Solid Earth Dynamics

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Lecture 25

continuing with

Glacial Dynamics

Part 1

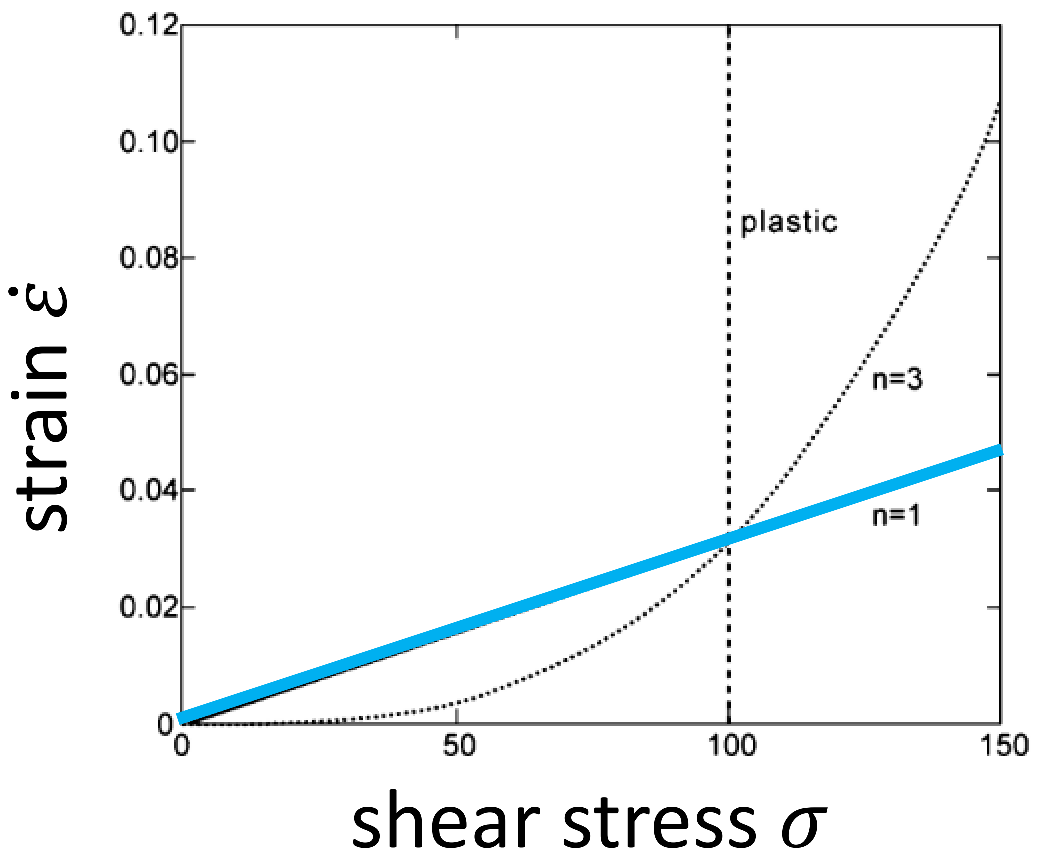
rheology of ice

viscous fluid

stress proportional to strain rate

not a particularly good model
of deformable solids like ice

viscous fluid
linear relationship

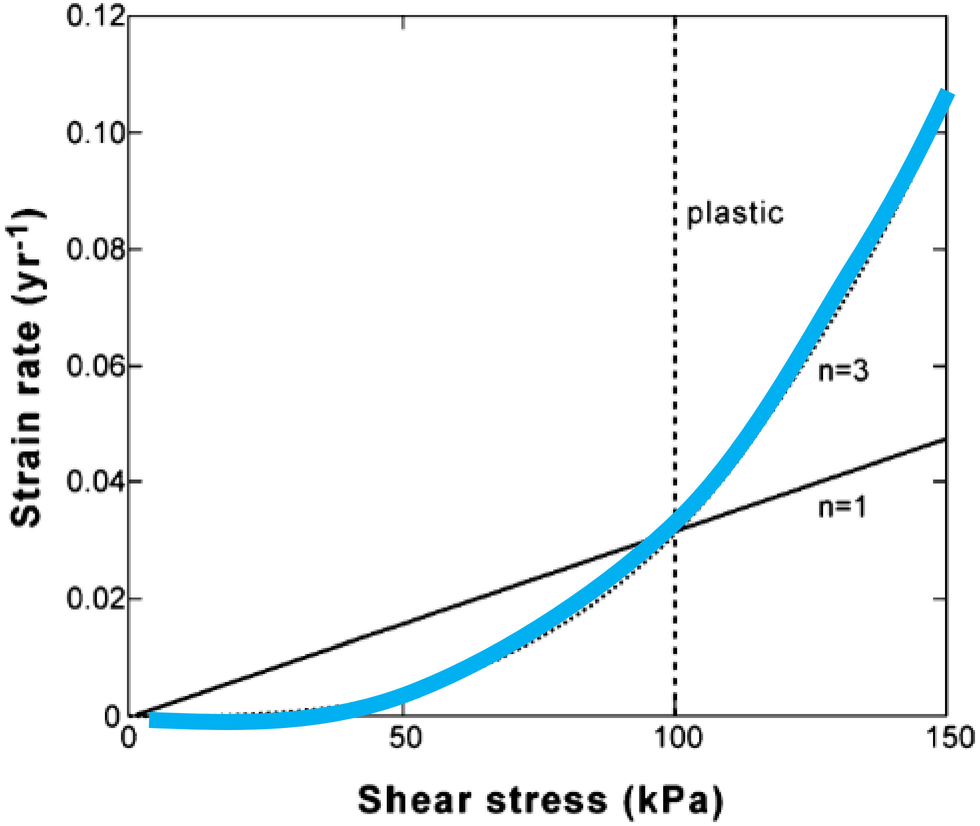


Glen's law is the most commonly used flow law for ice in glaciers and ice sheets.

$$\dot{\varepsilon} = A\tau^n$$

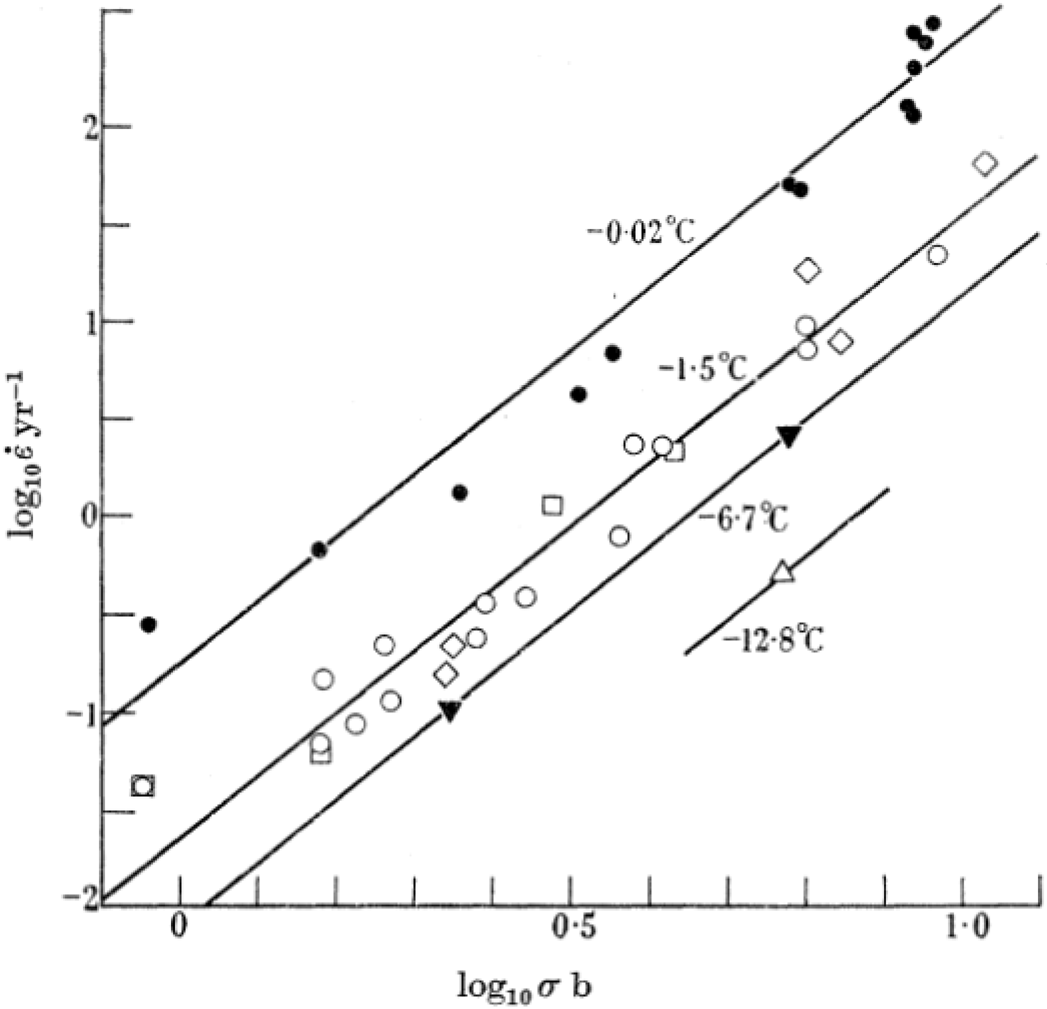
Usually $n \approx 3$ and $A \approx 2.4 \times 10^{-24}\ \mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$ at $0°\ \mathrm{C}$

But the most appropriate values in reality may depend on temperature, stress regime, grain size, etc



