for Weitte 4-5-96
MRN 044

Proof that the correlation coefficient

$$r_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\left[\Sigma(x_i - \bar{x})^2 \, \Sigma(y_i - \bar{y})^2\right]^{1/2}}$$

see CRC math tables p577 #2

is invariant under a linear transformation

$$u = ax + b$$
$$v = cy + d$$

First note that $\bar{u} = \frac{1}{N}\Sigma u_i = \frac{1}{N}\Sigma(ax_i + b) = a\frac{1}{N}\Sigma x_i + \frac{1}{N}Nb$ $= a\bar{x} + b$ and similarly $\bar{v} = c\bar{y} + d$. Then by direct substitution:

$$r_{uv} = \frac{\Sigma(u_i - \bar{u})(v_i - \bar{v})}{\left[\Sigma(u_i - \bar{u})^2 \, \Sigma(v_i - \bar{v})^2\right]^{1/2}}$$

$$= \frac{\Sigma(ax_i + b - a\bar{x} - b)(cy_i + d - \bar{y} - d)}{\left[\Sigma(ax_i + b - a\bar{x} - b)^2 \, \Sigma(cy_i + d - \bar{y} - d)^2\right]^{1/2}}$$

$$= \frac{ac\,\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\left[a^2\,\Sigma(x_i - \bar{x})^2 \, c^2\,\Sigma(y_i - \bar{y})^2\right]^{1/2}}$$

$$= \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2 \, \Sigma(y_i - \bar{y})^2}$$

$$= r_{xy}$$

Q.E.Q.