MRN 032

Kalman Filter

state - space model

$$X_t = \underline{h}_t^T \underline{\theta}_t + n_t$$

t time

X_t observed timeseries

θ_t true state vector

$\underline{h}_t$ known coefficients

n_t $N(0, \sigma_n^2)$ noise

$$\underline{\theta}_t = \underline{\underline{G}}_t \underline{\theta}_{t-1} + \underline{w}_t$$

$\underline{\underline{G}}_t$ known operator

w_t $\langle w \rangle = 0$

$[\mathrm{cov}\, w]$ known

problem: estimate $\underline{\theta}$ given X

suppose $\hat{\theta}_{t-1}$ is minimum error estimator for θ_{t-1} and has covariance $[\mathrm{co}\underline{\underline{v}}\, \hat{\theta}_{t-1}]$. Then estimator

$$\hat{\theta}_{t|t-1} = G_t \hat{\theta}_{t-1} \quad \text{with} \quad [\mathrm{co}\underline{\underline{v}}\, \hat{\theta}_{t|t-1}] = G_t [\mathrm{co}\underline{\underline{v}}\, \hat{\theta}_{t-1}] G_t^T + [\mathrm{co}\underline{\underline{v}}\, w_t]$$

now suppose X_t becomes available. error is

$$e_t = X_t - \underline{h}^T \underline{\theta}_{t|t-1} \quad , \quad \Delta\underline{\theta} = \hat{\underline{\theta}}_t - \hat{\underline{\theta}}_{t|t-1}$$

minimizing $\sigma_n^{-2} e_t^2 + \Delta\underline{\theta}^T [\mathrm{co}\underline{\underline{v}}\, \hat{\theta}_{t|t-1}]^{-1} \Delta\theta$

$$\hat{\underline{\theta}}_t = \hat{\underline{\theta}}_{t|t-1} + e_t \underline{K}_t$$

$$[\mathrm{cov}\, \hat{\theta}_t] = [\mathrm{cov}\, \hat{\theta}_{t|t-1}] - \underline{K}_t h_t^T [\mathrm{cov}\, \hat{\theta}_{t|t-1}]$$

$$K_t = \frac{[\mathrm{cov}\, \hat{\theta}_{t|t-1}]\, \underline{h}_t}{h^T [\mathrm{cov}\, \hat{\theta}_{t|t-1}] h_t + \sigma_n^2}$$