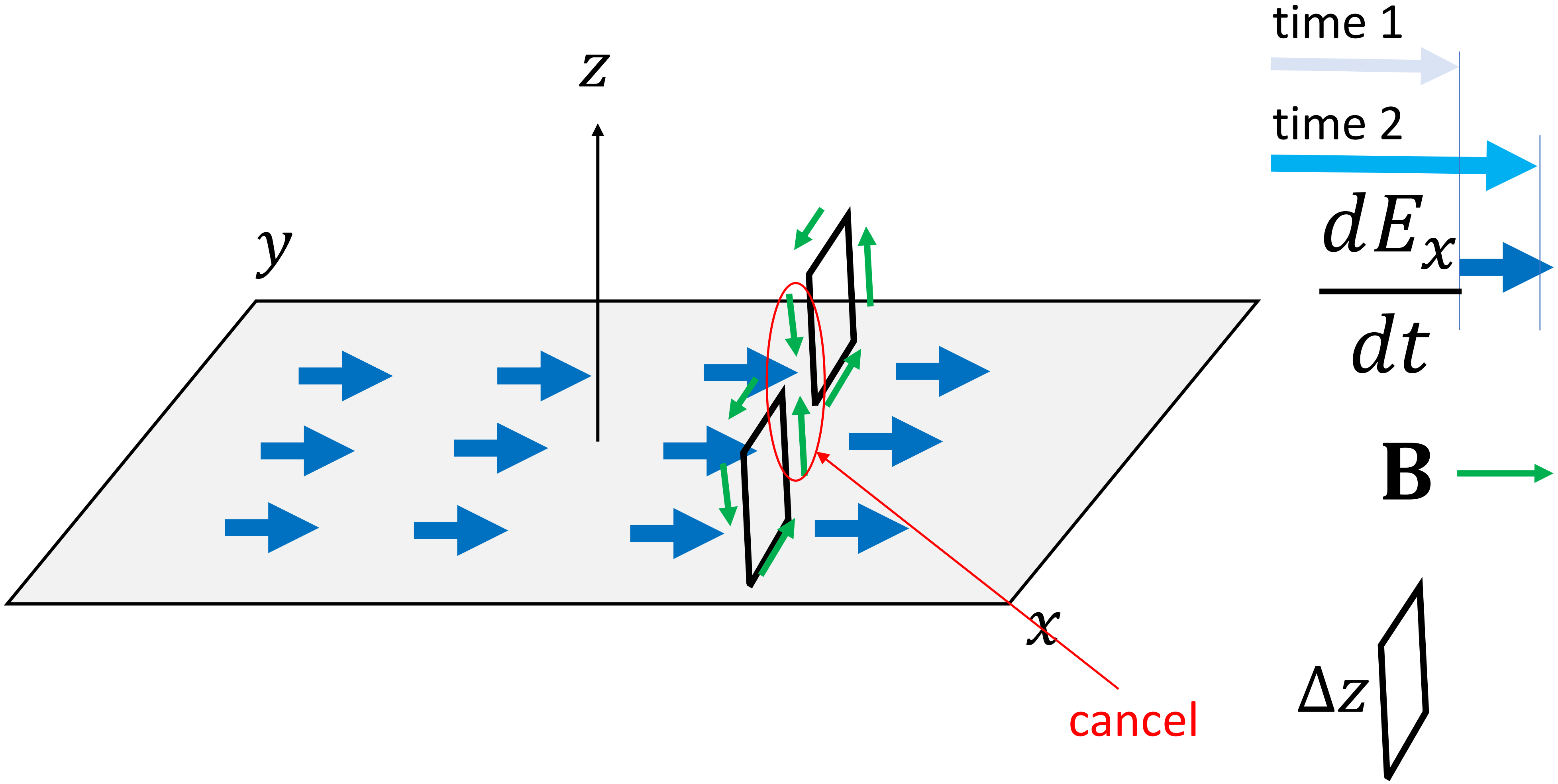
z
y
x
time 1
time 2
$\frac{d\mathbf{E}}{dt}$



