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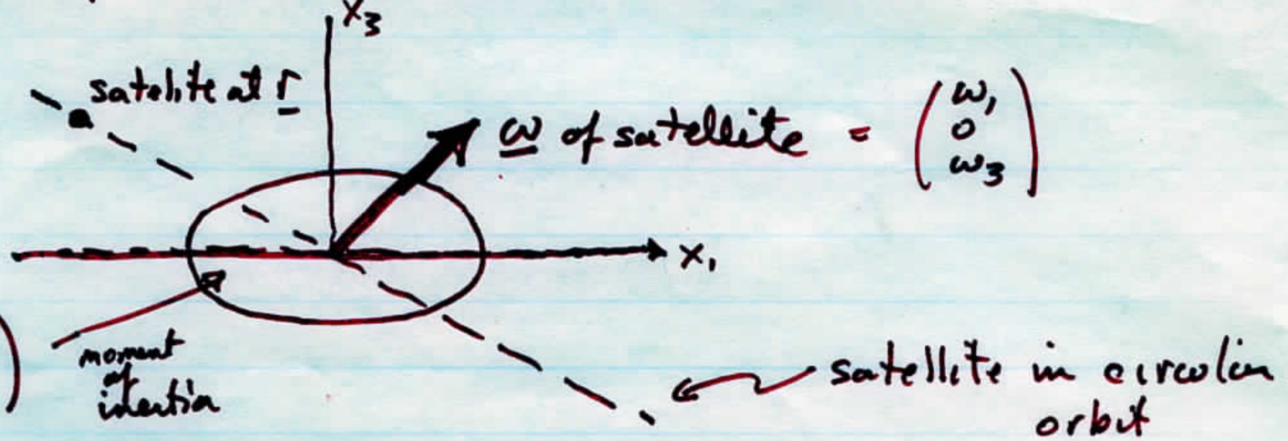
Precession of Satellite = regression of the nodes



$\underline{\underline{M}} = \begin{pmatrix} A & & \\ & A & \\ & & C \end{pmatrix}$ moment of inertia

Following Menke & Abbott 4.3.16

$$\underline{t} \propto (\underline{\underline{M}} \cdot \underline{r}) \times \underline{r} = \begin{bmatrix} (A-C)\, r_2 r_3 \\ (C-A)\, r_1 r_3 \\ 0 \end{bmatrix}$$

newton's law $\quad \tau = \frac{d}{dt} j$

$j \propto r \times \dot{r}$ and $\dot{r} = \underline{\omega} \times \underline{r}$ $\quad$ ω = angular velocity of satellite

$$\tau = \frac{d}{dt}\Big(r \times (\omega \times r) \Big) = \frac{d}{dt}\Big[r^2 \underline{\omega} - \cancel{(\underline{r} \cdot \omega)}\, \underline{r} \Big]$$ since $A \times (B \times C) = (a \cdot c) b - (a \cdot b) c$

0 since $\underline{r} \perp \underline{\omega}$

suppose orbit circular, so $\dot{r} = 0$. Then

$$\underline{\dot{\omega}} = \frac{\tau}{r^2}$$

Then note $\quad \omega \cdot \dot{\omega}$ = component of $\dot{\omega} \parallel \omega$ $= \begin{pmatrix} (A-C)\, r_2 r_3 \omega_1 \\ + \\ 0 \\ + \\ 0 \end{pmatrix}$

but $\langle \omega \cdot \dot{\omega} \rangle$ cancel out along orbit

note $\dot{\omega}_2 \propto$ orbital precession rate $\propto (C-A)\, r_1 r_3$. but $\langle \dot{\omega}_2 \rangle \neq 0$ since signs of r_1 and r_3 anticorrelate $r_1 r_3 < 0$ everywhere on orbit