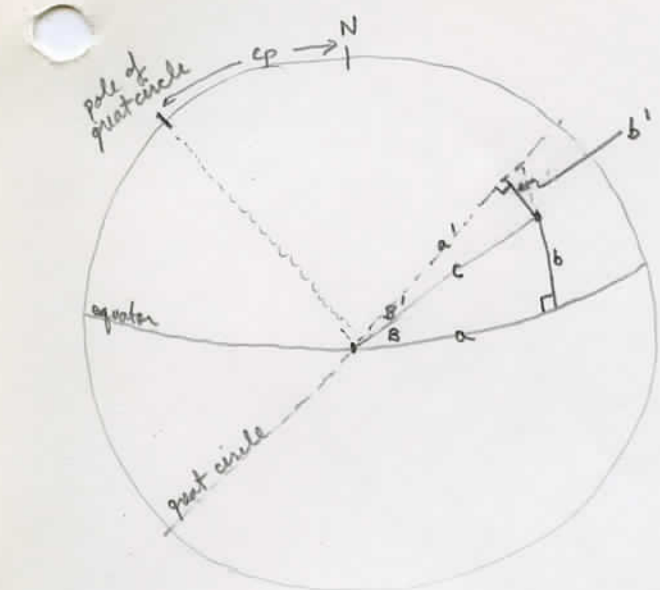
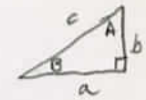
Perpendicular Distances

1. consider



note $B' = C_p - B$

from spherical trigonometry:



$\cos c = \cos a \cos b$

$\sin b = \sin B \sin c$

$\cos B = \tan a \cot c$

$\sin a = \tan b \cot B$

OBJECT is to get distance b'

2. START with triangle $a'b'c$, note that

$\sin b' = \sin B' \sin c$ but $B' = cp - B$ so

$\sin b' = \sin c \left[\sin cp \cos B - \sin B \cos cp \right]$ or

$\sin b' = \sin cp \sin c \cos B - \cos cp \sin c \sin B$

3. now note that $\cos B = \tan a \cot c$ and $\cos c = \cos a \cos b$

$$\cos B = \tan a \cot c = \tan a \frac{\cos c}{\sin c} = \frac{\tan a \cos a \cos b}{\sin c} = \frac{\sin a \cos b}{\sin c}$$

also note that $\sin a = \tan b \cot B$ or $\tan B = \tan b \frac{1}{\sin a}$

$\tan B = \tan b \frac{1}{\sin a}$ multiply by $\cos B$

$$\sin B = \frac{\tan b \cos B}{\sin a} = \frac{\tan b \sin a \cos b}{\sin c \sin a} = \frac{\sin b}{\sin c}$$

now we have relations for $\cos B$ and $\sin B$

4. plug into 2. given

$$\sin b' = \sin c_p \cancel{\sin c} \frac{\sin a \cos b}{\cancel{\sin c}} - \cos c_p \cancel{\sin c} \frac{\sin b}{\cancel{\sin c}}$$

$$\sin b' = \sin c_p \sin a \cos b - \cos c_p \sin b$$

5. now we note ~~that~~

$$a = l_g - l_p - 90$$

$$b = \lambda_g$$

c_p = colatitude of Pole

b' = perpendicular distance = d

$$\sin d = \sin c_p \cos \lambda_g \sin(l_g - l_p - 90) - \cos c_p \sin \lambda_g$$

$$\sin d = -\sin c_p \cos \lambda_g \cos(l_g - l_p) - \cos c_p \sin \lambda_g$$

$$\sin d = -\left[\sin c_p \cos \lambda_g \cos(l_g - l_p) + \cos c_p \sin \lambda_g\right]$$

6. so we seek to minimize the function

$$f(c_p, l_p) = \sin c_p \cos \lambda_g \cos(l_g - l_p) + \cos c_p \sin \lambda_g]$$

$$\frac{\partial f}{\partial c_p} = \cos c_p \cos \lambda_g \cos(l_g - l_p) - \sin c_p \sin \lambda_g$$

$$\frac{\partial f}{\partial l_p} = \sin c_p \cos \lambda_g \sin(l_g - l_p)$$

$$f(c_p, l_p) \approx [\sin c_p^\circ \cos \lambda_g \cos(l_g - l_p^\circ) + \cos c_p^\circ \sin \lambda_g]$$

$$+ [\cos c_p^\circ \cos \lambda_g \cos(l_g - l_p^\circ) - \sin c_p^\circ \sin \lambda_g]\, \Delta c_p + [\sin c_p^\circ \cos \lambda_g \sin(l_g - l_p^\circ)]\, \delta l_p$$

7. and the linearized equation $f = 0$ becomes

$$\left[\cos c_p^\circ \cos \lambda_g \cos (l_g - l_p^\circ) - \sin c_p^\circ \sin \lambda_g \right] \Delta c_p$$

$$+ \left[\sin c_p^\circ \cos \lambda_g \sin (l_g - l_p^\circ) \right] \Delta l_p$$

$$= - \left[\sin c_p^\circ \cos \lambda_g \cos (l_g - l_p^\circ) + \cos c_p^\circ \sin \lambda_g \right]$$