Glen's law

Strain rate

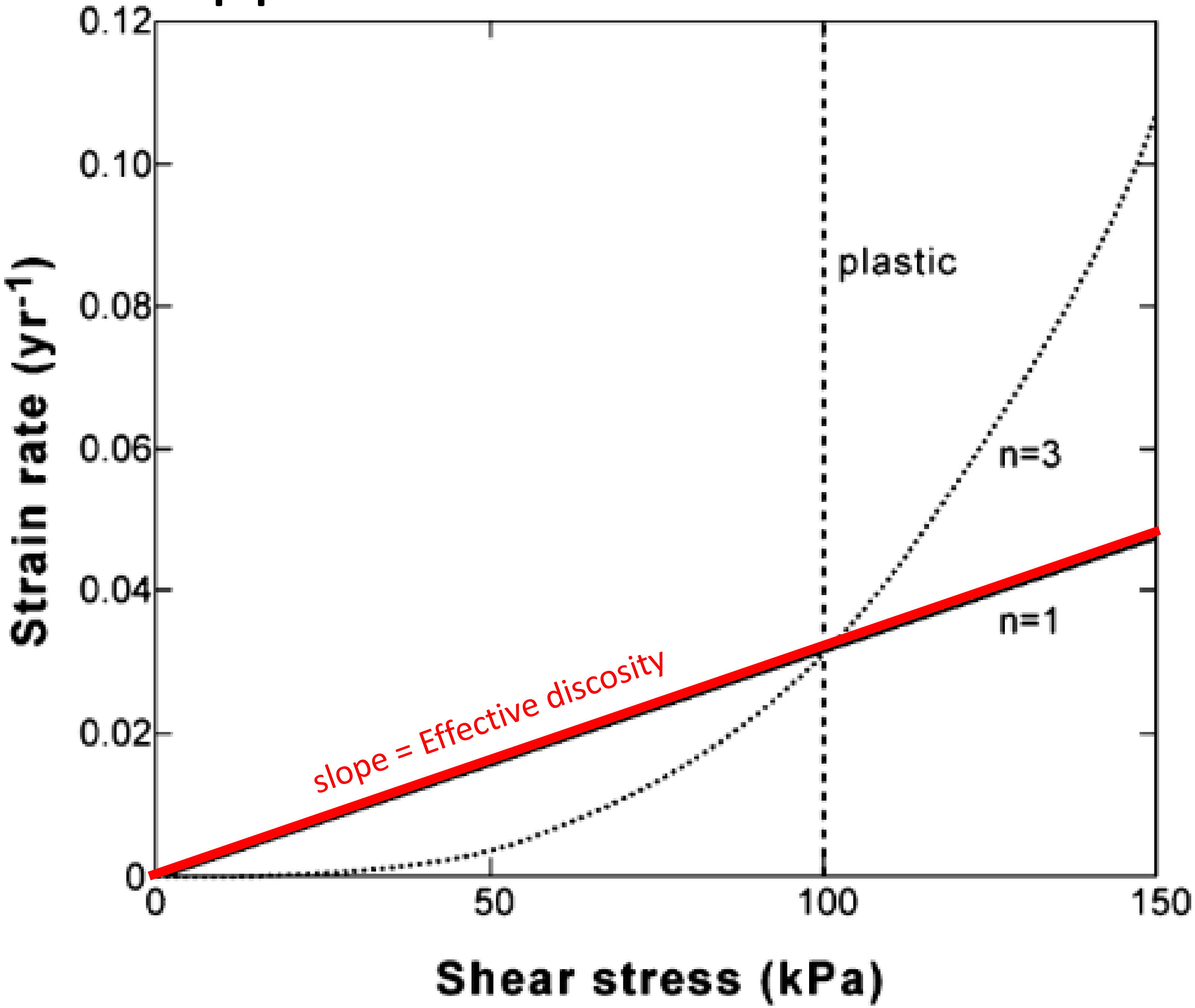


Stress

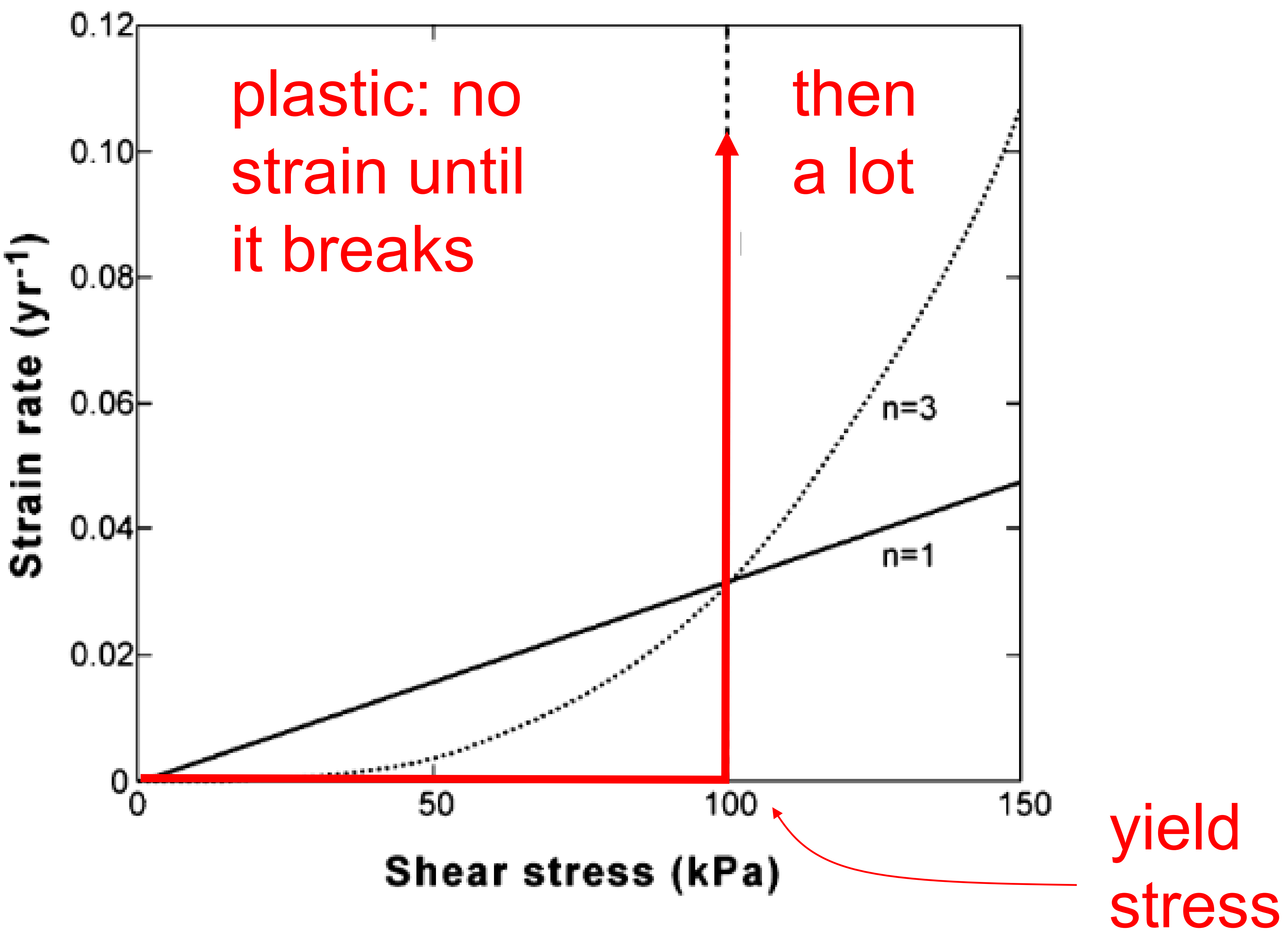
Slope indicates $n \approx 3$

(different lines for different temperatures)

One possible approximation



another

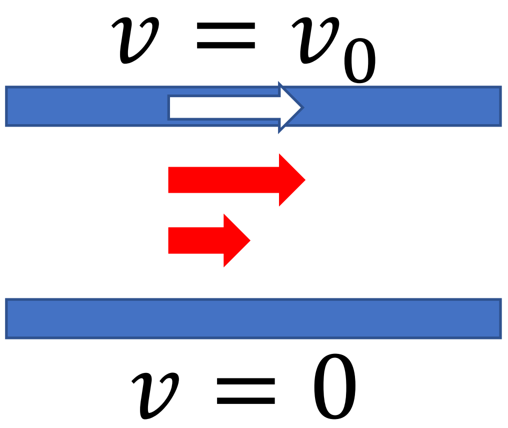


plastic: no
strain until
it breaks
then
a lot
yield
stress
Strain rate (yr-1)
Shear stress (kPa)
0.12
0.10
0.08
0.06
0.04
0.02
0
0
50
100
150
n=3
n=1

Or one can solve equations numerically

$$\frac{d\sigma}{dz} + f = \rho\frac{dv}{dt}$$

Newton's Law



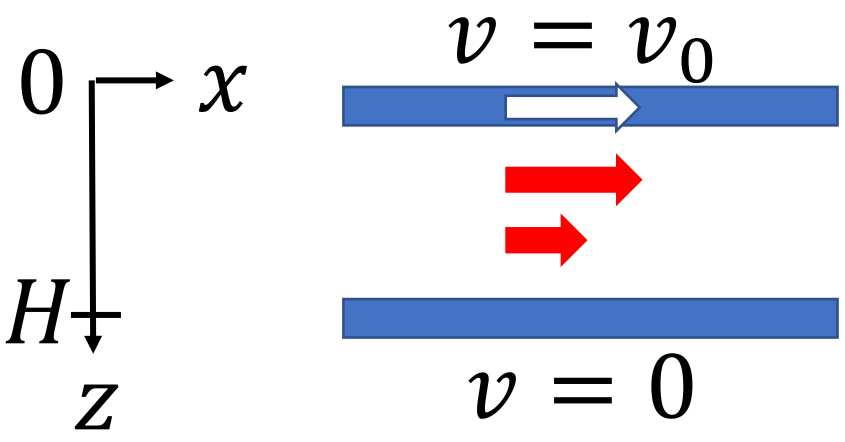
$$\frac{d\sigma}{dz} + \cancel{f} = \rho\cancel{\frac{dv}{dt}}$$

$$\frac{d\sigma}{dz} = 0 \qquad \sigma = constant = \sigma_0$$

$\sigma = \sigma_0$ Newton's Law

unknown

$$\frac{dv}{dz} = A{\sigma_0}^3$$ Glen's Law



unknown

$$v = A{\sigma_0}^3(c - z)$$

$v(H) = 0$ so $c = H$

$$v = A{\sigma_0}^3(H - z)$$

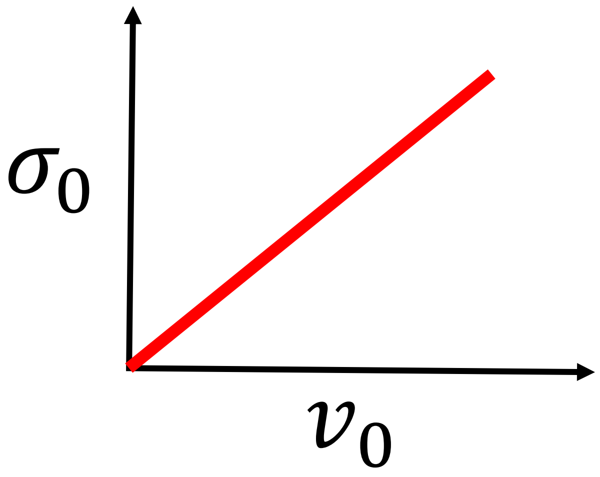
$v(0) = v_0$ so $\frac{v_0}{H} = A{\sigma_0}^3$

$$\sigma_0 = (A/H)^{1/3}{v_0}^{1/3}$$

Viscous

$$v = \frac{v_0}{H}(H - z)$$

$$\sigma_0 = -\frac{\mu}{H} v_0$$



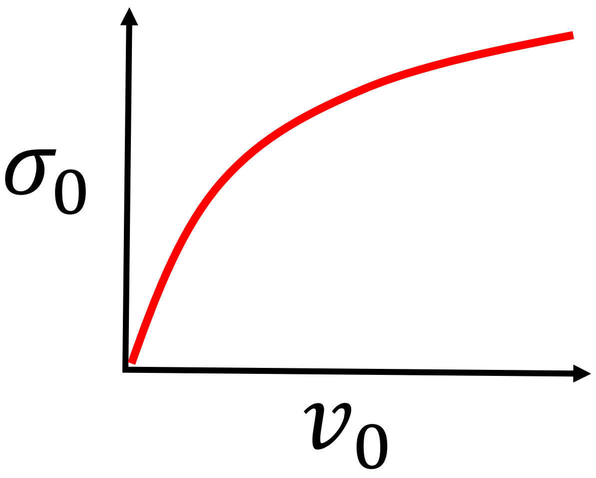
same flow

different stress

Glen's Law

$$v = \frac{v_0}{H}(H - z)$$

$$\sigma_0 = -\left(\frac{A}{H}\right)^{1/3} {v_0}^{1/3}$$



$$\frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

Newton's Law

$$\frac{dv}{dz} = A\sigma^3$$

$$\sigma = A^{-1/3} \left(\frac{dv}{dz}\right)^{1/3}$$

$$\frac{d\sigma}{dz} = (1/3)A^{-1/3} \left(\frac{dv}{dz}\right)^{-2/3} \frac{d^2v}{dz^2}$$

