Solid Earth Dynamics

Bill Menke, Instructor

Lecture 24

Glaciology

Kalfaf
1
Reynivellir
1
Jökulsárlón
1
alshnúkur
Layers
Terrain
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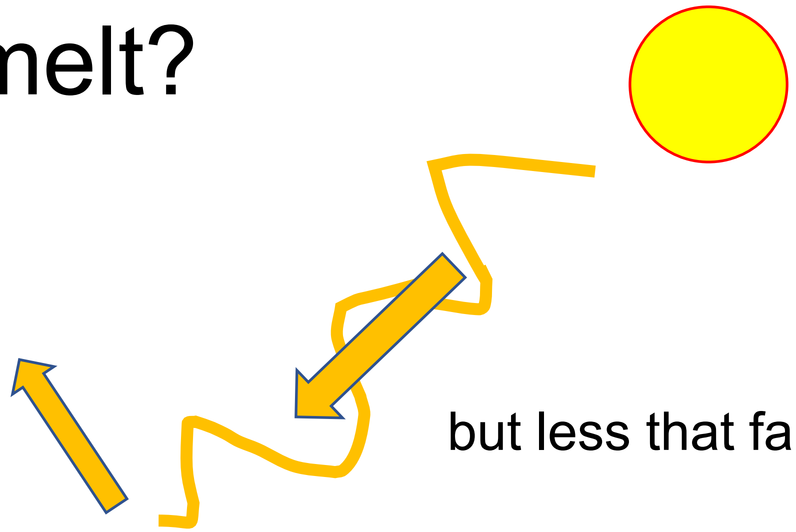
5 km

How fast does ice melt?



$$1360 \frac{J}{m^2 s}$$

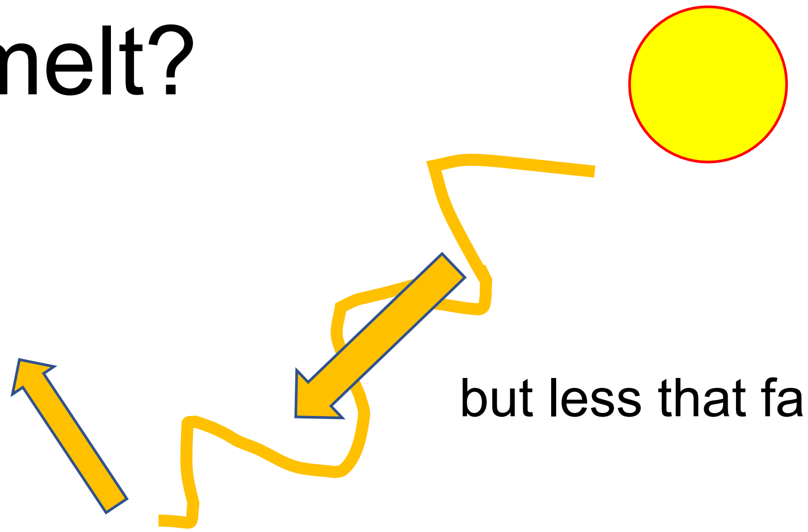
albedo $\alpha = 0.9$

but less that far north, say $1000 \frac{J}{m^2 s}$

glacier at 0 degC

Heat of fusion of ice $3.3 \times 10^5 \frac{J}{kg} \approx 3 \times 10^8 \frac{J}{m^3}$

How fast does ice melt?



$$1360 \frac{J}{m^2 s}$$

albedo $\alpha = 0.7$

but less that far north, say $1000 \dfrac{J}{m^2 s}$

rate of melting

$$\frac{(1-0.7)1000 \dfrac{J}{m^2 s}}{3 \times 10^8 \dfrac{J}{m^3}} = 1 \times 10^{-6} \frac{m}{s} = 9 \ \frac{\text{cm}}{\text{day}}$$

What do we want to know?

melting of glacier
flow velocity of glacier
shear stress on base
effect of temperature on flow
shape of glacier

What do we want to know?

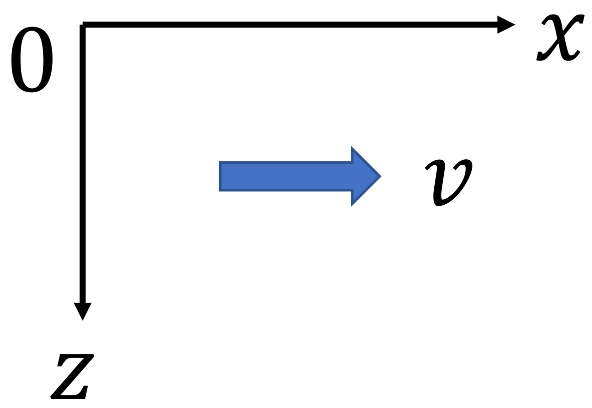
melting of glacier
flow velocity of glacier
shear stress on base
effect of temperature on flow
shape of glacier

