MRN022

Partial factor reconstruction for Spiegelman 3-4-98

$\underline{c} = \underline{\underline{F}}\,\underline{\alpha}$ $\qquad \underline{\underline{M}} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \\ & & & & 0 \\ & & & & & 0 \end{pmatrix}$

$$\underbrace{\underline{\underline{M}}\,\underline{c}}_{\downarrow\ \underline{d}} = \underbrace{\underline{\underline{M}}\,\underline{\underline{F}}}_{\downarrow\ \underline{\underline{G}}}\ \underbrace{\alpha}_{\downarrow\ \underline{m}}$$

$$\underline{d} = \underline{\underline{G}}\,\underline{m}$$

SVD: $\underline{\underline{G}} = \underline{\underline{U}}\,\underline{\underline{\Lambda}}\,\underline{\underline{V}}^T$ $\qquad \underline{\underline{\Lambda}} = \begin{pmatrix} \Lambda_p & 0 \\ 0 & 0 \end{pmatrix}$ $\quad \underline{\underline{V}} = \left(\underline{\underline{V}}_p, \underline{\underline{V}}_0\right)$ $\quad \underline{\underline{U}} = \left(\underline{\underline{U}}_p, \underline{\underline{U}}_0\right)$

$$\underline{m} = \underline{\underline{V}}_p \underline{\underline{\Lambda}}_p^{-1} \underline{\underline{U}}_p^T \underline{d} + \underline{\underline{V}}_0 \underline{n} \qquad \underline{n} = \text{any vector}$$

$$= \underline{\underline{G}}_p^{-g} \underline{d} + \underbrace{\underline{\underline{V}}_0 \underline{n}}_{\text{unknown additive part of } \underline{m}\text{, say } \underline{\underline{m}}_n} \qquad \left(\underline{\underline{G}}_p^{-g} = \underline{\underline{V}}_p \underline{\underline{\Lambda}}_p^{-1} \underline{\underline{U}}_p^T\right)$$

$$= \underline{\underline{G}}_p^{-g} d + \underline{\underline{m}}_n$$

$$\underline{d}^{est} = \underline{\underline{G}}\,\underline{\underline{G}}_p^{-g} \underline{d} + \underline{\underline{G}}\,\underline{\underline{V}}_0 \underline{n} = \underline{\underline{G}}\,\underline{\underline{G}}_p^{-g} d \qquad \left(\text{since } \underline{\underline{V}}_p^T \underline{\underline{V}}_0 = 0\right)$$

$$\underline{c} = \underline{\underline{F}}\,\underline{\underline{G}}_p^{-g} \underline{d} + \underline{\underline{F}}\,\underline{\underline{V}}_0 \underline{n} \qquad \left(\text{since } \underline{\underline{F}}\,\underline{\underline{V}}_0 \neq 0\right)$$

suppose $[\mathrm{cov}\, \underline{\underline{n}}]$ known. Then

$$[\mathrm{cov}\, \underline{\underline{c}}] = \underline{\underline{F}} V_0 [\mathrm{cov}\, \underline{\underline{n}}] V_0^T F^T$$

we ought to choose $[\mathrm{cov}\, \underline{\underline{n}}]$ to be uncorrelated, $[\mathrm{cov}\, \underline{\underline{n}}] = \sigma_n^2 \underline{\underline{I}}$, with σ_n^2 chosen so that $\underline{m}_n = \underline{\underline{V}}_0 \underline{n}$ has variance σ_m^2 that agrees with our prior notion of how α's vary. Thus we use

$$[\mathrm{cov}\, m_n] = V_0 [\mathrm{cov}\, n] V_0^T$$