Putting Glens law into Newton’s Law

$$\frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

Newton's Law

$$\frac{dv}{dz} = A\sigma^3$$

$$\sigma = A^{-1/3} \left(\frac{dv}{dz}\right)^{1/3}$$

a mess

$$\frac{d\sigma}{dz} = (1/3) A^{-1/3} \left(\frac{dv}{dz}\right)^{-2/3} \frac{d^2 v}{dz^2}$$

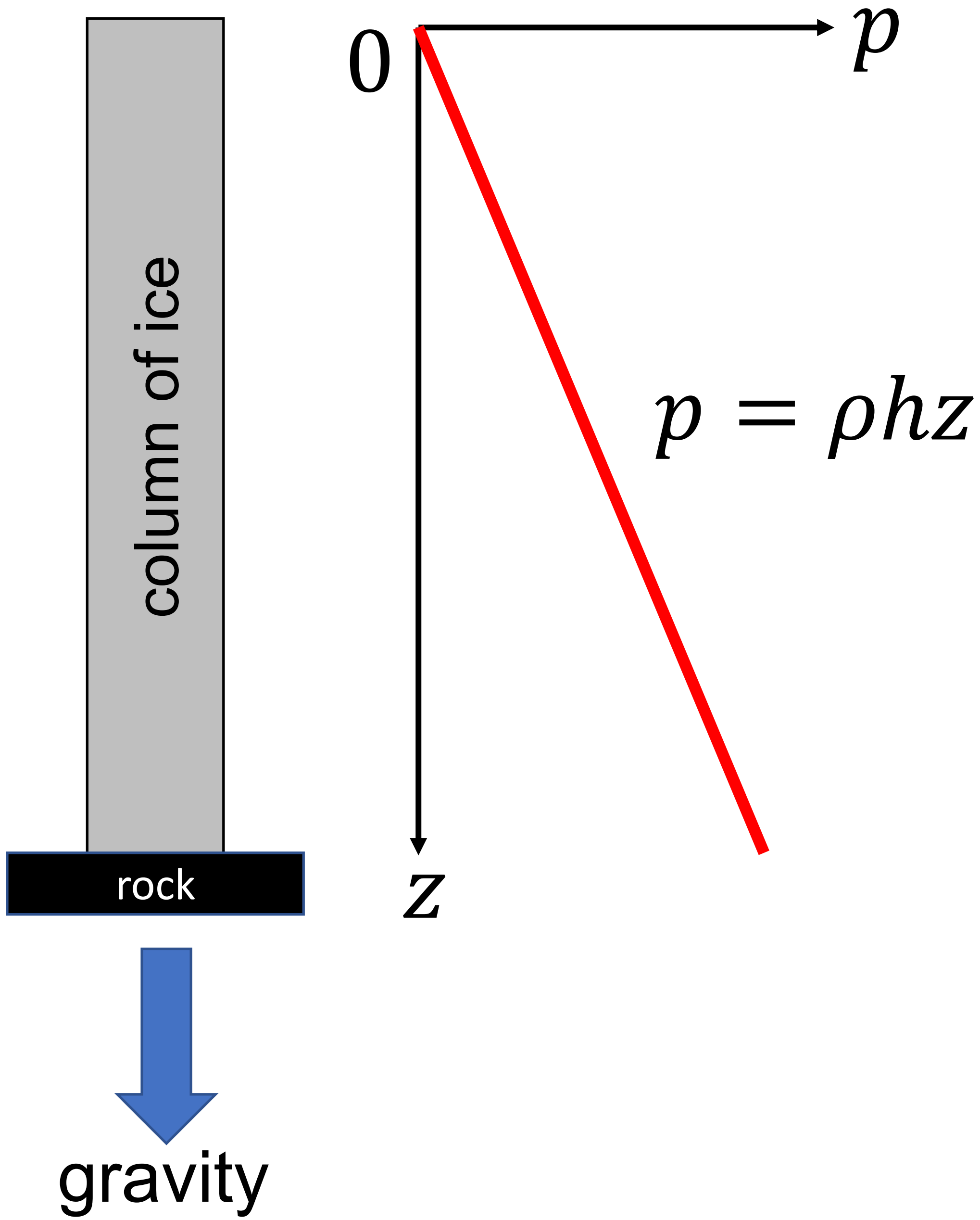
$$\frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