today

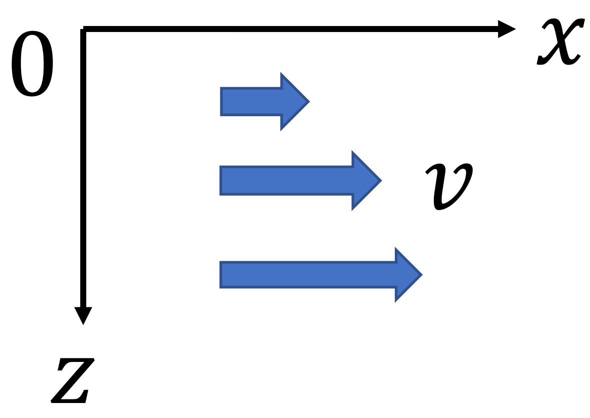
Part 1

flow of viscous fluid between plates

coordinate system



dynamical quantities



shear stress – strain rate law

$$\sigma = \underbrace{\mu}_{\text{viscosity}} \frac{dv}{dz}$$

Newton's Law

$$\frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

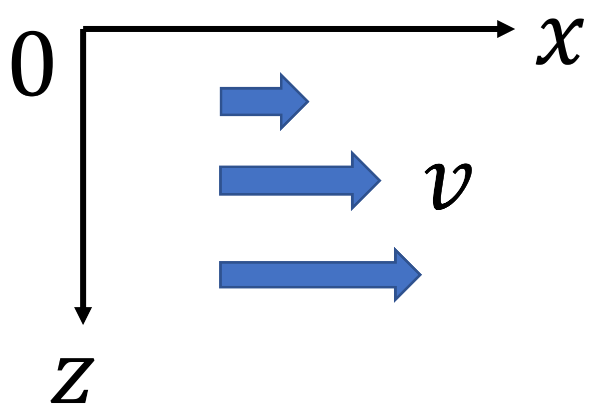
welded boundary

v same as object

free boundary

$$\sigma = 0$$

dynamical quantities



shear stress – strain rate law

$$\sigma = \mu \frac{dv}{dz}$$

Newton's Law

$$\mu \ \frac{d^2 v}{dz^2} + f = \rho \frac{dv}{dt}$$

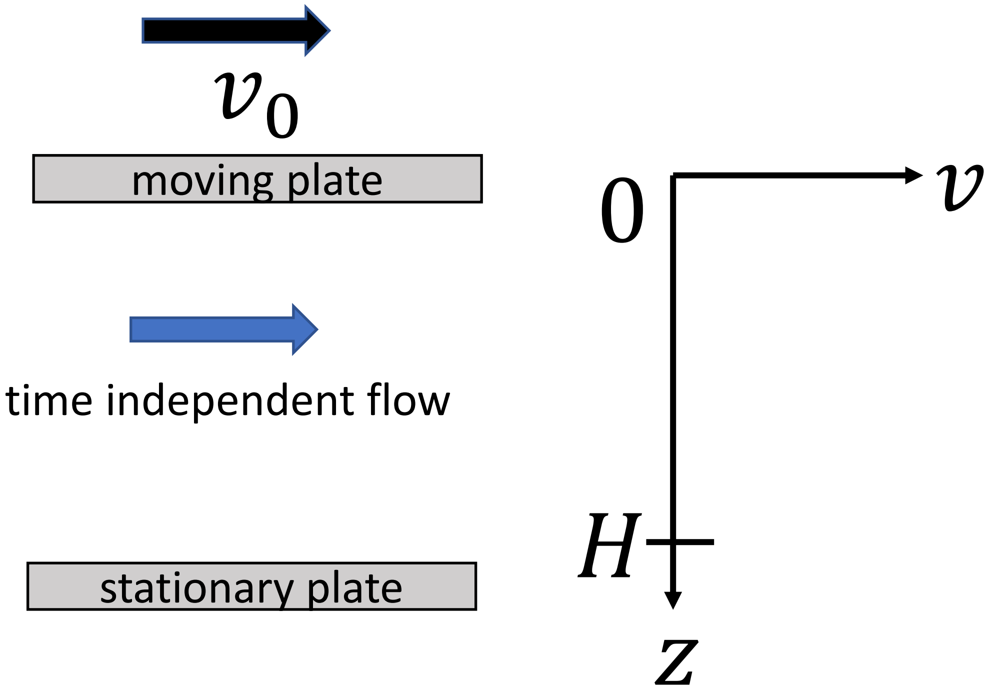
welded boundary

v same as object

free boundary

$$\sigma = 0$$

Newton's Law



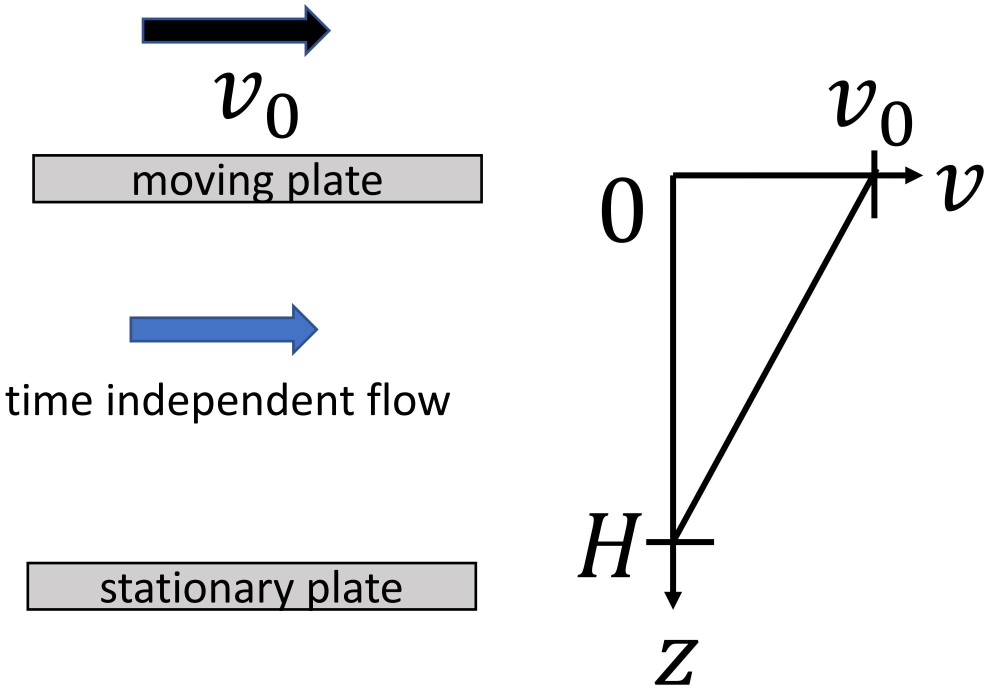
$$\mu \frac{d^2v}{dz^2} + \cancel{f} = \rho \cancel{\frac{dv}{dt}}$$

