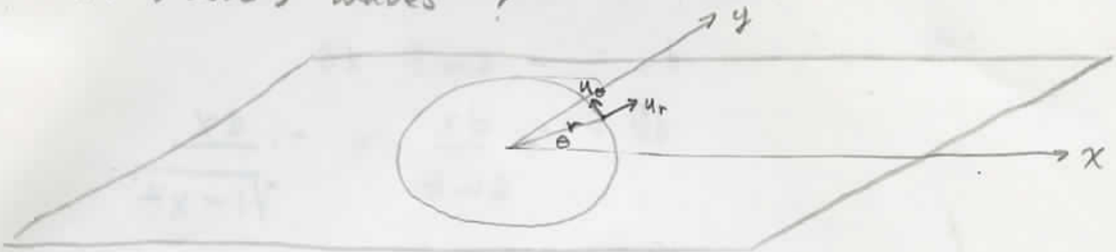
Roger Bilham has a gadget that senses angular accellerations, essentially by measuring differential pressure in a toroidal pipe of fluid. What would the accelleration of such a pipe, welded to the ground, be for SH (LOVE) waves?



THE OVERALL ACCELLERATION OF THE TORUS IS THE INTEGRAL OF THE RADIAL ACCELLERATION OF A POINT ON THE SURFACE, INTEGRATED AROUND THE TORUS.

SUPPOSE THE SH DISPLACEMENT IS $U_x = U_z = 0 \quad U_y = A \sin(\omega(px - t))$

WE CAN EXPAND THIS AS FOLLOWS

$$U_y = A \cos \omega t \ \sin \omega p x - A \sin \omega t \ \cos \omega p x$$

we note that $U_\theta = U_y \cos\theta - U_x \sin\theta = U_y \cos\theta$ if U_θ is measured in units of distance. we divide by r to get units of angle. since $x = r\cos\theta$ we have

$$U_\theta = \left\{ \frac{A}{r} \cos \omega t \ \sin(\omega p r \cos\theta) - \frac{A}{r} \sin \omega t \ \cos(\omega p r \cos\theta) \right\} \cos\theta$$

TAKING TIME DERRIVATIVES

$$U_{\theta,t} = -\frac{A}{r} \omega \sin \omega t \ \sin(\omega p r \cos\theta) \cos\theta - \frac{A}{r} \omega \cos \omega t \ \cos(\omega p r \cos\theta) \cos\theta$$

AGAIN TO GET ACCELLERATION

$$U_{\theta,tt} = -\frac{A}{r} \omega^2 \cos \omega t \ \sin(\omega p r \cos\theta) \cos\theta + \frac{A}{r} \omega^2 \sin \omega t \ \cos(\omega p r \cos\theta) \cos\theta$$

WE MUST NOW INTEGRATE THESE AROUND THE TORUS ie by performing $\int_0^{2\pi} d\theta$ or equivantly $\int_{-\pi}^{\pi} d\theta$ NOTE THE BASIC INTEGRAL looks like

$$I = \int_{-\pi}^{\pi} \begin{matrix}\sin \\ \cos\end{matrix} (\omega p r \cos\theta) \cos\theta \, d\theta$$

BUT SINCE THIS IS EVEN IN θ it can be WRITTEN

$$I = 2\int_0^{\pi} \begin{matrix}\sin\\ \cos\end{matrix} (wp r \cos\theta) \cos\theta \, d\theta .$$

making the substitution $x = \cos\theta$

$$dx = - \sin\theta \, d\theta$$

$$d\theta = - \frac{dx}{\sin\theta} = - \frac{dx}{\sqrt{1-x^2}}$$

$\theta = 0 \quad x = 1$

$\theta = \pi \quad x = -1$

SO THE INTEGral can be WRITTEN:

$$I = 2\int_{-1}^{1} \frac{\begin{matrix}\sin\\ \cos\end{matrix} (wpr\,x) \; x}{\sqrt{1-x^2}} \, dx$$

now if we choose the sine the integral is again even in x and we can write

$$I_s = 4\int_0^1 x \sin(wprx) \sqrt{1-x^2}^{\,-1} dx$$

But if we choose the cosine it is odd in x and The integral is Therefore zero

$$I_c = 0$$

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so now Gradstein + Rysin give $\int_0^1 \frac{x \sin a x}{\sqrt{1-x^2}} dx$ to equal $\frac{\pi}{2} J_1(a)$ so:

$$I_s = 2\pi J_1(wpr)$$

$$I_c = 0$$

SO WE CAN NOW COMPUTE THE AVERAGE OR NET displacement, velocity and ACCELLERATION AROUND THE TORUS

$$\frac{1}{2\pi}\int_0^{2\pi} u_\theta \, d\theta = \text{Average Angular displacement} = \frac{A}{r} J_1(\omega p r)\cos \omega t$$

$$\frac{1}{2\pi}\int_0^{2\pi} u_{\theta,\tau}\, d\theta = \text{AVERAGE ANGULAR VELOCITY} = -\frac{A}{r}\,\omega\, J_1(\omega p r)\sin \omega t$$

$$\frac{1}{2\pi}\int_0^{2\pi} u_{\theta,\tau\tau}\, d\theta = \text{AVERAGE Angular ACCELLERATION} = -\frac{A}{r}\,\omega^2 J_1(\omega p r)\cos \omega t$$

NOTE That at small values of $\omega p r = kr$ that $J_1(\omega p r) \approx \frac{\omega p r}{2}$. kr is small when the wavelength OF The LOVE WAVES is much larger THAN the radius of the torus. Then

$$\text{AVER. Angular ACCELERATTion} \approx -\frac{A}{2}\,\omega^2 k = -\frac{A}{2}\,\frac{\omega^3}{V_H}\cos \omega t$$ (independent of r)

where V_H is the Horizontal velocity of the Love waves.

(EXAMPLE) suppose $A = 2 \times 10^{-2}$ micron

$\omega = \frac{2\pi}{20} = .31/\text{sec}$

$V_H = 4\ \text{km/sec} = 4 \times 10^9\ \text{micron/sec}$

peak to peak Angular ACCELLERATion =

$$2\,(10^{-2})\,(.31)^3\,(4)^{-1}\,10^{-9} = 1.5 \times 10^{-12}\ \text{rad/sec}^2$$

Roger: HERE'S A QUICK CALCULATION TO MAKE MY EXACT CALCULATION REASONABLE. (ACTUALLY ITS THE CORRECT VERSION OF THE ONE WE TRIED before)

ASSUME THAT THE ANGULAR displacement of the torus is the SAME EVERYwhere on the torus. This gives then a MAXIMUM ANSWER.

IF The displacement at some time is $u_y = A \sin kx$ and two points on the torus are at $x = -r$ and $x = +r$. the angular displacement of each point, relative to the toroids center will be $A \sin kr$ which for small kr is about Akr. This is in units of dist. to get units of radians we divide by the radius of the circle

Angular displacement $< Ak$

now lets assume time dependence $\cos \omega t$ (this is exact for a standing Love wave, so I'LL approximate a travelling wave using it). Differentiating to get

Angular accelleration $< -Ak\omega^2 = -\dfrac{A\omega^3}{V_H}$

This is in fact twice the exact result.