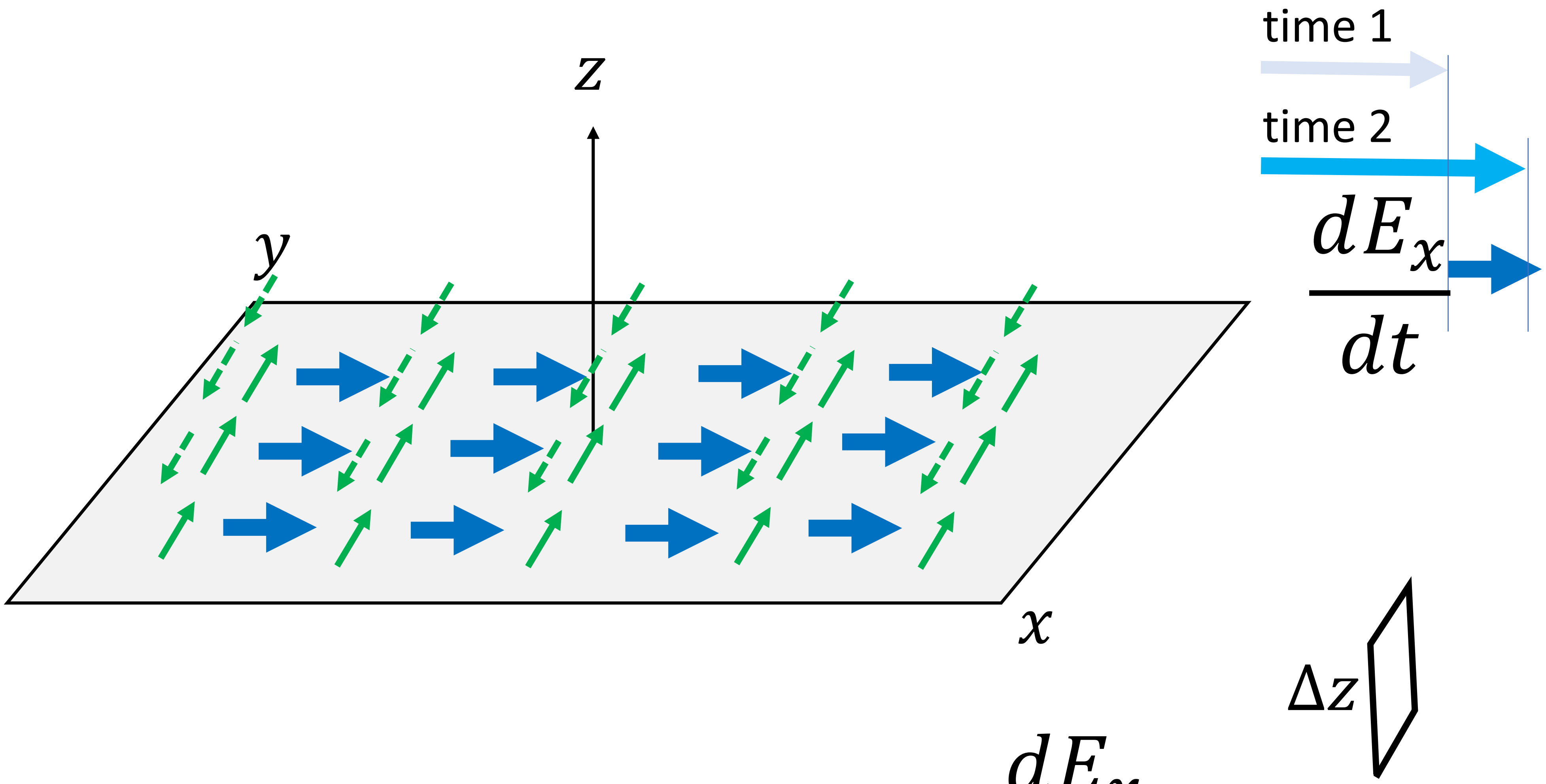
time 1
time 2
$\frac{d\mathbf{E}}{dt}$
z
y
x
Δz
Δz
Δz



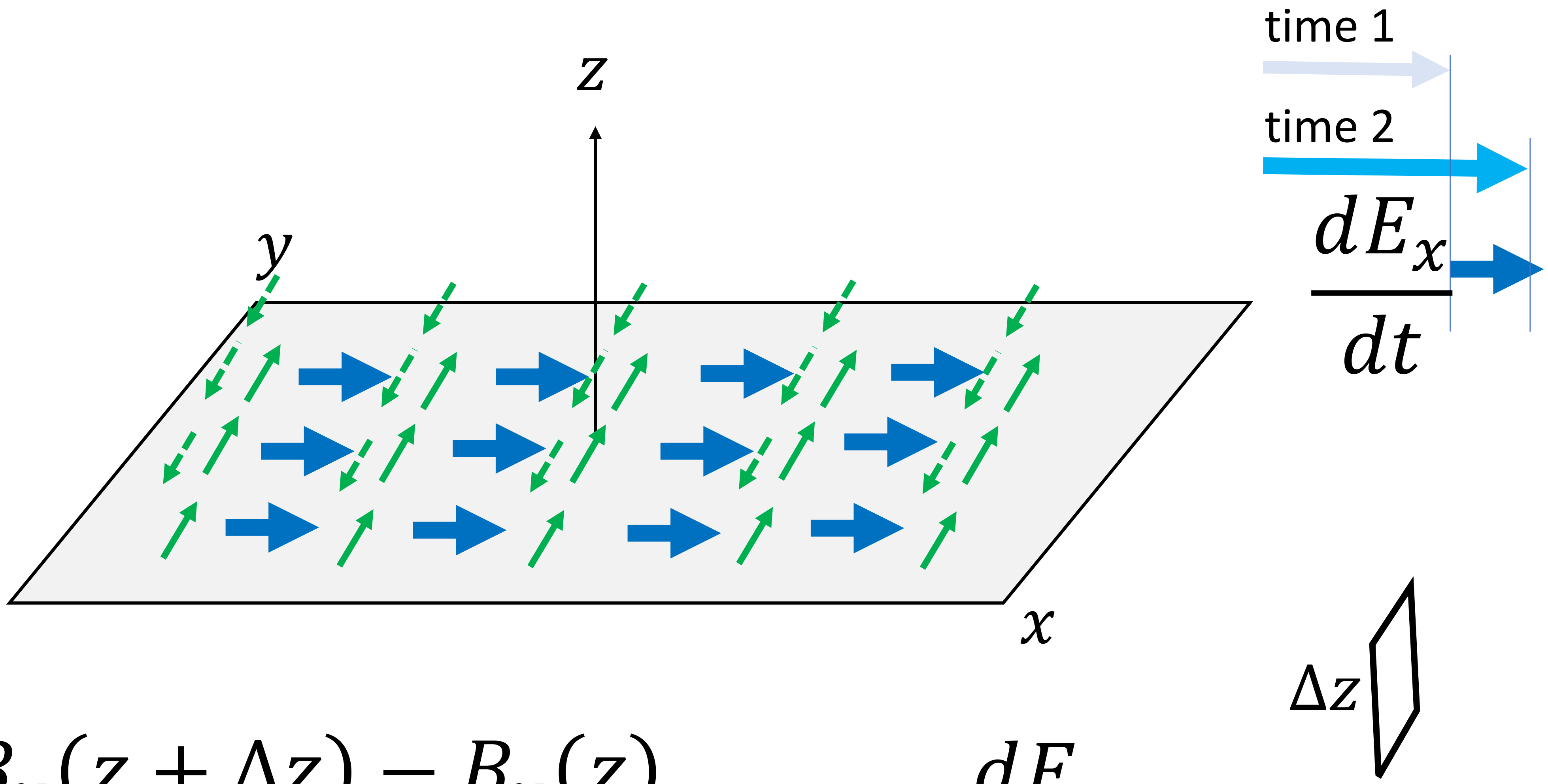
time 1
time 2
$\frac{d\mathbf{B}}{dt}$
z
y
x
Δz