$$\frac{d^2v}{dz^2} = 0$$

$$v(z = 0) = v_0$$

$$v(z = H) = 0$$

Newton's Law



$$\mu \frac{d^2 v}{dz^2} + \cancel{f} = \rho \cancel{\frac{dv}{dt}}$$

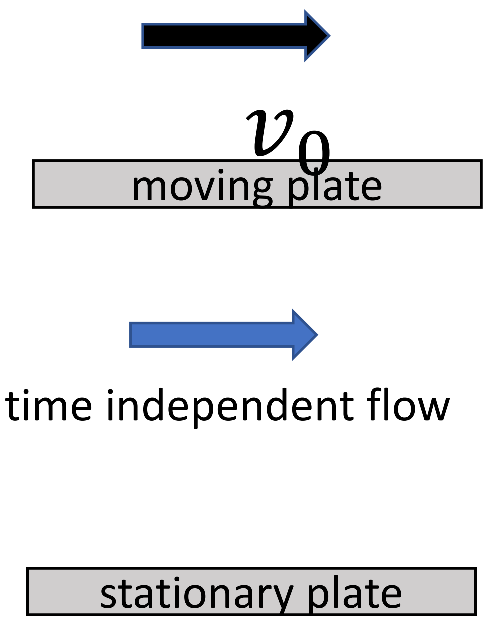
$$\frac{d^2 v}{dz^2} = 0$$

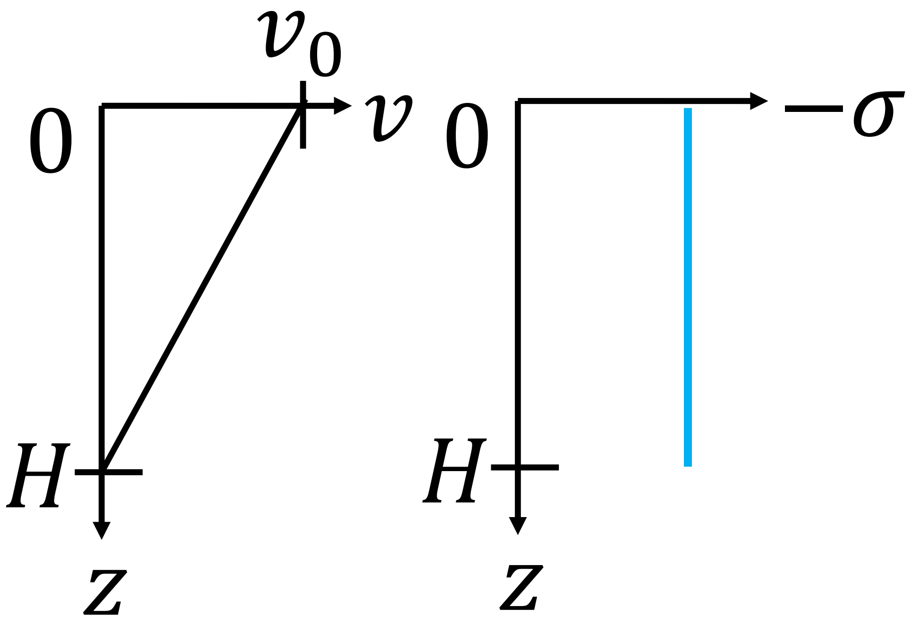
$$v(z = 0) = v_0$$

$$v(z = H) = 0$$

$$\frac{d^2 v}{dz^2} = 0$$ implies $v(z)$ linear

$$v(z) = \frac{v_0}{H}(H - z)$$





$$v(z) = \frac{v_0}{H}(H - z)$$

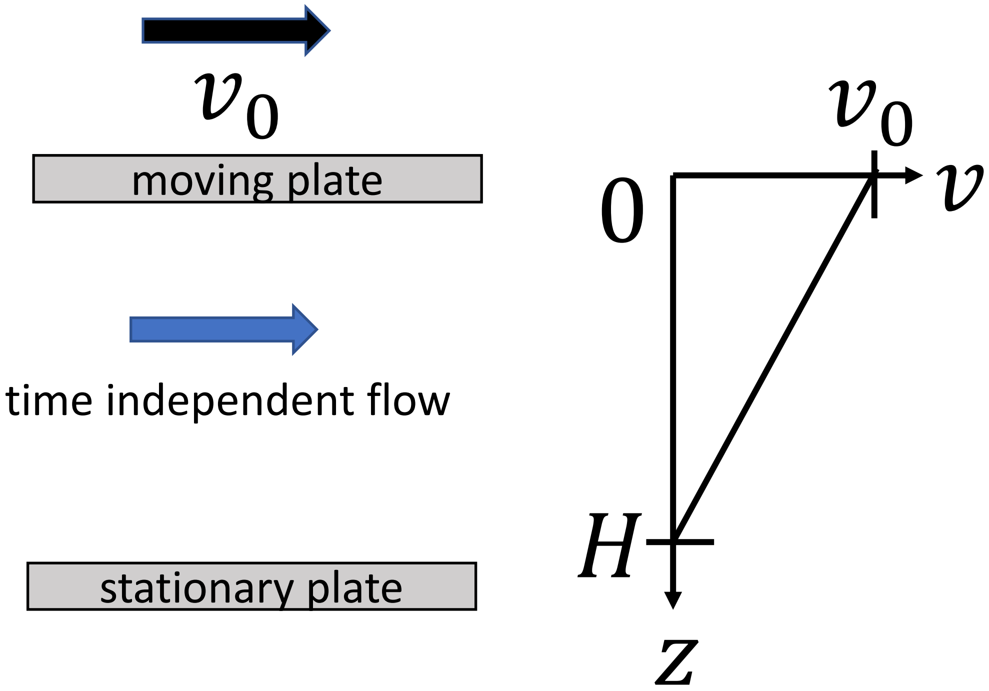
Lessons

linear velocity profile

constant shear stress

$$\sigma = \mu \frac{dv}{dz}$$

$$= -\frac{\mu v_0}{H}$$



$$v(z) = \frac{v_0}{H}(H - z)$$

Lessons

linear velocity profile

constant shear stress

$$\sigma = \mu \frac{dv}{dz} = -\frac{\mu v_0}{H}$$

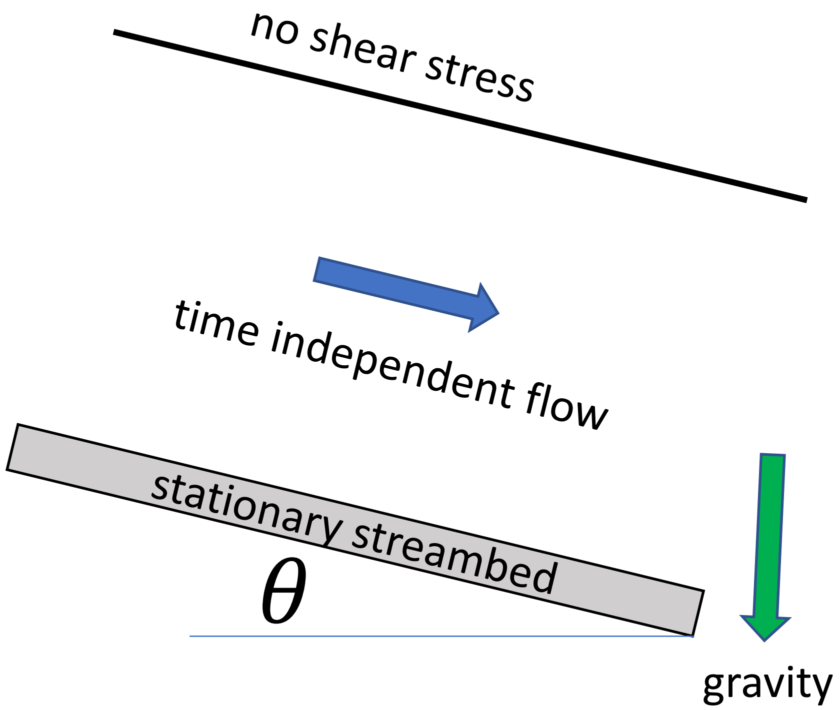
lubrication

- to reduce stress on plates
  - use inviscid oil
  - make oil layer thick

Part 2

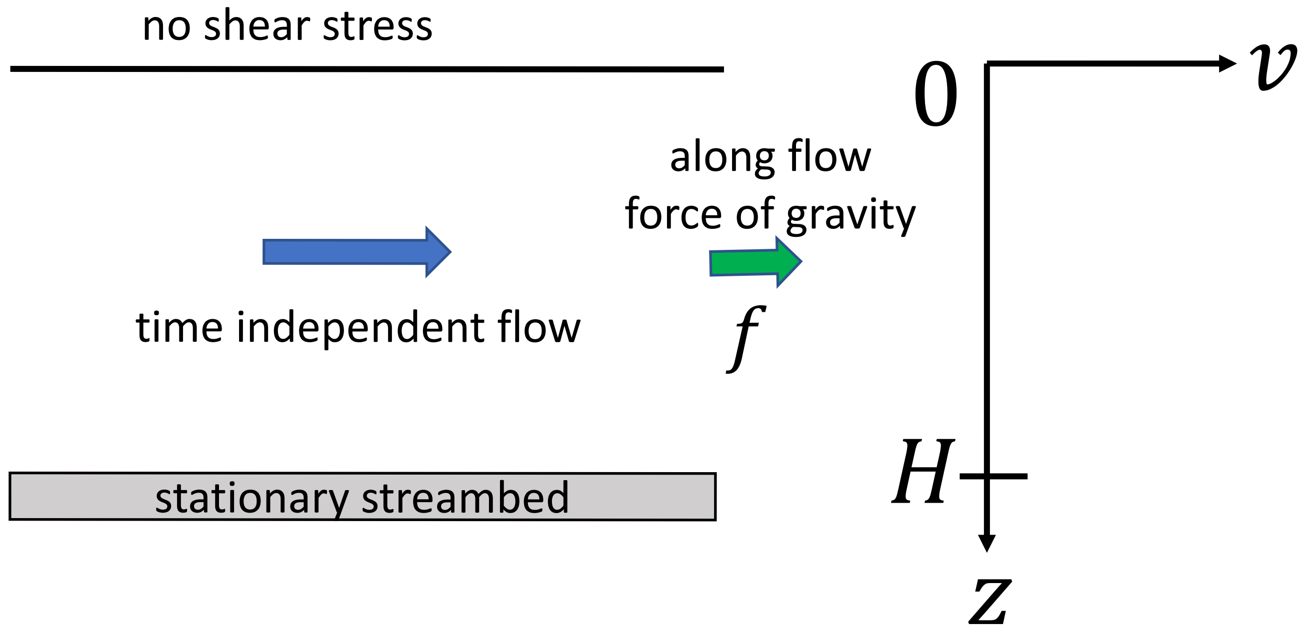
flow of viscous fluid in a wide stream

stream



no shear stress
time independent flow
stationary streambed
θ
gravity

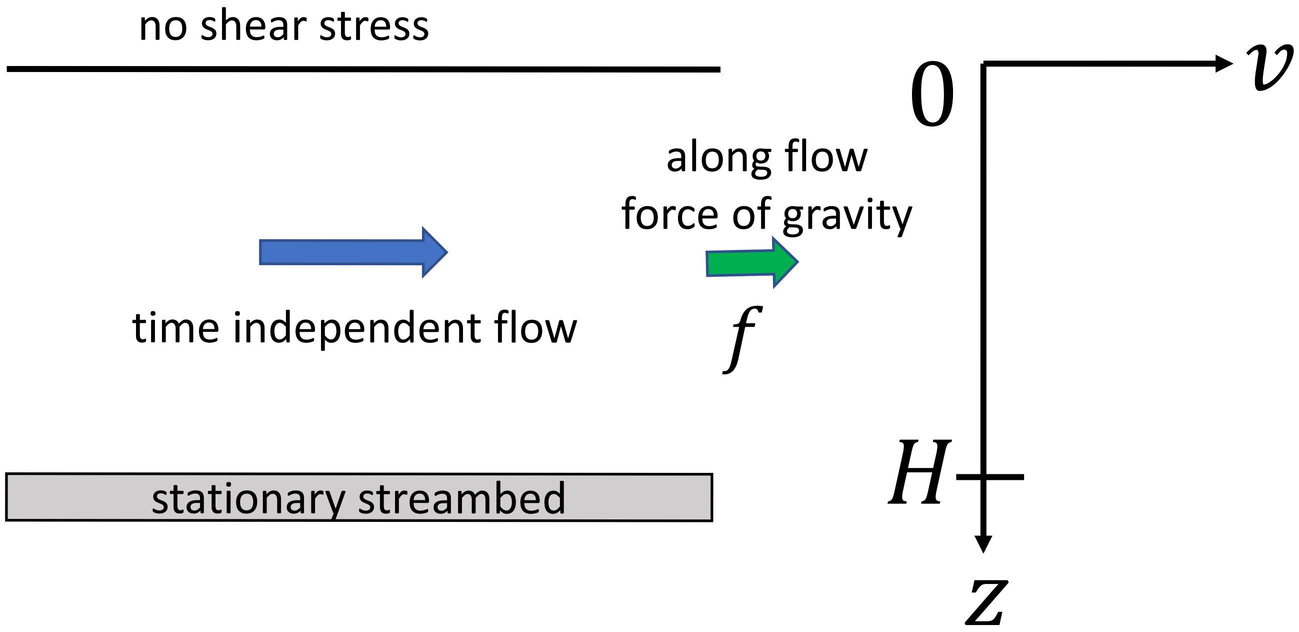
stream



$$f = \sigma g \sin \theta$$

stream

Newton’s Law