So solve numerically

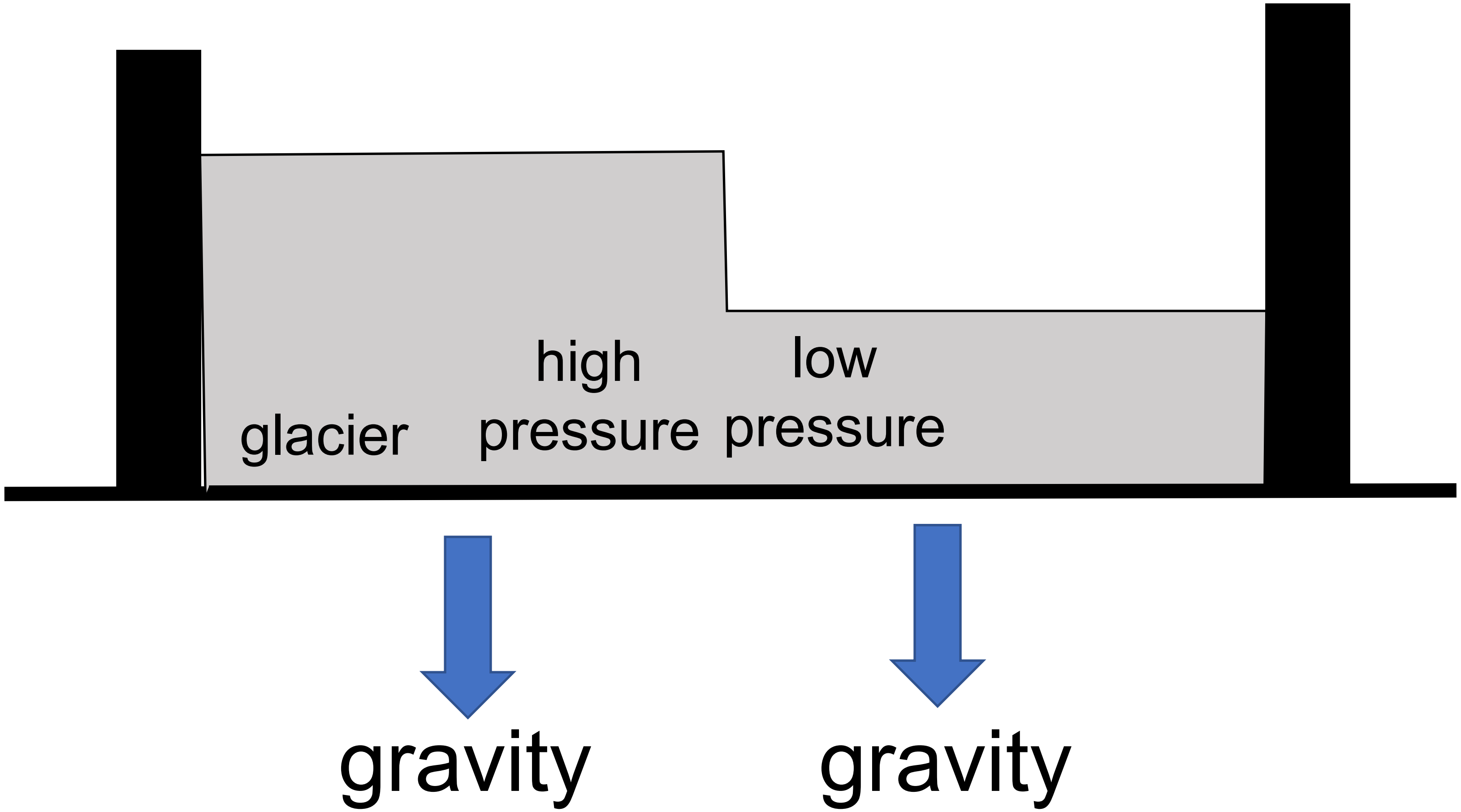
Part 2

effect of topography

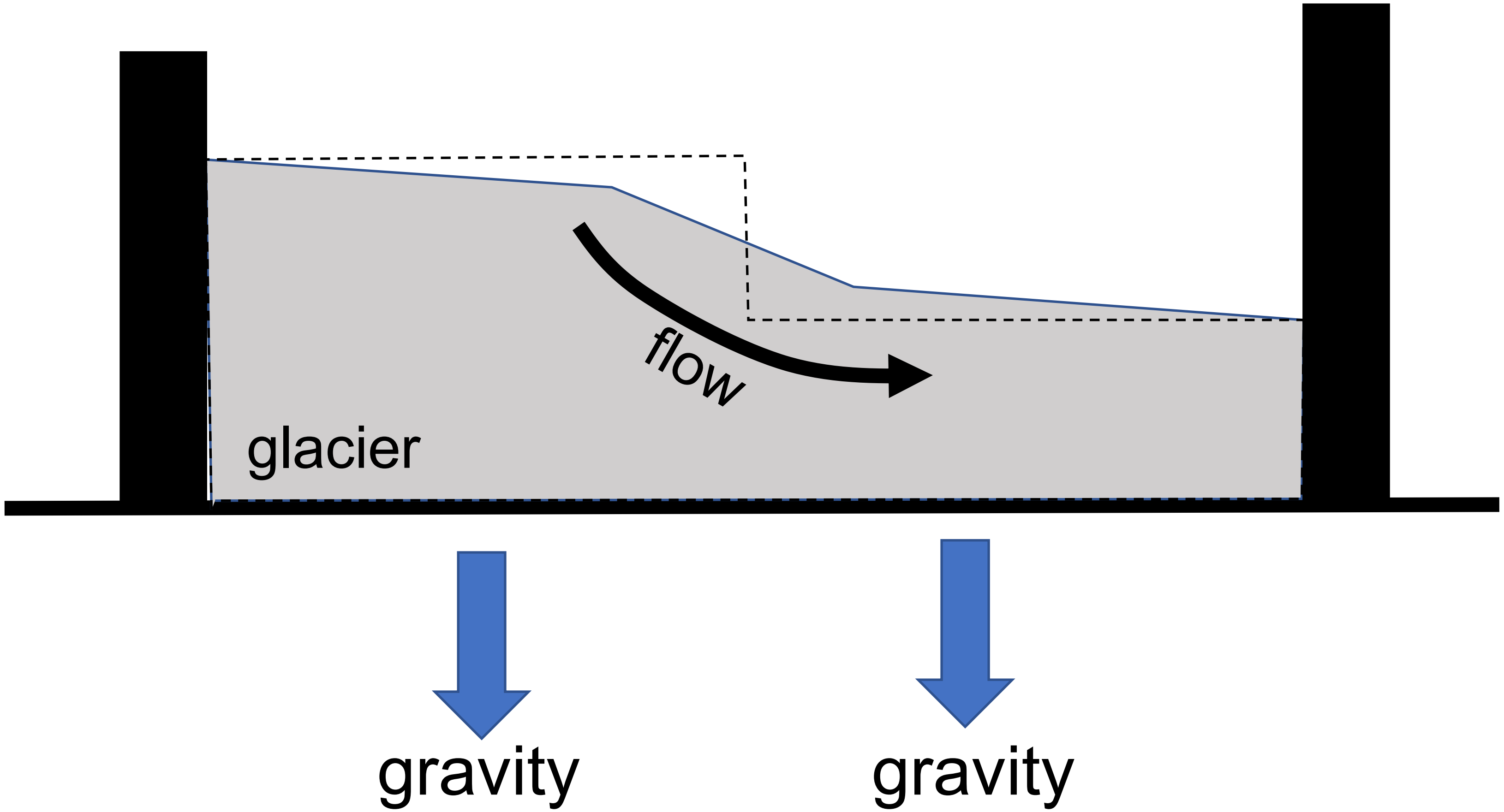
pressure due to weight of overlying rock