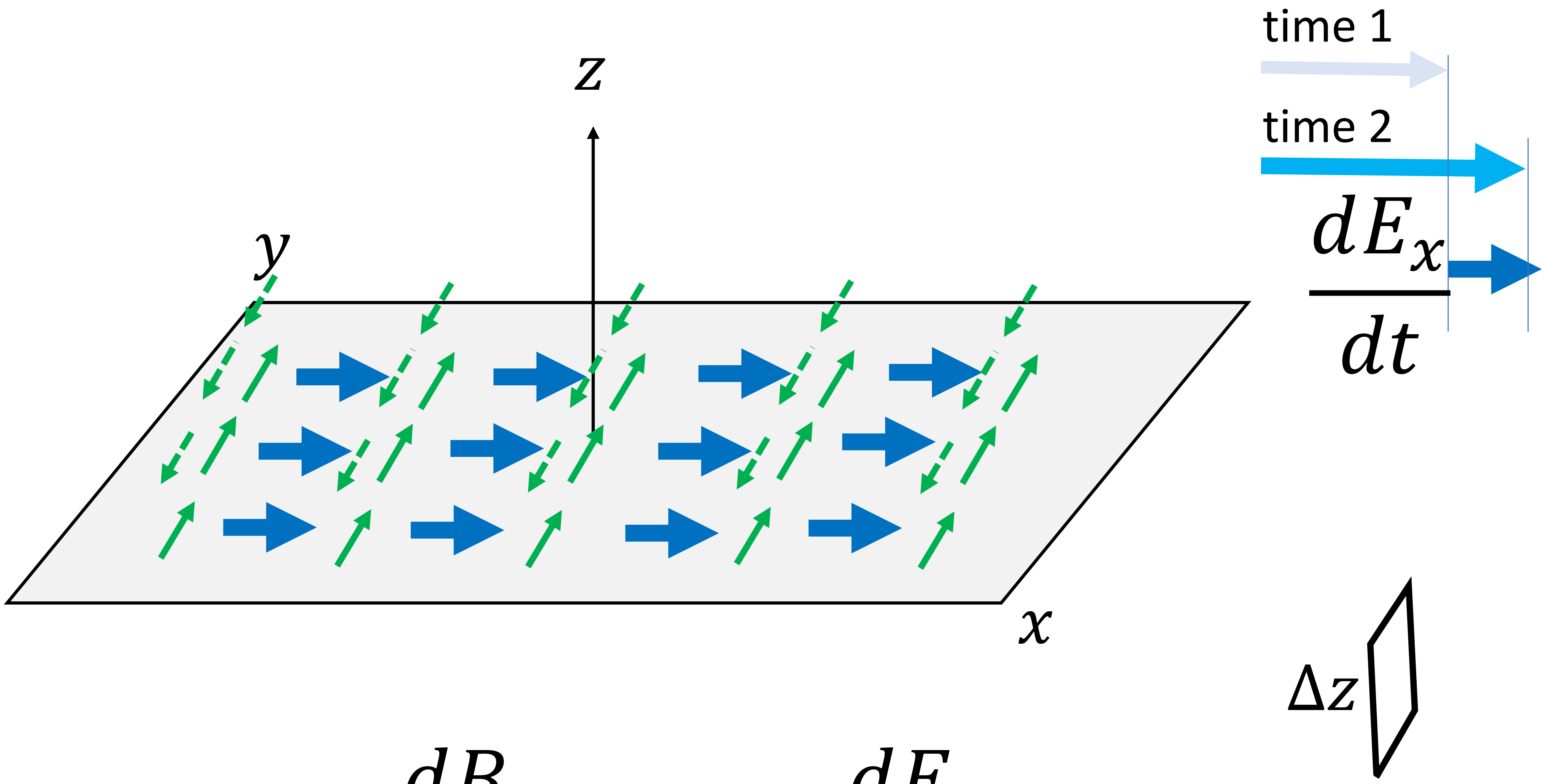
time 1
time 2
$\frac{dE_x}{dt}$
$\mathbf{B}$
z
y
x
cancel
Δz



$$B_y(z) - B_y(z + \Delta z) \propto \Delta z \varepsilon_0 \mu_0 \frac{dE_x}{dt}$$



