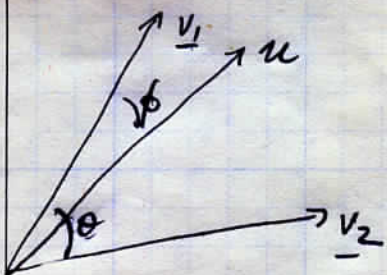


interpolating unit vectors

MRN056

given \underline{v}_1 and \underline{v}_2 (unit vectors)

find \underline{u} $\phi < \theta$ away
from \underline{v}_1 in direction of \underline{v}_2
with $|\underline{u}| = 1$

any vector between \underline{v}_1 and \underline{v}_2 is given by

$$\underline{u} = c (a \underline{v}_1 + (1-a) \underline{v}_2) ; \quad c > 0 \quad 0 \leq a \leq 1$$

since \underline{u} is unit vector $\underline{u} \cdot \underline{u} = 1$

$$\underline{u} \cdot \underline{u} = c^2 (a^2 + (1-a)^2 + 2a(1-a) \underline{v}_1 \cdot \underline{v}_2) = 1$$

$$c^{-2} = a^2 + 1 + a^2 - 2a + 2a \cos \theta - 2a^2 \cos \theta$$

$$c^{-2} = (2 - 2 \cos \theta) a^2 + (2 \cos \theta - 2) a + 1$$

$$\text{Then } \underline{v}_1 \cdot \underline{u} = \underline{v}_1 \cdot c(a \underline{v}_1 + (1-a) \underline{v}_2) = \cos \phi$$

$$\cos^2 \phi / c^2 = [a + (1-a) \cos \theta]^2 = a^2 + (1-a)^2 \cos^2 \theta + 2a(1-a) \cos \theta$$

$$= a^2 + \cos^2 \theta + a^2 \cos^2 \theta - 2a \cos^2 \theta + 2a \cos \theta - 2a^2 \cos \theta$$

$$= (1 + \cos^2 \theta - 2 \cos \theta) a^2 + (-2 \cos^2 \theta + 2 \cos \theta) a + \cos^2 \theta$$

$$a^2 : \cos^2 \phi (2 - 2 \cos \theta) - (1 + \cos^2 \theta - 2 \cos \theta)$$

$$a : \cos^2 \phi (2 \cos \theta - 2) - (-2 \cos^2 \theta + 2 \cos \theta)$$

$$1 : \cos^2 \phi - \cos^2 \theta$$

which is a quadratic eqn for a , choose root in range $[0, 1]$