

phase, group velocity formula written in terms of wavelength

$$c(k) = \frac{\omega(k)}{k}$$

$$u(k) = \frac{d\omega(k)}{dk}$$

$$u(k) = \frac{d}{dk} (c(k)k) = k \frac{dc(k)}{dk} + c(k)$$

$$u(k) = c(k) + \frac{2\pi}{\lambda} \frac{dc(k)}{dk}$$

$$k = 2\pi \lambda^{-1}$$

$$\frac{dk}{d\lambda} = -2\pi \lambda^{-2}$$

$$\text{and } \frac{d}{dk} = \frac{d\lambda}{dk} \frac{d}{d\lambda}$$

$$\frac{d}{dk} = -\frac{\lambda^2}{2\pi} \frac{d}{d\lambda}$$

$$u(\lambda) = c(\lambda) - \lambda \frac{dc}{d\lambda}$$

note that $\frac{dc}{d\lambda}$ is positive for cases shown at right so that $u(\lambda) \leq c(\lambda)$, where $\frac{dc}{d\lambda}$ is maximum, i.e. at the inflection in the phase velocity curve, $u(\lambda)$ has a minimum.