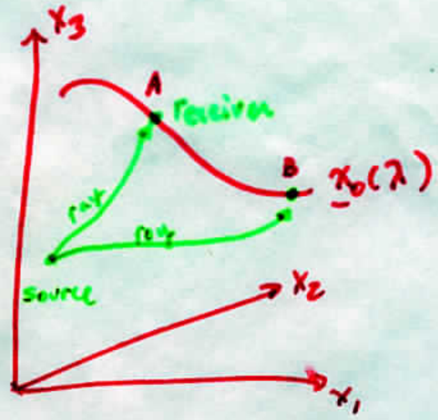
Notes on Mosen 15 Dec 93 SAVE

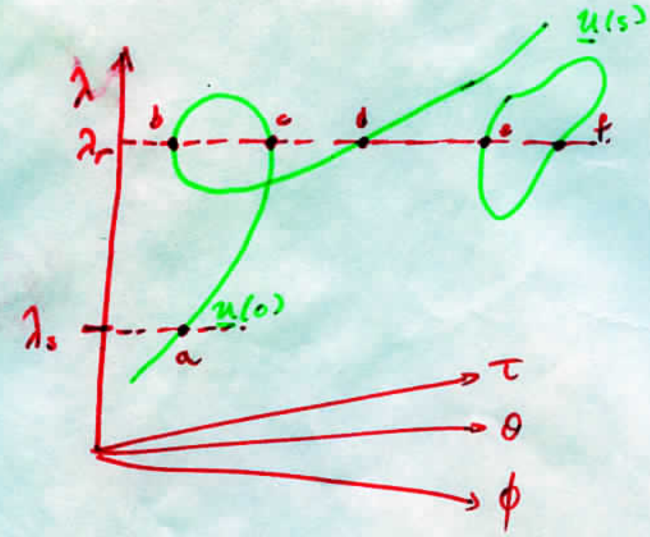
Ray tracing



A: receiver point at $\underline{x}_0(\lambda_r)$
has ray $\underline{x}(\theta, \phi, \tau, \lambda_r)$

B. starting point $x_0(\lambda_s)$

$x_0(\lambda)$ is some curve connecting starting point and receiver point.



$\underline{x}(\theta, \phi, \tau) - \underline{x}_0(\lambda) = 0 \equiv \underline{g}(\underline{u})$
3 eqns in 4 unknowns
defines a curve

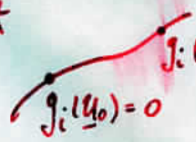
$\underline{u}(s) = [\theta(s), \phi(s), \tau(s), \lambda(s)]^T$

a: starting point, $s=0$, is known.
eg. $\underline{u}(0) = [\theta_s, \phi_s, \tau_s, \lambda_s]^T$

b-f: rays that connect source and receiver, solve $\lambda(s) = \lambda_r$

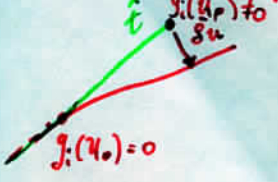
Step 1

track curve $\underline{u}(s)$ with predictor-corrector method. Predictor uses tangent



$g_i(\underline{u}_0 + \delta\underline{u}) = 0 \simeq g_i(\underline{u}_0) + \frac{\partial g_i}{\partial u_j}\delta u_j$ so tangent solves

$\partial g_i / \partial u_j \, \delta u_j = 0$ (3 eqns in 4 unknowns give direction). Corrector



$g_i(\underline{u}_p + \delta\underline{u}) = 0 = g_i(\underline{u}_p) + \frac{\partial g_i}{\partial u_j}\delta u_j = 0$. Again, ~~[illegible]~~ these are 3 eqns in 4 unknowns, so find shortest δu_i, which gives minimum length soln. if $A_{ij} = \partial g_i / \partial u_j$ then $\delta\underline{u} = -A^T(AA^T)^{-1}\underline{g}(u_p)$.

Step 2 locate all extrema along curve $\lambda(s) - \lambda_r = 0$

Step 3 find roots between extrema by Newton search procedure.

$\underline{x}(\lambda)$ is parabola in $z=0$ plane
symmetric
about radial line
just touching
cusp boundary