topography causes differences in pressure that can drive flow

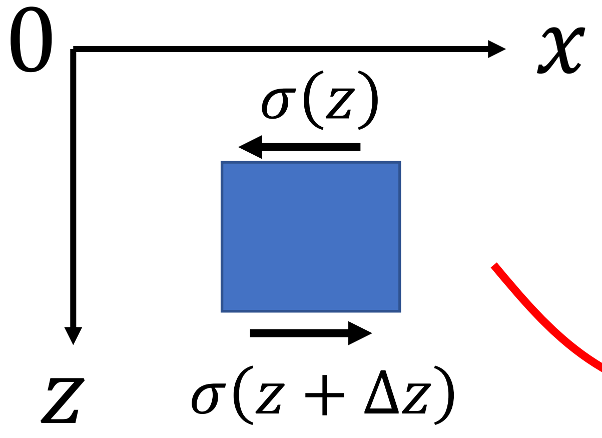


topography causes differences in pressure that can drive flow



Newton's Law from last lecture

dynamics

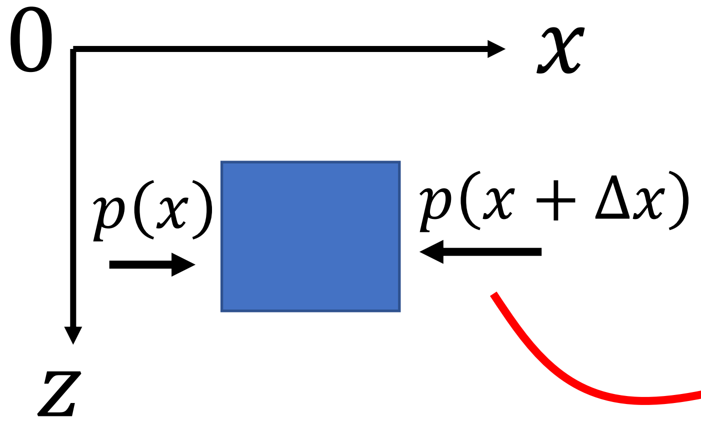


$$F = m\,A$$

$$\frac{d\sigma}{dz} + f = \rho\frac{dv}{dt}$$

Newton's Law today

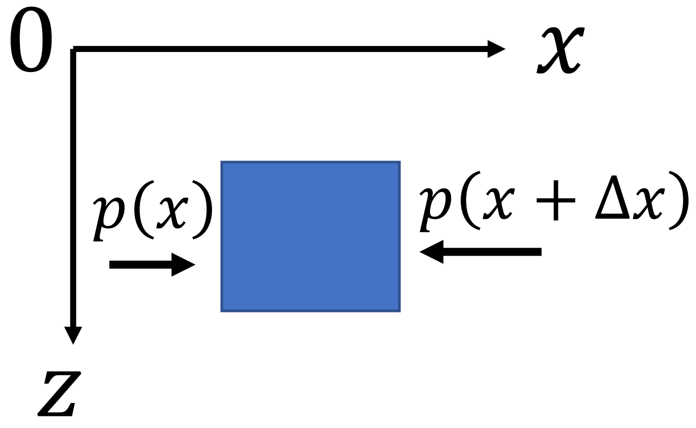
dynamics



$$F = m\,A$$

$$-\frac{dp}{dx} + \frac{d\sigma}{dz} + f = \rho\,\frac{dv}{dt}$$

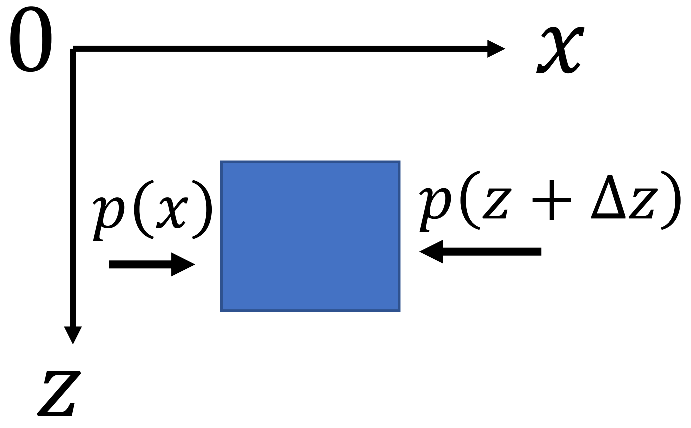
dynamics



Newton's Law today

$$-\frac{dp}{dx} + \frac{d\sigma}{dz} + f = \rho \frac{dv}{dt}$$

dynamics



Newton’s Law today

$$-\frac{dp}{dx} + \frac{d\sigma}{dz} + f = \rho\frac{dv}{dt}$$

when small

$$\frac{dp}{dx} = \frac{d\sigma}{dz}$$

pressure balances shear stress

Part 3

ultra-simplified model of equilibrium shape of glacial topography

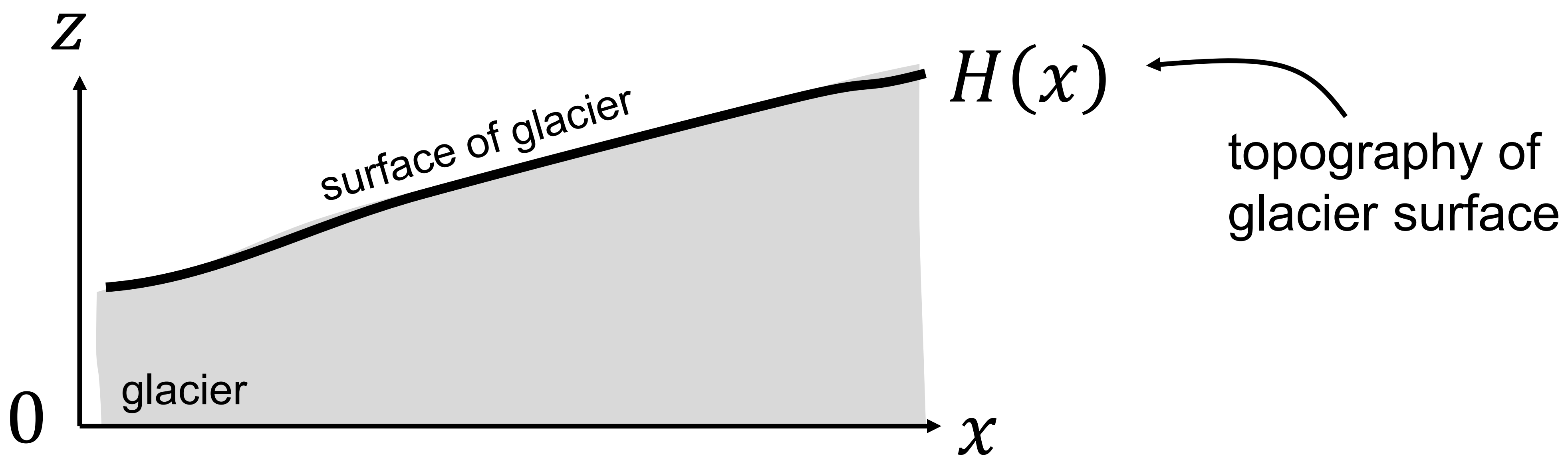
combines ideas

take vertical averages to “get rid of” vertical dimension

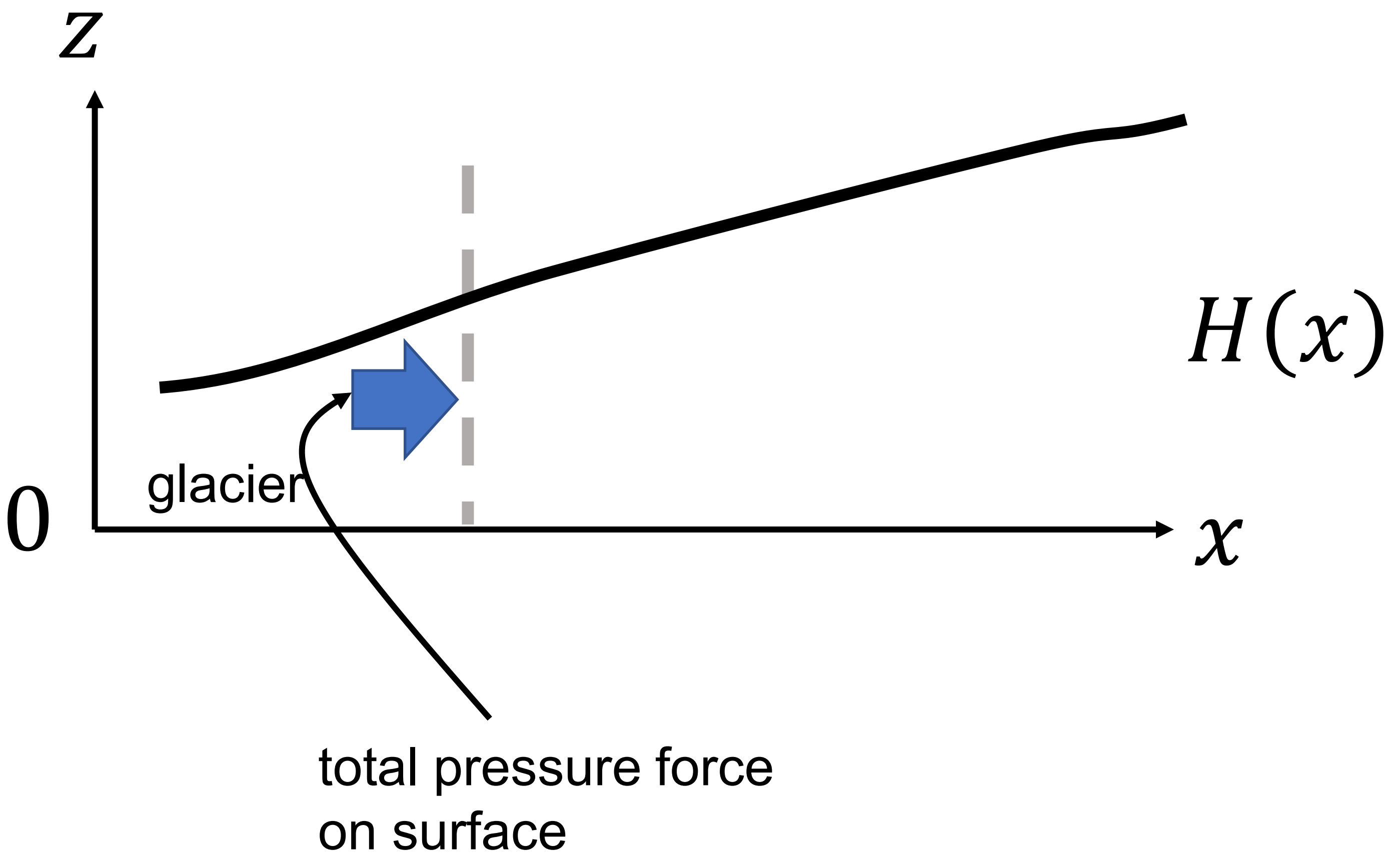
assume plastic rheology, ice just short of flowing

assume shear stress is biggest at bottom
(which is true for the stream model from last lecture)

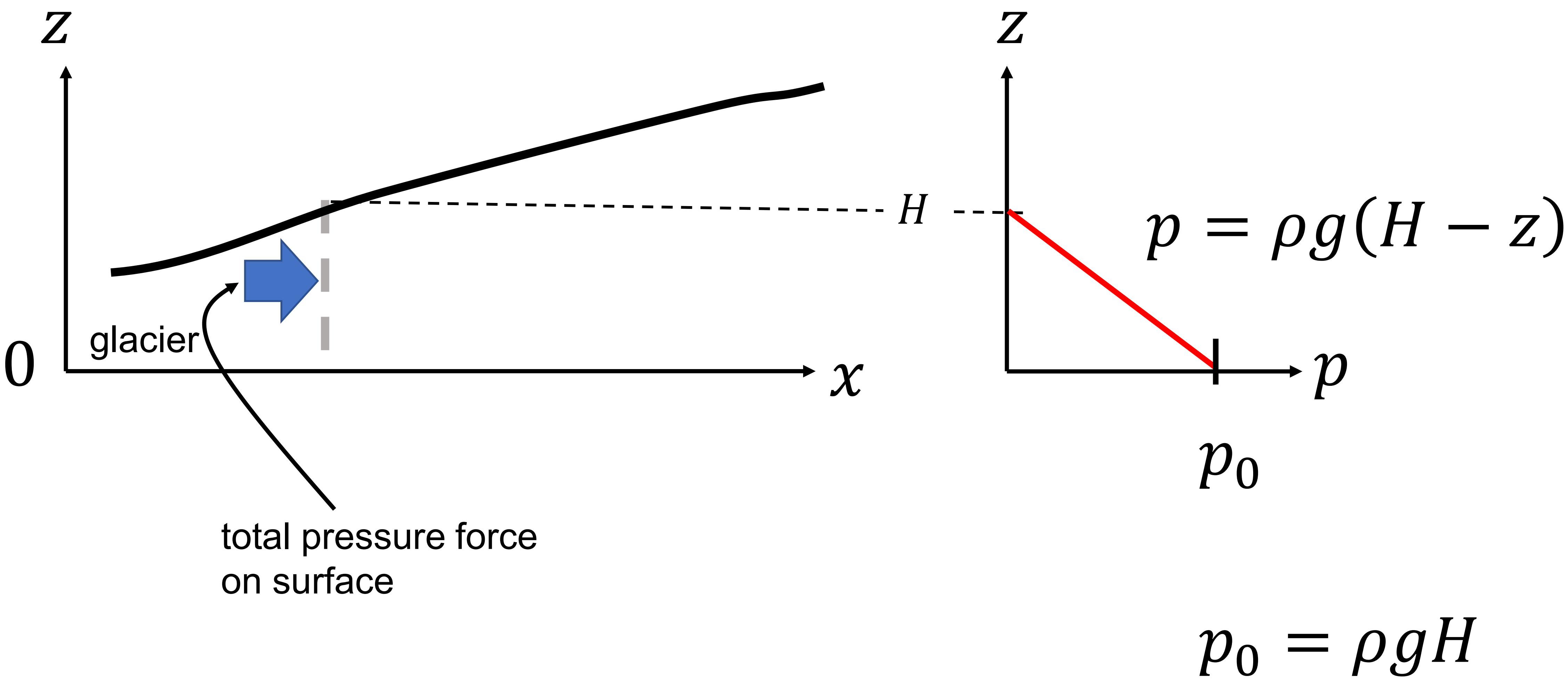
basal stress is just at yield stress of ice



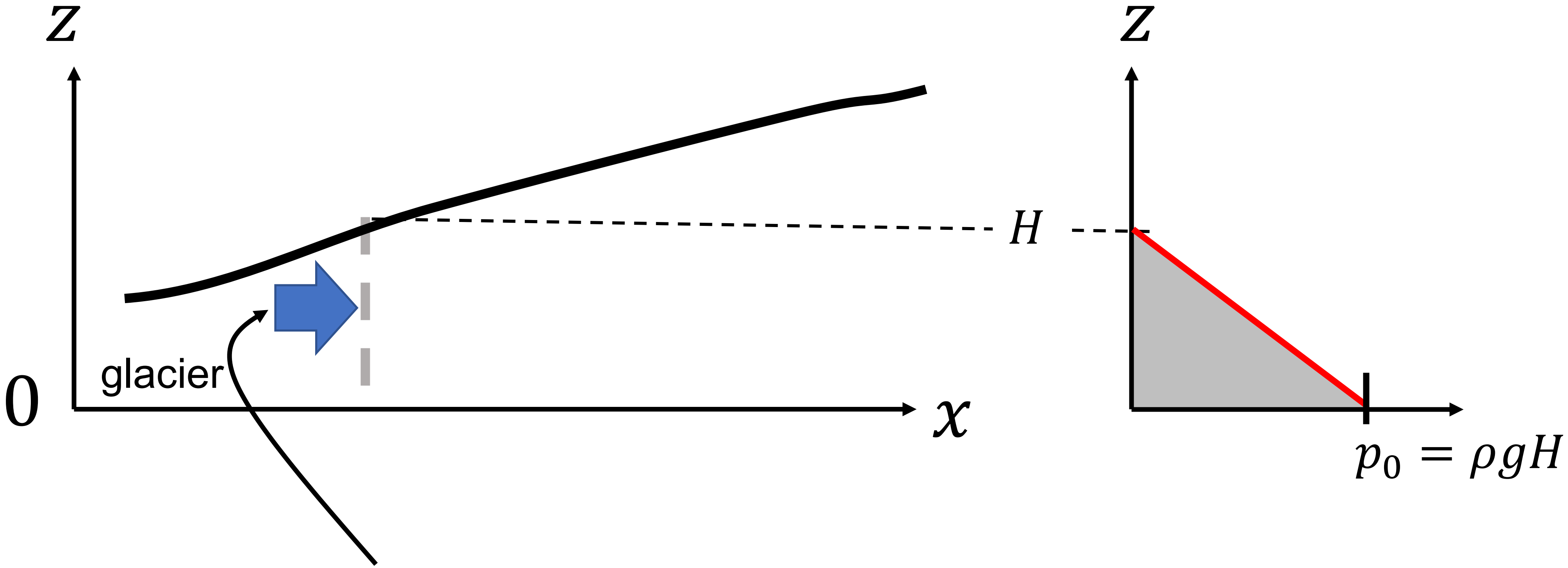
z
$H(x)$
surface of glacier
topography of glacier surface
glacier
0
x



