MRN020 Menke 10-27-4

matrix as Row vector + col vector + residual

$$M_{ij} = a_i + b_j + D_{ij} \qquad i=1,N \quad j=1,M$$

min $E = \sum_i \sum_j (D_{ij})^2$ wrt a, b

$$E = \sum_i \sum_j (M_{ij} - a_i - b_j)^2$$

$$\frac{\partial E}{\partial a_p} = \sum_i \sum_j (M_{ij} - a_i - b_j)(-2)\underbrace{\left(\frac{\partial a_i}{\partial a_p}\right)}_{\delta_{ip}}$$

$$0 = \sum_i \sum_j M_{ij}\delta_{ip} - \sum_i \sum_j a_i \delta_{ip} - \sum_i \sum_j b_j \delta_{ip}$$

note $\sum_i \delta_{ip} = 1$ $\quad \sum_j 1 = M$

$$0 = \sum_j M_{pj} - M\hat{a}_p - \sum b_j$$

$$a_p = \frac{1}{M}\left(\sum_j M_{pj} - \sum_j b_j\right) \quad (1)$$

similarly

$$b_p = \frac{1}{N}\left(\sum_i M_{ip} - \sum_i a_i\right) \quad (2)$$

let $u_p = \frac{1}{M}\sum_j M_{pj}$ $\quad p = 1 \dots N$

$v_p = \frac{1}{N}\sum_i M_{ip}$ $\quad p = 1 \dots M$

$$u_p = a_p + \frac{1}{M}\sum_j b_j$$

$$v_p = \frac{1}{N}\sum_i a_i + b_p$$

$$\begin{bmatrix} 0\,0\,0\,1\,0\,0\,0 & \frac{1}{M}\frac{1}{M}\cdots\frac{1}{M} \\ & \\ \frac{1}{N}\frac{1}{N}\cdots & 1\,0\,0\,0\,1\,0\,0\,0 \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \\ \tilde{b} \end{bmatrix} = \begin{bmatrix} \hat{u} \\ \\ \hat{v} \end{bmatrix}$$

[illegible] matrix eqn

note
Warren & Shearer, ~~J~~GR 2002
iterate (1) and (2)
to achieve soln