$$\mu \frac{d^2 v}{dz^2} + f = \rho \cancel{\frac{dv}{dt}}$$

$$\frac{d^2 v}{dz^2} = -\frac{f}{\mu} = -B$$

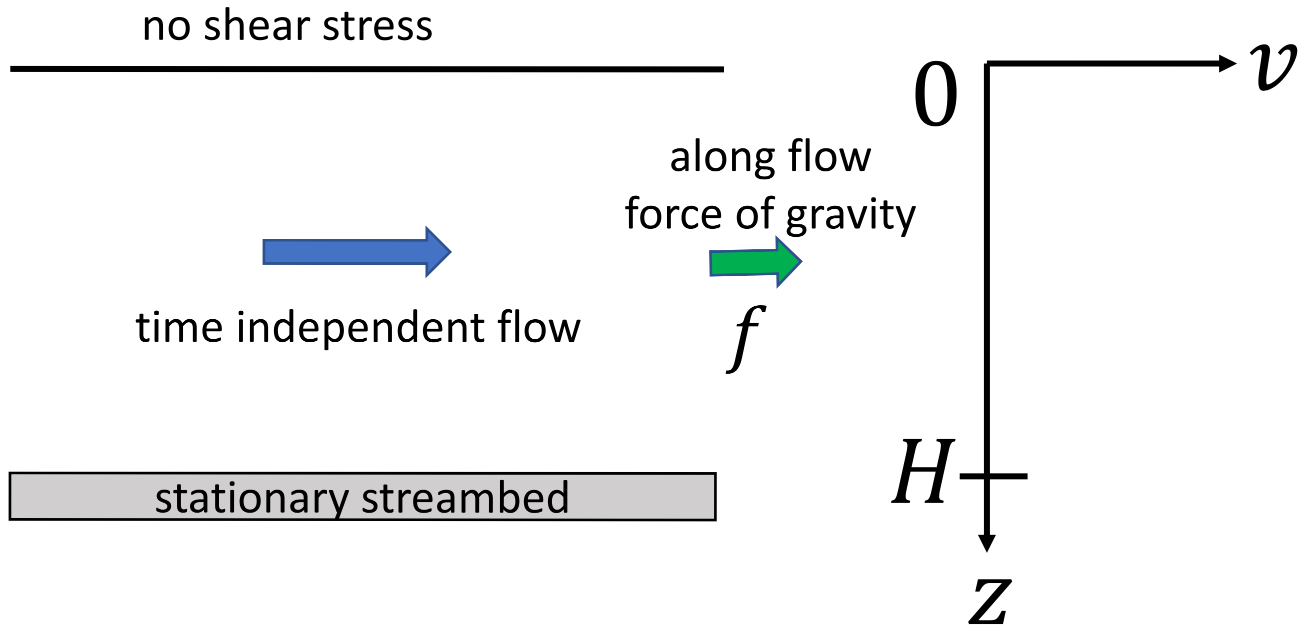
$$\frac{dv}{dz}(z = 0) = 0$$

$$f = \rho g \sin\theta$$

$$v(z = H) = 0$$

stream

Newton's Law



$$\mu \frac{d^2 v}{dz^2} + f = \rho \cancel{\frac{dv}{dt}}$$

$$\frac{d^2 v}{dz^2} = -\frac{f}{\mu} = -B$$

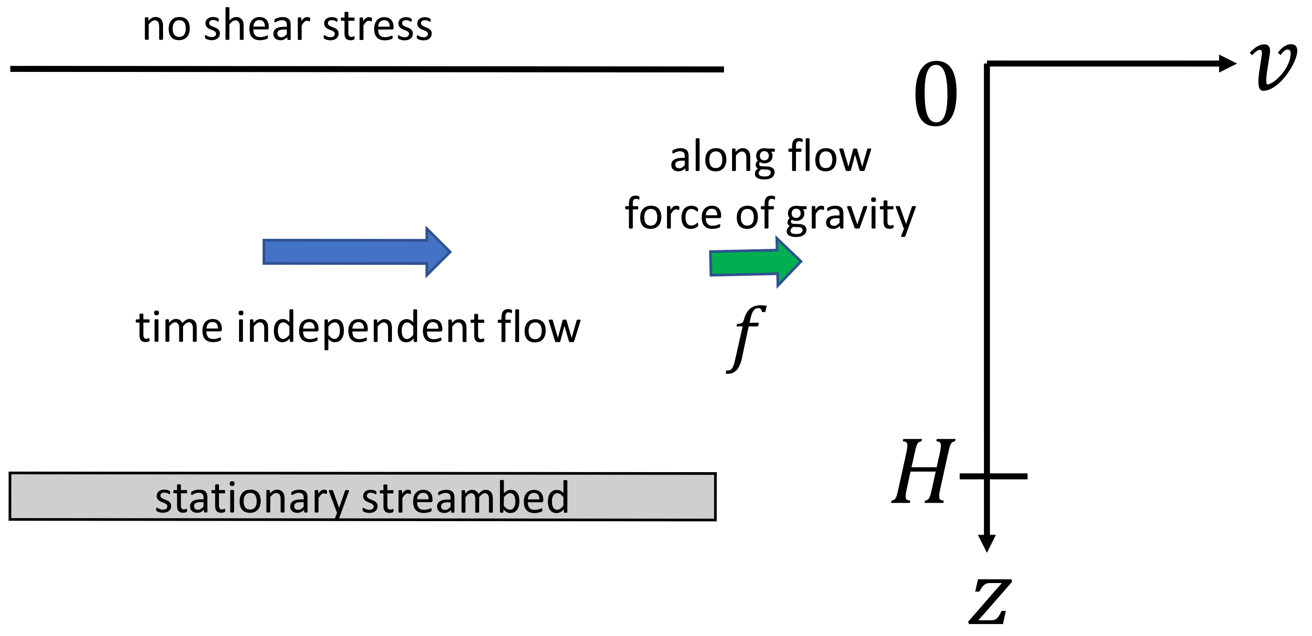
$$\frac{dv}{dz}(z = 0) = 0$$

$$v(z = H) = 0$$

$\dfrac{d^2 v}{dz^2} = -B$ implies $v(z)$ quadratic

stream

Newton's Law



$$\frac{d^2 v}{dz^2} = -B$$

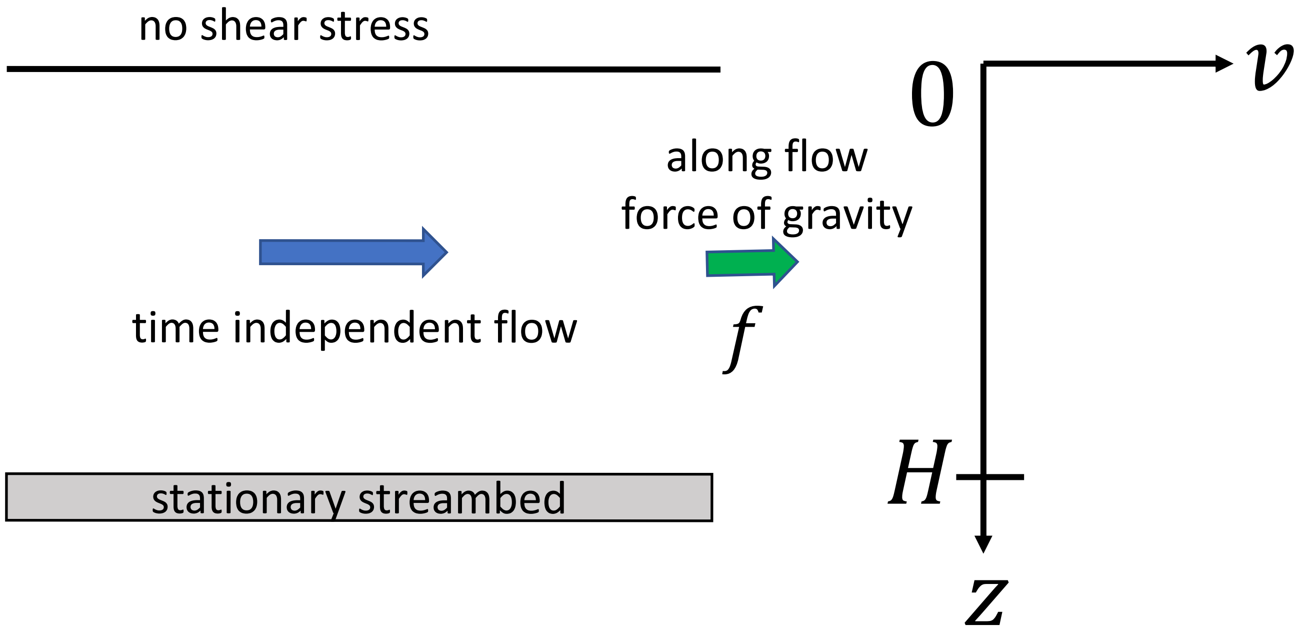
$$\frac{dv}{dz}(z = 0) = 0$$

$$v(z = H) = 0$$

$$v(z) = c_0 + c_1(H - z) + c_2(H - z)^2$$

stream

Newton's Law



$$\frac{d^2 v}{dz^2} = -B$$

$$\frac{dv}{dz}(z = 0) = 0$$

$$v(z = H) = 0$$

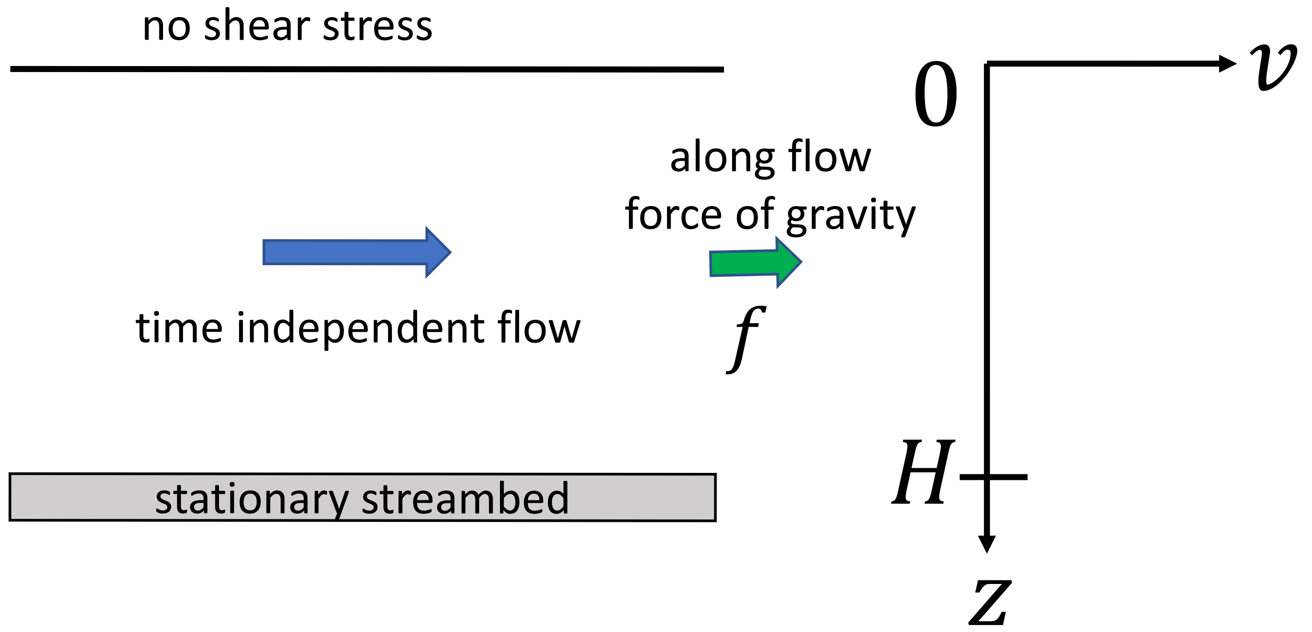
$$v(z) = c_0 + c_1(H - z) + c_2(H - z)^2$$

$$dv/dz\ = -c_1 - 2c_2(H - z)$$

$$d^2v/dz^2\ = 2c_2$$

stream

Newton's Law



A $\dfrac{d^2v}{dz^2} = -B$

B $\dfrac{dv}{dz}(z = 0) = 0$

C $v(z = H) = 0$

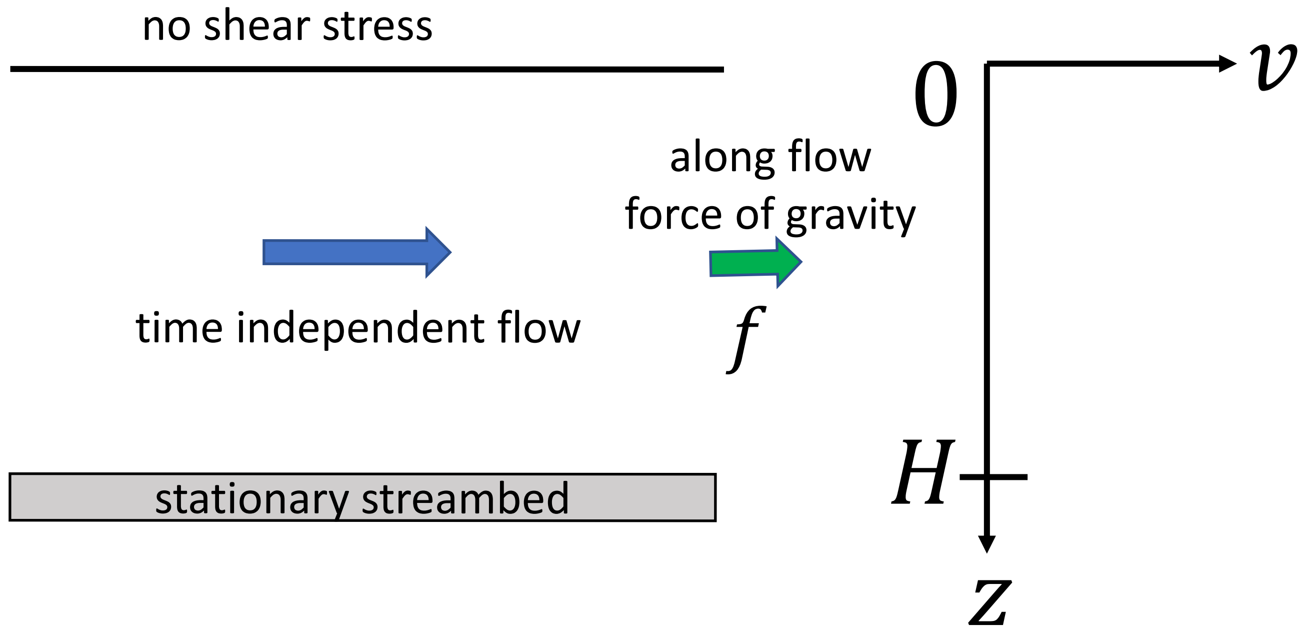
$$v(z) = c_0 + c_1(H - z) + c_2(H - z)^2 \xrightarrow{\text{C implies}} c_0 = 0$$