z
$H(x)$
glacier
0
x
total pressure force
on surface



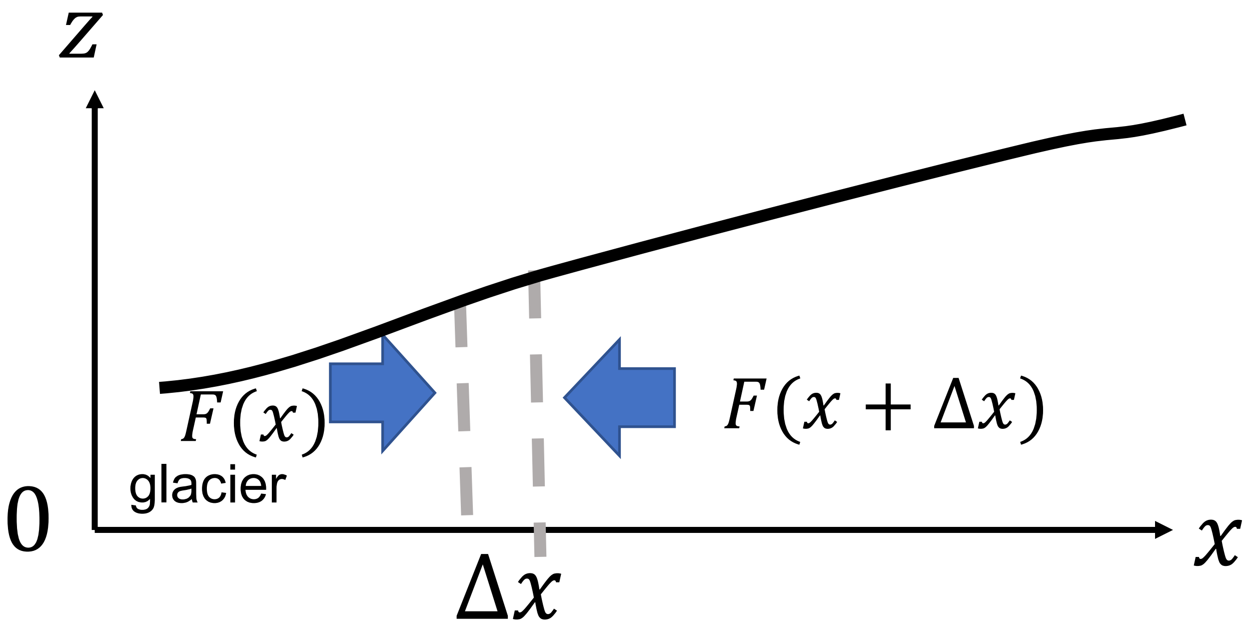
z
0
glacier
x
H
total pressure force
on surface
z
p
p_0
$p = \rho g(H - z)$
$p_0 = \rho g H$



total pressure force on surface = area under curve

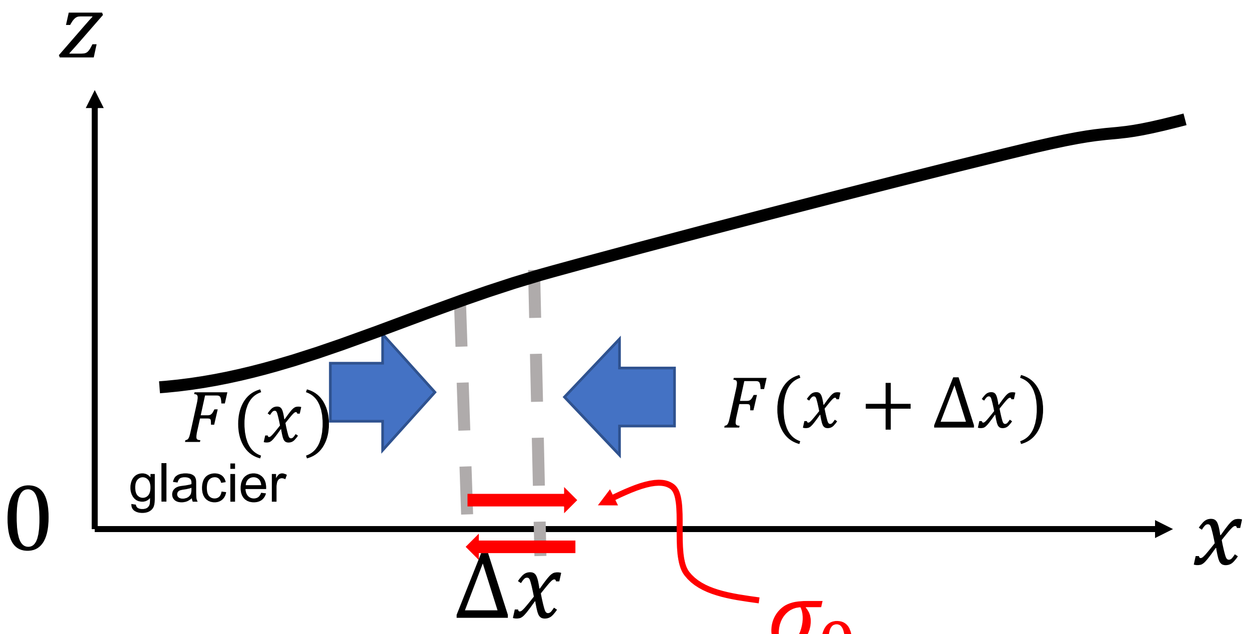
$$F = \tfrac{1}{2} H p_0$$

$$= \tfrac{1}{2} \rho g H^2$$



force imbalance

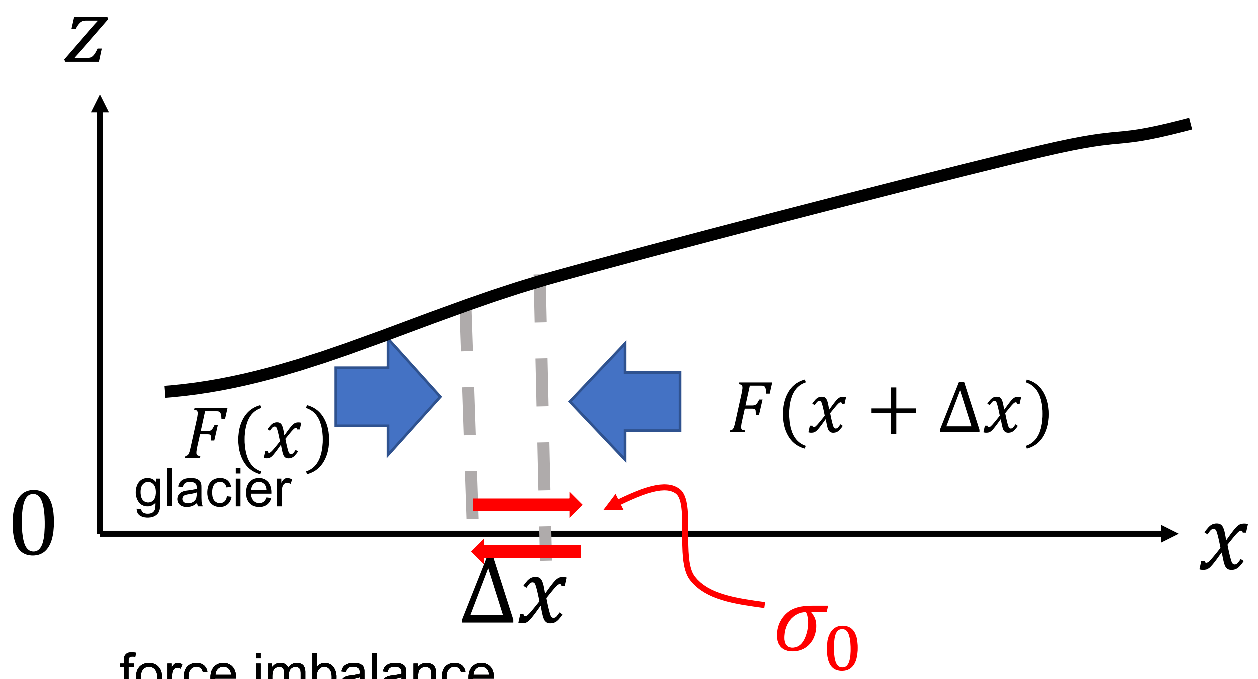
$$\Delta F = F(x + \Delta x) - F(x + \Delta x)$$