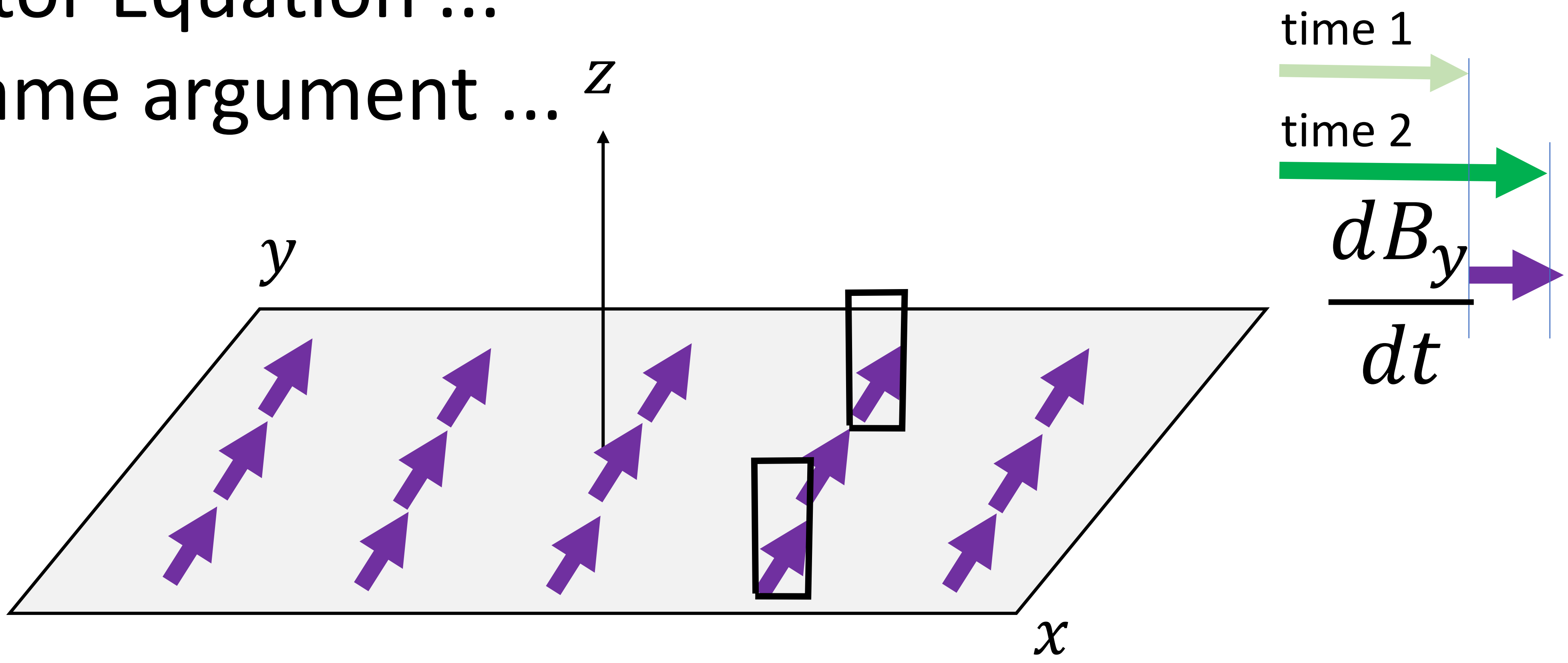
$$-\frac{B_y(z+\Delta z) - B_y(z)}{\Delta z} \propto \varepsilon_0 \, \mu_0 \, \frac{dE_x}{dt}$$



$$-\frac{dB_z}{dz} \propto \varepsilon_0\,\mu_0\,\frac{dE_x}{dt}$$

Generator Equation ...

Exact same argument ...



$$\frac{dE_x}{dz} \propto -\frac{dB_y}{dt}$$

$$\frac{dB_y}{dz} \propto -\varepsilon_0 \, \mu_0 \, \frac{dE_x}{dt}$$

$$\frac{dE_x}{dz} \propto -\frac{dB_y}{dt}$$

... actually, the proportionality factors are 1

$$\frac{dB_y}{dz} = -\varepsilon_0\,\mu_0\,\frac{dE_x}{dt}$$

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt}$$

$$\frac{dB_y}{dz} = -\varepsilon_0\,\mu_0\,\frac{dE_x}{dt} \quad \xrightarrow{\frac{d}{dz}} \quad \frac{d^2B_y}{dz^2} = -\varepsilon_0\,\mu_0\,\frac{d^2E_x}{dzdt}$$

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt} \quad \xrightarrow{\frac{d}{dt}} \quad \frac{d^2E_x}{dzdt} = -\frac{d^2B_y}{dt^2}$$

$$\frac{dB_y}{dz} = -\varepsilon_0 \mu_0 \frac{dE_x}{dt} \quad \xrightarrow{\frac{d}{dz}} \quad \frac{d^2 B_y}{dz^2} = -\varepsilon_0 \mu_0 \frac{d^2 E_x}{dzdt}$$