$$dv/dz = -c_1 - 2c_2(H - z)$$

$$d^2v/dz^2 = 2c_2$$

stream

Newton's Law



A $$\frac{d^2v}{dz^2} = -B$$

B $$\frac{dv}{dz}(z=0) = 0$$

C $$v(z=H) = 0$$

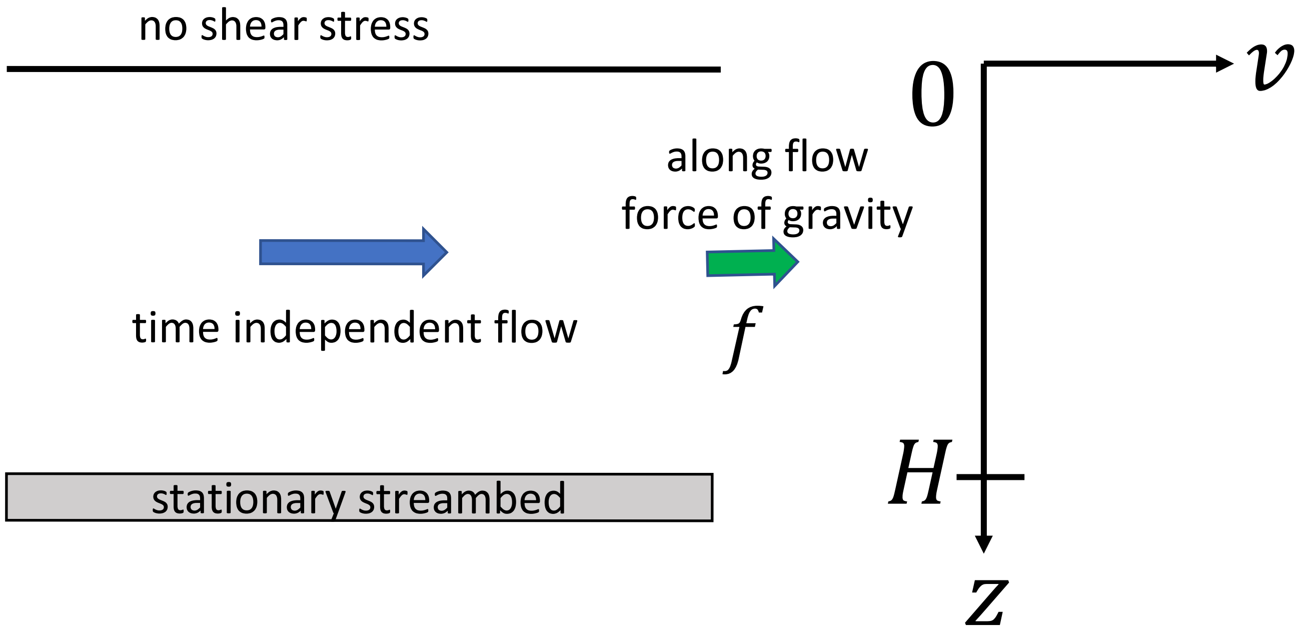
$$v(z) = c_0 + c_1(H-z) + c_2(H-z)^2$$ C implies $$c_0 = 0$$

$$dv/dz \ = -c_1 - 2c_2(H-z)$$

$$d^2v/dz^2 \ = 2c_2$$ A implies $$c_2 = -½B$$

stream

Newton's Law



A $\dfrac{d^2v}{dz^2} = -B$

B $\dfrac{dv}{dz}(z=0) = 0$

C $v(z=H) = 0$

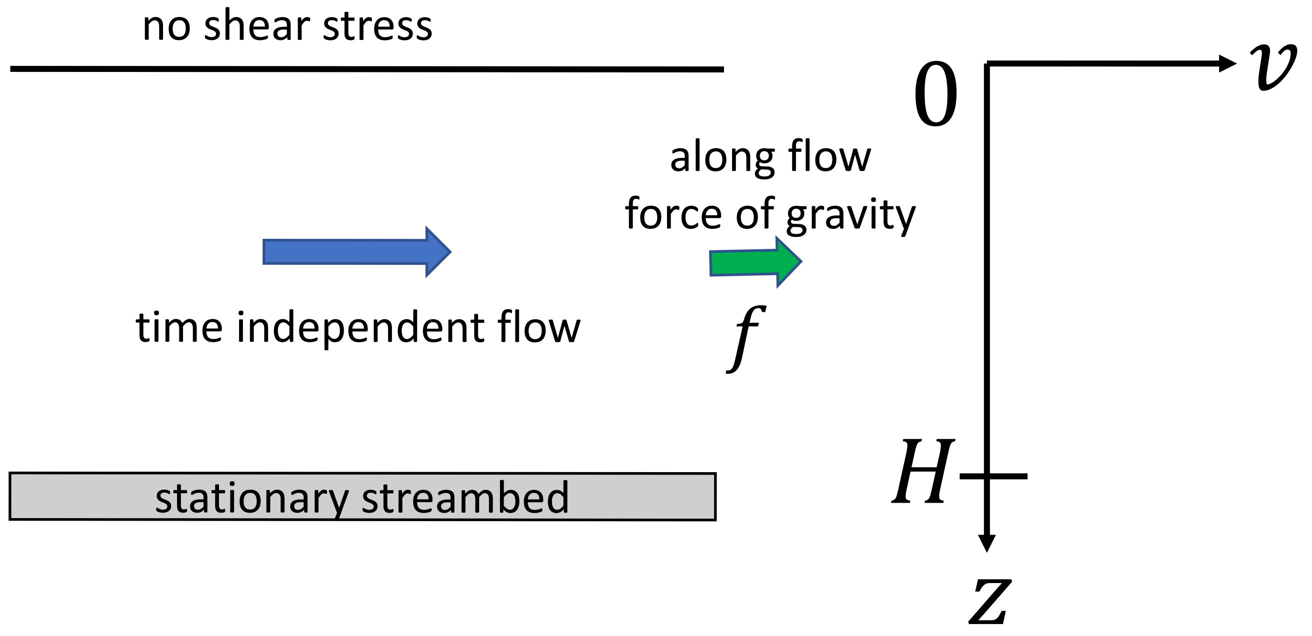
$$v(z) = c_0 + c_1(H-z) + c_2(H-z)^2 \xrightarrow{\text{C implies}} c_0 = 0$$

$$dv/dz = -c_1 - 2c_2(H-z) \xrightarrow{\text{B implies}} c_1 = -2c_2H = BH$$

$$d^2v/dz^2 = 2c_2 \xrightarrow{\text{A implies}} c_2 = -½B$$

stream

Newton's Law



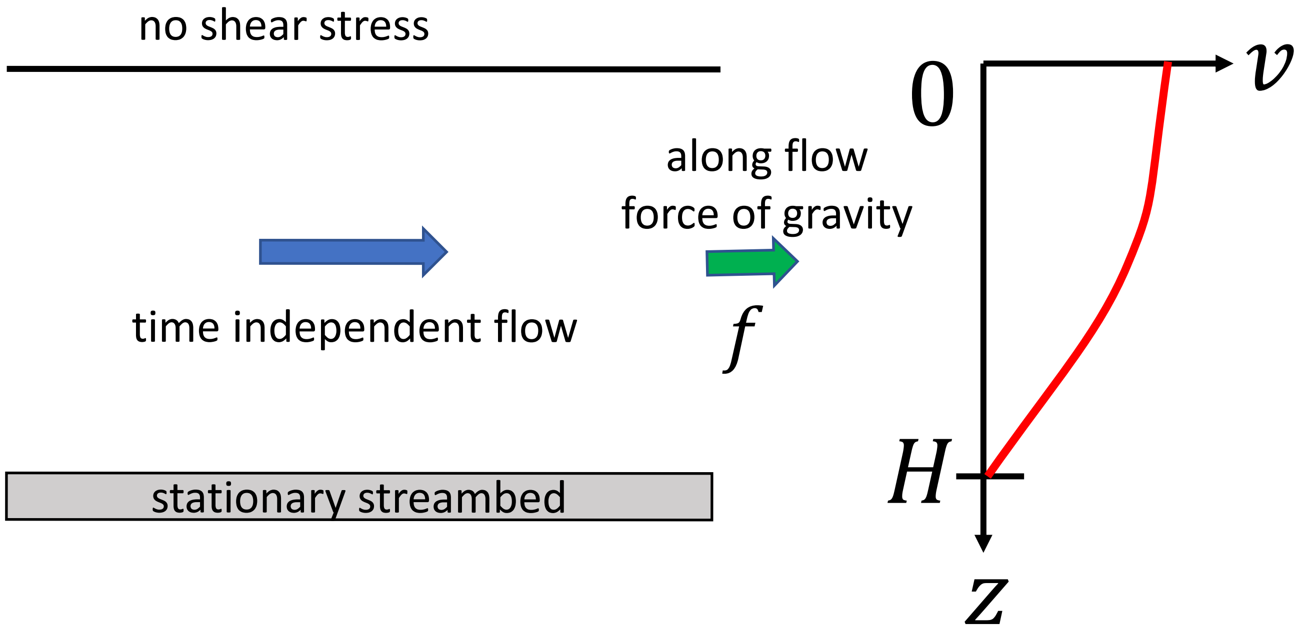
A $\quad \dfrac{d^2v}{dz^2} = -B$

B $\quad \dfrac{dv}{dz}(z=0) = 0$

C $\quad v(z=H) = 0$

$$v(z) = BH(H-z) - \tfrac{1}{2}B(H-z)^2$$

stream



maximum velocity

$$v_0 = v(0) = ½BH^2$$

maximum shear stress

$$\sigma_H = \sigma(H) = -\mu BH$$
$$= -2\frac{\mu v_0}{H}$$

$$v(z) = BH(H - z) - ½B(H - z)^2$$

$$\sigma(z) = -\mu BH + \mu B(H - z)$$

$$\sigma_H = \sigma(H) = -2\frac{\mu v_0}{H}$$

$$H = 1000 \text{ m}$$

$v_0 = 1 \times 10^{-5} \text{ m/s}$ (~ one meter per day)

