TO:

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FR: William Menke

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Dear Dr. Singer:

I read your May 1990 Science article on imaging with interest:

- 1. If you have a more technical article discussing the details of your calculations, I would appreciate a copy. I am especially interested in knowing more about the way in which you solved the forward problem. Did you in effect sum all possible paths thru the object (that is, to what order of multiples are included?). Do you have a clever way of parallelizing the algorithm?
- 2. As you hint in your article, your method can be viewed an an approximation to some integral equation that embodies energy conservation in a scattering medium. Let me bring to your attention Chandrasekhar's equation, which I learned about from an article in the geophysical literature by Wu (Multiple Scattering ..., Geophys. J.R. astr Soc 82, 57-80, 1985). This equation applys conservation of energy to the energy flux per unit area , $I(x,\Omega)$, in the Ω direction at position x:

increase in energy flux, I with distance, s =

- + loss due to intrinsic absorption
- + loss due to scattering of energy into other directions
- + gain due to scattering of energy into this direction

or

$$\begin{split} \frac{dI(x,\Omega)}{ds} &= -c_s I(x,\Omega) - I(x,\Omega) \! \int_{4\pi} S(\Omega,\Omega_0) d\Omega_0 \\ &+ \int_{4\pi} S(\Omega,\Omega_0) I(x,\Omega_0) d\Omega_0 \end{split}$$

where c_S is the intrinsic absorption coefficient and $S(\Omega,\Omega_0)$ is the scattering cross section from one direction to another. There is a clear correspondence between c_S and your absorption probability, and between $S(\Omega,\Omega_0)$ and your forward/back/side scattering probabilities.

- 3. One of the limitations of energy flux methods is that they do not retain phase information, that is, they do not account for constructive of destructive interference phenomena. This severely limits their application in many areas of acoustics, especially when the scatterers (or voxels?) are smaller than the wavelength of the acoustic energy. The problem is that interference between the many first order multiples is very important, and changes the ratio of net forward/back scattering very significantly. I am not aware of any work to include interference effects in the Chandrasekhar equation, though Wu suggests that it is possible.
- 4. One simple extension of your method is to consider the data to be energy flux as a function of time leaving the surface of the object, in contrast to the total energy that you now use. This would be more in line with the practices of acoustics (or seismology), where one typically measures a pressure time series instead of just a single energy flux measurement. Presumably the variation of energy flux with time provides more information about the location of scatters than the integrated flux.

Sincerely,

Bill Menke