force imbalance

balanced by shear stress on base of glacier

$$\Delta F = \Delta x \sigma_0$$



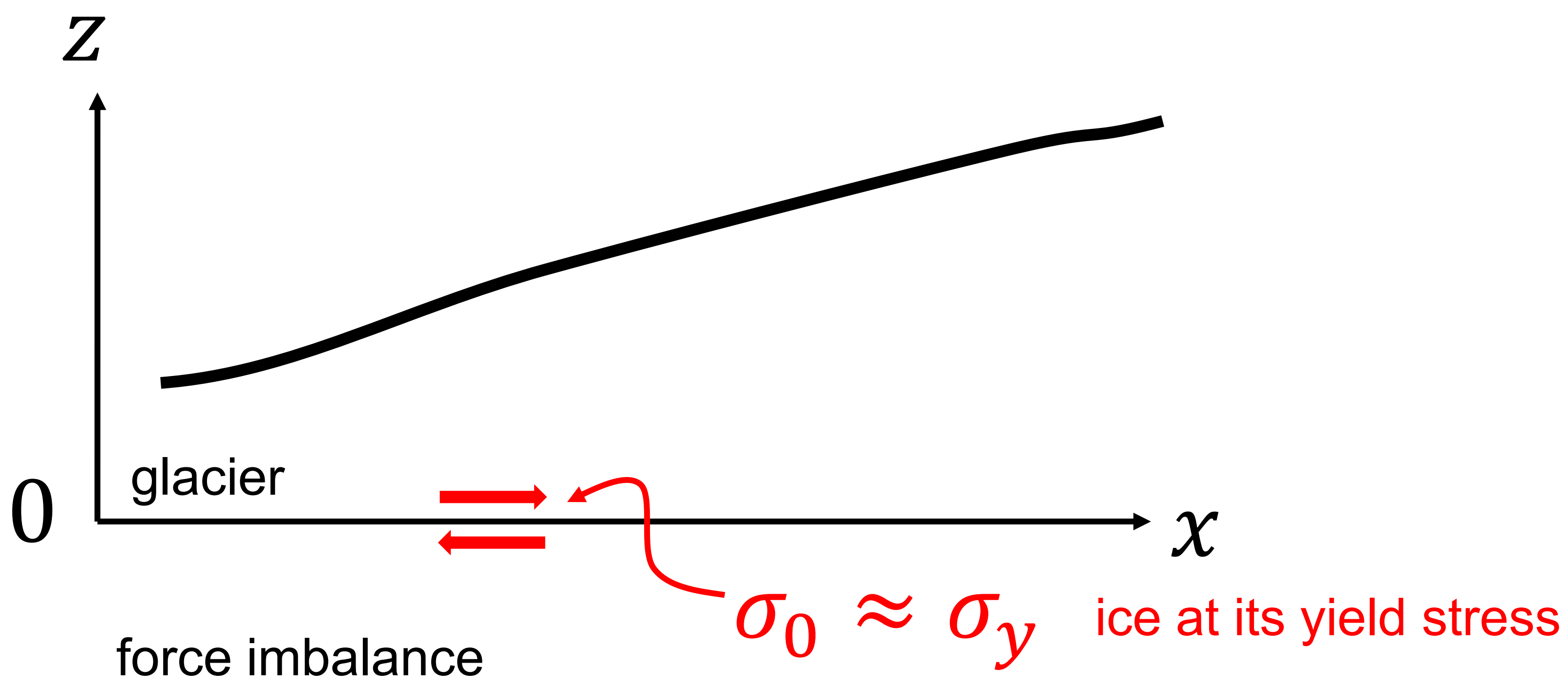
force imbalance

balanced by shear stress on base of glacier

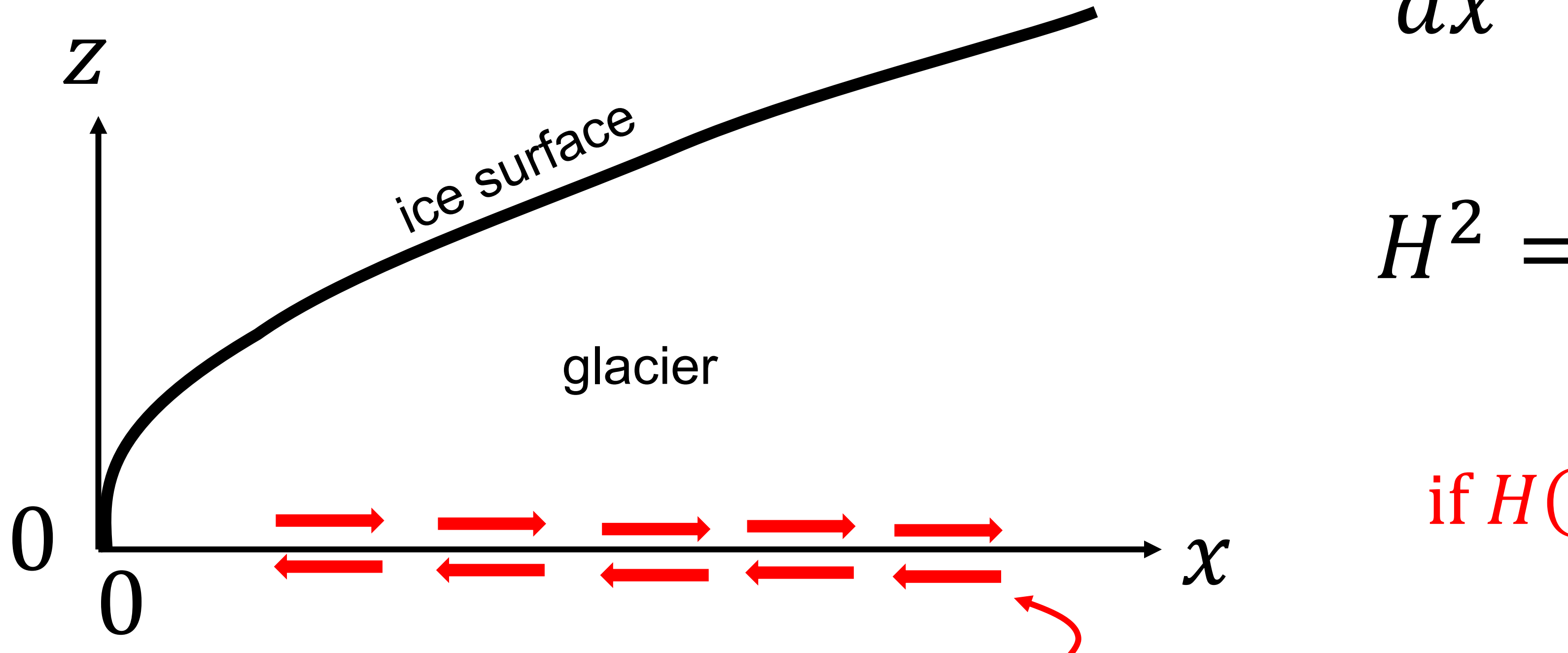
$$\Delta F = \Delta x \sigma_0$$

or

$$\frac{\Delta F}{\Delta x} = \sigma_0$$



$$\frac{d}{dx}H^2 = \frac{2\sigma_y}{\rho g}$$

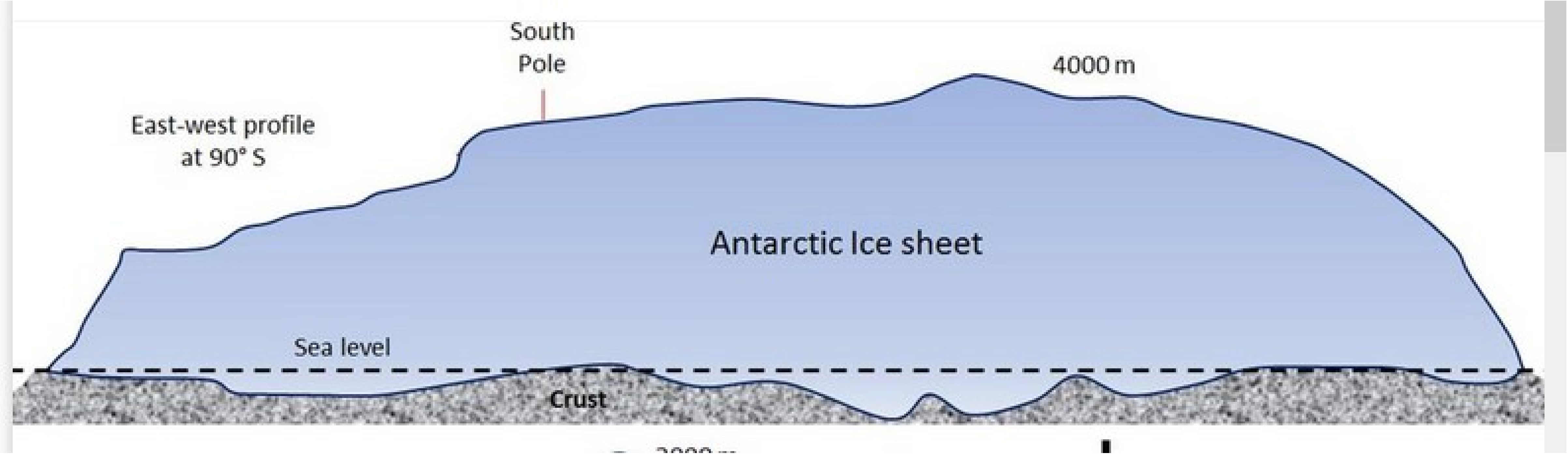


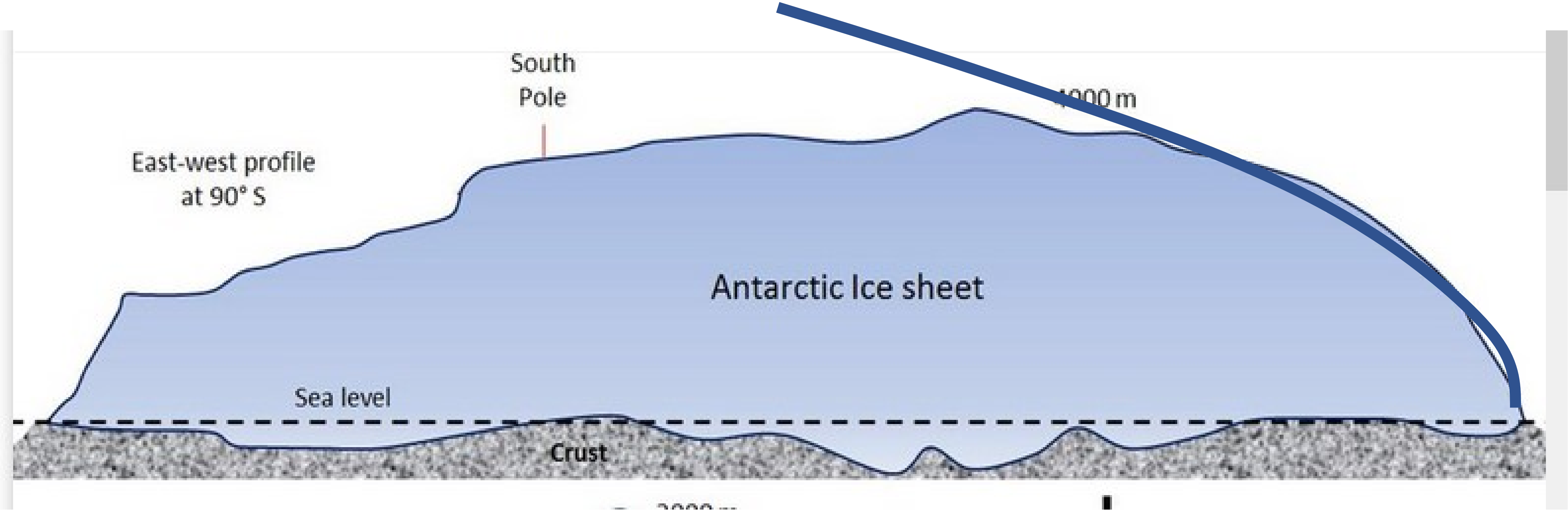
$$\frac{d}{dx}H^2 = \frac{2\sigma_y}{\rho g}$$

$$H^2 = \frac{2\sigma_y}{\rho g}(x + c)$$

if $H(0) = 0$ then $c = 0$

$$H = \sqrt{\frac{2\sigma_y x}{\rho g}}$$


South
Pole
4000 m
East-west profile
at 90° S
Antarctic Ice sheet
Sea level
Crust


South
Pole
East-west profile
at 90° S
Antarctic Ice sheet
Sea level
Crust

Part 3

ultra-simplified model of change in bed character

modeled as a change in basal shear stress

$$\frac{d}{dx}H^2 = \frac{2\sigma_y}{\rho g}$$

$\sigma_y(x)$ position-dependent yield stress

$H = H_0 + \delta H$ topography a small perturbation on top of flat

$$\frac{d}{dx}H^2 = \frac{2\sigma_y}{\rho g}$$

$\sigma_y(x)$ position-dependent yield stress

$H = H_0 + \delta H$ topography a small perturbation on top of flat

$$H^2 = (H_0 + \delta H)^2 \approx H_0^2 + 2H_0\delta H$$

$$\frac{d}{dx}H^2 = 2H_0\frac{d\delta H}{dx}$$

$$\frac{d}{dx}H^2 = \frac{2\sigma_y}{\rho g}$$

and

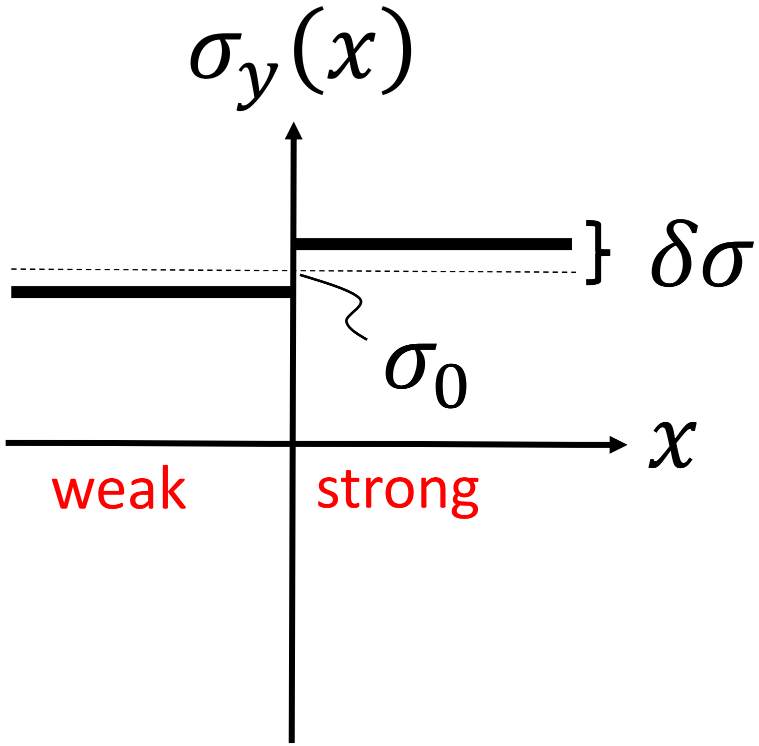
$$\frac{d}{dx}H^2 = 2H_0\frac{d\delta H}{dx}$$