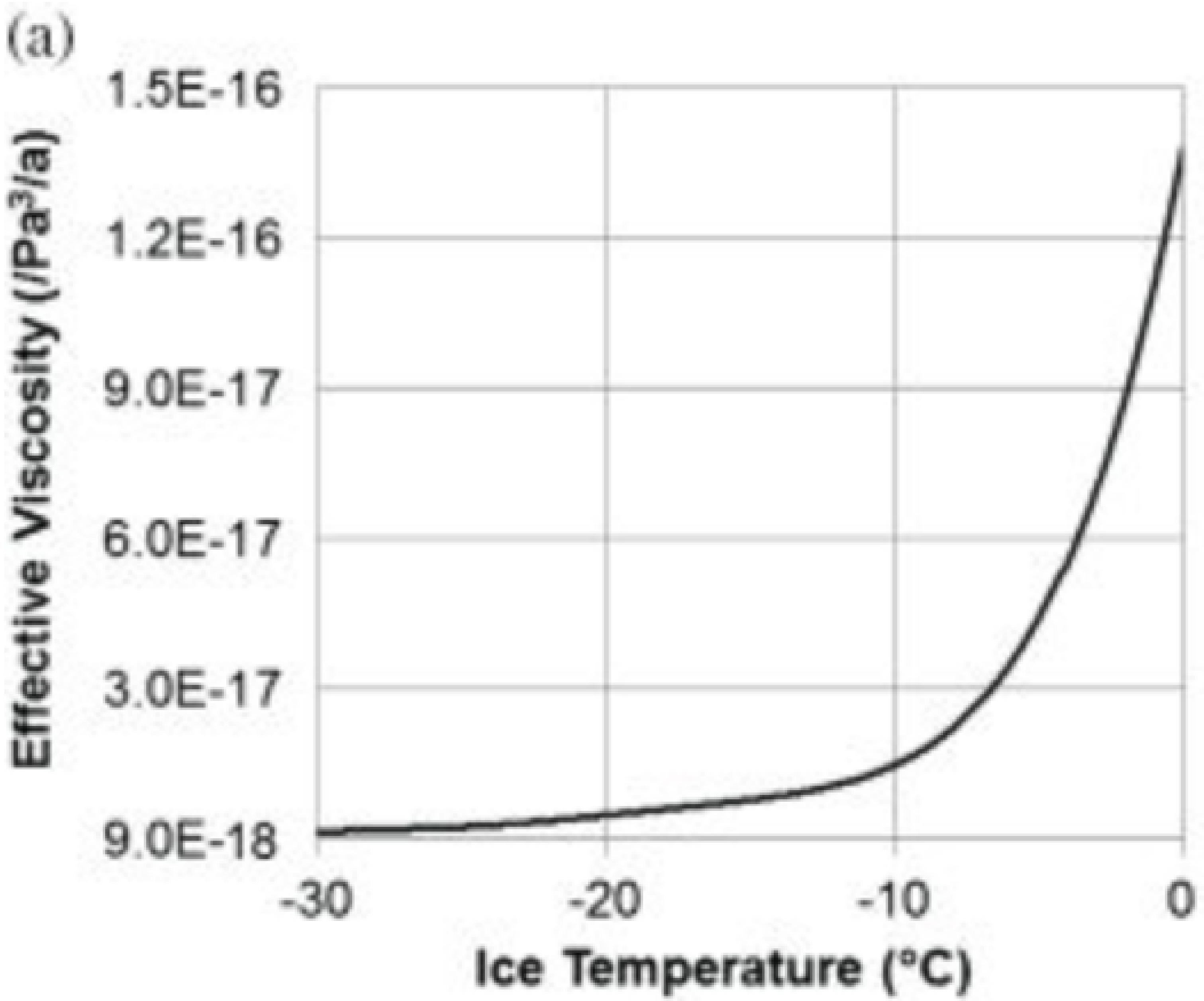
$$\mu = 10^{12} \text{ Pa} - \text{s}$$

$$\sigma_H = -2\frac{\mu v_0}{H} = -2 \times 10^4 \text{ Pa} = -20 \text{ kPa}$$

similar to the strength of sands and gravels

Part 3
Glaciers
hotter at the bottom

effect of variable (temperature dependent) viscosity


(a)
1.5E-16
1.2E-16
9.0E-17
6.0E-17
3.0E-17
9.0E-18
Effective Viscosity (/Pa³/a)
-30
-20
-10
0
Ice Temperature (°C)

Newton's Law

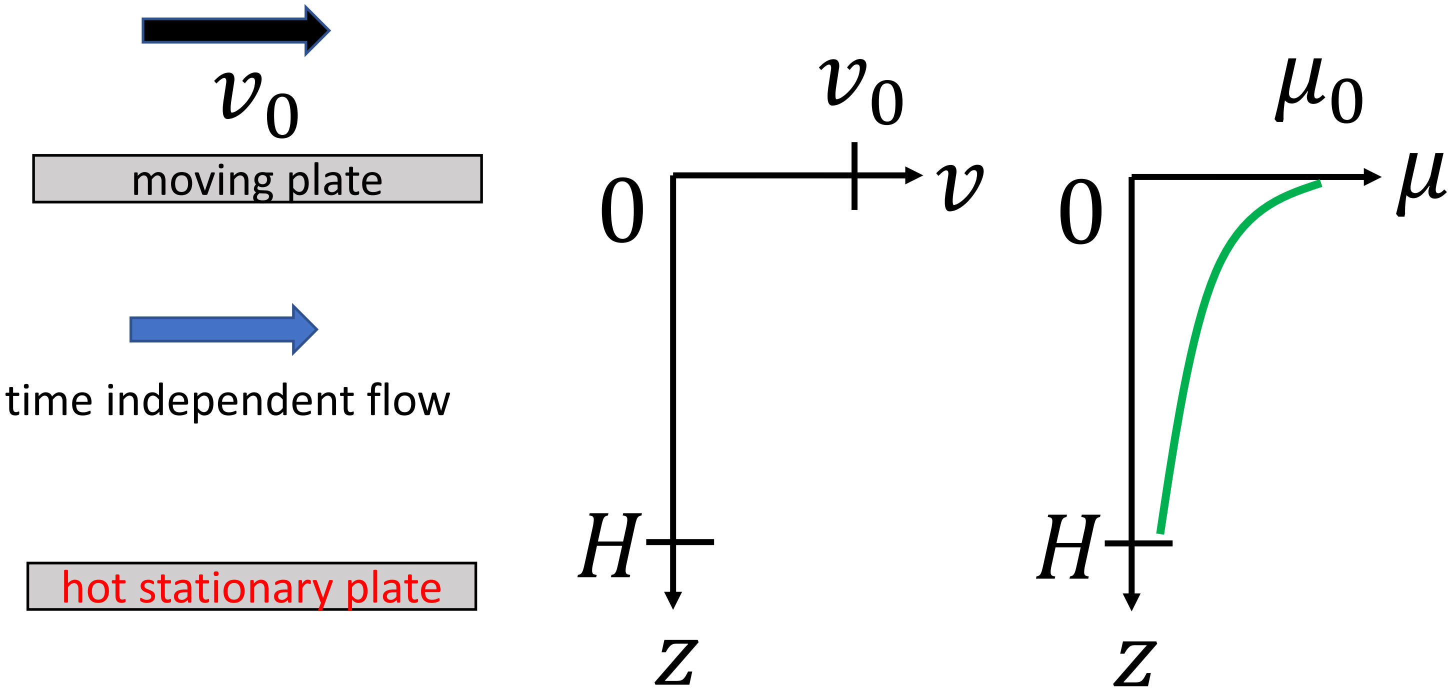
$$\frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

Viscous Flow Law

$$\sigma = \mu(z) \frac{dv}{dz}$$

so using chain rule

$$\mu \frac{d^2 v}{dz^2} + \frac{d\mu}{dz}\frac{dv}{dz} + f = \rho \frac{dv}{dt}$$



