Algorithm to fit least square great circle

1. input data
2. input trial pole
3. rotate data 45° from trial pole

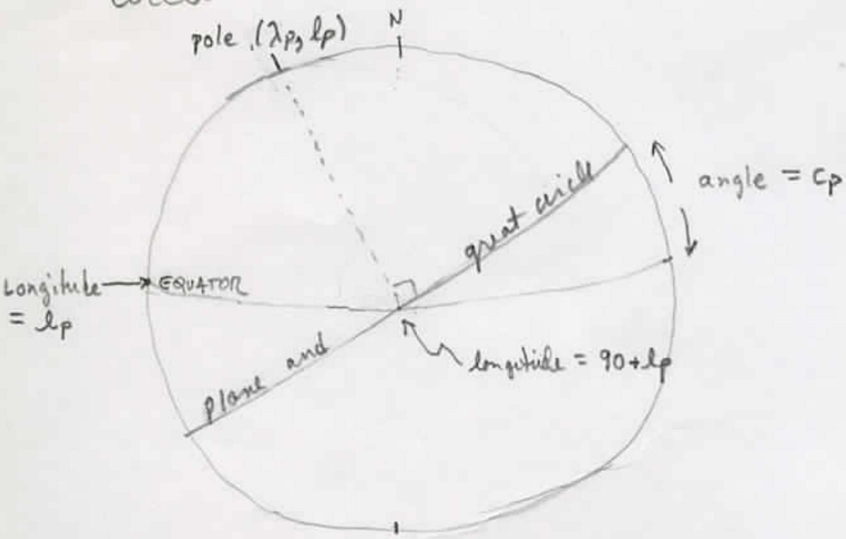
4. build matrix equation for correction factors
5. solve matrix equation using householder transformations
6. use correction factors to improve trial pole
7. test for convergence. goto 4. if not sufficiently converged.

8. compute problem standard deviation
9. build matrix equation using final pole position
10. decompose using S.V.D. theorem
11. compute pole's standard dev. make little box containing pole
12. transform box back into original coord system
13. write out pole position and standard deviation.
14. rotate data to new pole. residuals are just latitude in this reference frame.
15. write out original data and distances from great circle

16. STOP

great circle

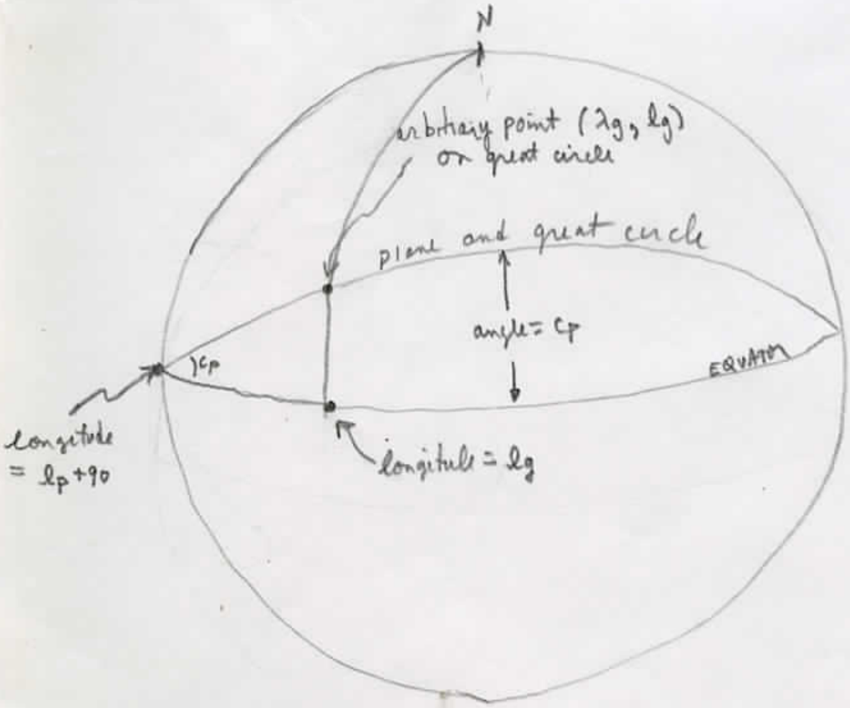
1. relation between (Latitude, longitude) on a great circle given (Latitude, longitude) of pole of plane defining great circle



