MRN037

1. There are no isotropic, symmetric third order tensors. Proof by contradiction:

hypothesize a tensor, K_{ijk} that is isotropic. Then it is invariant under coordinate rotations (using rotation matrix $\underline{\underline{S}}$)

$$K_{ijk} = S_{ip} S_{jq} S_{kr} K_{pqr}$$

or equivalently

$$K_{jik} = S_{jp} S_{iq} S_{kr} K_{pqr}$$

Now if K_{ijk} is also symmetric, then $K_{ijk} = K_{jik}$ so, from above

$$(S_{ip} S_{jq})(S_{kr} K_{pqr}) = (S_{jp} S_{iq})(S_{kr} K_{pqr})$$

or

$$S_{ip} S_{jq} = S_{jp} S_{iq}$$

now contract indices p and j by multiplying by δ_{pj}

$$S_{ip} S_{jq} \delta_{pj} = S_{jp} S_{iq} \delta_{pj} \Rightarrow S_{ip} S_{pq} = S_{pp} S_{iq}$$

or in matrix notation

$$\underline{\underline{S}}\,\underline{\underline{S}} = \mathrm{trace}(\underline{\underline{S}})\,\underline{\underline{S}} \quad \text{or} \quad \underline{\underline{S}} = \mathrm{trace}(\underline{\underline{S}})\,\underline{\underline{I}}$$

but then $\underline{\underline{S}}$ is not a rotation.

note that this proof fails for antisym matrices, since then the inverse of $S_{kr} K_{pqr}$ $\nexists$

2. There is no piezoelectric effect in isotropic crystals. Since $\tau_{ij} = K_{ijk} E_k$, and stress τ_{ij} is symmetric, K_{ijk} must be symmetric in (ij). But no such isotropic tensor exists.