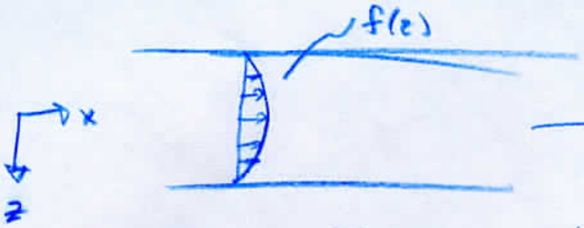
Darcy's law from navier stokes

N.-S. $\quad \frac{\partial v}{\partial t} + (\nabla v)\cdot v = \nu \nabla^2 \underline{v} + \frac{\underline{f} - \nabla p}{\rho}$

<u>assume</u> 1) no inertia, $\frac{\partial v}{\partial t}$ negligible

2) analogue of flow thru pipe



$\longrightarrow \underline{v} = \begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \bar{v}_x f(z) \\ 0 \\ 0 \end{pmatrix}$

3) $\nabla \underline{v} = \begin{pmatrix} \partial v_x/\partial x & \cdots & \partial v_x/\partial z \\ \partial v_z/\partial x & & \partial v_z/\partial v \end{pmatrix} = \begin{pmatrix} 0 & 0 & \bar{v}_x f'(z) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

4) $(\nabla \underline{v})\cdot \underline{v} = 0$

5) $\nabla^2 \underline{v} = \nabla \cdot \nabla v$

$[\nabla^2 \underline{v}]_1 = v_{1,jj} = \bar{v}_x f''(z)$, other cmp. are zero.

6) so $\nabla^2 v \approx \bar{v}_x f''(z)\, \hat{n} = \bar{v}_x \frac{v}{|v|} f''(z)$

↘ direction of v

7) $\nu \frac{\bar{v}_x}{|v|} f''(z)\, \underline{v} = \frac{\underline{f} - \nabla p}{\rho}$