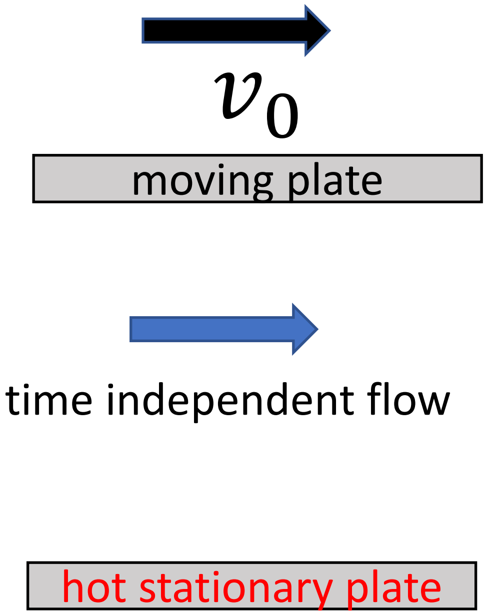
$$v(z=0)=v_0$$

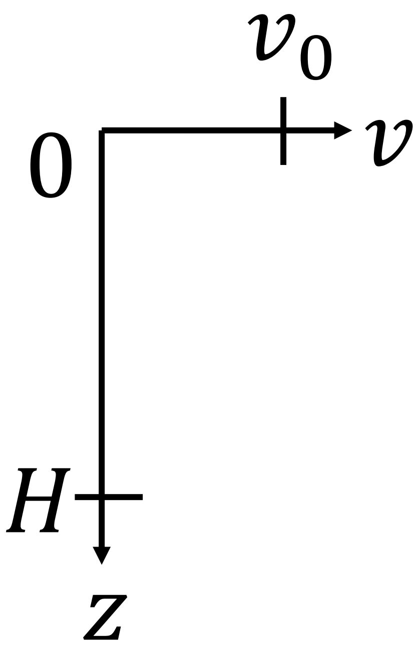
$$v(z=H)=0$$

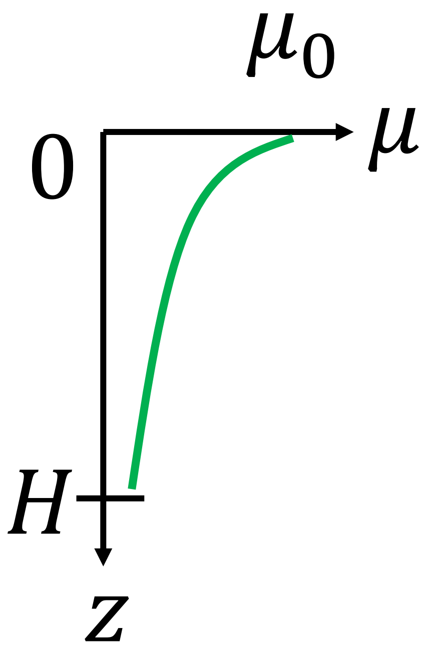
$$\mu=\mu_0\exp(-cz)$$

Newton's Law

$$\mu\frac{d^2v}{dz^2}+\frac{d\mu}{dz}\frac{dv}{dz}+\cancel{f}=\rho\cancel{\frac{dv}{dt}}$$







$$v(z=0)=v_0$$

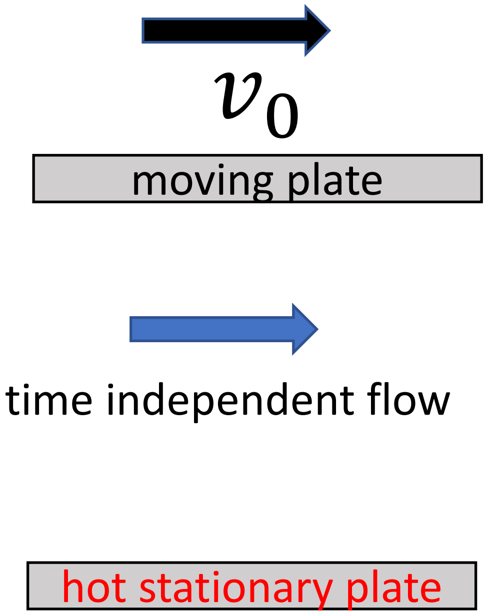
$$v(z=H)=0$$

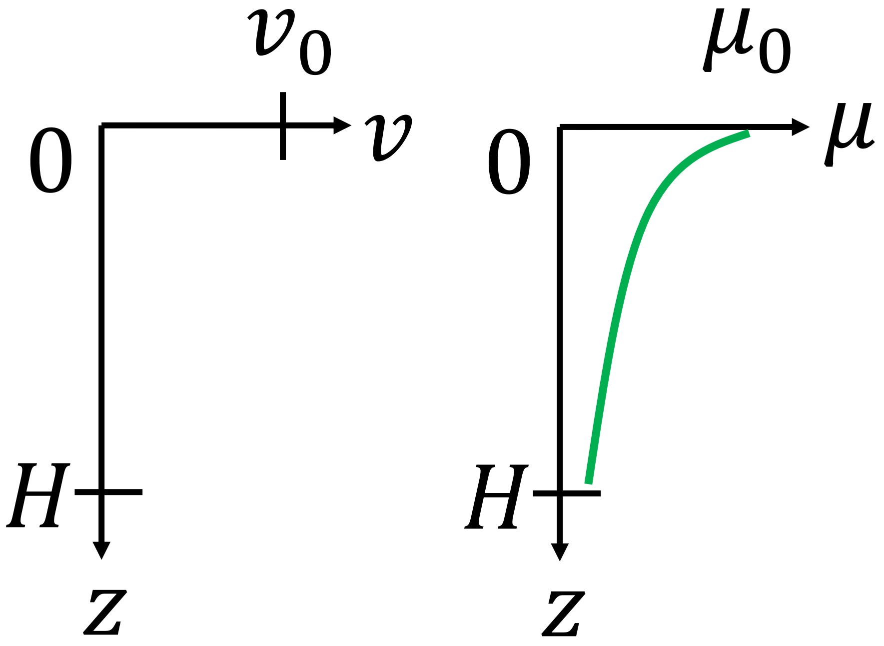
$$\mu=\mu_0\exp(-cz)$$

$$\frac{d\mu}{dz}=-c\mu_0\exp(-cz)$$

Newton's Law

$$\mu\frac{d^2v}{dz^2}+\frac{d\mu}{dz}\frac{dv}{dz}=0 \qquad \mu_0\exp(-cz)\frac{d^2v}{dz^2}-c\mu_0\exp(-cz)\frac{dv}{dz}=0$$





$$v(z=0) = v_0$$

$$v(z=H) = 0$$

$$\mu = \mu_0 \exp(-cz)$$

$$\frac{d\mu}{dz} = -c\mu_0 \exp(-cz)$$

Newton's Law

$$\mu \frac{d^2 v}{dz^2} + \frac{d\mu}{dz}\frac{dv}{dz} = 0$$

$$\frac{d^2 v}{dz^2} - c\frac{dv}{dz} = 0$$

$$\frac{d^2 v}{dz^2} - c\frac{dv}{dz} = 0$$

let $Z = \frac{dv}{dz}$

$$\frac{dZ}{dz} - cZ = 0 \quad \text{so} \quad \frac{dZ}{dz} = cZ \quad \text{so} \quad Z = Z_0 \exp(cz)$$