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt} \quad \xrightarrow{\frac{d}{dt}} \quad \frac{d^2 E_x}{dzdt} = -\frac{d^2 B_y}{dt^2}$$

$$\frac{d^2 B_y}{dz^2} = \varepsilon_0 \mu_0 \frac{d^2 B_y}{dt^2}$$

$$\frac{d^2 B_y}{dz^2} = \varepsilon_0 \, \mu_0 \, \frac{d^2 B_y}{dt^2}$$

$$\frac{d^2 B_y}{dz^2} = \varepsilon_0 \, \mu_0 \, \frac{d^2 B_y}{dt^2}$$

… do the same for E_x

$$\frac{d^2 E_x}{dz^2} = \varepsilon_0 \, \mu_0 \, \frac{d^2 E_x}{dt^2}$$

we've seen this one before, any shape $s(z)$ moves as

$$\frac{d^2 B_y}{dz^2} = \varepsilon_0 \, \mu_0 \, \frac{d^2 B_y}{dt^2}$$

$$B_y = C \, s(z - vt)$$

$$v = (\varepsilon_0 \, \mu_0)^{-½}$$

$$\frac{d^2 E_x}{dz^2} = \varepsilon_0 \, \mu_0 \, \frac{d^2 E_x}{dt^2}$$

$$E_x = D \, s(z - vt)$$

$$\frac{d^2 B_y}{dz^2} = \varepsilon_0\,\mu_0\,\frac{d^2 B_y}{dt^2}$$

$$B_y = C\,s(z - vt)$$

$$v = (\varepsilon_0\,\mu_0\,)^{-½}$$

$$\frac{d^2 E_x}{dz^2} = \varepsilon_0\,\mu_0\,\frac{d^2 E_x}{dt^2}$$

$$E_x = D\,s(z - vt)$$

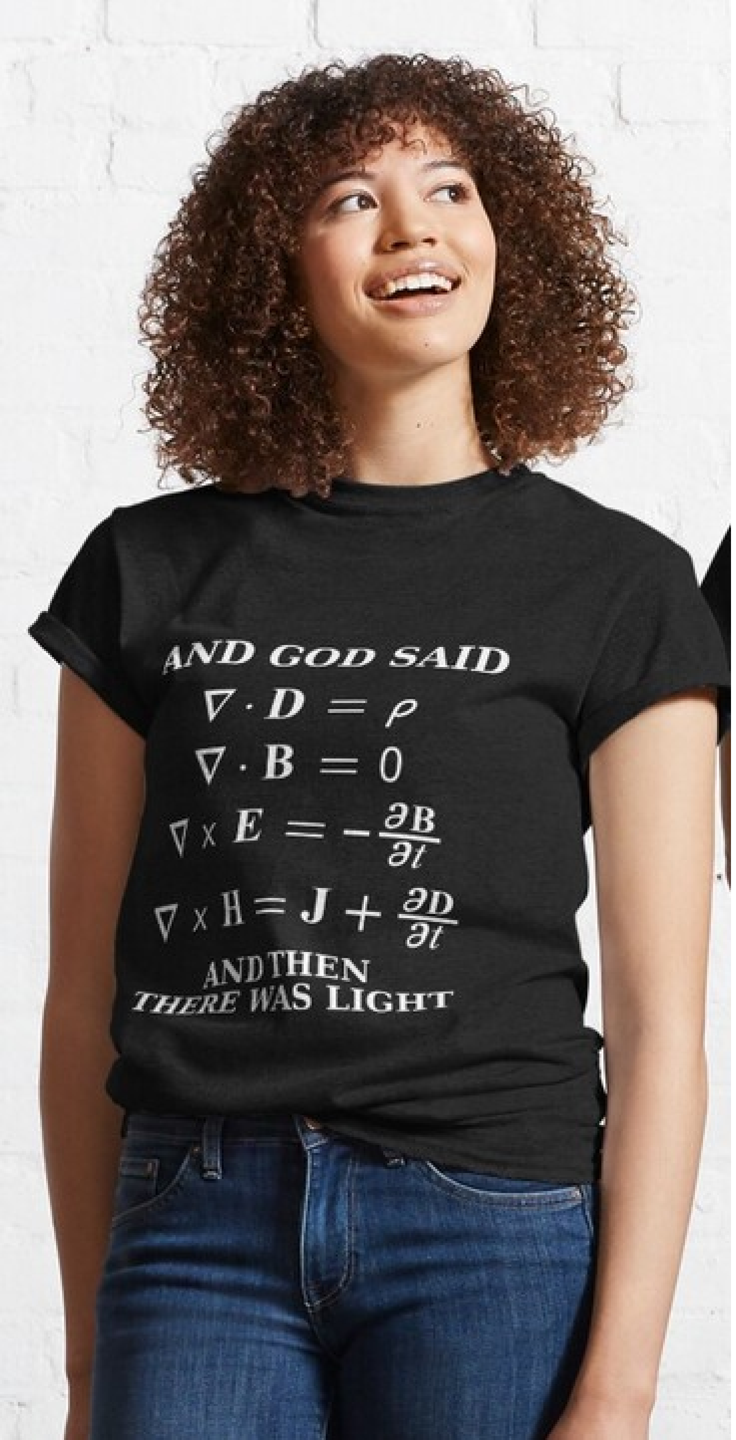
need cosine to satisfy

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt}$$

which also requires

$$D = vC$$

and the shapes to be the same



Back in the 70's
when I was an undergrad at MIT