so $$\cancel{2}H_0\frac{d\delta H}{dx} = \frac{\cancel{2}\sigma_y}{\rho g}$$

$$\frac{d\delta H}{dx} = \frac{1}{\rho g H_0} \sigma_y(x)$$

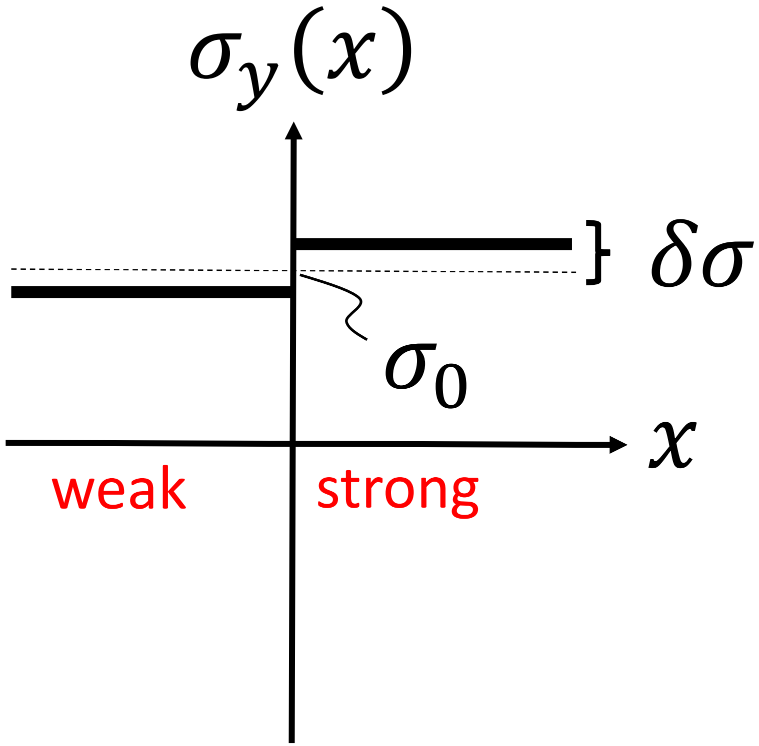
$$\frac{d\delta H}{dx} = \frac{1}{\rho g H_0} \begin{cases} \sigma_0 - \delta\sigma & x < 0 \\ \sigma_0 + \delta\sigma & x > 0 \end{cases}$$

along with $\delta H(x = 0) = 0$



$$\frac{d\delta H}{dx} = \frac{1}{\rho g H_0} \sigma_y(x)$$

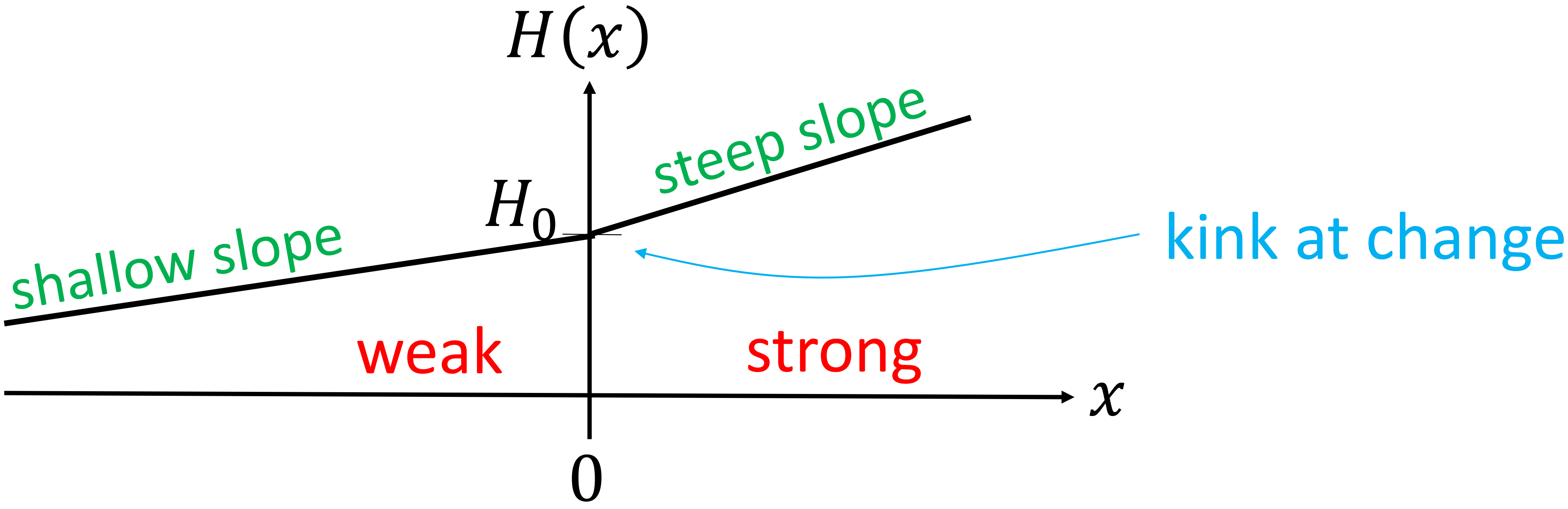
$$\frac{d\delta H}{dx} = \frac{1}{\rho g H_0} \begin{cases} \sigma_0 - \delta\sigma & x < 0 \\ \sigma_0 + \delta\sigma & x > 0 \end{cases}$$



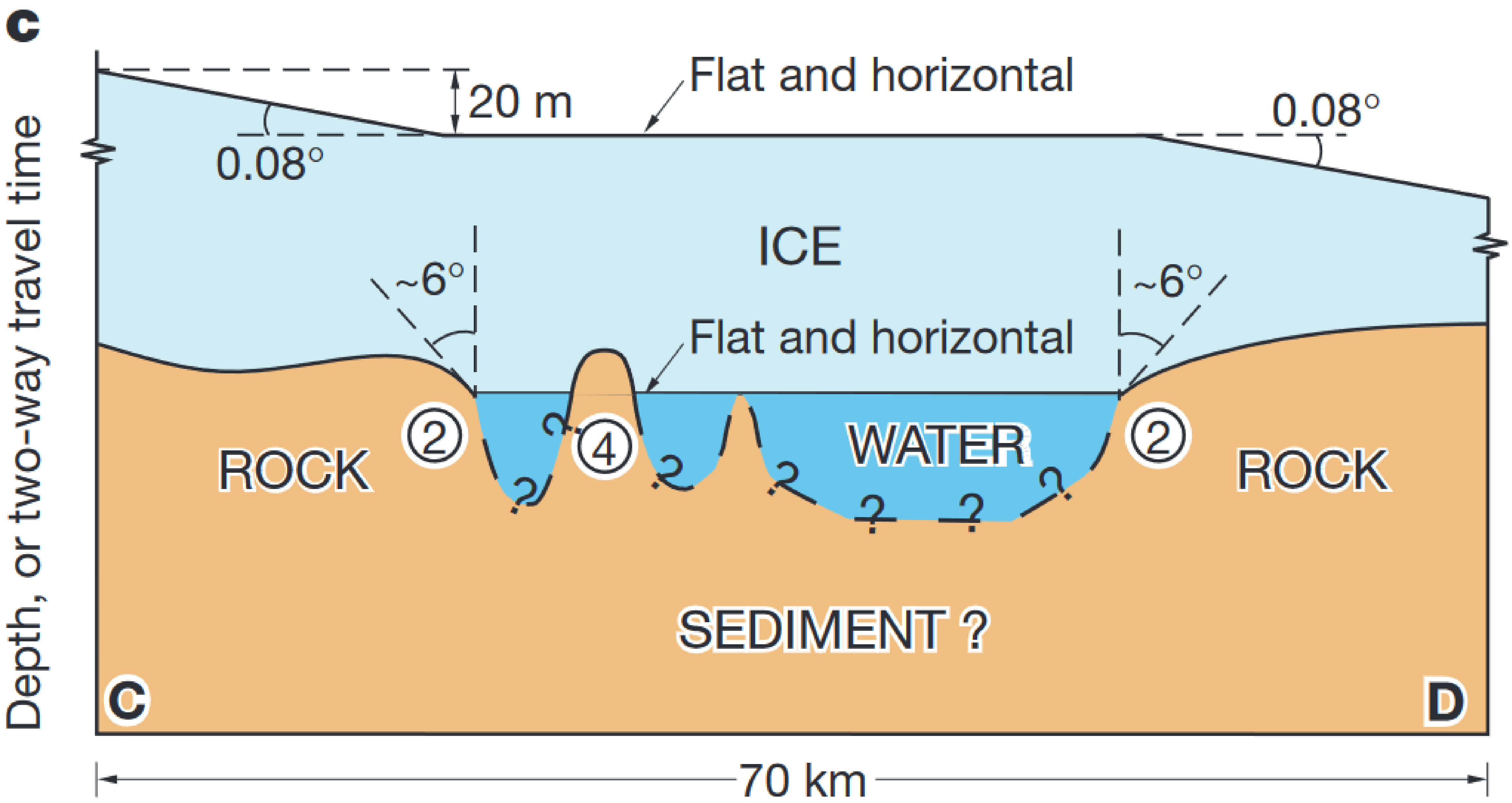
along with $\delta H(x = 0) = 0$

$$\delta H = \frac{1}{\rho g H_0} \begin{cases} (\sigma_0 - \delta\sigma)x + c_1 & when \quad x < 0 \\ (\sigma_0 + \delta\sigma)x + c_1 & when \quad x > 0 \end{cases}$$

but when $c_1 = 0$ for $\delta H(x = 0) = 0$



$$\delta H = \frac{x}{\rho g H_0}\begin{cases}(\sigma_0 - \delta\sigma) & when \quad x < 0 \\ (\sigma_0 + \delta\sigma) & when \quad x < 0\end{cases}$$



c
Flat and horizontal
20 m
0.08°
0.08°
ICE
~6°
~6°
Flat and horizontal
2
4
2
WATER
ROCK
ROCK
SEDIMENT ?
C
D
Depth, or two-way travel time
70 km