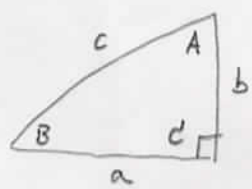
SIDE VIEW

Let (λ_p, l_p) = (latitude, longitude) of pole ; c_p = co-latitude of pole = $90-\lambda$

Front View



rule of spherical triangles



$\sin a = \tan b \cot B$

from rule

$a = l_g - l_p - 90$

$b = \lambda_g$

$B = c_p$

so equation for great circle is:

$$\sin(l_g - l_p - 90) = \tan\lambda_g \cot c_p$$

or since $\sin(\alpha - 90) = -\cos\alpha$

rearranging we have

$$\boxed{\tan\lambda_g = -\tan c_p \cos(l_g - l_p)}$$

notice when $l_g = l_p$ $\lambda_g = -c_p$ which is sensible.

2. least squares problem. we are given a bunch of $(\lambda_g, l_g)_i$ pairs, and we want to choose a c_p and l_p so that the error is minimized

$$\text{error} = \sum_{i=1}^{N} \left[\tan\lambda_g^i + \tan c_p \cos(l_g^i - l_p)\right]^2$$

$$= \sum_{i=1}^{N} \left[f(c_p, l_p, \lambda_g^i, l_g^i)\right]^2 = E$$

the error is minimized when $\frac{\partial E}{\partial c_p} = 0$ and $\frac{\partial E}{\partial l_p} = 0$

3. linearization

Basically we want to solve a bunch of simultanious equation of the form

$$f(c_p, l_p, \lambda_g^i, l_g^i) = 0$$

to get c_p and l_p. Unfortunately f is not linear in c_p and l_p. If however we have a initial guess at (c_p, l_p) say c_p^0, l_p^0 we can use taylor's Theorum to linearize f.

namely near (c_p^0, l_p^0) $\quad f \approx f(c_p^0, l_p^0) + \left.\frac{\partial f}{\partial c_p}\right|_{c_p^0, l_p^0} \Delta c_p + \left.\frac{\partial f}{\partial l_p}\right|_{c_p^0, l_p^0} \Delta l_p$

where $\Delta c_p = c_p - c_p^0 \qquad c_p = c_p^0 + \Delta c_p$

$\Delta l_p = l_p - l_p^0 \qquad l_p = l_p^0 + \Delta l_p$

4. calculation of derivatives and approx. for $f(c_p, l_p^0)$

$$f = \tan \lambda_g^i + \tan c_p \cos(l_g^i - l_p)$$

$$\frac{\partial f}{\partial c_p} = \sec^2 c_p \cos(l_g^i - l_p)$$

$$\frac{\partial f}{\partial l_p} = \tan c_p \sin(l_g^i - l_p)$$

$$0 = f(c_p, l_p) = \tan \lambda_g^i + \tan c_p^0 \cos(l_g^i - l_p^0) + \sec^2 c_p^0 \cos(l_g^i - l_p^0)\, \Delta c_p + \tan c_p^0 \sin(l_g^i - l_p^0)\, \Delta l_p$$

5. so the linearized equation $f(c_p, l_p) = 0$ becomes

$$\left[\sec^2 c_p^0 \cos(l_g^i - l_p^0)\right] \Delta c_p + \left[\tan c_p^0 \sin(l_g^i - l_p^0)\right] \Delta l_p = -\left[\tan \lambda_g^i + \tan c_p^0 \cos(l_g^i - l_p^0)\right]$$

6. notice that $\sec^2 c_p = (\cos c_p)^{-2}$ singular when $c_p = 90$ $\quad \lambda_p = 0$
however when $c_p = 0$ the coefficient of Δl_p is zero
and the matrix is singular. This is because of the coord sing
at the North pole. The formulae should work best
when $c_p \approx 45^\circ$