these shirts were popular

solenoid equation

$$\frac{dB_y}{dz} = -\varepsilon_0\, \mu_0\, \frac{dE_x}{dt}$$

generator equation

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt}$$

solenoid equation

$$\frac{dB_y}{dz} = -v^{-2}\frac{dE_x}{dt} - \mu_0 J$$

generator equation

$$\frac{dE_x}{dz} = -\frac{dB_y}{dt}$$

solenoid equation

$$\frac{dB_y}{dz} = -v^{-2}\,\frac{dE_x}{dt} - \mu_0\,\sigma E_x$$

generator equation

$$\frac{dE_x}{dz} = -\,\frac{dB_y}{dt}$$

solenoid equation

$$\frac{d^2 B_y}{dzdt} = -v^{-2}\,\frac{d^2 E_x}{dt^2} - \mu_0\,\sigma\,\frac{dE_x}{dt} =$$

generator equation

$$\frac{d^2 E_x}{dz^2} = -\frac{d^2 B_y}{dzdt}$$

solenoid equation

$$\frac{d^2 B_y}{dz dt} = -v^{-2} \frac{d^2 E_x}{dt^2} - \mu_0 \, \sigma \frac{dE_x}{dt} =$$

generator equation

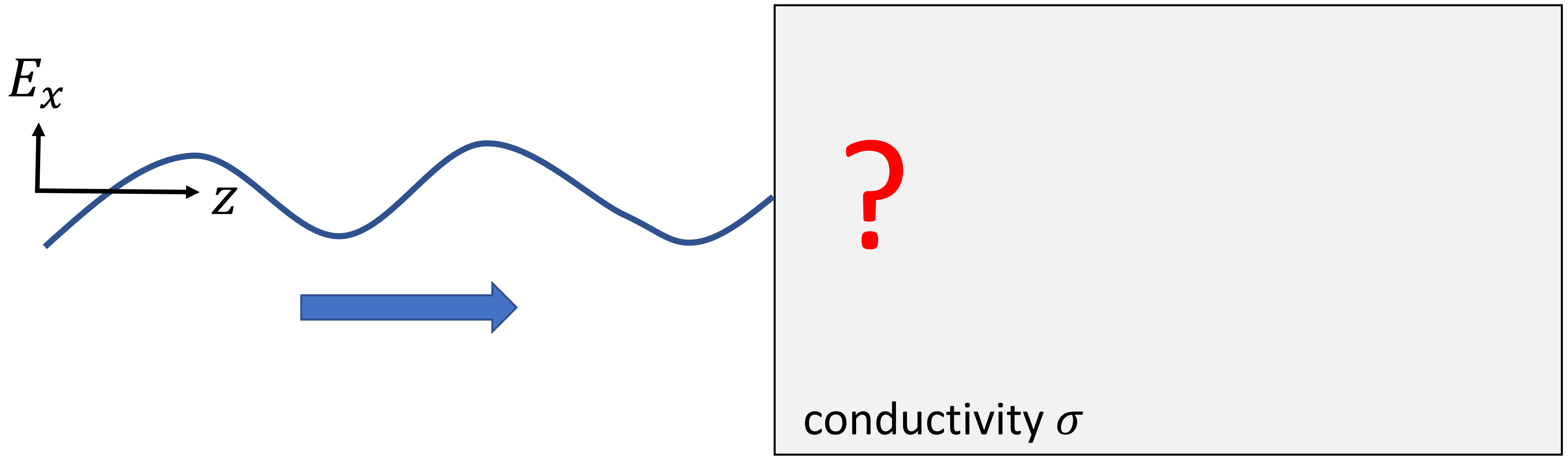
$$\frac{d^2 E_x}{dz^2} = -\frac{d^2 B_y}{dz dt}$$

$$\frac{d^2 E_x}{dz^2} = v^{-2} \frac{d^2 E_x}{dt^2} + \mu_0 \, \sigma \frac{dE_x}{dt}$$

$$\frac{d^2 E_x}{dz^2} = \varepsilon_0 \mu_0 \frac{d^2 E_x}{dt^2} + \mu_0 \sigma \frac{dE_x}{dt}$$

solution for large conductivity

$$\frac{d^2 E_x}{dz^2} = \varepsilon_0 \mu_0 \frac{d^2 E_x}{dt^2} + \mu_0 \sigma \frac{dE_x}{dt}$$