$$Z = \frac{dv}{dz} \quad \text{so} \quad v = \frac{Z_0}{c}\exp(cz) - C$$

$$v = \frac{Z_0}{c}\exp(cz) - C$$

A $v(z = 0) = v_0$

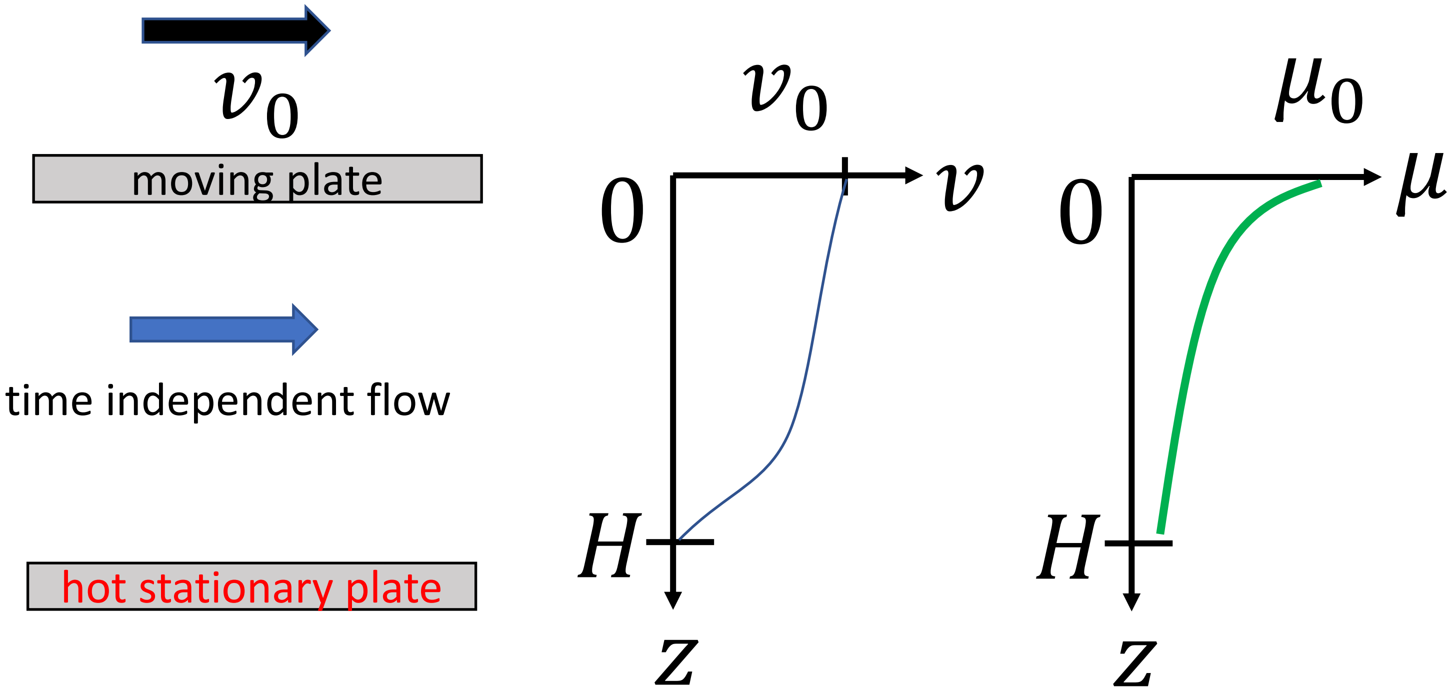
B $v(z = H) = 0$

B $v = \frac{Z_0}{c}\exp(cH) - C = 0$ so $C = \frac{Z_0}{c}\exp(cH)$

A $v = \frac{Z_0}{c}[\exp(0) - \exp(cH)] = v_0$

so $Z_0 = \frac{cv_0}{[1 - \exp(cH)]}$

$$v = v_0\frac{[\exp(cz) - \exp(cH)]}{[1 - \exp(cH)]}$$



$$v = v_0 \frac{[\exp(cz) - \exp(cH)]}{[1 - \exp(cH)]}$$

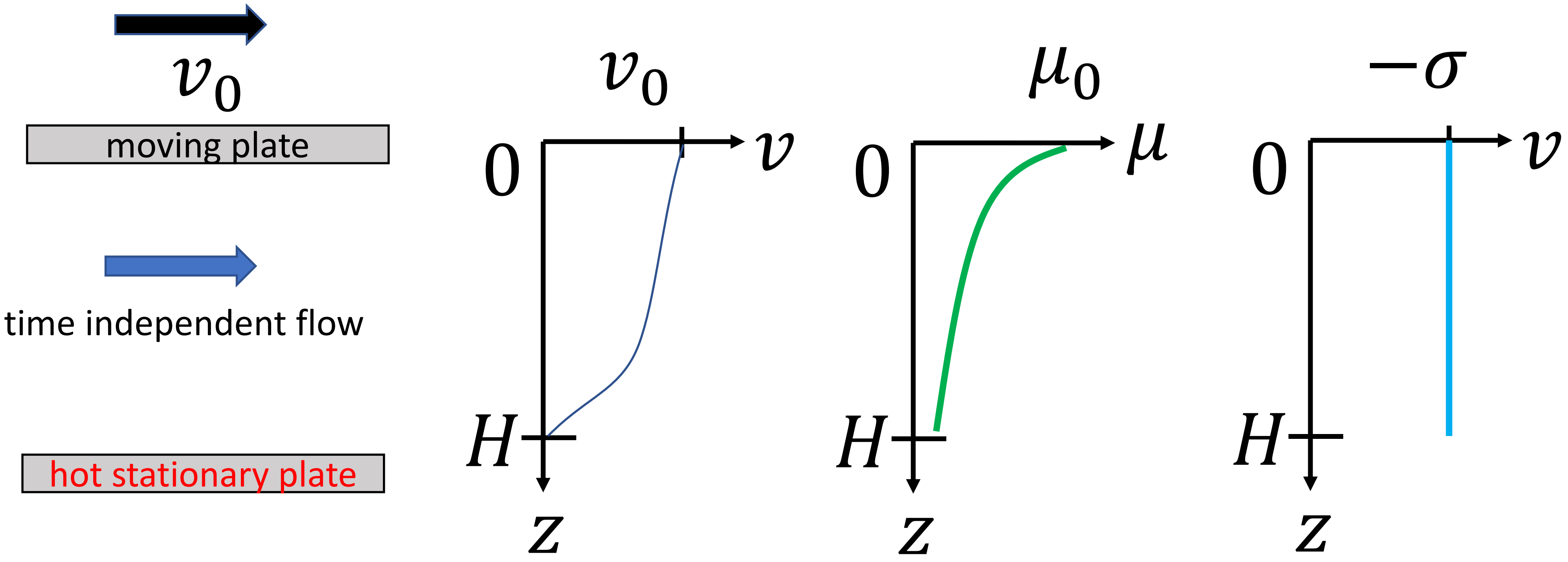
What about the shear stress

Newton's Law

$$\frac{d\sigma}{dz} + \cancel{f} = \rho \cancel{\frac{dv}{dt}}$$

so $\frac{d\sigma}{dz} = 0$

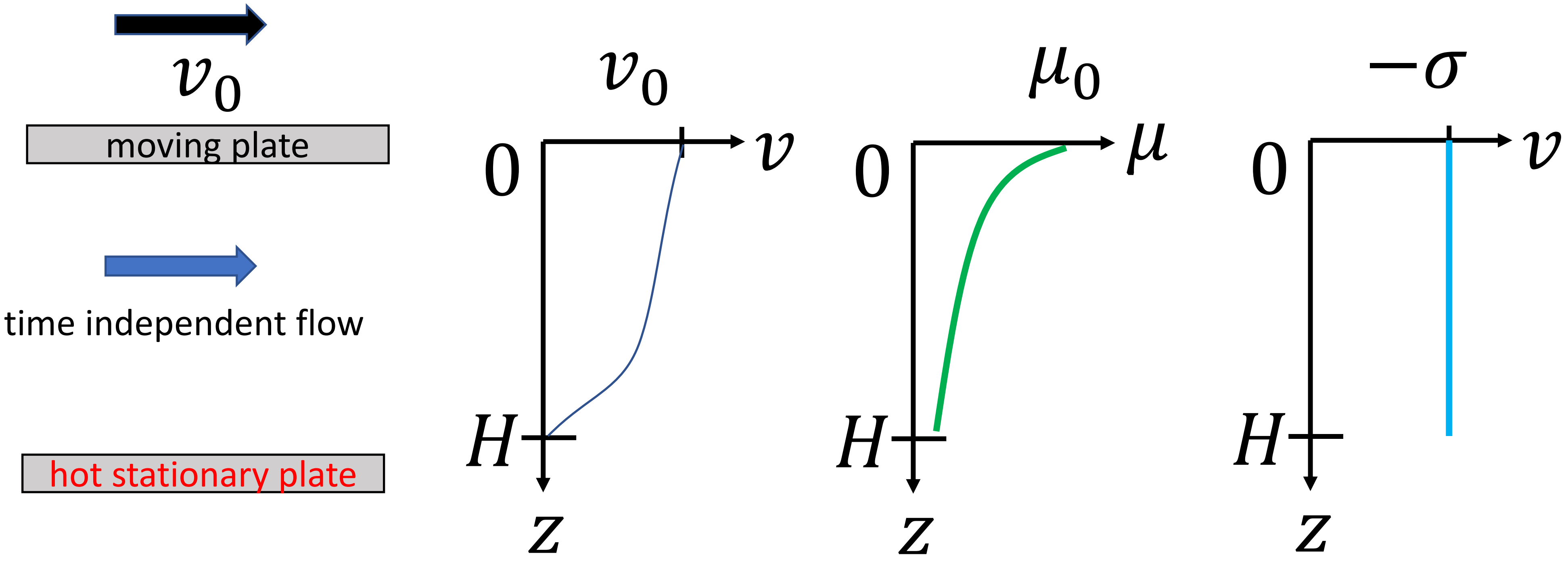
and $\sigma = \text{constant}$



$$v = v_0 \frac{[\exp(cz) - \exp(cH)]}{[1 - \exp(cH)]}$$

$$\sigma = \mu \frac{dv}{dx} = \frac{\mu_0 c v_0}{[1 - \exp(cH)]}$$

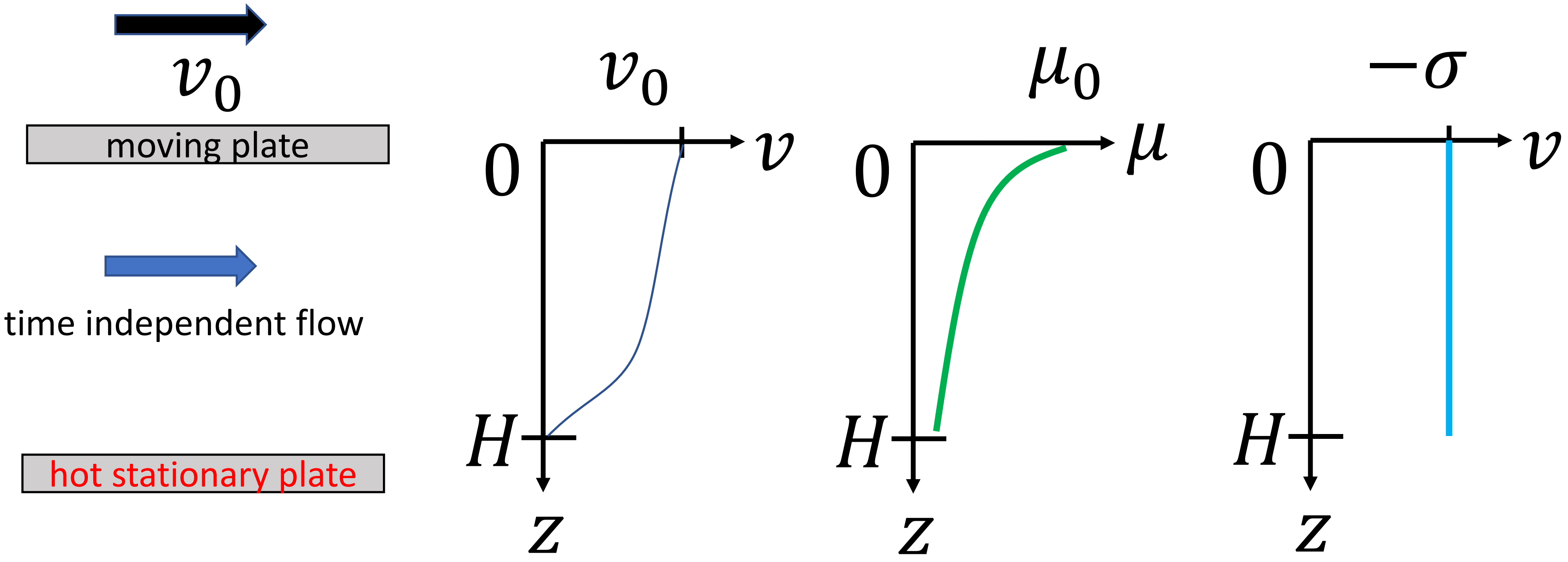
$$\frac{dv}{dx} = v_0 \frac{c \exp(cz)}{[1 - \exp(cH)]}$$



small c

$$\sigma = \mu \frac{dv}{dx} = \frac{\mu_0 c v_0}{[1 - (1 + cH + ½c^2H^2)]} = \frac{\mu_0 v_0}{H(1 + ½c^2H)}$$

stress less than constant viscosity case



Lessons

Top part has more uniform velocity

Stress went down (compared to uniform viscosity)