$$\frac{d^2 E_x}{dz^2} = +\mu_0\,\sigma\frac{dE_x}{dt}$$

solution for large conductivity

and when

$$E_x(z=0) = C\sin(2\pi f t)$$

$$E_x(z=0) = C\sin(2\pi f t - rz)\exp(-rz)$$

$$r = \sqrt{\pi f \sigma \mu_0}$$

$$\frac{d^2 E_x}{dz^2} = + \mu_0 \, \sigma \frac{dE_x}{dt}$$

solution for large conductivity

and when

$$E_x(z = 0) = C \sin(2\pi f t)$$

$$E_x(z = 0) = C \sin(2\pi f t - rz) \exp(-rz)$$

$$r = \sqrt{\pi f \sigma \mu_0}$$

electromagnetic waves decays with depth in a conductor

$$\frac{d^2 E_x}{dz^2} = +\mu_0 \, \sigma \frac{dE_x}{dt}$$

solution for large conductivity

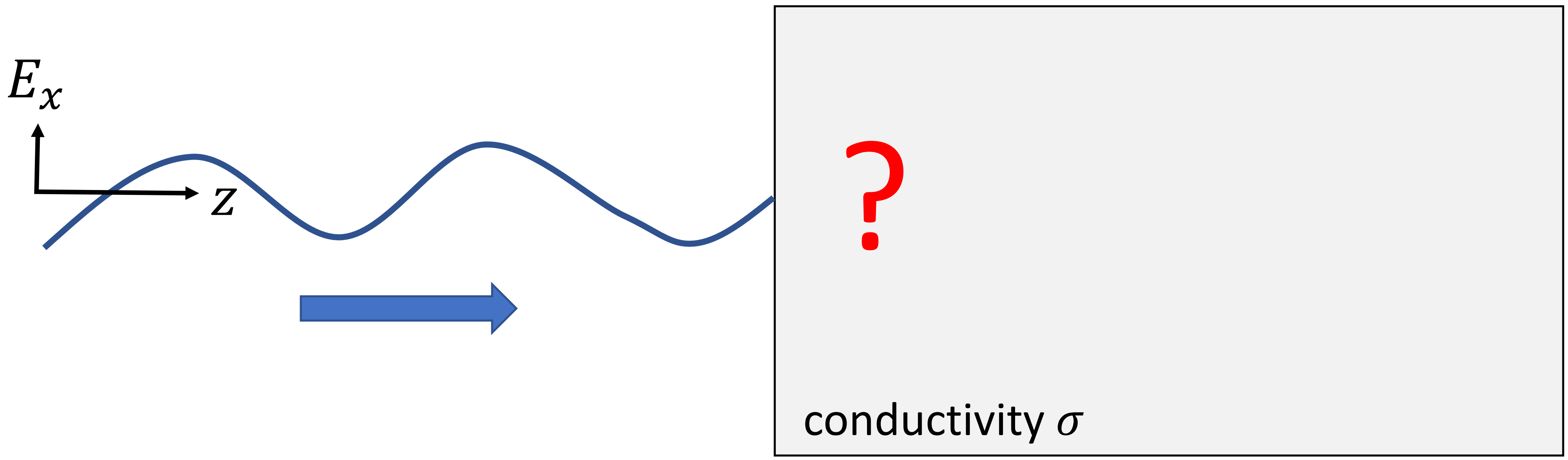
and when

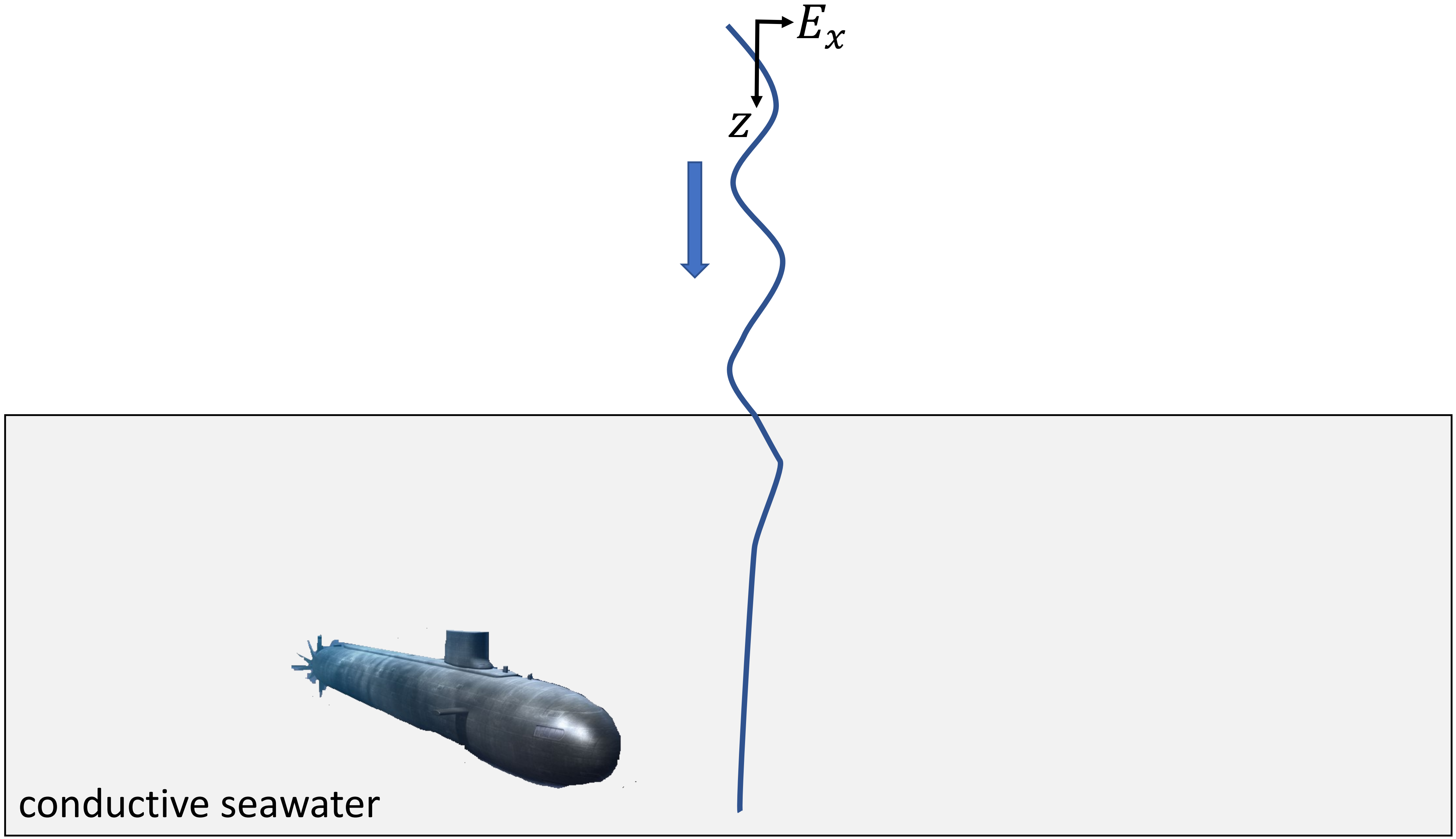
$$E_x(z=0) = C\sin(2\pi f t)$$

$$E_x(z=0) = C\sin(2\pi f t - rz)\exp(-rz)$$

$$r = \sqrt{\pi f \sigma \mu_0}$$

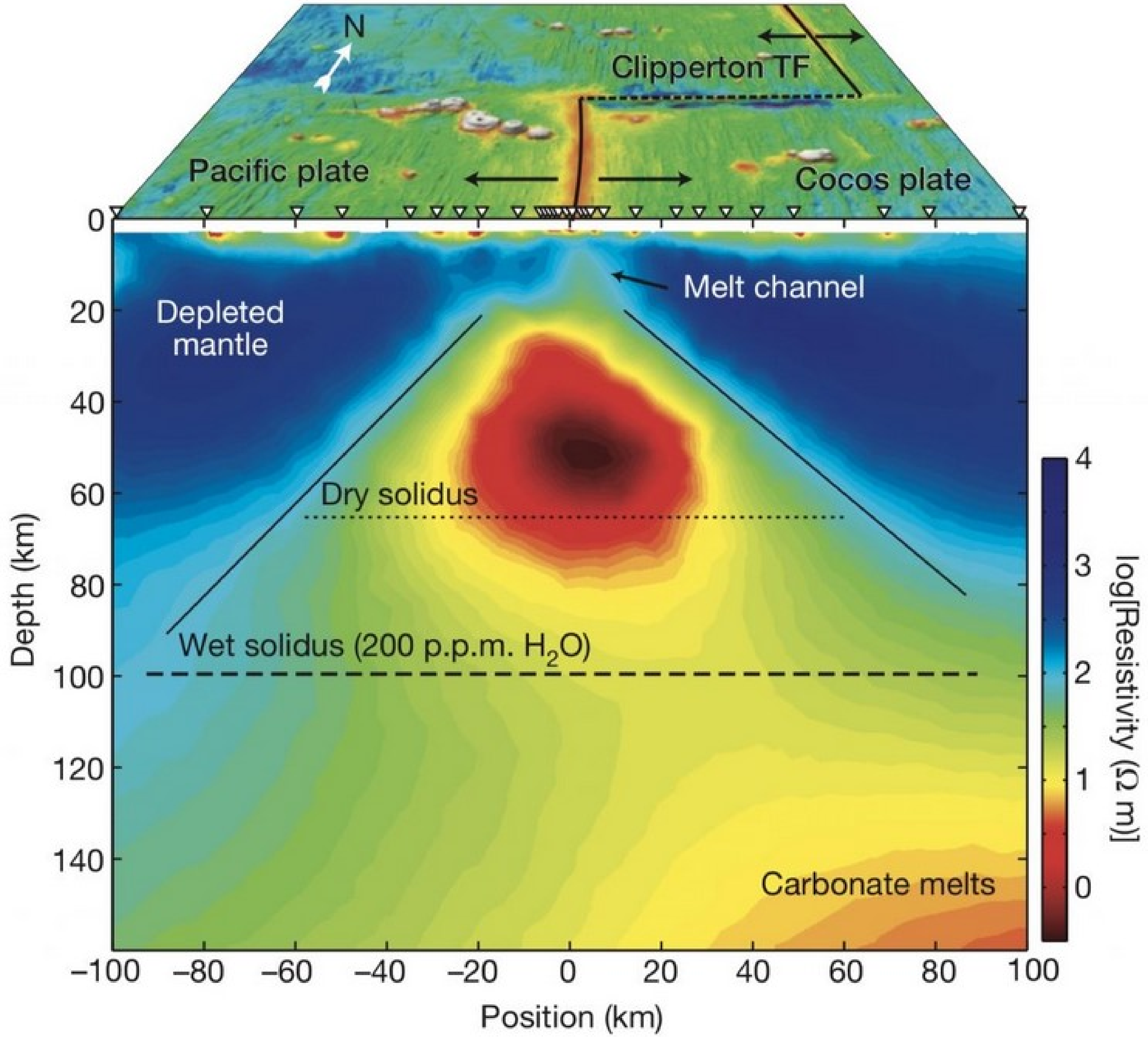
electromagnetic waves decay in a conductor
at a rate that increases with
frequency of the wave
conductivity of the medium


E_x
z
?
conductivity σ



E_x
z
conductive seawater

Navy VLF Transmitter, Cutler, Maine



can exploit the effect to measure the conductivity of the Earth